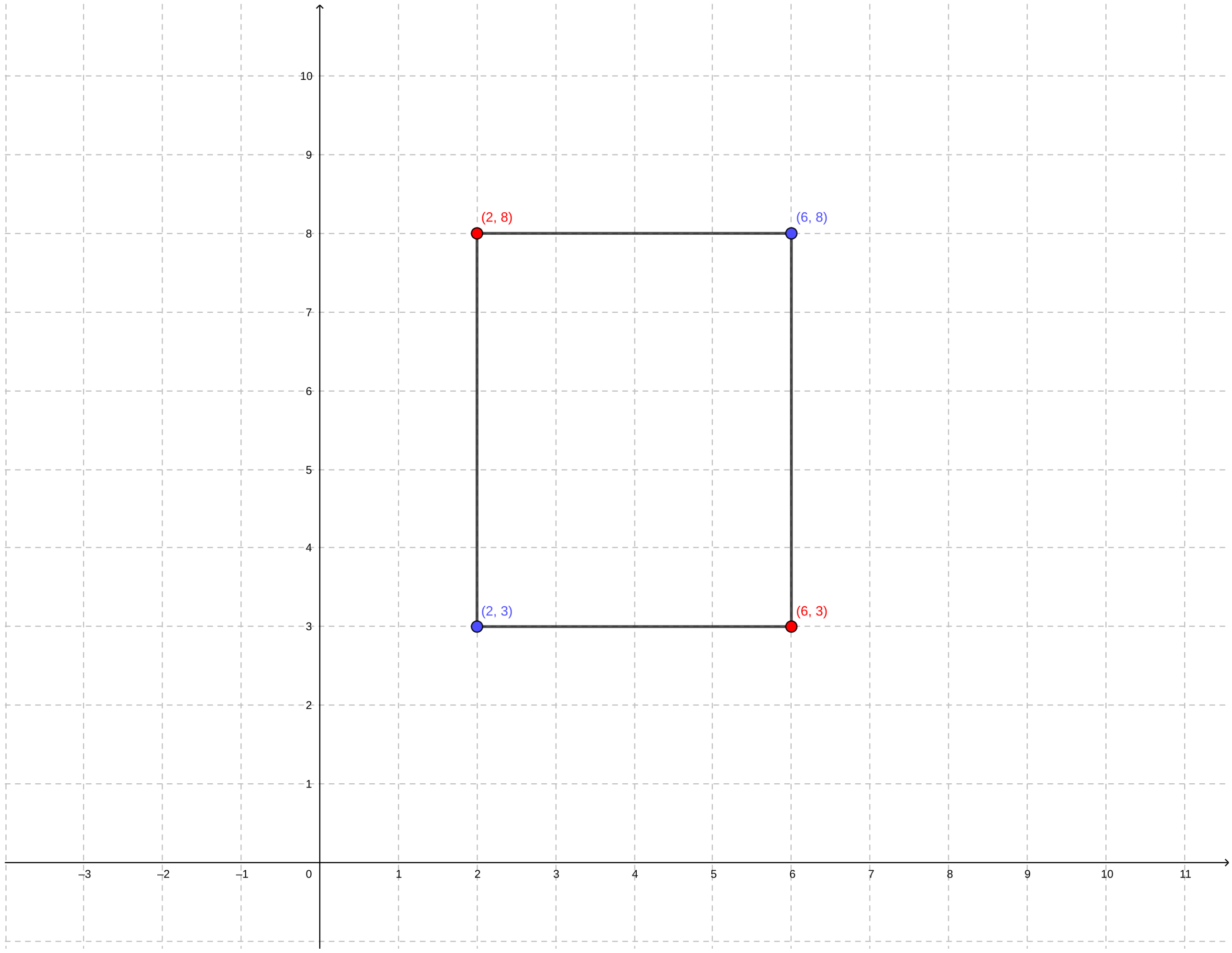
World and Object Representation

Exercises

1. **Bounding Box Definition**

- Given a 2D bounding box defined by $(x_{min}, y_{min}, x_{max}, y_{max}) = (2, 3, 6, 8)$, list all four corner coordinates.

  The bounding box corner will be given by
  $p_1 = (2, 3), ; p_2 = (2, 8), ; p_3 = (6, 8); and; p_4 = (6, 3)$.



- Compute the area of the bounding box.

  The area of the bounding box is,

$$box_{area} = (x_{max} - x_{min}) \times (y_{max} - y_{min})$$

  Then,

$$box_{area} = (6 - 2) \times (8 - 3) = 20$$

  The $box_{area} = 20$.

2. **Bounding Boxes and Occupied Space**

- (a) Given a 3D bounding box with parameters $(x, y, z, l, w, h, \Psi) = (5, 3, 0, 4, 2, 2, 45^\circ)$, compute the volume occupied by the object.

  The volume box_{volume} occupied by the object is given by:

  $$box_{volume} = l \times w \times h = 4 \times 2 \times 2 = 16$$

- (b) If the bounding box in (a) is rotated by $\Psi = 45^\circ$, sketch (or describe) how the occupied space differs compared to $\Psi = 0^\circ$.

  The heading Ψ is the measured angle of object local reference relative to the global origin coordinate $X - axis$, using the right hand rule. Which means that the rotation will be along the $Z - axis$. The Z rotation matrix is given by:

  $$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  Assuming the initial object direction is in $X - axis$, we can write,

  $$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

  The rotation is,

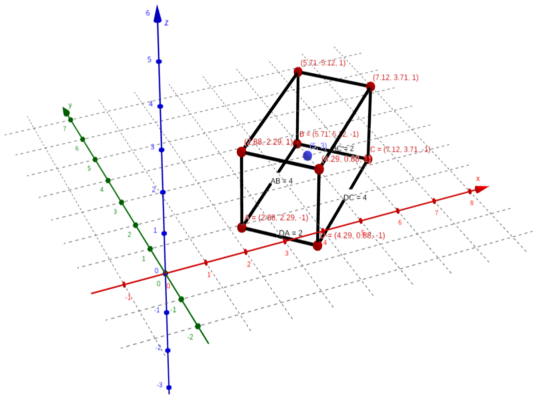
  $$P_r = R_z(\theta) \times P = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

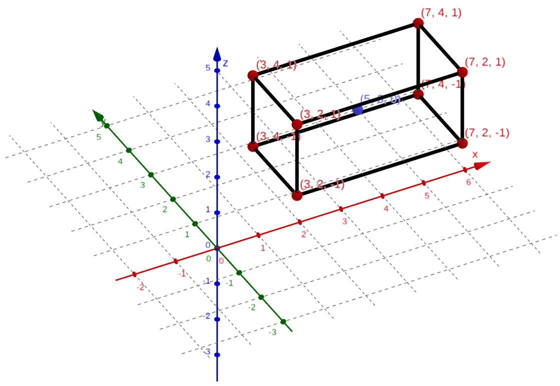
  $$P_r = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{bmatrix}$$

  The translation matrix is,

  $$T = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  Applying the rotation and translation for each corner, $P' = T \times R_z \times P$,





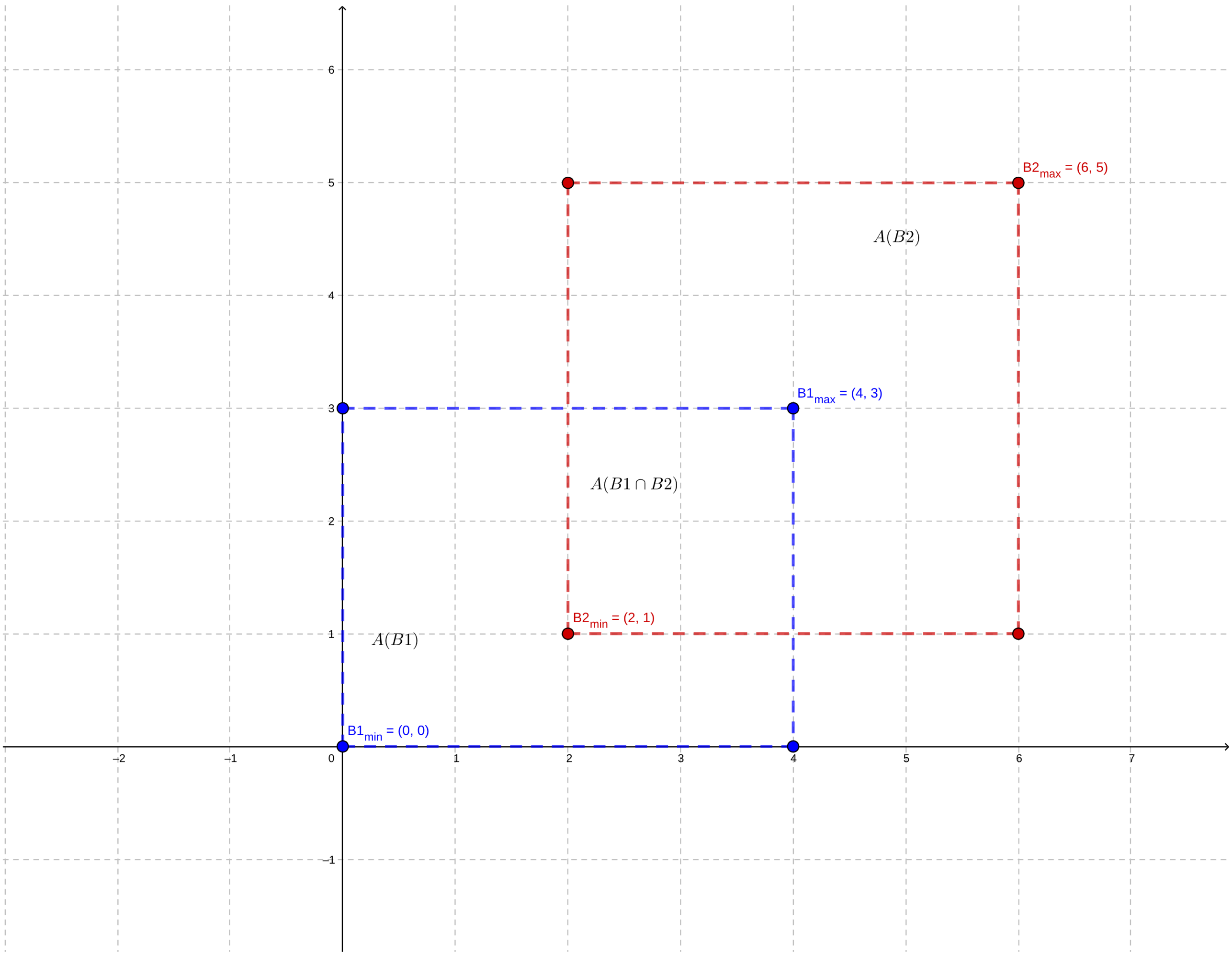
Show bounding box when $\Psi = 45^\circ$	*Show bounding box when* $\Psi = 0^\circ$

- (c) Two 2D bounding boxes $B_1 ; and ; B_2$ are defined as:

$$B_1 : (x_{min}, y_{min}, x_{max}, y_{max}) = (0, 0, 4, 3), ; B_2 : (2, 1, 6, 5).$$

Compute their intersection-over-union (IoU).

Given the two bounding boxes depicted in the image bellow, we will calculate the $A(B1 \cap B2)$ first,



The intersection are, denoted by $A(B1 \cap B2)$ can be calculated as,

$$A(B1 \cap B2) = (b1_{x_{max}} - b2_{x_{min}}) \times (b1_{y_{max}} - b2_{y_{min}})$$

$$A(B1 \cap B2) = (4 - 2) \times (3 - 1) = 4$$

The area for each bounding box is,

$$A(B1) = 4 \times 3 = 12,\ A(B2) = (5 - 1) \times (6 - 2) = 4 \times 4 = 16$$

The area of the union,

$$A(B1 \cup B2) = A(B1) + A(B2) - A(B1 \cap B2)$$
$$A(B1 \cup B2) = 12 + 16 - 4 = 24$$

Finally, we can calculate the IoU,

$$IoU = \frac{A(B1 \cap B2)}{A(B1 \cup B2)} = \frac{4}{24} = \frac{1}{6} \approx 0.1666$$

Reference

[1]. Images generated using the online tool available in https://geogebra.org.