

Assignment 2

CS 2813 - Discrete Structures

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Question 1: *Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. first, let the domain consist of the students in your class and second, let it consist of all people.*

- a) Everyone in your class has a cellular phone.
- b) Somebody in your class has seen a foreign movie.
- c) There is a person in your class who cannot swim.
- d) All students in your class can solve quadratic equations.
- e) Some student in your class does not want to be rich.

Let x be a person and $C(x)$ is a propositional function for "x is in your class."

- a) Let $F(x)$ be "x has a cellular phone." Then $\exists x F(x)$ for the first domain and $\forall x (C(x) \rightarrow F(x))$ for the second domain.
- b) Let $G(x)$ be "x has seen a foreign movie." Then $\exists x G(x)$ for the first domain and $\exists x (C(x) \wedge G(x))$ for the second domain.
- c) Let $H(x)$ be "x can swim." Then $\exists x \neg H(x)$ for the first domain and $\exists x (C(x) \wedge \neg H(x))$ for the second domain.
- d) Let $J(x)$ be "x can solve quadratic equations." Then $\forall x J(x)$ for the first domain and $\forall x (C(x) \rightarrow J(x))$ for the second domain.
- e) Let $K(x)$ be "x wants to be rich." Then $\exists x \neg K(x)$ for the first domain and $\exists x (C(x) \wedge \neg K(x))$ for the second domain.

Question 2: *Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y , where x is 1, 2, or 3 and y is 1, 2, or 3. Using these values, write out these propositions using disjunctions and conjunctions.*

- a) $\exists x P(x, 3)$
 $P(1,3) \vee P(2,3) \vee P(3,3)$
- b) $\forall y P(1, y)$
 $P(1,1) \wedge P(1,2) \wedge P(1,3)$
- c) $\exists y \neg P(2, y)$
 $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

- d) $\forall x \neg P(x, 2)$
 $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

Question 3: Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\forall x \exists y (x^2 = y)$
 T The square of a real number is a real number.
- b) $\forall x \exists y (x = y^2)$
 F There are no real numbers that can be squared to equal a negative number.
- c) $\exists x \forall y (xy = 0)$
 T x can be equal to zero to for all values of y to make this statement true.
- d) $\exists x \exists y (x + y \neq y + x)$
 F By the Associative Law of arithmetic, this statement is false.
- e) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
 F Both of these statements represent the equation of a line and these two line equations are parallel to each other.

Question 4: Show that if A , B , and C are sets, then $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

$$|A| = x, |B| = y, |C| = z$$

First include all of the cardinality of the sets

$$x + y + z$$

Then remove the redundant intersections

$$x \cap y + x \cap z + y \cap z$$

Then account for intersection of all three sets that was removed when accounting for redundant intersections

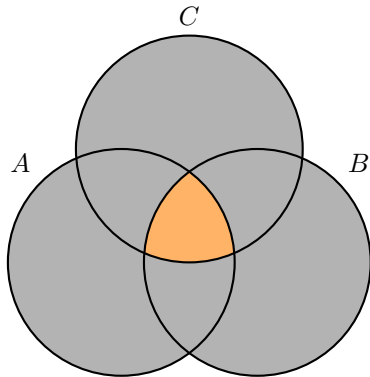
$$x \cap y \cap z$$

Combining all these together forms

$$x + y + z - x \cap y - x \cap z - y \cap z + x \cap y \cap z$$

Substituting the values back in and we get the same results in the beginning

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



Question 5: Suppose that the universal set is $U = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. Express each of these sets with bit strings where the i th bit in the string is 1 if i is in the set and 0 otherwise.

a) 3, 4, 5

0011100000

b) 1, 3, 6, 10

1010010001

c) 2, 3, 4, 7, 8, 9

01110001110

Question 6: Given the following Algorithm 1:

a) Can we check if $x == a_j$ instead of if $x == a_i$? Why?

Yes, if we check for a_j instead of a_i then the loop will end when $a_j == a_i$ already.

b) Can we check if $x == a_m$ instead of if $x == a_i$? Why?

No, if check for a_m instead of a_i then the loop will end prematurely before getting the correct location.

Question 7: For n pairs of socks ($2n$ total socks), how many iterations (pick up a sock, pick up another and check if they match to pair them) are in the best- and worst-case scenarios?

Best-case scenerio is one iterations to find a matching pair. Worst-case scenerio is $2n - 1$ iterations.

Question 8: Show that

$\forall x P(x) \vee \forall x Q(x)$ and

$\forall x \forall y (P(x) \vee Q(y))$,

where all quantifiers have the same nonempty domain, are logically equivalent. (The new variable y is used to combine the quantifications correctly).

if $\forall x P(x) \vee \forall x Q(x) = \forall x \forall y (P(x) \vee Q(y))$

Then substituting $y = x$, we get

$\forall x \forall y (P(x) \vee Q(x))$