

Homework Assignment 8*October 31, 2024*

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Exercise 1**(a)**

i. Yes.



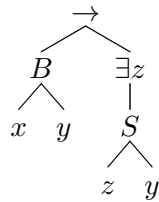
ii. Yes.

iii. No, because $f(m)$ is a term.

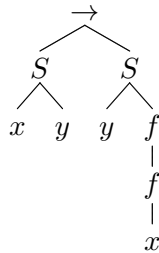
iv. No, because we cannot nest predicates in predicate logic.

v. No, because we cannot nest predicates in predicate logic.

vi. Yes.



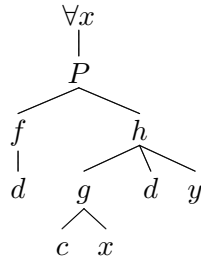
vii. Yes.



viii. No, because we cannot nest predicates in predicate logic.

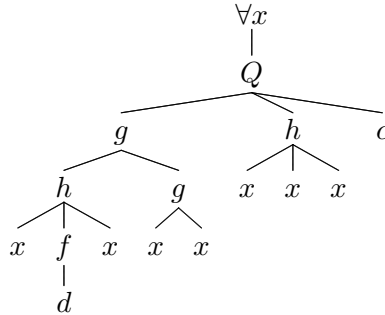
(b)

i. Yes.



ii. No, because we cannot nest predicates in predicate logic.

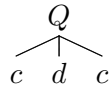
iii. Yes.



iv. No, because P has three arguments and only one was used.

v. No, because $g(x, y)$ is a function symbol not a predicate (violating the BNF of predicate logic).

vi. Yes.



Exercise 2

Exercise 2.3.2

This formula holds in a model iff the model has exactly two distinct elements.

Exercise 2.3.3

(a)

The formula would be:

$$\exists x \exists y \exists z (\neg(x \approx y) \wedge \neg(y \approx z) \wedge \neg(x \approx z) \wedge \forall w (w \approx x \vee w \approx y \vee w \approx z))$$

(b)

The formula would be:

$$\begin{aligned} & \exists x \forall w (w \approx x) \\ & \vee \exists x \exists y (\neg(x \approx y) \wedge \forall w (w \approx x \vee w \approx y)) \\ & \vee \exists x \exists y \exists z (\neg(x \approx y) \wedge \neg(y \approx z) \wedge \neg(x \approx z) \wedge \forall w (w \approx x \vee w \approx y \vee w \approx z)) \end{aligned}$$

Intuitively, the first part of the formula is the sentence for exactly one distinct element, the second is for exactly two distinct elements, and the third for exactly three distinct elements. Then, we just have to use a \vee between each sentence.

(c)

The formula would be:

$$\exists x \exists y \exists z (\neg(x \approx y) \wedge \neg(y \approx z) \wedge \neg(x \approx z))$$

This formula guarantees that exists at least three distinct elements.

Alternatively, we can also negate the formula for “at most two distinct elements” (thus we have at least three).

$$\begin{aligned} & \neg(\exists x \forall w (w \approx x) \\ & \vee \exists x \exists y (\neg(x \approx y) \wedge \forall w (w \approx x \vee w \approx y))) \end{aligned}$$

Exercise 3

The Lean template file with the solutions is available on [GitHub](#).

Exercise 4

The Lean template file with the solutions is available on [GitHub](#).

Problem 2

The Lean template file with the solutions is available on [GitHub](#).