Math 490: Mathematics of Social Choice

Anonymity: Does the method treat each voter the same way? A social welfare function is *anonymous* if swapping the ballots of any two voters can never change who wins.

Neutrality: Does the method treat each candidate the same way? A social welfare function is *neutral* if swapping the locations of two candidates on all ballots ultimately swaps what happens to those candidates: if x wins the first time and we swap x with y everywhere, then y wins the second time, and vice versa.

Decisive: A social welfare function satisfies the *no ties* criterion if there is always exactly one winner.

Unanimity: A social welfare function satisfies the *unanimity* criterion if, whenever every single voter ranks candidate x at the top of their ballot, x must be the unique winner.

Condorcet: A Condorcet candidate, is a candidate that in every pairwise comparison is ranked above the alternative in a majority of the ballots. A social welfare function satisfies the *Condorcet criterion* if, whenever there is a Condorcet candidate, that candidate must be the unique winner.

Pareto: A social welfare function satisfies the *Pareto* condition if, whenever every voter prefers candidate x over candidate y, candidate y must not beat candidate x.

Up Monotonicity: A social welfare function is *up monotone* if the following always holds: suppose we have a profile and then the profile is modified by moving x up one spot on somebody's ballot. If the social choice function is run on this new profile, then x does no worse then they did before. (A social choice function that isn't monotone is sometimes called *perverse*.)

Add Monotonicity: A social choice function is *add monotone* if the following always holds: suppose we have a profile and then the profile is modified by adding a new voter with x at the top of their ballot. If the social choice function is run on this new profile, then x does no worse then they did before.

IIA: A social welfare function is *independent of irrelevant alternatives* (or IIA) if the following always holds: suppose we are given a profile and then the profile is modified by changing some ballots, but nobody changes their opinion on x relative to y. In other words, if some voter had x over y the first time, then she still does the second time, and vice versa. If this happens and the election is run again, then IIA guarantees that the relative ranking of x and y is unchanged, (i.e. if it was $x > y$ then it is still $x > y$.