

## 0.1 Model 1

Consider the simplest model of multi-product firms with switching costs, we assume that there are two periods and two products. For example one can think that initially a consumer opens a checking account (product 1) and later on she may take a loan (product 2). We denote the period/product by  $t = 1, 2$ .

There are  $J$  firms, indexed by  $j$ .

**Consumer problem** The per period utility of the consumer is given by:

$$u_{ijt} = \beta_t - \alpha p_{ijt} + \xi_{ijt} + \epsilon_{ijt} \quad (1)$$

where  $\beta_t$  is the valuation for the product,  $p_{ijt}$  is the price and  $\xi_{ijt}$  is a demand shock.

If the consumer buys from differnt firm in each period, she incurs a switching cost  $s$ . Denote by  $j_t$  the firm chosen in period  $t$ , then the total utility across both periods is given by:

$$U_i(j_1, j_2) = u_{ij_11} + u_{ij_22} - s \cdot \mathbb{I}(j_1 \neq j_2) \quad (2)$$

and the optimal choice is given by:

$$(j_1^*, j_2^*) = \arg \max_{j_1, j_2} U_i(j_1, j_2) \quad (3)$$

Denote by  $D_{1j}(p_1) = \Pr(j_1^* = j; p_1)$  and  $D_{2j}(j_1, p_2) = \Pr(j_2^* = j; j_1, p_2)$  the demand functions for period 1 and period 2 respectively.

Then:

$$D_{2j}(j_1, p_2) = \begin{cases} \frac{\exp(\delta_{ij_1t})}{\exp(\delta_{ij_1t}) + \sum_{j' \neq j_1} \exp(\delta_{ij't} - \alpha s)} & ; j = j_1 \\ \frac{\exp(\delta_{ij_2t})}{\exp(\delta_{ij_2t}) + \sum_{j' \neq j_1} \exp(\delta_{ij't} - \alpha s)} & ; j \neq j_1 \end{cases} \quad (4)$$

Denote by  $p_2(p_1)$  the vector of prices in period 2 given the prices in period 1 and by  $S_2(p_2; j_1)$  the surplus function of period 2 given the prices and the firm chosen in period 1. Then in the first period the consumer maximizes:

$$\max_{j_1} u_{ij_11} + S_2(p_2(p_1); j_1) \quad (5)$$

where  $p_2(p_1)$  will be determined in equilibrium.

**Firm problem** In the second period the firm chooses prices  $p_{j2}(j_1)$  taking as given the firm chosen in period 1, to maximize profits:

$$\max_p D_{2j}(j_1, (p, p_{-j_2}^*(j_1)))(p - c_2) \quad (6)$$

In the first period each firm chooses  $p_{j1}$  according to:

$$\max_{p_{j1}} D_{1j}((p_{j1}, p_{-j_1}^*)) (p_{j1} - c_1) + \sum_{j_1} D_{2j}(j_1, (p_2^*(j_1)))(p_{j2}^*(j_1) - c_2) \quad (7)$$

This model is essentially the same as Dube et al. (2009), but in a two-period setting.