Daily Prep Assignment for March 9th

Overview

In previous sections we have learned about partial derivatives that measure the rate of change of a function as the input moves in the x or y direction. In this section we start by considering rates of change in other directions. This will lead us to investigating the *gradient* of a function, a tool that will show up again and again for the rest of the semester.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Feel comfortable computing partial derivatives and interpreting them in context as rates of change in a certain direction.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with practice**.

- Compute directional derivatives.
- Understand the importance of unit vectors in the directional derivative.
- Compute the gradient of a function.
- Understand the meaning of the gradient magnitude and direction.
- Understand how the gradient relates to level curves of a function.

To prepare for class

Preview activities: Read the example preview activity solution on the course website then,

- Preview activity 10.6.1

Reading:

- read section 10.6

Watching: Watch these additional resources if you need support reading the text.

1. Overview of extra 10.6: `https://youtu.be/WPa60m8VFzc`

During and after class

- Activity 10.6.2
- Activity 10.6.3
- Activity 10.6.4
- Activity 10.6.5
- Activity 10.6.6
- I want to point out that there are all sorts of connections to be made between things we are learning about. Here is a cool one connecting tangent planes with the gradient vector.

  First I need to tell you what a *level surface* is. Recall that a level curve is the curve defined by setting a function $f(x, y)$ equal to a constant. For example if $f(x, y) = x^2 + y^2$, then $f(x, y) = 1$ defines the unit circle as a level curve. We can do the same thing with functions of three variables, $f(x, y, z)$, only now the equation $f(x, y, z) = c$ define a surface in space. For example, if $f(x, y, z) = x^2 + y^2 + z^2$ then $f(x, y, z) = 1$ gives us the unit sphere as a level surface.

  (a) Consider $f(x, y, z) = x^2 + y^2 + z^2$. Describe the level curves, $f(x, y, z) = c$, of this function for different values of c.

  (b) Compute the gradient of $f(x, y, z)$. Use appropriate technology to sketch a level curve and a gradient vector on that level curve and notice that they are orthogonal. Make sure that the point you use to calculate your gradient vector does in fact lie on the level surface you depict. (in calcplot3d you can use implicit surfaces and vectors to do this).

  (c) Argue that in general, for any level surface the gradient is orthogonal to it, just like the gradient of a function of two variables is orthogonal to the level curves.

  (d) Use the fact that the gradient is normal to level surfaces to compute the tangent plane of the level surface $x^2 + y^2 + z^2 = 1$ at the point $(1/\sqrt{2}, 1/\sqrt{2}, 0)$.

  (e) Note that any graph $z = f(x, y)$ can be thought of as a level surface of the function $g(x, y, z) = f(x, y) - z$ with value $c = 0$. Use this idea and your method from part (d) to find a formula for the tangent plane to the surface $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

  (f) Does the equation for a tangent plane you found in part (e) agree with the other equation we discussed in section 10.4?