10.6 Exercises

1. **Bounding Box Definition**
   - Given a 2D bounding box defined by $(x_{min}, y_{min}, x_{max}, y_{max}) = (2,3,6,8)$, list all four corner coordinates.
   - Compute the area of the bounding box.
2. **Bounding Boxes and Occupied Space**
   (a) Given a 3D bounding box with parameters $(x,y,z,l,w,h,\Psi) = (5,3,0,4,2,2,45^\circ)$, compute the volume occupied by the object.
   (b) If the bounding box in (a) is rotated by $\Psi = 45^\circ$, sketch (or describe) how the occupied space differs compared to $\Psi = 0^\circ$.
   (c) Two 2D bounding boxes B_1 and B_2 are defined as:

   $$B_1 : (x_{min}, y_{min}, x_{max}, y_{max}) = (0,0,4,3), \quad B_2 : (2,1,6,5).$$

   Compute their intersection-over-union (IoU).
3. **Confidence and Uncertainty**
   - A detector outputs a bicycle at 0.65 confidence. Discuss whether this uncertainty is more likely to be aleatoric or epistemic in the following scenarios:
     (a) The bicycle is partially hidden behind a bus.
     (b) The detector was trained mostly on car and pedestrian classes, with few examples of bicycles.
   - A detector identifies a pedestrian at 0.85 confidence under foggy conditions. Which type of uncertainty (aleatoric or epistemic) is dominant, and why?
   - An object is detected at 0.40 confidence. Suggest two possible ways to reduce epistemic uncertainty for this case.
   - Suppose the detector assigns high confidence (0.95) to a phantom detection caused by sensor reflection. Discuss why confidence alone may be misleading in safety-critical contexts.
4. **Coordinate Transformation**
   - An object is observed in the local frame of vehicle O_1 at position $(p_x, p_y) = (10,5)$ m.
   - The origin of O_2 is 4 m east and 3 m north of O_1.
   - Compute the position of the object relative to O_2 assuming both vehicles use ENU coordinates.
   - An observer O_3 detects an object at $(p_x, p_y) = (15,-3)$ m. Observer O_4 is located 10 m east and 2 m north of O_3. Express the object's coordinates in O_4's frame.
   - Generalize the previous problem by deriving the transformation matrix $T_{O_3 \to O_4}$ for arbitrary displacements $(\Delta x, \Delta y)$ between observers.
   - If observer O_4 is additionally rotated by 30° with respect to O_3, update the transformation and compute the new object coordinates.
5. **Motion Vector Propagation**
   - A car is modeled with the CTRV state vector:

   $$x_k = [p_x, p_y, v, \Psi, \dot{\Psi}] = [0,0,20,30^\circ,0]$$

   where speed is $20,$m/s, heading 30°, and yaw rate 0.
   - Compute its position after $\Delta t = 2$ s.
6. **Comparing Motion Models**
   - Using the same initial conditions as Exercise 5, describe qualitatively how the predicted position would differ if the CTRA model were used with a longitudinal acceleration of $a = 2$ m/s^2.

- Likewise, compare with the CV model.

7. **Geopositioning and GNSS**
   - A vehicle reports its position as $(\text{lat}, \text{lon}) = (52.5200°N, 13.4050°E)$ in WGS84. Explain how this can be approximated as a Cartesian coordinate in UTM.
   - Two vehicles measure positions using GNSS, but both suffer from a 2 m random error. Discuss the implications of this error when: (i) estimating absolute global positions, and (ii) estimating relative distance between the two vehicles when they are only 5 m apart.
   - A CAV network chooses to exchange relative Cartesian coordinates instead of geodetic coordinates. Justify this design decision in terms of uncertainty and computational cost.
8. **Time derivatives in rotating reference frames**
   - Consider a scenario in which an autonomous vehicle has an IMU mounted at its center of rotation. Derive a differential equation that relates the measurements of the IMU, expressed in the vehicle's body frame, to the longitudinal acceleration of the vehicle.
   - Consider a scenario in which an autonomous vehicle has an IMU that is not mounted at its center of rotation, but is displaced with respected to it by a vector r_{IMU}. Derive a differential equation that relates the measurements of the IMU, expressed in the vehicle's body frame, to the longitudinal acceleration of the vehicle.