HW 11: Section 5.6 and 7.1

Due: Thursday, October 17th in SQRC by 9pm

Learning Goals:

- Use integrals to compute work.
- Use differential equations to model problems.
- Solve differential equations using initial conditions using integration.
- Sketch slope fields of differential equations.

Questions:

1. A water tower is spherical in shape with radius 50 meters, extending from 10 meters to 110 meters above ground.

   a) Compute the work done in filling the tank from the ground.

   b) Compute the work done in filling it halfway.

   On this problem, feel free to use a computer to help evaluate the integrals.

2. Use integration to find the solution to the differential equation $\frac{dy}{dt} = -y$ with the initial condition $y(0) = 12$.

3. Use integration to find the solution to the differential equation $\frac{dy}{dt} = ry - 50$ with the initial condition $y(0) = 0$ and where r is some constant.

4. sketch the slope field of the differential equation $\frac{dy}{dx} = 2 - xy$ on the region $[-3, 3] \times [-3, 3]$ (i.e. a square of side length 6 centered at the origin). Sketch a few curves that are possible solutions for $y(t)$. What do you think happens to the value of y as t goes to ∞?

5. In 1902 there were 21 bison in Yellowstone national park. In 1915 there were 250.

   a) Using the model that population growth is proportional to current population write down a differential equation and initial conditions that models this situation. In an English sentence clearly state what the variables you are using mean.

   b) Solve the differential equation to find the population of bison at any time.

   c) Using this model, how many bison would there be today? Do you think the model is accurate? If so why, and if not what do you think is wrong with it? How could you try to fix it?

6. The half-life (time it takes for the amount to decrease by a factor of 1/2) of uranium 235 is approximately 0.7×10^9 years. If 50 grams are buried at a nuclear waste site, how much will remain after 100 years? What about 1000 years?