Assignment 3

CS 2813 - Discrete Structures

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Question 1: *Consider the following quantified statement: For every real number x, there exists a positive real number y such that $y < x^2$.*

(a) Express this quantified statement in symbols.

$\forall$x $\in \mathbb{R}$, $\exists$y $\in \mathbb{R}_{\geq 0}$, y < x^2

(b) Express the negation of this quantified statement in symbols.

$\exists$x $\in \mathbb{R}$, $\forall$y $\in \mathbb{R}_{\geq 0}$, y $\geq$ x^2

(c) Express the negation of this statement in words.

There exists a real number x for every positive real number y such that y is greater than or equal to x squared.

Question 2: *Prove that if r and s are rational numbers, then r - s is a rational number.*

Let r and s equal to $\frac{a}{b}$ and $\frac{c}{d}$ respectively where a, b, c, and d are all integers.
Then r - s = $\frac{ad-bc}{cd}$
ad - bc and cd are both rational numbers which makes r - s is a rational number a true statement.

Question 3: *Let x and y be integers. Prove that if $x + y \geq 9$, then either $x \geq 5$ or $y \geq 5$.*

Let x and y equal to 4.5
x + y = 9
This disproves the statement that if x + y $\geq$ 0, then x $\geq$ 5 or y $\geq$ 5 as 4.5 < 5.

Question 4: *Let m and n be two integers. Prove that mn and m + n are both even if and only if m and n are both even.*

mn = $2(\frac{mn}{2})$ = 2l where l is any integer.
This statement is only true if m and n are even numbers.
m + n = $2(\frac{m+n}{2})$ = 2l where l is any integer.
This statment is only true if m and n are even numbers.
Both statements are only true if m and n are even numbers which proves the original statement.

Question 5: *Disprove: Let A, B, and C be sets. If $A \cup B = A \cup C$, then $B = C$.*

Let A = {1, 2, 3}, B = {1, 2}, and C = {2, 3}
A $\cup$ B = A $\cup$ C = {1, 2, 3}
But B $\neq$ C which disproves the original statement.

Question 6: *Prove that if a and b are positive real numbers, then $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$*

Let a and b equal 1
$\sqrt{1} + \sqrt{1} \neq \sqrt{2}$
From this statement this is in line with the original statement[1]

Question 7: *Let $r \geq 2$ be an integer. Prove that $1 + r + r^2 + \ldots + r^n = \frac{r^{n+1}-1}{r-1}$ for every positive integer n.*

Let the above summation be summarized to $\sum_1^n r^{n-1}$
This is a geometric sequence which has a formula equal to $\frac{r^{n+1}-1}{r-1}$
This formula is equal to the original formula which proves the orignal statement

Question 8: *Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n+1}$ for every integer $n \geq 3$*

Let the above summation be summarized to $\sum_1^n n^{-\frac{1}{2}}$
This is a geometric sequence which has a formula equal to 2ln(n) - 1
2ln(n) - 1 ¿ $\sqrt{n+1}$ which proves the original statement

Question 9: *A sequence a_1, a_2, a_3, ... is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$*

(a) Detemine a_2, a_3, a_4, and a_5

7, 15, 31, 63

(b) Based on the values obtained in (a), make a guess for a formula for every positive integer n and us induction to verify that your guess is correct.

$a_n = 2^{n+1} - 1$

Question 10: *A sequence $\{a_n\}$ is defined recursively by $a_1 = 5$, $a_2 = 7$ and $a_n = 3a_{n-1} - 2a_{n-2} - 2$ for $n \geq$. Prove that $a_n = 2n + 3$ for every positive integer n.*

For $a_1 = 2(1) + 3 = 5$
For $a_n = 2n + 3$
For $a_{n+1} = 2(n + 1) + 3 = 2n + 3 + 2$
$a_{n+1} = a_n + 2$
Through inductive reasoning the original statement holds true[2]

REFERENCES

[1] ChatGPT. Prove a and b are real numbers then sqrt(a) + sqrt(b) $\neq$ sqrt(a+b), Retrieved : 2023, October 12. [2] ChatGPT. Prove $a_n = 2n + 3$ by mathematical induction, Retrieved : 2023, October 12.