Checkpoint 1

Directions:

- You will have 3 hours to complete as many of the following questions as you can. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- This last direction might feel strange or contrived but it has actually been shown to help you do better on assessments like these. Before you start please write out the following statement or rephrase it in your own word:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. When I get stuck I try different strategies. This assessment might help me reflect on my current understanding but it does not define my potential. I learn from my mistakes. Learning is my superpower.

Chapter 9: I can use and interpret multivariable and vector functions.

☐ F.1 I can identify, evaluate and interpret functions of two variables using formulas, tables, graphs, and contour maps.

☐ F.2 I can compute dot products of vectors as well as use dot products to find lengths, angles, and projections. I can compute cross products of vectors and interpret them geometrically.

☐ F.3 I can find equations of lines and planes in space in various forms. I can determine the intersections, angles between, and distance between lines and planes.

☐ F.4 I can draw curves in space and define vector-valued functions for space curves.

☐ F.5 I can evaluate and interpret derivatives and integrals of vector-valued functions.

☐ F.6 ** I can find arc length of space curves.

F.1 (a) For the surface defined by $z = xy$

i. sketch the contour plot, including at least five labeled level curves.

ii. do your best job to sketch the surface in 3D on a labeled xyz-axes.

(b) For the surface defined by $z^2 = x^2 + y^2 + 1$

i. sketch the contour plot, including at least five labeled level curves.

ii. do your best job to sketch the surface in 3D on a labeled xyz-axes.

F.2 Consider the vectors $\mathbf{u} = \langle 1, 2, 3\rangle$ and $\mathbf{v} = \langle 1, 0, 1\rangle$.

(a) What is the angle between $\mathbf{u}$ and $\mathbf{v}$? *Feel free to use a calculator on this problem to compute the angle.*

(b) What is the area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$?

(c) Compute the projection of $\mathbf{u}$ onto $\mathbf{v}$.

F.3 (a) Find the equation of a plane passing through the point $(2, 3, 5)$ with normal vector $\langle 3, 4, 5\rangle$.

(b) Find a parametric equation for the line passing through the points $(1, 1, 1)$ and $(6, 0, -2)$.

(c) Do the plane and line you found in parts (a) and (b) intersect? Explain in a full sentence.

F.4 (a) On an xy-axis sketch the graph of the parametric equation

$$\mathbf{r}(t) = \langle 2\cos(t) + 2, \sin(t)\rangle.$$

(b) In a complete English sentence describe the motion of an object in space parameterized by

$$\mathbf{r}(t) = \langle \cos(-t), t\sin(-t), t\rangle.$$

as t ranges from 0 to ∞. Draw your best sketch of this curve on a 3D-axis.

F.5 Suppose that a robot (like Wall-E from the Pixar movie) is jetting around in outer space and its velocity at time t is given by

$$v(t) = \left\langle 2e^t, 4t, \frac{1}{t}\right\rangle.$$

(a) If the robot is at position $(1, 1, 1)$ at time $t = 0$, where is the robot at time $t = 2$?

(b) Suppose that at time $t = 2$ the robot runs out of fuel. This means that it loses all acceleration and keeps whatever velocity it had at time $t = 2$ as it drifts into space (this problem got sad quick...). Find a parametric equation for the position of the robot at time t where $t \geq 2$.

F.6 Find the length of the portion of the curve

$$\mathbf{r}(t) = \left\langle t^2, \frac{2}{3}(2t + 1)^{3/2}\right\rangle,$$

where $t \geq 0$, that lies between the lines $x = 0$ and $x = 16$.(*hint: you can do this one by hand, you will just need to do a little factoring at some point*)