0.1 Model 1

Consider the simplest model of multi-product firms with switching costs, we assume that there are two periods and two products. For example one can think that initially a consumer opens a checking account (product 1) and later on she may take a loan (product 2). We denote the period/product by $t = 1, 2$.

There are J firms, indexed by j.

Consumer problem The per period utility of the consumer is given by:

$$u_{ijt} = \beta_t - \alpha p_{ijt} + \xi_{ijt} + \epsilon_{ijt} \tag{1}$$

where β_t is the valuation for the product, p_{ijt} is the price and ξ_{ijt} is a demand shock.

If the consumer buys from differnt firm in each period, she incurs a switching cost s. Denote by j_t the firm chosen in period t, then the total utility across both periods is given by:

$$U_i(j_1, j_2) = u_{ij_1 1} + u_{ij_2 2} - s \cdot \mathbb{I}(j_1 \neq j_2) \tag{2}$$

and the optimal choice is given by:

$$(j_1^*, j_2^*) = \arg\max_{j_1, j_2} U_i(j_1, j_2) \tag{3}$$

Denote by $D_{1j}(p_1) = \Pr(j_1^* = j; p_1)$ and $D_{2j}(j_1, p_2) = \Pr(j_2^* = j; j_1, p_2)$ the demand functions for period 1 and period 2 respectively.

Then:

$$D_{2j}(j_1, p_2) = \begin{cases} \frac{\exp(\delta_{ijt})}{\exp(\delta_{ijt}) + \sum_{j' \neq j} \exp(\delta_{ijt} - \alpha s)} & ; j = j_1 \\ \frac{\exp(\delta_{ijt} - \alpha s)}{\exp(\delta_{ij_1 t}) + \sum_{j' \neq j_1} \exp(\delta_{ijt} - \alpha s)} & ; j \neq j_1 \end{cases} \tag{4}$$

Denote by $p_2(p_1)$ the vector of prices in period 2 given the prices in period 1 and by $S_2(p_2; j_1)$ the surplus function of period 2 given the prices and the firm chosen in period 1. Then in the firt period the consumer maximizes:

$$\max_{j_1} u_{ij_1 1} + S_2(p_2(p_1); j_1) \tag{5}$$

where $p_2(p_1)$ will be determined in equilibrium.

Firm problem In the second period the firm chooses prices $p_{j2}(j_1)$ taking as given the firm chosen in period 1, to maximize profits:

$$\max_p D_{2j}(j_1, (p, p_{-j2}^*(j_1)))(p - c_2) \tag{6}$$

In the first period each firm chooses p_{j1} according to:

$$\max_{p_{j1}} D_{1j}((p_{j1}, p_{-j1}^*))(p_{j1} - c_1) + \sum_{j_1} D_{2j}(j_1, (p_2^*(j_1)))(p_{j2}^*(j_1) - c_2) \tag{7}$$

This model is essentially the same as Dube et al. (2009), but in a two-period setting.