There is one neighbor firm and an arbitrary number of non-neighbor firms. Let X and Z denote, respectively, the private and public signals on V, the unknown value of the representative drainage tract. The neighbor firm observes the realizations of X and Z prior to bidding on the tract, while non-neighbor firms observe only the realization of Z. Realizations of the random variables will be denoted by lowercase letters. In what follows, we treat z as given, and are explicit about the dependence of the distributions on its value. However, for notational convenience, we will suppress the dependence of bidding strategies on z.

The essential feature of our model is that the information revealed by on-site drilling of an adjacent tract by a neighbor firm is a sufficient statistic for the information non-neighbor firms acquire from seismic surveys. The assumption that this information is known to the neighbor firm is made in order to obtain a precise characterization of the equilibrium and its properties. A more realistic, but less tractable, assumption is that the non-neighbor firms have noisy, but private, estimates of tract value. However, as long as the estimates of the non-neighbor firms are not too informative, we can use the result by Milgrom and Weber (1985) on the upper hemicontinuity of the equilibrium correspondence to argue that the behavioral implications of this descriptively more accurate model are approximately the same as those of a model in which the estimates of non-neighbor firms are based on public information.

The strategy of non-neighbor firm i is a distribution function $G_i(\cdot)$ over the non-negative real numbers. Adopting the approach of Engelbrecht-Wiggans, Milgrom, and Weber, we summarize the information of the neighbor firm by the real-valued random variable $H = E[V \mid X, z]$. We shall assume that H has an atomless distribution $F(\cdot \mid z)$, with finite mean $\bar{H}$. The strategy of the neighbor firm can then be defined as a function a which maps realizations of H, which are associated with the realizations of X, into the non-negative real numbers. We shall assume that $a(h)$ is a differentiable, strictly increasing function on the range (R, ∞), where R is the reservation price, and denote its inverse function on this interval by $T(b)$.

Define $G(b) = G_1(b) \cdots G_n(b)$ to be the distribution function of the maximum of the bids submitted by the uninformed firms on the tract. Given the strategy combination $(a, G_1, \ldots, G_n)$, the payoff to the neighbor firm, when its estimate of V is h, is the product of the probability that its bid is highest and its expected value of the tract less its bid:

$$G(a(h))\big(h - a(h)\big). \tag{1}$$

If the drainage tract contains any oil, it is usually part of a pool which the neighbor firm has discovered on the adjacent tract. This makes the value of the drainage tract to each firm contingent upon the manner in which production is allocated among the firms. If the firms bargain to an efficient allocation, tract valuations are identical across firms. In many instances, however, competition leads to some dissipation of rents. In these cases, the neighbor firm is likely to have a higher tract valuation than the non-neighbor firm, since it can take the externality into account and internalize its effects.

We parameterize the possible difference in tract valuations by letting the expected value of the drainage tract to the non-neighbor firm be equal to $E[H \mid z] - c$, where c is a fixed, non-negative constant. The expected payoff to non-neighbor firm i which submits a bid b greater than R is

$$E[H - b - c \mid b > a(h); z]\, F(T(b) \mid z) \prod_{j \neq i} G_j(b). \tag{2}$$

The first term in equation (2) is expected profits, conditional on winning the tract (and hence

$b > a(h)$). The remaining terms represent the probabilities of outbidding the neighboring firm and the other non-neighbors. Ties at the reservation price are assumed to be settled by randomization.

A Bayesian Nash equilibrium for the bidding game is an $(n+1)$-tuple of strategies $(a^*, G_1^*, \ldots, G_n^*)$ such that the expected payoff to each firm conditional on its information is maximized, given the strategies employed by the other firms.

We turn next to a characterization of the equilibrium bid distributions. Define

$$\phi(h) = \exp\left(- \int_h^\infty \frac{f(s \mid z) \int_h^s F(u \mid z)\, du}{cF(s \mid z)^2 + F(s \mid z) \int_h^s F(u \mid z)\, du}\, ds \right). \tag{3}$$

Note that if c is equal to zero, then $\phi(h)$ is equal to $F(h \mid z)$. Our theorem is a restatement of the theorem proved by Engelbrecht-Wiggans, Milgrom, and Weber, extended to auctions with asymmetric tract valuations. The proof is essentially the same as the one given by Engelbrecht-Wiggans, Milgrom, and Weber, and is given in the Appendix.

Theorem 1. *The $(n+1)$-tuple $(a^*, G_1^*, \ldots, G_n^*)$ is an equilibrium point if and only if*

$$G^*(b) = \begin{cases} 1, & b > H - c, \\ \phi(T(b)), & R < b < H - c, \\ \phi(R), & 0 < b < R, \end{cases} \tag{4}$$

and

$$a^*(h) = \begin{cases} E[H \mid H < h; z] - c, & h > \bar{h}, \\ R, & \bar{h} \geq h \geq R, \\ 0, & h < R, \end{cases} \tag{5}$$

where $\bar{h}$ solves

$$E[H \mid H < \bar{h}; z] - c = R.$$

The theorem states that the supports of the equilibrium distribution functions are identical, and consist of $\{0\}$ and the interval $[R, H - c]$. We interpret a zero bid as no bid. The equilibrium strategy of the neighbor firm on $(R, H - c)$ is uniquely determined by the condition that, in equilibrium, non-neighbor firms must earn zero profits. That is, suppose a non-neighbor firm submits an equilibrium bid b. Then, since a^* is strictly increasing at b, there is a unique h' such that $b = a^*(h')$. The expected profit of the tract to the non-neighbor firm conditional on the event that it wins is $E[H \mid H < h'; z] - b - c$. Setting this equation equal to zero implies that

$$a^*(h') = E[H \mid H < h'; z] - c.$$

The equilibrium strategies of non-neighbor firms are indeterminate. However, the equilibrium distribution function of the maximum uninformed bid is unique. It is chosen in order to induce the neighbor firm to bid according to the function given above. The two distributions differ in the probability of the events of no bid, and of a reservation bid R. G^* possesses a mass point equal to $F(\bar{h})$ at $\{0\}$, and is constant at this value on the interval $(0, R]$. The distribution of the neighbor bid also possesses a mass point at $\{0\}$, but it is equal to $F(R)$, which is less than $F(\bar{h})$. The distribution is constant at $F(R)$ on the open interval $(0, R)$, and then jumps discontinuously upward at R. The value of the mass point at R is equal to $F(\bar{h}) - F(R)$. If c is equal to zero, G^* is identical to the distribution of the informed bid on (R, H).

The randomized strategies of non-neighbor firms are a direct consequence of the assumption that the neighbor firm knows their estimates. If non-neighbor firms bid according to a pure (and hence predictable) strategy which specifies a bid for each realization of the public information variables, the optimal response of the neighbor firm is to bid slightly above the maximum non-neighbor bid if the tract is worth more than this number, and not bid otherwise. But this implies that on average the non-neighbor firm is certain to lose, since it will win only those tracts whose expected value is less than its bids. By randomizing, non-neighbor firms can induce the neighbor firm to bid according to a strategy in which it will lose profitable tracts some of the time. As a result, non-neighbor firms will earn positive expected profits on some tracts, and it is only on average that their expected profits are zero.