# Assignment 1

## CS 2813 - Discrete Structures

Lucas Ho

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**Question 0**: *This book[1] has the the answer to everything. What is the question?*

What is the meaning of life?

**Question 1**: *The propositions p NAND q and p NOR q are denoted by $p \mid q$ and $p \downarrow q$, respectively.*

a. Construct a truth table for the logical operators NAND and NOR.

| p | q | $p \mid q$ | $p \downarrow q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | T | F |
| F | T | T | F |
| F | F | T | T |

b. Show that $p \mid q$ is logically equivalent to $\neg(p \wedge q)$.

| p | q | $\neg(p \wedge q)$ | $p \mid q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | T | T |
| F | T | T | T |
| F | F | T | T |

**Question 2**: *A proposition is satisfiable if some setting of the variables makes the proposition true. For example, $p \wedge \neg q$ is satisfiable because the expression is true if p is true and q is false. On the other hand, $p \wedge \neg p$ is not satisfiable because the expression as a whole is false for both settings of p. But determining whether or not a more complicated proposition is satisfiable is not so easy. How about this one?*

$$(p \vee q \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee \neg r) \wedge (\neg r \vee \neg q)$$

a. Find whether or not the previous compound proposition is satisfiable?

| p | q | r | $p \vee q \vee r$ | $\neg p \vee \neg q$ | $\neg p \vee \neg r$ | $\neg r \vee \neg q$ |
|---|---|---|---|---|---|---|
| T | F | F | T | T | T | T |

The previous compound proposition is satisfiable.

b. Given the following disjunctions, determine whether each is satisfiable. Use a different approach to how you solved a).

i) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
This proposition is satisfiable for any p or q as long as r and s are false.

ii) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$
This proposition is not satisfiable as it is not possible for both $(\neg p \vee \neg q \vee r)$ and $(p \vee q \vee \neg r)$ to both occur.

iii) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$
This proposition is not satisfiable because p is unable to be both true and false. p is of note here because after expanding the compound proposition with logical equivalences, there are propositions of $(p \wedge \neg p)$ which help determine satiability.

c. Alice claims she wrote an 'efficient' algorithm for determining whether a compound proposition is satisfiable or not. Bob believes Alice; can Bob use Alice's algorithm to determine whether a compound proposition p is a tautology or not?

Yes

d. Why is it not easy to generalize a truth table for any complex proposition? In other words, is there a more efficient solution to this problem? Do you think Alice from c) is lying?

You can use logical equivalences to solve any complex propositions so Alice is not lying.

**Question 3**: *Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent. Use Logical Equivalences not truth tables.*

$p \vee (q \rightarrow r)$
$p \vee \neg q \vee r$
$\neg q \vee p \vee r$
$q \rightarrow (p \vee r)$

REFERENCES

[1] D. Adams. *The Hitchhiker's Guide to the Galaxy.* San Val, 1995.