Champs quantiques en interaction

Formules pour les calculs

Formule de Feynman $\dfrac{1}{AB} = \displaystyle\int_0^1 \mathrm{d}x\, \frac{1}{(Ax+B(1-x))^2}$

Formule LSZ – Bosons (formule de réduction de Lehmann Symanzik Zimmermann), dans l'hypothèse (très forte !) où les champs sont libres en $\pm\infty$,

$$\langle f|S|i\rangle = \underbrace{\prod_i \left(-\int \mathrm{d}^4x_i\, \mathrm{e}^{-ip_ix_i}(\Box_{x_i}+m^2)\right)}_{\text{particules de l'état initial}} \underbrace{\prod_f \left(-\int \mathrm{d}^4y_f\, \mathrm{e}^{-ip_fy_f}(\Box_{y_f}+m^2)\right)}_{\text{particules de l'état final}} \underbrace{\langle 0\,|\,T\phi(x_1)\ldots\phi(x_I)\phi(y_1)\ldots\phi(y_F)\,|\,0\rangle}_{\boxed{G(x_1,\ldots,x_I,y_1,\ldots,y_F)}}$$

$$= i^{I+F}\prod_{n\in i,f}(p_n+m^2)\tilde{G}(-p_{f1},\ldots,-p_{fF},p_{i1},\ldots,p_{iI}).$$

plus les contributions non connexes.

Formule LSZ – Fermions

$$\langle f|S|i\rangle = \langle 0|d^{\lambda'}_{\text{out}}(k')b^{\lambda}_{\text{out}}(k)b^{\dagger\sigma}_{\text{out}}(p)d^{\dagger\sigma'}_{\text{out}}(p')|0\rangle = i^{\text{nb. ferm.}}i^{\text{nb. a.ferm.}}\int \mathrm{d}^4x\,\mathrm{d}^4x'\,\mathrm{d}^4y\,\mathrm{d}^4y'\,\mathrm{e}^{-ipx}\mathrm{e}^{-ip'x'}\mathrm{e}^{iky}\mathrm{e}^{ik'y'}$$

$$\underbrace{\bar{U}_\lambda(k)(i\partial\!\!\!/_y-m)}_{\text{fermion final}}\underbrace{\bar{V}_{\sigma'}(p')(i\partial\!\!\!/_{x'}-m)}_{\text{antifermion initial}}\langle 0|T\bar{\psi}_{\alpha'}(y')\psi_\alpha(y)\bar{\psi}_\beta(x)\bar{\psi}_{\beta'}(x')|0\rangle\underbrace{(i\overleftarrow{\partial\!\!\!/}_x-m)U_\sigma(p)}_{\text{fermion initial}}\underbrace{(i\overleftarrow{\partial\!\!\!/}_{y'}-m)V_{\lambda'}(k')}_{\text{antifermion final}}$$

plus les contributions non connexes.

Formule de Gell-Mann Low $G(x_1,\ldots,x_n) = \dfrac{\big\langle 0\,\big|\,T\phi_{\text{in}}(x_1)\ldots\phi_{\text{in}}(x_n)\exp\big(i\int \mathrm{d}^4x\,\mathcal{L}_{\text{int}}(x)\big)\,\big|\,0\big\rangle}{\big\langle 0\,\big|\,T\exp\big(i\int \mathrm{d}^4x\,\mathcal{L}_{\text{int}}(x)\big)\,\big|\,0\big\rangle}$

Théorème de Wick, *pairing* Pour le T-Π à $2n$ points, $\frac{2n!}{2^n\,n!}$ termes à trouver.

Calculs d'amplitudes $U^\dagger = \bar{U}\gamma^0$.

Règles de Feynman

Règles de Feynman en espace de configuration

- Vertex (th. $\frac{g}{3!}\phi^3$) : $\frac{ig}{3!}\int \mathrm{d}^4y_i$.
- Propagateur : $D_F(\text{start}-\text{end})$.

Règles de Feynman en espace des impulsions (exemples p. 13, p. 21-22)

- Vertex (th. $\frac{g}{3!}\phi^3$) : $\frac{ig}{3!}(2\pi)^4\delta^{(4)}(\Sigma k_i)$.
- Vertex (QED) : $-ie\gamma^\mu_{\alpha\beta}$.
- Propagateur boson : $\frac{i}{k_j^2-m^2+i\epsilon}$.
- fermion α in : $U_\sigma(p)_\alpha$.
- anti-fermion α in : $\bar{V}_\sigma(p)_\alpha$.
- fermion α out : $\bar{U}_\sigma(p)_\alpha$.
- anti-fermion α out : $V_\sigma(p)_\alpha$.
- photon in : $\varepsilon_\mu(p)$
- photon out : $\varepsilon^*_\mu(p)$
- fermion interne $\alpha\to\beta$: $\frac{i(p\!\!\!/+m)_{\alpha\beta}}{p^2-m^2+i\epsilon} = \frac{i}{(p\!\!\!/-m+i\epsilon)_{\beta\alpha}}$
- photon interne $-\frac{i}{p^2+i\epsilon}\left(\eta_{\mu\nu}+(\xi-1)\frac{p_\mu p_\nu}{p^2}\right)$

Expressions des champs libres, relations de commutations et lagrangiens associés

Spin 0

Lagrangien, équation de Klein-Gordon $\mathcal{L} = \partial_\mu \hat{\phi} \partial^\mu \hat{\phi}^\dagger - m^2 \hat{\phi}\hat{\phi}^\dagger, \quad \left(\Box + m^2\right) \phi = 0$

Champ scalaire $\hat{\phi}(x) = \int \mathrm{d}\tilde{k} \left(\hat{a}(\vec{k})\mathrm{e}^{-ikx} + \hat{b}^\dagger(\vec{k})\mathrm{e}^{ikx}\right), \quad \underbrace{\mathrm{d}\tilde{k} = \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{2\omega_k}}_{\text{invariant de Lorentz}}, \quad kx = k_\mu x^\mu, \quad \omega_k = k_0 = \sqrt{\vec{k}^2 + m^2}$

Moment conjugué et relations de commutation

$$\hat{\pi} = \frac{\partial \mathcal{L}}{\partial \partial_0 \phi} = \partial_0 \phi^*, \quad \left[\hat{\phi}(x), \hat{\pi}(y)\right] = i\hbar \delta^{(4)}(x-y) \Leftrightarrow \left[\hat{a}(\vec{k}), \hat{a}^\dagger(\vec{k}')\right] = \left[\hat{b}(\vec{k}), \hat{b}^\dagger(\vec{k}')\right] = (2\pi)^3\, 2\omega_k\, \delta^{(3)}(\vec{k} - \vec{k}')$$

$$\hat{a}^\dagger(\vec{k})\,|0\rangle = |k;\rangle\,, \quad \hat{b}^\dagger(\vec{k})\,|0\rangle = |;k\rangle\,, \quad \hat{a}^\dagger(\vec{k}_1)\hat{b}^\dagger(\vec{k}_2)\hat{a}^\dagger(\vec{k}_3)\,|0\rangle = |k_1 k_3; k_2\rangle = |k_3 k_1; k_2\rangle\,, \quad \langle k_1 | k_2 \rangle = (2\pi)^3\, \underbrace{2\omega_k\, \delta^{(3)}(\vec{k}_1 - \vec{k}_2)}_{\text{invariant de Lorentz}}$$

Propagateurs, commutateurs

$$D(x-y) = \int \mathrm{d}\tilde{k}\, \mathrm{e}^{-ik(x-y)} = \langle 0|\phi(x)\phi^\dagger(y)|0\rangle = \langle 0|\phi^\dagger(x)\phi(y)|0\rangle = \begin{cases} \dfrac{m}{4\pi^2 |x|} \displaystyle\int_0^\infty \frac{u\,\mathrm{d}u}{\sqrt{1+u^2}} \sin(|x|\, mu), & x_\mu x^\mu < 0 \\ \sim \exp(im\,|x|), \quad x_\mu x^\mu > 0 \end{cases}$$

Invariant de Lorentz : $D(x) = D(\Lambda x)$.

$$\left[\phi(x), \phi^\dagger(y)\right] = D(x-y) - D(y-x) = -i\Delta(x-y) \qquad = 0 \text{ si } (x-y)^2 < 0.$$

$$D_F(x-y) = \langle 0|T\phi(x)\phi^\dagger(y)|0\rangle = i \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{-ik(x-y)}}{k^2 - m^2 + i\epsilon} = \Theta(x^0 - y^0) D(x-y) + \Theta(y^0 - x^0) D(y-x)$$

Spin 1/2

Lagrangien, équation de Dirac $\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi, \quad \left(i\partial\!\!\!/ - m\right)\psi = 0$

Champ spinoriel $\hat{\psi}_\alpha(x) = \int \mathrm{d}\tilde{k}\, 2m \sum_{\lambda=1}^{2} \left(\hat{b}_\lambda(\vec{k}) U_\alpha^{(\lambda)}(k)\mathrm{e}^{-ikx} + \hat{d}_\lambda^\dagger(\vec{k}) V_\alpha^{(\lambda)}(k)\mathrm{e}^{ikx}\right),$

$$(p\!\!\!/ - m) U^{(\lambda)}(p) = 0, \quad (p\!\!\!/ + m) V^{(\lambda)}(p) = 0, \quad \sum_\lambda U^\lambda(p) \bar{U}^\lambda(p) = \frac{p\!\!\!/ + m}{2m}$$

Moment conjugué et relations d'anti-commutation $\left\{\hat{b}_\lambda(\vec{k}), \hat{b}_\sigma^\dagger(\vec{k}')\right\} = \left\{\hat{d}_\lambda(\vec{k}), \hat{d}_\sigma^\dagger(\vec{k}')\right\} = (2\pi)^3 \frac{k_0}{m} \delta^{(3)}(\vec{k} - \vec{k}') \delta_{\lambda\sigma}$

Propagateurs

$$S(x-y) = \langle 0|T\psi(x)\bar{\psi}(y)|0\rangle = (i\partial\!\!\!/ + m) D_F(x-y), \quad T\psi(x)\bar{\psi}(y) = \Theta(x^0 - y^0)\psi(x)\bar{\psi}(y) - \Theta(y^0 - x^0)\bar{\psi}(y)\psi(x)$$

$$S(x-y) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-ik(x-y)} \frac{i(k\!\!\!/ + m)}{k^2 - m^2 + i\epsilon} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-ik(x-y)} \frac{i}{k\!\!\!/ - m + i\epsilon}$$

Spin 1

Lagrangien, équations « de Maxwell » en jauge de Lorentz,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \partial_\mu F^{\mu\nu} = 0$$

Champ vectoriel et solution de Gupta et Bleuler

$$A_\mu(x) = \int \mathrm{d}\tilde{k} \sum_{\lambda=0}^{3} \left(\varepsilon_\mu^{(\lambda)}(k) a_\lambda(k) \mathrm{e}^{-ikx} + \bar{\varepsilon}_\mu^{(\lambda)}(k) a_\lambda^\dagger(k) \mathrm{e}^{ikx} \right) = A_\mu^{(+)}(x) + A_\mu^{(+)^\dagger}(x), \quad A_\mu^{(+)}(x) = \int \mathrm{d}\tilde{k} \sum_{\lambda=0}^{3} \varepsilon_\mu^{(\lambda)}(k) a_\lambda(k) \mathrm{e}^{-ikx}$$

Moment conjugué et relations de commutation

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \partial^0 A_\mu} = -F^{0\mu} - \eta^{\mu 0} \partial^\nu A_\nu, \quad [A_\mu(x), \pi_\nu(y)] = i\eta_{\mu\nu} \delta^{(4)}(x-y) \Leftrightarrow \left[a_{(\lambda)}(\vec{k}), a_{(\lambda')}^\dagger(\vec{k}') \right] = -2k_0 (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}') \eta_{\lambda\lambda'}$$

Propagateurs $D_{\mu\nu}(x-y) = -i \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-ik(x-y)} \frac{1}{k^2 + i\epsilon} \left(\eta_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)$

Matrices de Dirac ou matrices gamma

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

$$\not{p} = \gamma^\mu p_\mu, \quad \{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} I_4, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Identités $\gamma^\mu \gamma_\mu = 4I_4, \quad \gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu, \quad \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4\eta^{\nu\rho} I_4, \quad \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu = -2\gamma^\sigma \gamma^\rho \gamma^\nu,$

$$\gamma^\mu \gamma^\nu \gamma^\rho = \eta^{\mu\nu} \gamma^\rho + \eta^{\nu\rho} \gamma^\mu - \eta^{\mu\rho} \gamma^\nu - i\epsilon^{\sigma\mu\nu\rho} \gamma_\sigma \gamma^5, \quad (\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$$

Traces trace of any product of an odd number of γ^μ is zero. Be carefull with γ^5.

$$\mathrm{Tr}(\gamma^\mu) = 0, \quad \mathrm{tr}(\gamma^\mu \gamma^\nu) = 4\eta^{\mu\nu}, \quad \mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}), \quad \mathrm{tr}(\gamma^5) = \mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^5) = 0,$$
$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}, \quad \mathrm{tr}(\gamma^{\mu 1} \ldots \gamma^{\mu n}) = \mathrm{tr}(\gamma^{\mu n} \ldots \gamma^{\mu 1}).$$

QED

L'utilisation des dérivées covariantes fait apparaître les termes d'interaction. La dérivée covariante D_μ se transforme comme A_μ, ce qui laisse invariant le lagrangien par symétrie de jauge *locale*.

$$\mathcal{L}_{\mathrm{int}} = -eA^\mu \bar{\psi} \gamma_\mu \psi, \quad D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi \Rightarrow \mathcal{L} = \psi(i\not{D} - m)\psi = \psi(i\not{\partial} - m)\psi - eA^\mu \bar{\psi} \gamma_\mu \psi$$

Méthodes fonctionnelles – lien avec les intégrales de chemin

Bosons

$$\left.\begin{aligned} \langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle &= \frac{\int \mathcal{D}\phi\, \phi(x_1)\ldots\phi(x_n) \mathrm{e}^{i\int \mathrm{d}^4x \mathcal{L}(x)}}{\int \mathcal{D}[\phi] \exp\{i \int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{tot}}(x)\}} \\ \mathcal{Z}[J] = \frac{Z[J]}{Z[0]} &= \frac{\int \mathcal{D}[\phi] \exp\{i \int \mathrm{d}^4x\, (\mathcal{L}_{\mathrm{tot}}(x) + J(x)\phi(x))\}}{\int \mathcal{D}[\phi] \exp\{i \int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{tot}}(x)\}} \end{aligned}\right\} \langle 0|T\phi(\tau_1)\ldots\phi(\tau_n)|0\rangle = \frac{(-i)^n}{Z[0]} \frac{\delta^n}{\delta J(x_1)\ldots\delta J(x_n)} Z[J]\Big|_{J=0},$$

$$Z[J] = \mathrm{e}^{i\int \mathrm{d}^4x \mathcal{L}_{\mathrm{int}}\left(\frac{\delta}{\delta J}\right)} Z_0[J], \quad Z_0[J] = \int \mathcal{D}[\phi] \exp\left(i \int \mathrm{d}^4x \left(\underbrace{\mathcal{L}_0(x)}_{\text{libre}} + J(x)\phi(x) \right) \right) = \mathrm{e}^{-\frac{1}{2}\int \mathrm{d}^4x \mathrm{d}^4y J(x) D_F(x-y) J(y)}.$$

$Z[J]$ reproduit les règles de Feynman : pour $\mathcal{L}_{\text{int}} = \lambda\phi^3$,

$$Z[J] \simeq \left(1 - \lambda\left(\frac{\delta}{\delta J}\right)^3 + \frac{1}{2}\lambda^2\left(\frac{\delta}{\delta J}\right)^6 + \dots\right)\left(\sum_{n\geqslant 0}\frac{1}{n!}(\tfrac{-1}{2}JDJ)^n\right), \quad \lambda \equiv \text{vertex}, \quad D \equiv \text{ligne}.$$

$$Z[J] \simeq 1 - \underbrace{\frac{1}{2}JDJ}_{\text{propag. simple}} + \dots - \lambda\Bigg(\underbrace{\frac{4!}{2\times 2^2}D^2J}_{\text{no contrib.}} + \underbrace{\dots}_{\text{contrib ?}}\Bigg) + \frac{\lambda^2}{2}\Bigg(-\underbrace{\frac{6!}{3!\times 2^3}D^3}_{\text{2 vertex, trois lignes}} + \dots\Bigg) + \dots$$

et avec thm de Wick pour les paires.

Fermions et variables de Graßmann $\quad \mathcal{L}_0 = \bar\psi(i\partial\!\!\!/ - m)\psi$.

$$\left.\begin{aligned}\langle 0|T\psi(x)\bar\psi(y)|0\rangle &= \frac{\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\psi(x)\bar\psi(y)\exp\{i\int \mathrm{d}^4x\,\mathcal{L}_{\text{tot}}(x)\}}{\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\exp\{i\int \mathrm{d}^4x\,\mathcal{L}_{\text{tot}}(x)\}}\\ \mathcal{Z}[\eta,\bar\eta] = \frac{Z[\eta,\bar\eta]}{Z[0,0]} &= \frac{\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\exp\{i\int \mathrm{d}^4x\,(\mathcal{L}_{\text{tot}}(x) + \bar\eta\psi + \bar\psi\eta)\}}{\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\exp\{i\int \mathrm{d}^4x\,\mathcal{L}_{\text{tot}}(x)\}}\end{aligned}\right\} \langle 0|T\psi(x)\bar\psi(y)|0\rangle = \frac{-i}{Z[0,0]}\,\frac{\delta}{\delta\eta(y)}\frac{\delta}{\delta\bar\eta(x)}Z[\eta,\bar\eta]\Big|_{\eta=\bar\eta=0},$$

$$Z[\eta,\bar\eta] = \langle 0|T\mathrm{e}^{i\int \mathrm{d}^4x\left(\bar\psi(x)\eta(x) + \bar\eta(x)\psi(x)\right)}|0\rangle = \mathrm{e}^{i\int \mathrm{d}^4x\left(\mathcal{L} + \bar\psi(x)\eta(x) + \bar\eta(x)\psi(x)\right)} = \mathrm{e}^{-\int \mathrm{d}^4x\mathcal{L}_{\text{int}}\left(\frac{\delta}{\delta\eta},\frac{\delta}{\delta\bar\eta}\right)}Z_0[\eta,\bar\eta],$$

$$Z_0[\eta,\bar\eta] = \int \mathcal{D}[\bar\psi]\mathcal{D}[\psi]\exp\left(i\int \mathrm{d}^4x\,\big(\underbrace{\mathcal{L}_0(x)}_{\text{libre}} + \bar\psi(x)\eta(x) + \bar\eta(x)\psi(x)\big)\right) = \mathrm{e}^{-\int \mathrm{d}^4x\mathrm{d}^4y\bar\eta(x)S(x-y)\eta(y)}.$$

Attention, avec les variables de Graßmann, $\int \mathrm{d}\bar\xi\,\mathrm{d}\xi\,\mathrm{e}^{-\bar\xi A\xi} = \det A$ (cas bosonique : $\int \mathrm{d}J\,\mathrm{e}^{-JAJ} = \pi^{n/2}(\det A)^{-1/2}$).

Renormalisation p. 26-32

On se place en dimension $D = 4 - \epsilon$ et on obtient

$$\begin{aligned}\psi = \psi_0 &= \sqrt{Z_2}\psi_R, \quad Z_2 = 1 + \Delta_2 = 1 - \frac{e^2}{8\pi^2\epsilon} + \dots\\ m = m_0 &= Z_m m_R, \quad Z_m = 1 + \Delta_m = 1 - \frac{3e^2}{8\pi^2\epsilon} + \dots\\ A^\mu = A_0^\mu &= \sqrt{Z_3}A_R^\mu, \quad Z_3 = 1 + \Delta_3 = 1 - \frac{e^2}{6\pi^2\epsilon} + \dots\\ e = e_0 &= \frac{Z_1}{Z_2\sqrt{Z_3}}e_R, \quad Z_1 = 1 - \frac{e^2}{8\pi^2\epsilon} + \dots = Z_2\end{aligned}$$

$$\begin{aligned}\tilde G &= \frac{i}{p\!\!\!/\,(1 + \frac{e^2}{8\pi^2\epsilon}) - m\,(1 + \frac{e^2}{2\pi^2\epsilon})} + \mathcal{O}(e^4) = \left(1 - \frac{e^2}{8\pi^2\epsilon}\right)\frac{i}{p\!\!\!/ - m\,(1 + \frac{3e^2}{8\pi^2\epsilon})} + \mathcal{O}(e^4)\\ \tilde G_R &= \frac{i}{p\!\!\!/ - m_R - \Sigma_R(p\!\!\!/)} + \mathcal{O}(e^4), \quad \Sigma_R(p\!\!\!/) = \Sigma_2(p\!\!\!/) - \Delta_2 p\!\!\!/ - (\Delta_2 + \Delta_m)m_R \equiv \text{partie à l'ordre } \epsilon^0 \text{ dans } \Sigma_2(p\!\!\!/).\end{aligned}$$

Degré superficiel de divergence $\quad d = DL - 2P_i - E_i$, D la dimension (4 ?), L le nombre de boucles, P_i les nombre de lignes internes de photons, E_i le nombre de ligne interne d'électrons.

Action effective $\quad Z[J] = \exp(iW[J])$. $iW[J]$ est la fonctionnelle génératrice des fonctions de corrélations connexes. Soit $A(x) = \frac{\delta W}{\delta J(x)}$. Alors, $\Gamma[A] = W - \int \mathrm{d}^4x\,JA$ est la fonctionnelle génératrice des diagrammes à une particule irréductible (1PI). Γ contient les blocs élémentaires de la théorie. Voir illustrations p. 31.

Identités de Ward voir p. 31-32.

Première partie

Champs quantiques libres, champ de Klein-Gordon, bosons

Hamiltonien, impulsion, charge, nombre de particules

$$\hat{H} = \int \mathrm{d}\tilde{k}\, \hbar\omega(\vec{k}) \left(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}\right) = \frac{1}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}\right),$$
$$\hat{\vec{P}} = \int \mathrm{d}\tilde{k}\, \vec{k} \left(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}\right), \quad \hat{Q} = \int \mathrm{d}\tilde{k} \left(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}\right), \quad \hat{N} = \int \mathrm{d}\tilde{k} \left(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}\right).$$

Fonctionnelle génératrice

$$Z[j,\bar{j}] = \left\langle 0 \middle| T \exp\left[i \int \mathrm{d}^4 x \left(j(x)\phi^\dagger(x) + \bar{j}(x)\phi(x)\right)\right] \middle| 0 \right\rangle \underset{\text{th. libre}}{=} \exp\left[-\int \mathrm{d}^4 x \mathrm{d}^4 y \bar{j}(x) D_F(x-y) j(y)\right]$$

$$G^{(2)}(x,y) = \left(-i \frac{\delta}{\delta \bar{j}(x)}\right) \left(-i \frac{\delta}{\delta j(y)}\right) Z[j,\bar{j}]\bigg|_{j=\bar{j}=0}$$

Deuxième partie

Interactions

Matrice S $S\,|p_1 p_2\rangle_{\text{out}} = |p_1 p_2\rangle_{\text{in}}$: changement de base.

$$P_{fi} = |{}_{\text{out}}\langle k_1 \dots k_J | p_1 p_2 \rangle_{\text{in}}|^2 = |{}_{\text{out}}\langle k_1 \dots k_J | S | p_1 p_2 \rangle_{\text{out}}|^2 = |S_{fi}|^2$$

Section efficace

$$\mathrm{d}\sigma = \frac{\text{Volume}}{\text{Temps}} \frac{1}{|\vec{v}_1|} \mathrm{d}P = \frac{V}{T} \frac{1}{|\vec{v}_1|} \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} \mathrm{d}\Pi = \frac{V}{T} \frac{1}{|\vec{v}_1|} \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle} \left[\prod_{j=1}^{J} \frac{V}{(2\pi)^3} \mathrm{d}^3 p_j\right]$$

Or, $S = 1 + i\tau$, soit $\langle f|S|i\rangle = \cancel{\langle F|i\rangle} + i\langle f|\tau|i\rangle$

$$|\langle f|S|i\rangle|^2 = |\langle f|\tau|i\rangle|^2 = (2\pi)^4 V T\, |\mathscr{M}|^2 \delta^{(4)}(p_1 + p_2 - \sum_j k_j)$$

$$\mathrm{d}\sigma = \frac{1}{4\omega_1 \omega_2 |\vec{v}_1|} |\mathscr{M}|^2 \mathrm{d}\pi_{\text{LIPS}}$$

$$\mathrm{d}\pi_{\text{LIPS}} = (2\pi)^4 \delta(p_1 + p_2 - \sum_j k_j) \prod_j \frac{1}{2\omega_{p_j}} \frac{\mathrm{d}^3 p_j}{(2\pi)^3} \quad \text{espace des phases invariant de Lorentz.}$$