Speed Measurement by Optical Techniques

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ABSTRACT: Here I will demonstrate the use of optical wavelength electromagnetic radiation from a helium-neon laser to obtain the relative velocity of various systems. Using optical interferometric techniques an expected Doppler shift is detected in the frequency of a recombined 632 nm laser beam to measure the moving objects velocity component along the line of sight of the incident laser. This technique allowed the measurement of rotation rate of a 8.5±0.1 cm radius wheel spinning at constant velocity (0.38π±.1 rad/s) as well as the average velocity of a toy train travelling along the source laser beam's line of sight (.29±.08 m/s). An attempt was also made to explore the velocity profile of a laminar flow of water mixed with scattering particulates, however this portion of the experiment had limited success. Finally, the rotation rate of a constant velocity spinning wheel of radius 5±0.1 cm was obtained using interference scattering techniques similar to Young's Double Slit Experiment [1] obtaining a value of 47±3 rotations per second.

INTRODUCTION: The ability to measure the speed of a moving object relative to an observer with minimal interaction with the object of interest has great practical importance across a broad range of applications. Military, industrial, and scientific endeavors of all kinds find value in knowing the velocity of a distant object to a high precision without noticeable impact on the moving system [2,3,4].

Among the most widely used method for measuring the speed of a moving object is to use the Doppler effect. The Doppler effect is a phenomenon in which an incident wave reflected from a moving object will be shifted in frequency in proportion to the line of sight velocity of the moving target. Modern humans are mostly familiar with examples such as police traffic radar which uses electromagnetic radiation in the radio wavelengths or with the apparent change in pitch of a passing ambulance siren. This paper aims to use the Doppler effect optically, with wavelengths much shorter than those from everyday experience.

Firstly, a discussion of the mathematics of the Doppler effect. The phenomenon was first described mathematically by Christian Doppler in 1842 [5]. The most basic generalization is that the Doppler effect or shift is the change in frequency or wavelength of a wave for an observer who is moving relative to the wave source [6]. The physical reason for the Doppler effect is that when the source of the wave is moving towards the observer each successive wave crest is emitted from a position closer to the observer than the previous wave. This means that the wave takes slightly less time to reach the observer than the previous wave. Thus, the time between the arrival of a successive wave's crest at the observer is reduced. This time reduction leads to an increased frequency. For these experiments we will be treating the medium of travel of the electromagnetic radiation emitted from the laser (air) as a vacuum, due to not only the relatively short distances the light will travel on each path of the interferometric arms (< 1 m) but also because the air has values very close to those of a vacuum.

This assumption allows one to only consider the relative difference in velocity between observer and source, simplifying the math. In terms of frequency this gives a frequency shifted Doppler formula:

$$f = \frac{c}{c+v} f_0 \tag{1}$$

Where f is the observed frequency, f_0 is the emitted frequency, c is the speed of light in vacuum (~$3\text{x}10^8$ m/s), and v is the velocity of the source relative to the observer. The source velocity v is by definition positive if moving away from the receiver. Of note is that this velocity is only the velocity along line of sight.

Experiment 1: Our first experiment begins with the construction of an optical interferometer as shown in Figure 1 below.

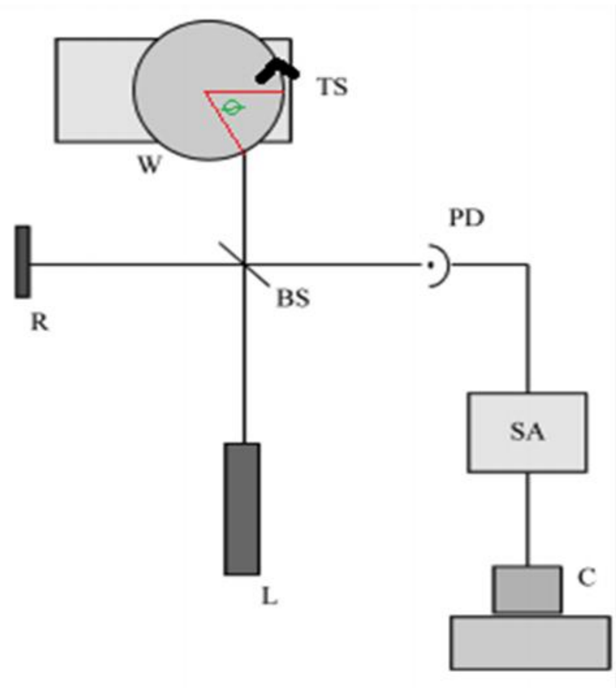

Figure 1: Slow wheel rotation rate measurement apparatus This figure shows the setup used for the determination of the wheel rotation rate. L is a Melles Griot 35mW He-Ne laser, R is a stationary reference mirror, BS is a beam splitter, W is the slowly rotating wheel, TS is a translating stage allowing measured transverse movement of the wheel, SA is a spectrum analyzer, PD is a Thorlabs PDA 36A photon detector, and C is a computer for spectra output from the spectrum analyzer.

Among the greatest challenges in setting up this experimental apparatus are proper positioning of the components and maximizing signal to noise ratio. Simple components are used for spatial filtering to ensure the photons emitted from the laser are following an expected path. These components include a focusing lens that can be seen directly to the right of the laser as well as an iris directly above the beam splitter. The source beam that is incident on the spinning wheel will emit photons that are not directed exactly back towards the beam splitter. These photons are blocked by a piece of paper used as a baffle positioned between the photon detector and wheel. An important first step in calibrating the arms of the interferometer is to ensure that the intensity of light from each arm is of equal magnitude and as far above noise as possible. This is easily accomplished by measurement of the output voltage of the photon detector when blocking one or both of the arms. For this experimental setup our noise was measured by blocking the laser completely which gave a baseline value of 20±3 mV. The arms of the interferometer were then calibrated to have matching intensity at the photon detector by using a polarization filter placed in the path of the reference beam to attenuate that beam only. This gave a value of 40±6.5 mV per arm. Configuration of the photon detector gain is very important. Since the frequency of interest falls in the 20 to 200 kHz range, a gain setting of 20 dB was chosen per the photon detectors manual this provides a bandwidth of 1 MHz.

The wheel's rotation rate can be estimated by removing a small slit of the reflected material the source beam is incident upon. This missing portion of reflective tape will cause a drop in intensity of the recombined beam measured by the photon detector. Thus, an estimate of the rotation period of the wheel can be deduced from a noticeable drop in voltage measured on the oscilloscope. This rough estimate gave a rotation period of 5 seconds which corresponds to an angular velocity of 0.4π rad/s for a wheel with radius 8.5 cm.

The goal of the experiment will be to use an optical Doppler technique to determine the rotation rate of the wheel. As stated earlier, the Doppler formula used can only detect a frequency shift proportional to the velocity component along the line of sight. For this reason, measurements are made at 1 cm spacings by translating the wheel perpendicular to the path of the source laser beam. This is accomplished by mounting the wheel on a moveable stage, as shown in figure 1. One expects to encounter no Doppler shift when the source beam is incident near the center of the wheel (as pictured in figure 1) and a shift of order f_0v/c when the wheel is translated by an amount just outside its radius. Measurement data for values starting at the near center and moving the wheel 'up' in the picture are shown below.

Offset (cm)	Run 1 (kHz)	Run 2 (kHz)	Run 3 (kHz)	Avg. (kHz)
1	27.4	28.05	28.05	27.8±0.37
2	67.7	65.7	67.7	67±1.15
3	107.3	108.65	110.6	108.8±1.65
4	146.35	145.05	141.8	144.4±2.34
5	195.1	203.55	204.85	201.1±5.29
6	225	236.1	235.4	232.1±6.21
7	264	259.4	269.2	264.2±4.9

Table 1: Beat frequency measurement for slow spinning wheel

To interpret the data contained in table 1 one needs to understand the notion of beat frequencies. Assuming the reference beam and source beam have approximately the same amplitude A we can write the signal S of the recombined beam as $S^2 =< Acos\omega_R t + Acos\omega_S >$ where ω_R and ω_S are the frequency of the reference and source beams respectively. The angled brackets indicate averaging done by the photon detector and other involved electronics. S can be rewritten using trigonometric identities to arrive at:

$$S^2 = A^2\{< cos^2\omega_R t > + < cos^2\omega_S t > + < cos^2(\omega_R - \omega_S t) > + < cos^2(\omega_R + \omega_S t) >\}$$

Working in optical frequencies makes the resolution of the frequency of the reference and source beams impractical. The detection system averaging will also make the first, second and last terms in the equation above unusable. However, the third term contains a difference of frequencies that can be resolved with modern detection equipment. This ‘beat frequency’ is what is measured as a strong peak on a spectrum analyzer that disappears when either arm of the interferometer is blocked. It is these values that are reported on table 1. The linear velocity of the wheel can be solved for by plotting the angular position linearly against the detected frequency, as in plot 1 at right.

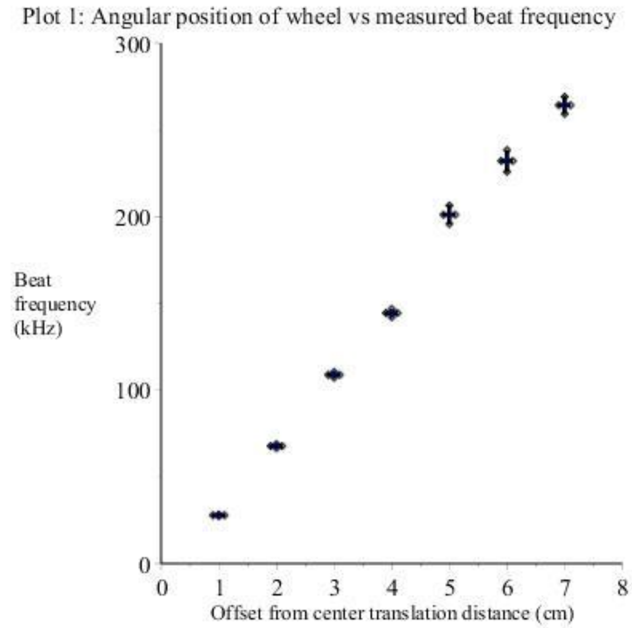

Figure 2:Plot of experiment 1 data with error bars indicating error in position of offset (±1 mm) and error in beat frequency average.

Notice the slope of the data points in figure 2 is fairly constant. This is the first indicator that the hypothesis of Doppler shift being linearly proportional to the angular position of the wheel relative to the source beam is correct. Combining the idea of the beat frequency in equation 2 with equation 1 it can be shown that:

$$|f - f_0| = \frac{2f_0 v}{c} = \frac{2v}{\lambda} \tag{2}$$

Where f and f_0 are the shifted and laser frequency respectively, c is the speed of light, λ is the wavelength of the laser light and v is the component of velocity along the laser line of sight. From the geometry of the diagram in figure 1 it is apparent the component of the linear velocity along the line of sight of the incident laser is the sin of angle theta. As the translating stage is varied in position relative to the laser this angle grows until the sin of the angle reaches its maximum value of 1.

A closer analysis of the measurement data allows a linear fit as shown in figure 3. The slope of this line reveals the angular velocity of the wheel according to the relation:

$$slope = \frac{2\omega}{\lambda} \quad (3)$$

The linear fit provides a slope of 38.7±2.6 kHz/cm, with a correlation coefficient of 0.999. This gives a value for the radial velocity of the wheel 0.38π rad/sec.

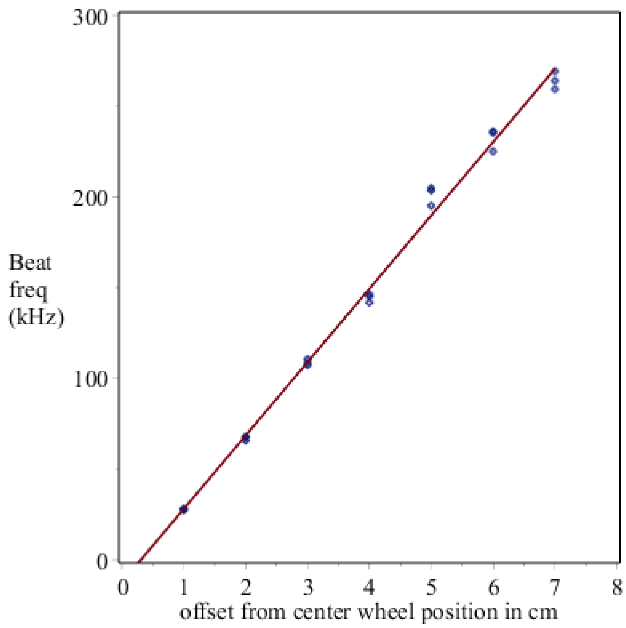

Figure 3: Linear least squares fit of experimental beat data. This plot is based on the assumption that the moving object is equivalent to a translating mirror. Three runs were completed, and all data points are shown. Red line is a linear fit with a zero-frequency intercept at ~.25cm.

Experiment 2: Next was an experiment involving a toy train moving on an electrified track along the line of sight of the laser. This setup has the advantage of the entire component of the train's velocity, instead of the velocity component changing with different angles. However, the train accelerates during its path along the track, making the statistical analysis more challenging. A similar reasoning to the first experiment will permit a measurement of the Doppler shift, again using the 'beat frequency', but only average velocity values will be solved for. Figure 4 contains a diagram of the experimental setup.

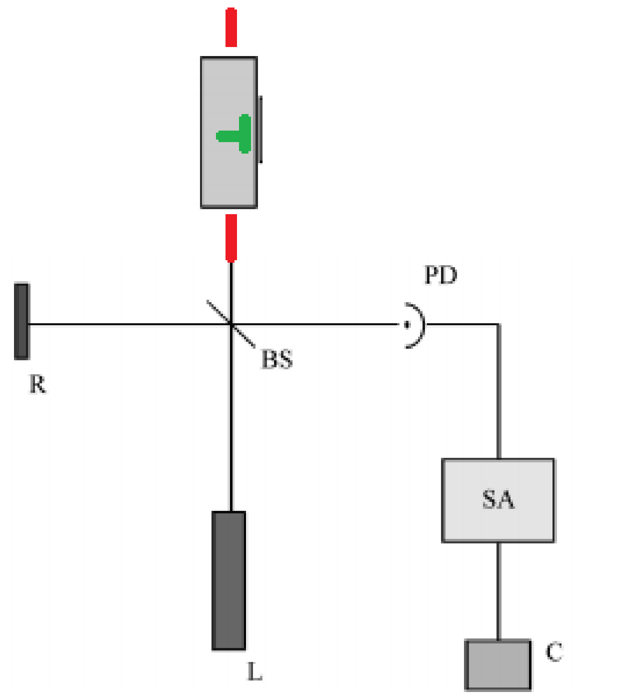

Figure 4: Train average velocity experiment setup. The common components with experiment one are labeled as such where SA is an Agilent N9320B 9kHz-3GHz spectrum analyzer connected to a computer C. The toy train, T, is mounted on a track and allowed to move along the laser line of sight over distances of about 1 meter.

The methodology used in obtaining measurements was as follows. First a 'baseline' was established to capture a noise spectrum. This was done my turning the spectrum analyzer into a max hold mode in which input spectra are peak held for a period of time of about ten minutes with the lights and laser off. This noise spectrum was then subtracted from a similar max hold measurement taken during ten runs of the toy train with the laser on. Each run of the train was positioned so that the laser light was incident on the reflective material attached to the back of the train. The train was placed at the same position for each ~1 m run and removed from the track immediately once it crossed a marked distance. The toy train was connected to a simple current source, or more accurately the train track was connected to the source, and great care was taken to give the train the same voltage. This was an attempt to have each run repeat the same motion, with the same acceleration when turned on. A rudimentary measurement of the trains average velocity using a stopwatch and ruler gave a value of ~1/3 m/s.

Figure 5 shows the trace obtained after subtraction of the baseline noise.

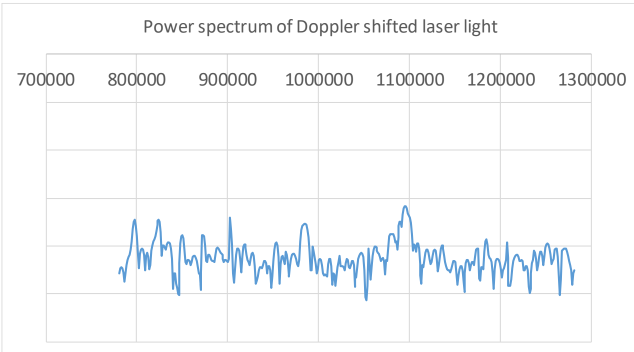

Figure 5: This trace shows the power spectrum obtained over ten runs of the toy train. The ordinate is arbitrary units and the abscissa is in units of frequency. We see several strong peaks of different widths indicating the changing velocity of the train during its one-meter journey. A fast Fourier transform is used by the spectrum analyzer to bring measurements from the time domain into the frequency domain.

The amplitude and width of the peaks are proportional to the time the train spent at a certain velocity. This velocity profile can be analyzed using the method of moments to obtain an average velocity of the train over all ten runs. This gives a range of velocities of the train during its one-meter journey. These values are of order .1 m/s shortly after the train initially accelerates to .5 m/s at the end of the track. The mean value, or first moment, provides the best estimation of the average velocity which is 0.29±0.08 m/s.

Experiment 3: For this experiment the velocity profile of water with light scattering particulates was examined using the Doppler effect. This experiment was met with very little success. The data obtained was riddled with noise even after many attempts to obtain frequency spectrum data. Due to this I will discuss how the Doppler effect could have been used to obtain the velocity of the flowing water at different positions in the flow tube. Figure 6 shows an idealized velocity profile for a laminar fluid flow.

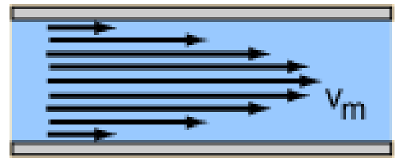

Figure 6: Laminar fluid flow (credit Wikipedia)

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We can see that in non-turbulent fluid flow such as this that the magnitude of velocity vectors of the water, indicated by v_m in the figure, are similar to a deck of cards with layers that slide over each other [7]. As the number of these idealized layers approaches infinity, the profile takes on a parabolic shape. Noting the length of velocity vectors shown to create this parabolic shape, it is assumed the water is moving slower near the edges of the tube and the velocity has a maximum at the tubes center. Figure 7 below shows the experimental setup for this experiment

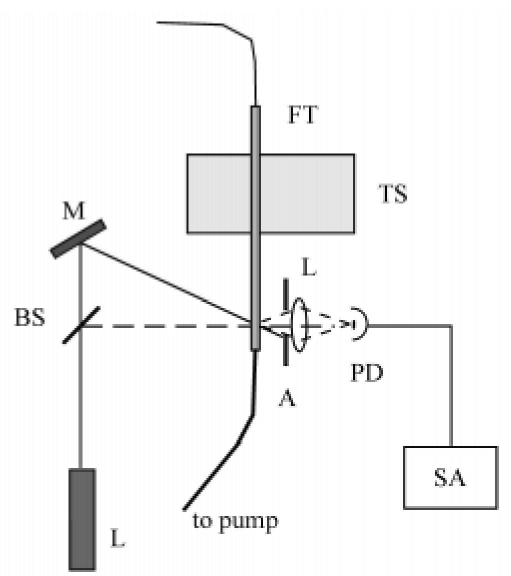

Figure 7: Block diagram of laminar water flow experiment. The labeled equipment is the same as the prior experiments however the translating stage, TS, is placed on a lift to allow vertical adjustment relative to the laser beam. A pump is connected to a flow tube, FT, that pushes water through a narrow tube. An aperture, A, is used as a baffle to prevent light from the stationary mirror M from reaching the detector.

A trace is viewed on the spectrum analyzer with the laser beam at different vertical positions of the flow tube. Figure 8 below shows the actual setup used. Note the position of the laser beam at the expected maximal flow velocity position in the center. Similar to the slow wheel experiment above an attempt was made to find a peak in the spectral trace that corresponded to the Doppler shift beat frequency due to scattered light off of particulates suspended in the water as they moved thorough the tube transverse to the laser beam. Even with many attempts at different positions a peak wasn't observed above noise that could be reasonably targeted as being a beat frequency from a Doppler shift. Repeated measurement with more scattering particulates would be a recommendation for future attempts as similar experiments have been done with a much more opaque fluid [8].

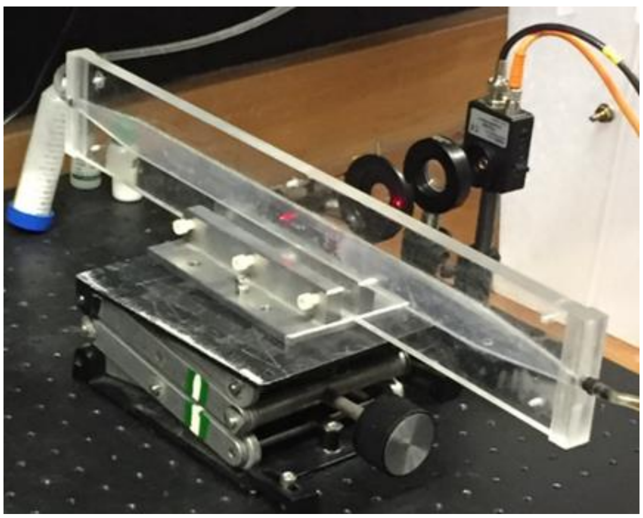

Figure 8: Experimental setup for laminar water flow.
This photograph shows the laser position when measuring Doppler shift due to laminar flow in the center of the flow tube.

Experiment 4: Finally, the rotation rate of a smaller and faster rotating wheel than that used in the first experiment was solved for using interference scattering techniques. The experimental setup is shown in simplified form in figure 10 below. Due to the angular separation between the two beams of light incident on the same point of the wheel this experiment has many parallels to Young's Double Slit experiment [1].

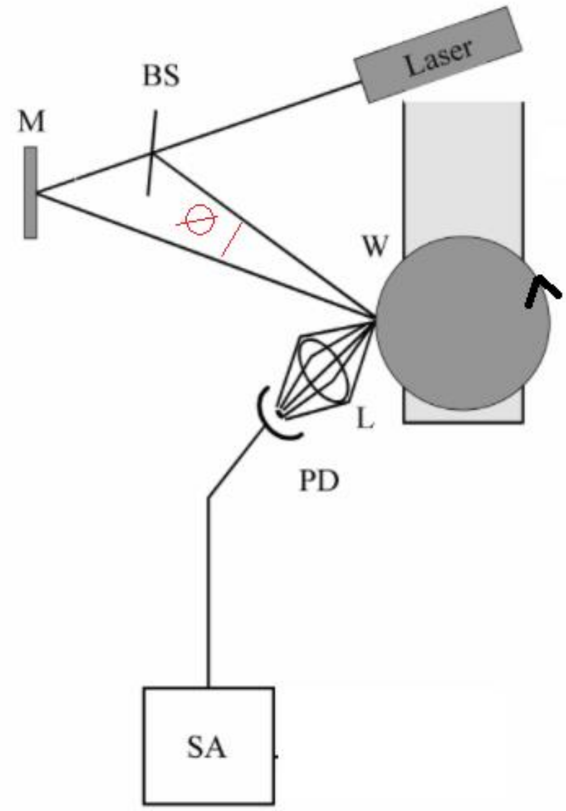

Figure 9: Block diagram of small wheel rotation rate experiment.
Components are the same as previously mentioned experiments however the wheel base remains in a fixed position and a focusing lens L is used to capture light scattered from the reflective tape on the rim of the wheel. The angle was minimized to a value of 10±2 degrees.

In following with Young's Double Slit experiment, when the laser light is incident on the wheel at the same time a peak will be recorded on the spectrum analyzer. These peaks correspond to moments in time when constructive interference occurs due to the maximum of the laser light's photonic wave hitting the wheel's rim in the same spot from both of the separated beams. This peak to peak length dL is given by:

$$dL = \frac{\lambda}{2sin\theta} \qquad (4)$$

Where λ is the wavelength of the emitted laser light and the angle is proportional to the path length difference of each beam. For the setup used an angle of 10° gives a value for dL of 1.8±.28 μm. This spacing of the interference fringes can then be related to a frequency peak on the spectrum analyzer. This frequency peak is directly proportional to the number of times the moving wheel causes a constructive maximum to occur. Equation 5 below gives this relation.

$$frequency\ peak = \frac{linear\ velocity}{dL} \qquad (5)$$

Where the linear velocity is the full velocity of the wheel, not just a component along line of sight as for the prior experiments that used Doppler shift techniques. An estimate of the expected frequency peak was obtained by viewing a period of rotation on an oscilloscope in a similar manner to the slower moving wheel. This allowed a tighter range of frequencies to be traced on the spectrum analyzer. Figure 10 below is the observed trace on the spectrum analyzer.

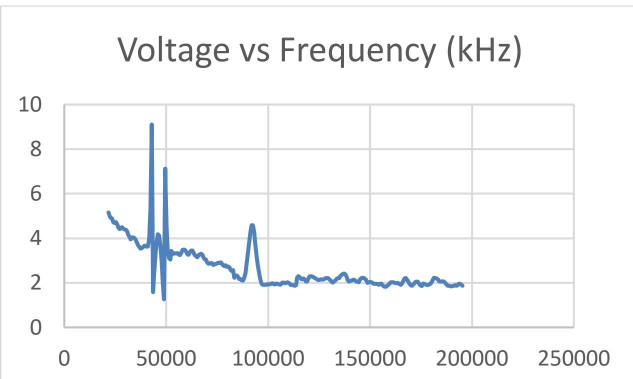

Figure 10: Spectrum analyzer trace for fast spinning wheel. By blocking components as in prior experiments, the two peaks near 50 kHz can be attributed to noise. The wider peak near the 900 kHz point was only present when both incident laser beams were present.

A full width half maximum analysis of the peak of interest gives a value of 0.9±.15 MHz for the frequency of fringe crossing. This can be used with the formula for linear velocity to solve for the period of one rotation, and hence the rotation rate.

$$period = \frac{2\pi * radius}{linear\ velocity} \qquad (6)$$

Inputting the measured values, a period of one rotation is found of 22±2 ms. This gives a rotation rate of 47 rotations per second.

Summary: This set of experiments has been enlightening not only for understanding the behavior of light and the phenomenon of Doppler shift, but also what can be accomplished with only simple lab equipment. In fact, if the experiments were performed again the number of optical components used could possibly be reduced, especially for the slow wheel experiment. This would require a more thought out placement of components, as well as more in depth knowledge of the tuning of the spectrum analyzer, however the benefit would be reduced noise and error due to these unneeded components.

The lack of results for the laminar water flow velocity profile is disappointing, however this feels like the most 'real world' experiment of the set. The wheel rotation rates for both wheels were easily solved for using a laser and oscilloscope, and the trains average velocity can be estimated to arbitrary precision with repeated measurement using a stopwatch. The water flow velocity is more challenging in that it cannot be solved by these simple means and even attempts to watch a particle travel through the tube and time it, somewhat similar to how the trains speed is estimated, prove fruitless as repeated measurements varied wildly.

Minimization of noise is essential to all these experiments. Upon reflection, the most pronounced error must surely be due to motional shifts in the various setups. Since the wavelengths being used are so small, even the tiniest movement of the laser's target will cause a huge uncertainty. This is the most difficult to overcome aspect of these experiments.

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