CS 511 Formal Methods, Fall 2024 — Instructor: Assaf Kfoury

Homework Assignment 01

September 12, 2024 — Lucas Miguel Tassis

Exercise 1

Exercise 1.2.1 – (h): $p \vdash (p \to q) \to q$

1.	p	premise
2.	$p \to q$	assumption
3.	q	$\to$e 1, 2
4.	$(p \to q) \to q$	$\to$i 2–3

Exercise 1.2.1 – (i): $(p \to r) \wedge (q \to r) \vdash p \wedge q \to r$

1.	$(p \to r) \wedge (q \to r)$	premise
2.	$p \to r$	$\wedge e_1$ 1
3.	$p \wedge q$	assumption
4.	p	$\wedge e_1$ 3
5.	r	$\to$e 2, 4
6.	$(p \wedge q) \to r$	$\to$i 3–5

Exercise 1.2.1 – (j): $q \to r \vdash (p \to q) \to (p \to r)$

1.	$q \to r$	premise
2.	$p \to q$	assumption
3.	p	assumption
4.	q	$\to$e 2, 3
5.	r	$\to$e 1, 4
6.	$p \to r$	$\to$i 3–5
7.	$(p \to q) \to (p \to r)$	$\to$i 2–6

Exercise 2

Exercise 1.4.2 – (g): $((p \to q) \to p) \to p$

p	q	$p \to q$	$(p \to q) \to p$	$((p \to q) \to p) \to p$
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**
F	**F**	**T**	**F**	**T**

Exercise 1.4.2 – (h): $((p \vee q) \to r) \to ((p \to r) \vee (q \to r))$

p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \vee (q \to r)$	$((p \vee q) \to r) \to ((p \to r) \vee (q \to r))$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	F	T	F	F	T	T	T
T	F	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	T	F	T	F	T	F	T	T
F	F	F	F	T	T	T	T	T

Exercise 1.4.2 – (i): $(p \to q) \to (\neg p \to \neg q)$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \to \neg q$	$(p \to q) \to (\neg p \to \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Exercise 3

Link to code on GitHub. The solution was:

```
-- Example 1.3.4
example {w : ℚ} (h1 : 3 * w + 1 = 4) : w = 1 :=
  calc
    w = ((3 * w + 1) / 3) - (1 / 3) := by ring
    _ = (4 / 3) - 1 / 3 := by rw [h1]
    _ = 1 := by ring
```

Exercise 4

Link to code on GitHub. The solution was:

```
-- Example 1.3.9
example {a b : ℚ} (h1 : a - 3 = 2 * b) : a ^ 2 - a + 3 = 4 * b ^ 2 + 10 * b + 9 :=
  calc
    a ^ 2 - a + 3 = ((a - 3) ^ 2 + 6 * a - 9) - a + 3 := by ring
    _ = (a - 3) ^ 2 + 5 * a - 6 := by ring
    _ = (a - 3) ^ 2 + 5 * ((a - 3) + 3) - 6 := by ring
    _ = (a - 3) ^ 2 + 5 * (a - 3) + 9 := by ring
    _ = (2 * b) ^ 2 + 5 * (2 * b) + 9 := by rw [h1]
    _ = 4 * b ^ 2 + 10 * b + 9 := by ring
```