

World and Object Representation

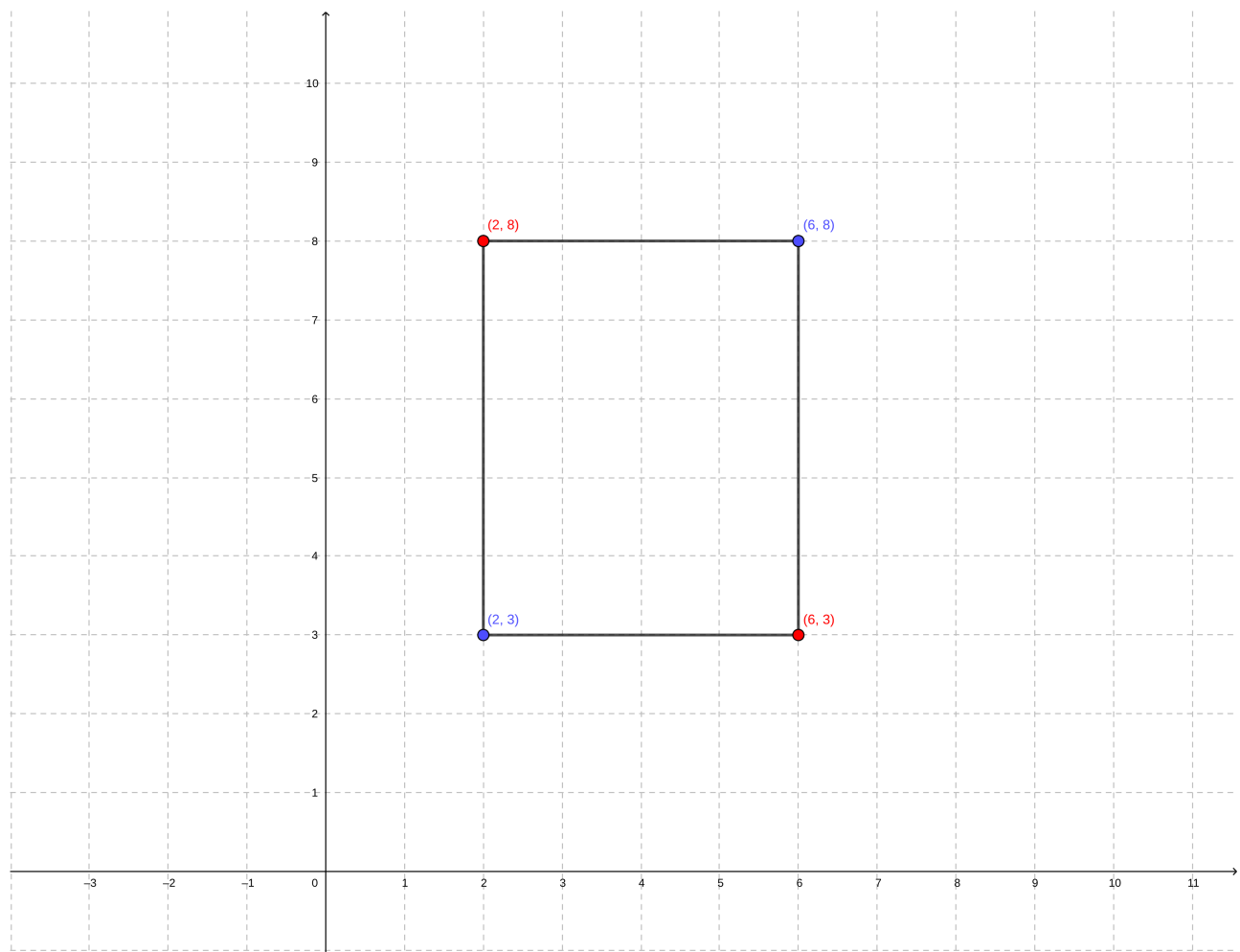
Exercises

1. Bounding Box Definition

- Given a 2D bounding box defined by $(x_{min}, y_{min}, x_{max}, y_{max}) = (2, 3, 6, 8)$, list all four corner coordinates.

The bounding box corner will be given by

$p_1 = (2, 3); p_2 = (2, 8); p_3 = (6, 8); \text{ and } p_4 = (6, 3)$.



- Compute the area of the bounding box.

The area of the bounding box is,

$$box_{area} = (x_{max} - x_{min}) \times (y_{max} - y_{min})$$

Then,

$$box_{area} = (6 - 2) \times (8 - 3) = 20$$

The $box_{area} = 20$.

2. Bounding Boxes and Occupied Space

- (a) Given a 3D bounding box with parameters $(x, y, z, l, w, h, \Psi) = (5, 3, 0, 4, 2, 2, 45^\circ)$, compute the volume occupied by the object.

The volume box_{volume} occupied by the object is given by:

$$box_{volume} = l \times w \times h = 4 \times 2 \times 2 = 16$$

- (b) If the bounding box in (a) is rotated by $\Psi = 45^\circ$, sketch (or describe) how the occupied space differs compared to $\Psi = 0^\circ$.

The heading Ψ is the measured angle of object local reference relative to the global origin coordinate $X - axis$, using the right hand rule. Which means that the rotation will be along the $Z - axis$. The Z rotation matrix is given by:

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

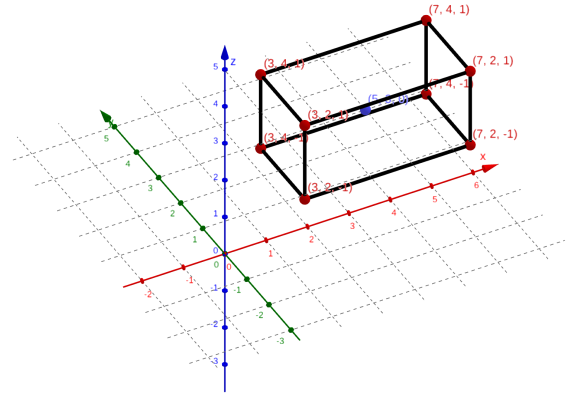
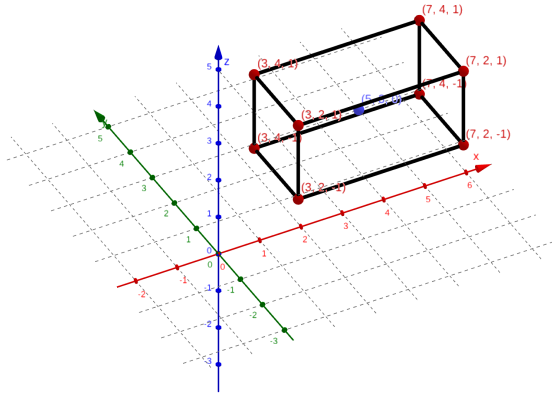
Assuming the initial object direction is in $X - axis$, we can write,

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The rotation is,

$$P_r = R_z(\theta) \times P = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Applying the rotation,



Show bounding box when $\Psi = 0^\circ$

Show bounding box when $\Psi = 45^\circ$

- (c) Two 2D bounding boxes B_1 ; and; B_2 are defined as:

$$B_1 : (x_{min}, y_{min}, x_{max}, y_{max}) = (0, 0, 4, 3), ; B_2 : (2, 1, 6, 5).$$

Compute their intersection-over-union (IoU).