CS 511 Formal Methods, Fall 2024 — Instructor: Assaf Kfoury

Homework Assignment 7

October 24, 2024 — Lucas Miguel Tassis

Exercise 1

To show that $\{\psi_1, \psi_2\} \models \varphi$, we can show that $\neg((\psi_1 \wedge \psi_2) \to \varphi)$ is a contradiction. Using the analytic tableaux:

$$\neg(\forall x \forall y \forall z(P(x,y) \wedge P(y,z) \to P(x,z)) \wedge \forall x \forall y(P(x,y) \to P(y,x)) \to \forall x \forall y \forall z(P(x,y) \wedge P(z,y) \to P(x,z)))$$

$$\forall x \forall y \forall z(P(x,y) \wedge P(y,z) \to P(x,z)) \wedge \forall x \forall y(P(x,y) \to P(y,x))$$

$$\neg \forall x \forall y \forall z(P(x,y) \wedge P(z,y) \to P(x,z)))$$

$$\exists x \exists y \exists z \neg(P(x,y) \wedge P(z,y) \to P(x,z)))$$

$$\forall x \forall y \forall z(P(x,y) \wedge P(y,z) \to P(x,z))$$

$$\forall x \forall y(P(x,y) \to P(y,x))$$

$$\neg(P(a,b) \wedge P(c,b) \to P(a,c))$$

$$P(a,b) \wedge P(c,b)$$

$$\neg P(a,c)$$

$$P(a,b)$$

$$P(c,b)$$

$$P(c,b) \to P(b,c)$$

- $\neg P(c,b)$
  - **X**
- $P(b,c)$
  - $P(a,b) \wedge P(b,c) \to P(a,c)$
    - $\neg(P(a,b) \wedge P(b,c))$
      - $\neg P(a,b)$
        - **X**
      - $\neg P(b,c)$
        - **X**
    - $P(a,c)$
      - **X**

Since all paths are closed, the negation of $\neg((\psi_1 \wedge \psi_2) \to \varphi)$ is a contradiction, thus $\{\psi_1, \psi_2\} \models \varphi$.

Exercise 2

To show that $\{\psi_1, \psi_2, \psi_3\} \models \varphi$, we can show that $\neg((\psi_1 \wedge \psi_2 \wedge \psi_3) \rightarrow \varphi)$ is a contradiction. Using the analytic tableaux (in the next page). Notice that I already start with all the premises $\psi_1, \psi_2, \psi_3, \neg\varphi$ in the beginning of the tableaux. This was due lack of space, since the tree would break if I showed all these steps explicitly. But the steps are simple to explain, since we only have to use the $\neg(\psi \rightarrow \varphi)$ expansion rule, and then follow by $\wedge$ expansion rule 2 times in order to separate all these rules.
Since all paths are closed in the tableaux, the negation of $\neg((\psi_1 \wedge \psi_2 \wedge \psi_3) \rightarrow \varphi)$ is a contradiction, thus $\{\psi_1, \psi_2, \psi_3\} \models \varphi$.

Exercise 3

The Lean template file with the solutions is available on GitHub.

Exercise 4

The Lean template file with the solutions is available on GitHub.

$\forall x(Q(a,x,x))$

$\forall x \forall y \forall z(Q(x,y,z) \to Q(x,s(y),s(z)))$

$\forall x \forall y \forall z(Q(x,y,z) \to Q(y,x,z))$

$\neg \exists x(Q(s^{(2)}(a),s^{(3)}(a),x))$

$\forall x \neg(Q(s^{(2)}(a),s^{(3)}(a),x))$

$Q(a,a,a)$

$Q(a,a,a) \to Q(a,s(a),s(a))$

- $\neg Q(a,a,a)$
  - **X**
- $Q(a,s(a),s(a))$
  - $Q(a,s(a),s(a)) \to Q(a,s^{(2)}(a),s^{(2)}(a))$
    - $\neg Q(a,s(a),s(a))$
      - **X**
    - $Q(a,s^{(2)}(a),s^{(2)}(a))$
      - $Q(a,s^{(2)}(a),s^{(2)}(a)) \to Q(s^{(2)}(a),a,s^{(2)}(a)))$
        - $\neg Q(a,s^{(2)}(a),s^{(2)}(a))$
          - **X**
        - $Q(s^{(2)}(a),a,s^{(2)}(a))$
          - $Q(s^{(2)}(a),a,s^{(2)}(a)) \to Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
            - $\neg Q(s^{(2)}(a),a,s^{(2)}(a))$
              - **X**
            - $Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
              - $Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a)) \to Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
                - $\neg Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
                  - **X**
                - $Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
                  - $\neg Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
                    - **X**