Assigment 5

CS 2813 - Discrete Structures

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Question 1: *Considering the following pseudocode from Figure 1 for the function procedure all_permutations:*

(a) Decribe what this procedure does.

Using S = {7, 2} as an example, the function all_permutations will check if the length of S is 1. Since S is not length 1, the code will proceed to the else conditional statement. A loop through each element of S in sorted order is implemented with each element referenced as "x." Sx will equal {2,7} - {2} = {7}. Then another loop is implemented for each element in Sx inputed into all_permutations with each element is referenced as "P." Since Sx has a length of 1, its element is returned as P for this specific iteration of the loop. x and P are added together and stored in all_perm. Then the next loop for S in sorted order is implemented. The same steps will be repeated except for this iteration x will be 7 and P will be 2. After finishing these loops, all_perm will be [{2,7}, {7,2}].

(b) What is the growth complexity of this procedure? Explain briefly.

O(n(n-1)!). This complexity stems from the first loop being O(n) for looping once through each element of S. The second loop uses a recursive formula which loops factorially by n-1.

Question 2: *Write all possible combinations that can be generated from elements of a set S {1,2,3} in the form of a set of sets C. how many elements should your new set have? Give a generalized formula in terms of n, the number of element in S, of how many elements the set C has.*

C = {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1,2,3}}
C has 8 elements in its set.

$$\sum_{i=0}^{n} \binom{i}{n}$$

is a generalized formula to determine the number elements in C.

Question 4: *Given the two pseudocodes A and B in Figure 2 that are used to evaluate a polynomial $a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$ at $x = c$.*

(a) Exactly how many multiplications and additions are used to evaluate a polynomial of degree n at x = c for the algorithm given in A?

n multiplications and additions

(b) Evaluate $3x^2 + x + 1$ at x = c by working through each step of the algorithm in B. Which algorithm do you think is more efficient? And Why?

y = 3

y = 3 * 2 + 1 = 7

y = 7 * 2 + 1 = 15

Algorithm B is the most efficient as both algorithms perform the same amount of loops but algorithm B performs less actions per iteration.

(c) What is the big O notation of both A and B?

O(n)

Question 5: *Given a list/set S with n elements, what does the function in figure 3 do? Analyze the time complexity for this function and dive the big O value.*

The function in figure 3 performs bubble sort and has a big O notation of $O(n^2)$.