Chapter 12 Checkpoint, version 2

Directions:

- You will have 30 minutes per question to complete whichever question you want. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- Write your own personal growth mindset statement. This really does help you do better on the checkpoint. If you have trouble thinking of a growth mindset statement you can use this one:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. I learn from my mistakes. Learning is my superpower.

Chapter 12: I can calculate, use, and interpret vector calculus

☐ VC.1 I can identify, evaluate, sketch and interpret vector fields in the plane and in space.

☐ VC.2 I can define and interpret line integrals of vector fields along oriented curves. I can use parametrizations to evaluate line integrals of vector fields along oriented curves.

☐ VC.3 I can use the Fundamental Theorem of Calculus for Line Integrals to evaluate line integrals of gradient fields.

☐ VC.4 ** I can define, evaluate, and interpret the divergence of vector fields. I can define, evaluate, and interpret the curl of vector fields.

☐ VC.5 ** I can use Green's Theorem to evaluate circulations of smooth vector fields along simple closed curves in the plane.

☐ VC.6 ** I can define, evaluate, and interpret flux integrals of vector fields across parametrized surfaces.

☐ VC.7 ** I can use Stokes' Theorem to evaluate circulations of smooth vector fields along simple closed curves in space.

☐ VC.8 ** I can use The Divergence Theorem to evaluate flux of continuous vector fields through closed surfaces in space.

VC.1 a) Sketch the vector field $\langle x^2, 1\rangle$ by hand. Make sure to include several vectors in all four quadrants of the plane.

b) Give three real life examples of vector fields.

c) Let $f(x, y) = x^2 + y^2$. Sketch the vector field grad f by hand. Make sure to include several vectors in all four quadrants of the plane.

VC.2 (a) For a give curve C and vector field F suppose that $\int_C F \cdot dr = 12$. Evaluate $\int_{-C} F \cdot dr$.

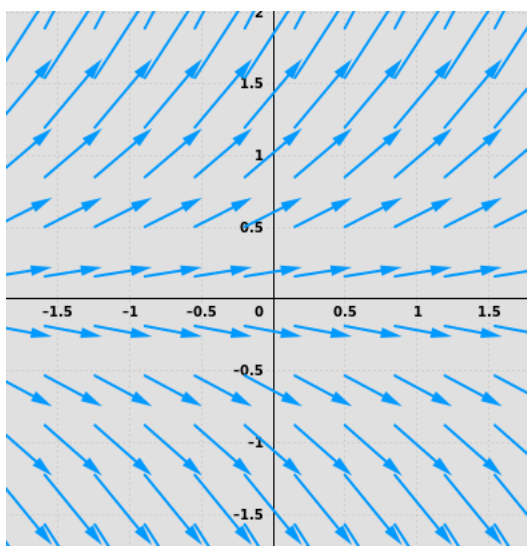
(b) Set up but do not evaluate the line integral $\int_C F \cdot dr$ where $F = \langle x, y\rangle$ and C is the half circle with radius 1 starting at $(0, 0)$ and ending at $(0, 2)$ staying in the first quadrant. (*note: this C and F are different than in part a)*)

VC.3 Consider the vector field $\mathbf{F} = \langle \cos(x)e^z + 6yx^2, 2x^3, \sin(x)e^z\rangle$. Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is parametrized by $r(t) = \langle t - t^2, (1 + t^2)\cos(\pi t), e^t\rangle$ starting at $t = 0$ and ending at $t = 1$.

VC.4 Consider the vector field F below. This vector field has the form $F = \langle 1, Q(x, y)\rangle$ for some function $Q(x, y)$.



(a) At the point (1,1), is the circulation density positive, zero, or negative?

(b) At the point (1,1), is the divergence positive, zero, or negative?

VC.5 Use Green's theorem to evaluate the integral

$$\oint_C \langle x^{x^2} \cos x, xy^2\rangle \cdot d\mathbf{r}$$

where C is the unit square between the points $(0, 0), (1, 0), (1, 1), (0, 1)$ oriented in the **clockwise** direction.

VC.6 Compute the flux integral $\iint_S F \cdot dS$ where $F = \langle y, -x, 2\rangle$ and S is the surface that is the portion of the sphere of radius 1 in the first octent (i.e. $x \geq 0$, $y \geq 0$, and $z \geq 0$) oriented so it's normal vectors face towards the origin.

VC.7 Use Stoke's Theorem to compute the line integral $\int_C \langle -y, -x, \sin^3(z^5)\rangle \cdot d\vec{r}$, where C is the square passing through the points $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 1)$, $(0, 1, 1)$ and then back to $(0, 0, 0)$. (this also specifies the orientation).

VC.8 Use the divergence theorem to compute the flux integral $\iint_S \langle xy, \sin(z), -z\rangle$, where S is every surface of the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 1, 0)$ and $(0, 1, 1)$ given the positive orientation (outwards pointing).