HW 16: Section 6.5 and 6.6

Due: Monday, November 11th in SQRC by 9pm

Learning Goals:

- Understand the difference between convergence and divergence for improper integrals.
- Evaluate improper integrals.
- Use the comparison test to decide if an integral converges or diverges.

Questions:

1. For each of the following improper integrals, decide if they converge or not and if they do converge compute what they converge to:

   a) $\int_0^\infty \frac{1}{1+x^2}\,dx$

   b) $\int_0^\infty \sin(x)\,dx$

   c) $\int_1^\infty ze^{-z}\,dz$

   d) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

   e) $\int_0^3 \frac{1}{x^2-1}\,dx$

2. For which values of p does the integral $\int_0^1 \frac{1}{x^p}\,dx$ converge?

3. For each of the following integrals use a comparison to another integral to decide if they converge or not. You do not need to compute what they converge to.

   a) $\int_1^\infty \sin(x)e^{-x}\,dx$

   b) $\int_2^\infty \frac{1}{t-1}\,dx$

   c) $\int_3^\infty \frac{1}{x+e^x}\,dx$

   d) $\int_0^\infty e^{-x^3}\,dx$

4. The force due to gravity between two objects is $F = \frac{Gm_1m_2}{r^2}$ where G is the universal constant of gravity, m_1 is the first mass and m_2 is the second mass. The universal constant G is approximately $G = 6.6 \times 10^{-11} Nm^2/kg^2$, the mass of the earth is $m_1 = 6 \times 10^{24} kg$ and the mass of a rocket ship is $m_2 = 2 \times 10^6 kg$. For simplicity we can treat the earth and the rocket as point-masses, so if the rocket starts on the surface of the earth the distance between the two masses is about $6.4km$.

   How much work does it take to move the rocket arbitrarily far away from the earth? In other words, how much work is need to escape the earths gravitational field?