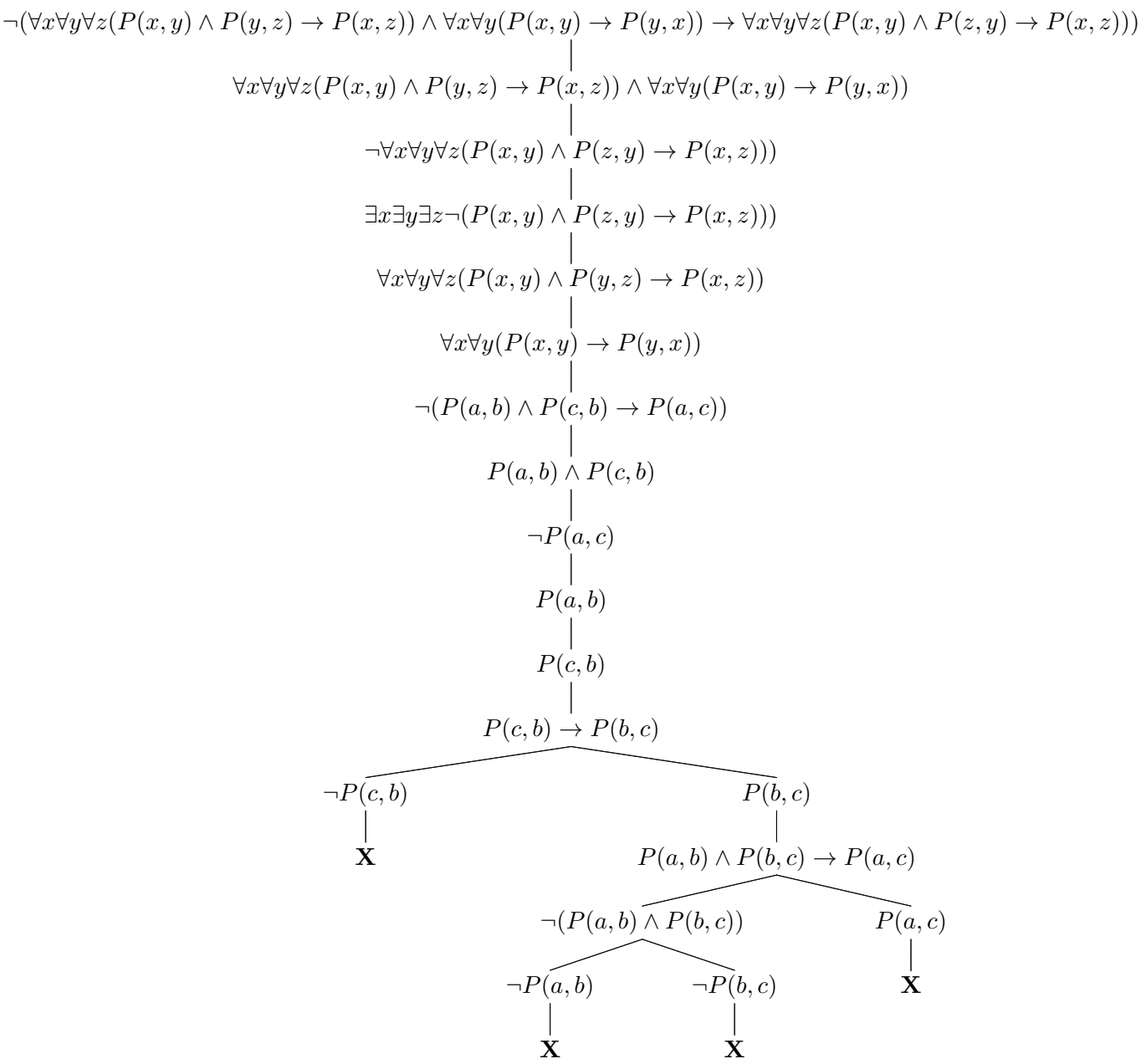
CS 511 Formal Methods, Fall 2024 — Instructor: Assaf Kfoury

Homework Assignment 7

October 24, 2024 — Lucas Miguel Tassis

Exercise 1

To show that $\{\psi_1, \psi_2\} \models \varphi$, we can show that $\neg((\psi_1 \wedge \psi_2) \to \varphi)$ is a contradiction. Using the analytic tableaux:



Since all paths are closed, the negation of $\neg((\psi_1 \wedge \psi_2) \to \varphi)$ is a contradiction, thus $\{\psi_1, \psi_2\} \models \varphi$.

Exercise 2

To show that $\{\psi_1, \psi_2, \psi_3\} \models \varphi$, we can show that $\neg((\psi_1 \wedge \psi_2 \wedge \psi_3) \rightarrow \varphi)$ is a contradiction. Using the analytic tableaux (in the next page). Notice that I already start with all the premises $\psi_1, \psi_2, \psi_3, \neg\varphi$ in the beginning of the tableaux. This was due lack of space, since the tree would break if I showed all these steps explicitly. But the steps are simple to explain, since we only have to use the $\neg(\psi \rightarrow \varphi)$ expansion rule, and then follow by $\wedge$ expansion rule 2 times in order to separate all these rules.
Since all paths are closed in the tableaux, the negation of $\neg((\psi_1 \wedge \psi_2 \wedge \psi_3) \rightarrow \varphi)$ is a contradiction, thus $\{\psi_1, \psi_2, \psi_3\} \models \varphi$.

Exercise 3

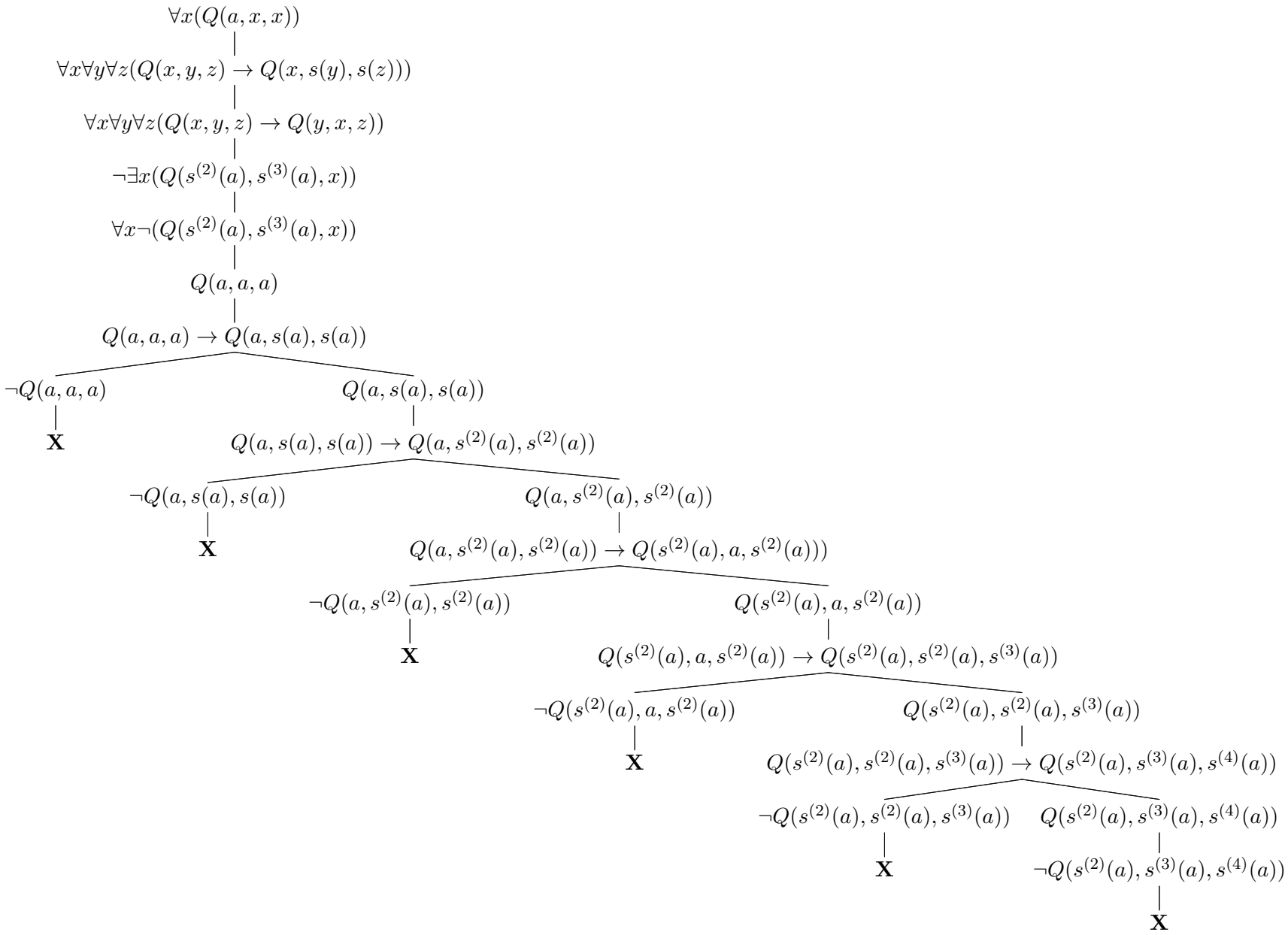
The Lean template file with the solutions is available on GitHub.

Exercise 4

The Lean template file with the solutions is available on GitHub.

Problem 2

The Lean template file with the solutions is available on GitHub.


$\forall x(Q(a,x,x))$
$\forall x\forall y\forall z(Q(x,y,z) \to Q(x,s(y),s(z)))$
$\forall x\forall y\forall z(Q(x,y,z) \to Q(y,x,z))$
$\neg\exists x(Q(s^{(2)}(a),s^{(3)}(a),x))$
$\forall x\neg(Q(s^{(2)}(a),s^{(3)}(a),x))$
$Q(a,a,a)$
$Q(a,a,a) \to Q(a,s(a),s(a))$
$\neg Q(a,a,a)$
$Q(a,s(a),s(a))$
$\mathbf{X}$
$Q(a,s(a),s(a)) \to Q(a,s^{(2)}(a),s^{(2)}(a))$
$\neg Q(a,s(a),s(a))$
$Q(a,s^{(2)}(a),s^{(2)}(a))$
$\mathbf{X}$
$Q(a,s^{(2)}(a),s^{(2)}(a)) \to Q(s^{(2)}(a),a,s^{(2)}(a)))$
$\neg Q(a,s^{(2)}(a),s^{(2)}(a))$
$Q(s^{(2)}(a),a,s^{(2)}(a))$
$\mathbf{X}$
$Q(s^{(2)}(a),a,s^{(2)}(a)) \to Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
$\neg Q(s^{(2)}(a),a,s^{(2)}(a))$
$Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
$\mathbf{X}$
$Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a)) \to Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
$\neg Q(s^{(2)}(a),s^{(2)}(a),s^{(3)}(a))$
$Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
$\mathbf{X}$
$\neg Q(s^{(2)}(a),s^{(3)}(a),s^{(4)}(a))$
$\mathbf{X}$