

Homework Assignment 3*September 26, 2024*

Lucas Miguel Tassis

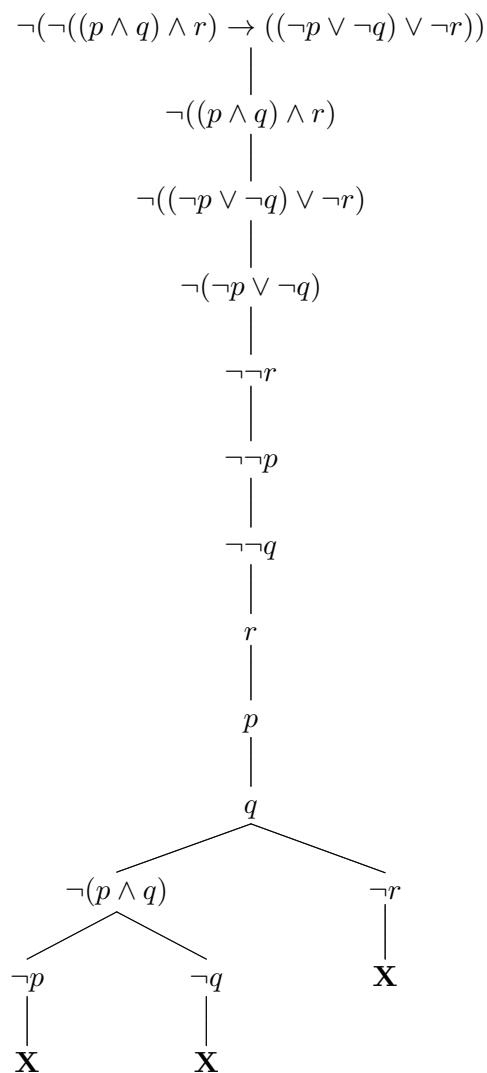
Exercise 1

$$\phi_1 \stackrel{\text{def}}{=} \neg((p \wedge q) \wedge r) \rightarrow ((\neg p \vee \neg q) \vee \neg r)$$

1.	$\neg((p \wedge q) \wedge r)$	assumption
2.	$\neg((\neg p \vee \neg q) \vee \neg r)$	assumption
3.	$\neg p$	assumption
4.	$\neg p \vee \neg q$	$\vee i$ 3
5.	$(\neg p \vee \neg q) \vee \neg r$	$\vee i$ 4
6.	\perp	$\neg e$ 2, 5
7.	$\neg\neg p$	$\neg i$ 3–6
8.	$\neg q$	assumption
9.	$\neg p \vee \neg q$	$\vee i$ 8
10.	$(\neg p \vee \neg q) \vee \neg r$	$\vee i$ 9
11.	\perp	$\neg e$ 2, 10
12.	$\neg\neg q$	$\neg i$ 8–11
13.	$\neg r$	assumption
14.	$\neg q \vee \neg r$	$\vee i$ 13
15.	$(\neg p \vee \neg q) \vee \neg r$	$\vee i$ 14
16.	\perp	$\neg e$ 2, 15
17.	$\neg\neg r$	$\neg i$ 13–16
18.	p	$\neg\neg e$ 7
19.	q	$\neg\neg e$ 12
20.	r	$\neg\neg e$ 17
21.	$p \wedge q$	$\wedge i$ 18, 19
22.	$(p \wedge q) \wedge r$	$\wedge i$ 20, 21
23.	\perp	$\neg e$ 1, 22
24.	$\neg\neg((\neg p \vee \neg q) \vee \neg r)$	$\neg i$ 2–23
25.	$((\neg p \vee \neg q) \vee \neg r)$	$\neg\neg e$ 24
26.	$\neg((p \wedge q) \wedge r) \rightarrow ((\neg p \vee \neg q) \vee \neg r)$	$\rightarrow i$ 1–25

Exercise 2

To show that the de Morgan's law for three variables $\phi_1 \stackrel{\text{def}}{=} \neg((p \wedge q) \wedge r) \rightarrow ((\neg p \vee \neg q) \vee \neg r)$ is valid, we need to show that its negation is a contradiction, thus:



Since all paths are closed, the negation of the de Morgan's law is a contradiction. Thus, we proved that the de Morgan's law for 3 variables is a tautology.

Exercise 3

The Lean template file with the solutions is available on [GitHub](#).

Exercise 4

The Lean template file with the solutions is available on [GitHub](#).