Adapting Engelbrecht-Wiggans et al. (1983)

Define h to be default probability, there is an informed bank (bank 1) and an uninformed bank (bank 2). The strategies are $r_1(h) = \sigma(h) : h \to r$ and $G(x) = \Pr(r_2 \leq x)$, which are interest rates.

Assume that in equilibrium σ is an increasing function, and denote by $\tau : r \to h$ its inverse, $\tau(\sigma(h)) = h$.

Then expected profits of bank 1 are:

$$\begin{aligned}
\pi_1(r_1(h)) &= \Pr(\sigma(h) \leq r_2) \cdot [(1-h)\sigma(h) - 1] \\
&= [1 - \Pr(\sigma(h) > r_2)] \cdot [(1-h)\sigma(h) - 1] \\
&= [1 - G(\sigma(h))] \cdot [(1-h)\sigma(h) - 1]
\end{aligned} \tag{1}$$

and the expected profits of bank 2 are:

$$\begin{aligned}
\pi_2 &= \Pr(r_2 \leq \sigma(h)) \cdot E[(1-h) \cdot r_2 - 1 \mid r_2 \leq \sigma(h)] \\
&= \Pr(\tau(r_2) \leq h) \cdot E[(1-h) \cdot r_2 - 1 \mid \tau(r_2) \leq h] \\
&= [1 - F(\tau(r_2)] \cdot E[(1-h) \cdot r_2 - 1 \mid \tau(r_2) \leq h] \\
&= [1 - F(\tau(r_2)] \cdot [E[(1-h) \mid \tau(r_2) \leq h] \cdot r_2 - 1]
\end{aligned} \tag{2}$$

Assume that bank 2 makes zero profits, then we have:

$$[1 - F(\tau(r_2)] \cdot [E[(1-h) \mid \tau(r_2) \leq h] \cdot r_2 - 1] = 0 \tag{3}$$

since the winning rpobability is not zero, then the expected profits have to be zero.

$$E[(1-h) \mid \tau(r_2) \leq h] \cdot r_2 - 1 = 0 \implies r_2 = \frac{1}{E[(1-h) \mid \tau(r_2) \leq h]} \tag{4}$$

Then we can use profit mazimization by the first firm, the FOC of equation 1 are:

$$\begin{aligned}
-g(\sigma(h))[(1-h)\sigma(h) - 1] + [1 - G(\sigma(h))][1-h] &= 0 \\
\frac{1-h}{[(1-h)\sigma(h) - 1]} = \frac{g(\sigma(h))}{[1 - G(\sigma(h))]} &= -\frac{d}{d\sigma}[\log(1 - g(\sigma(h)))] \\
\frac{1-\tau(r)}{[(1-\tau(r))r - 1]} &= -\frac{d}{d\sigma}[\log(1 - g(\sigma(h)))]
\end{aligned}$$

Integrating both sides from $\underline{r} = \sigma(\underline{h})$ to a given r, where $G(\underline{r}) = 0$ we have:

$$\begin{aligned}
-[\log(1 - G(r)) - \log(1 - \underbrace{G(\underline{r}))}_{=0}] &= \int_{\underline{r}}^{r} \frac{1-\tau(u)}{[(1-\tau(u))u - 1]} du \\
-\log(1 - G(r)) &= \int_{\underline{r}}^{r} \frac{1-\tau(u)}{[(1-\tau(u))u - 1]} du \\
G(r) &= 1 - \exp\left[-\int_{\underline{r}}^{r} \frac{1-\tau(u)}{[(1-\tau(u))u - 1]} du\right]
\end{aligned} \tag{5}$$

Let $k = \tau(r_2)$ the the default probability such that the home bank offers r_2, then we have $r_2 = \sigma(k)$ and we replace into equation 4

$$\sigma(h) = \frac{1}{E[(1-H) \mid H \leq h]} \equiv \frac{1}{\mu(h)} \tag{6}$$

Then we can obtain the mixed strategy of bank 2 by a change of variables, consider $u = \sigma(t) \implies \tau(u) = t, du = \sigma'(t)dt$, then the limits of integration change from $[\underline{r}, r]$ to $\underline{h}, \tau(r)]$. Substituting into equation 5 we have:

$$G(\sigma(h)) = 1 - \exp\left[-\int_{\underline{h}}^{h} \frac{1-t}{[(1-t)\sigma(t) - 1]} \sigma'(t)dt\right] \tag{7}$$

given that $\sigma(t) = 1/\mu(t)$, we have $\sigma'(t) = -\mu'(t)/\mu(t)^2$, replacing in the equation above:

$$G(\sigma(h)) = 1 - \exp\left[\int_{\underline{h}}^{h} \frac{1-t}{\frac{1-t}{\mu(t)} - 1} \frac{\mu'(t)}{\mu(t)^2} dt\right] = 1 - \exp\left[\int_{\underline{h}}^{h} \frac{(1-t)\mu'(t)}{(1-t-\mu(t))\mu(t)} dt\right] \tag{8}$$