

**Homework Assignment 01***September 12, 2024*

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**Exercise 1****Exercise 1.2.1 – (h):**  $p \vdash (p \rightarrow q) \rightarrow q$ 

1.  $p$  premise
2.  $p \rightarrow q$  assumption
3.  $q$   $\rightarrow$ e 1, 2
4.  $(p \rightarrow q) \rightarrow q$   $\rightarrow$ i 2–3

**Exercise 1.2.1 – (i):**  $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \wedge q \rightarrow r$ 

1.  $(p \rightarrow r) \wedge (q \rightarrow r)$  premise
2.  $p \rightarrow r$   $\wedge$ e<sub>1</sub> 1
3.  $p \wedge q$  assumption
4.  $p$   $\wedge$ e<sub>1</sub> 3
5.  $r$   $\rightarrow$ e 2, 4
6.  $(p \wedge q) \rightarrow r$   $\rightarrow$ i 3–5

**Exercise 1.2.1 – (j):**  $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$ 

1.  $q \rightarrow r$  premise
2.  $p \rightarrow q$  assumption
3.  $p$  assumption
4.  $q$   $\rightarrow$ e 2, 3
5.  $r$   $\rightarrow$ e 1, 4
6.  $p \rightarrow r$   $\rightarrow$ i 3–5
7.  $(p \rightarrow q) \rightarrow (p \rightarrow r)$   $\rightarrow$ i 2–6

**Exercise 2****Exercise 1.4.2 – (g):**  $((p \rightarrow q) \rightarrow p) \rightarrow p$ 

$p$	$q$	$p \rightarrow q$	$(p \rightarrow q) \rightarrow p$	$((p \rightarrow q) \rightarrow p) \rightarrow p$
<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>
<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>
<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>
<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>T</b>

**Exercise 1.4.2 – (h):**  $((p \vee q) \rightarrow r) \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$

$p$	$q$	$r$	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$((p \vee q) \rightarrow r) \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	F	T	F	F	T	T	T
T	F	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	T	F	T	F	T	F	T	T
F	F	F	F	T	T	T	T	T

**Exercise 1.4.2 – (i):**  $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$

$p$	$q$	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

### Exercise 3

Link to code on [GitHub](#). The solution was:

```
-- Example 1.3.4
example {w : ℚ} (h1 : 3 * w + 1 = 4) : w = 1 :=
  calc
    w = ((3 * w + 1) / 3) - (1 / 3) := by ring
    _ = (4 / 3) - 1 / 3 := by rw [h1]
    _ = 1 := by ring
```

### Exercise 4

Link to code on [GitHub](#). The solution was:

```
-- Example 1.3.9
example {a b : ℚ} (h1 : a - 3 = 2 * b) : a ^ 2 - a + 3 = 4 * b ^ 2 + 10 * b + 9 :=
  calc
    a ^ 2 - a + 3 = ((a - 3) ^ 2 + 6 * a - 9) - a + 3 := by ring
    _ = (a - 3) ^ 2 + 5 * a - 6 := by ring
    _ = (a - 3) ^ 2 + 5 * ((a - 3) + 3) - 6 := by ring
    _ = (a - 3) ^ 2 + 5 * (a - 3) + 9 := by ring
    _ = (2 * b) ^ 2 + 5 * (2 * b) + 9 := by rw [h1]
    _ = 4 * b ^ 2 + 10 * b + 9 := by ring
```