CS 511 Formal Methods, Fall 2024	Instructor: Assaf Kfoury

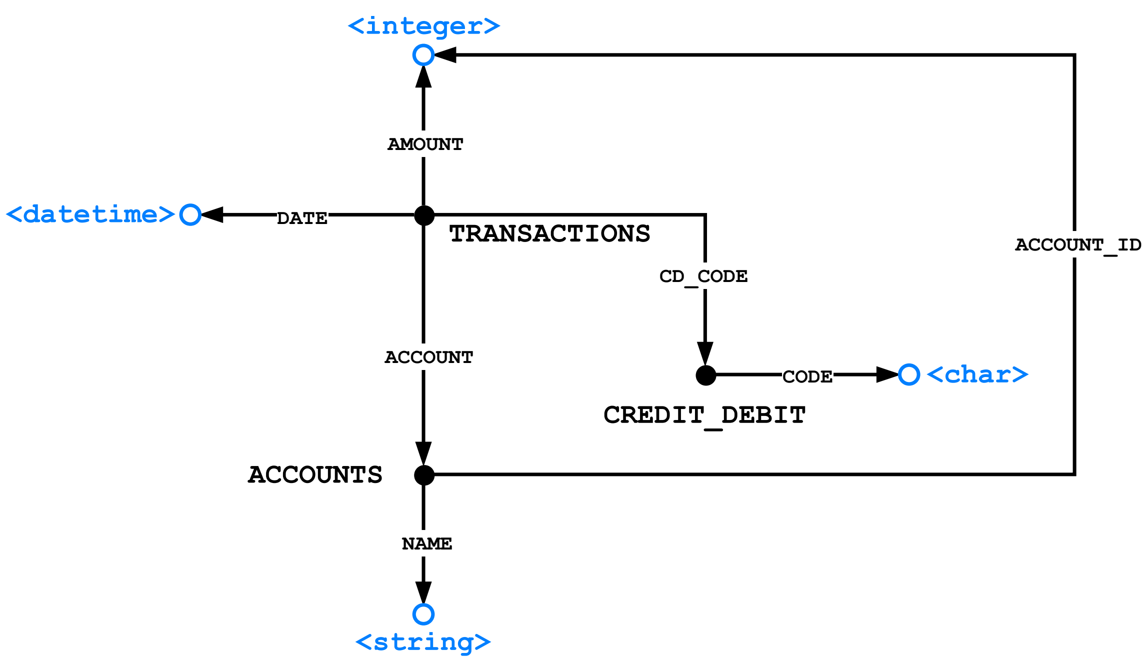
Homework Assignment 13

December 12, 2024 Lucas Miguel Tassis

Exercise 1

(a) First-order logic is sufficient for most of the basic SQL operations. Most of the SQL operations such as `SELECT`, `GROUPBY`, `JOIN`, in addition to the `EXISTS` operation mentioned in the exercise are first-order definable. More generally, relational algebra, which is the theoretical foundation of most relational databases is first-order definable[1]. However, some more complex operations present in newer database systems might need the use of second-order logic (such as operations over sets and their subsets, *e.g.* queries about all subsets).

(b) The diagram that represents the schema is:



Exercise 2

(a) The 19 morphisms of $\mathcal{K}$ are:

(1) $f : A \to B$	(6) $j : D \to A$	(11) $\mathrm{id}_D : D \to D$
(2) $h : A \to C$	(7) $\ell : D \to C$	(12) $f \circ k : A \to C$
(3) $i : A \to D$	(8) $\mathrm{id}_A : A \to A$	(13) $i \circ \ell : A \to C$
(4) $g : B \to A$	(9) $\mathrm{id}_B : B \to B$	(14) $g \circ h : B \to C$
(5) $k : B \to C$	(10) $\mathrm{id}_C : C \to C$	(15) $g \circ i : B \to D$

[1] Reference: Logic and Databases by Phokion G. Kolaitis

(16) $j \circ f : D \to B$

(17) $j \circ h : D \to C$

(18) $(g \circ i) \circ \ell : B \to C$

(19) $(j \circ f) \circ k : D \to C$

(b) The list of morphisms of $\mathcal{K}'$ is:

(1) $\mathrm{id}'_A : A \to A$

(2) $\mathrm{id}'_B : B \to B$

(3) $\mathrm{id}'_C : C \to C$

(4) $\mathrm{id}'_D : D \to D$

(5) $f' : A \to B$

(6) $h' : A \to C$

(7) $i' : A \to D$

(8) $g' : B \to A$

(9) $k' : B \to C$

(10) $j' : D \to A$

(11) $\ell' : D \to C$

(12) $(g \circ i)' : B \to D$

(13) $(j \circ f)' : D \to B$

Notice that these morphisms cover all the domains and codomains present in $\mathcal{K}$, just eliminating the redundancy.

(c) First, recall that the properties of a functor are:

(1) Objects to objects;

(2) Morphisms to morphisms;

(3) Respect the identity;

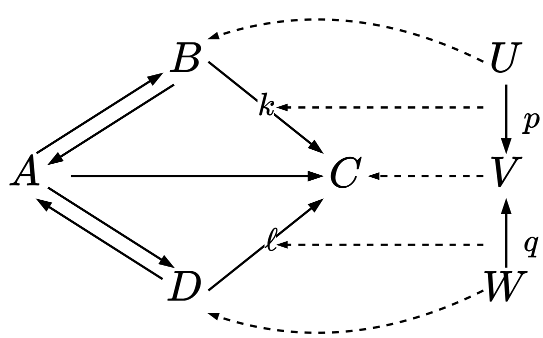
(4) Respect composition.

Therefore, we can define a functor $F : \mathcal{V} \to \mathcal{K}$:

(1) $F(U) = B$, $F(W) = D$, and $F(V) = C$;

(2) $F(p) = k$ and $F(q) = \ell$;

(3) $F(\mathrm{id}_U) = \mathrm{id}_B$, $F(\mathrm{id}_W) = \mathrm{id}_D$, and $F(\mathrm{id}_V) = \mathrm{id}_C$



(d) Because in this category we have four vertices and all vertices have outgoing connections. If we look at $\mathcal{K}$, the vertex C is a sink (*i.e.* it has no outgoing connections). Therefore, there cannot exist a functor from this category to $\mathcal{K}$, because there is not a mapping for C, violating the functor properties.

Exercise 3

The Lean template file with the solutions is available on GitHub.

Exercise 4

The Lean template file with the solutions is available on GitHub.

Problem 2

The Lean template file with the solutions is available on GitHub.