

## Problem Set 7

Due: Thursday, February 24th

**Instructions:** Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1.   a. Find the greatest common divisors of the following pairs of integers.
  - i. 52134, 312
  - ii. -324, 552
- b. Express the GCD's that you found in part (a) as a linear combination of the associated pairs of integers.
2. Here we practice the Division Algorithm and Euclidean Algorithm in the context of polynomials with *real* coefficients.

- a. Find the greatest common divisor of  $x^2 + x + 1$  and  $x^2 - x + 1$ .
- b. Find two polynomials  $a(x)$  and  $b(x)$  so that

$$\text{GCD}(x^2 + x + 1, x^2 - x + 1) = (x^2 + x + 1)a(x) + (x^2 - x + 1)b(x)$$

- c. Find two polynomials  $c(x)$  and  $d(x)$  so that

$$(x^2 + x + 1)c(x) + (x^2 - x + 1)d(x) = 2 - 2x$$

(hint: use part b.)

- d. Find the greatest common divisor of  $x^5 - 1$  and  $x^2 - 1$ .
- e. Find two polynomials  $a(x)$  and  $b(x)$  so that

$$\text{GCD}(x^5 - 1, x^2 - 1) = (x^5 - 1)a(x) + (x^2 - 1)b(x)$$