Chapter 11 Checkpoint, version 2

Directions:

- You will have 30 minutes per question to complete whichever question you want. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- Write your own personal growth mindset statement. This really does help you do better on the checkpoint. If you have trouble thinking of a growth mindset statement you can use this one:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. I learn from my mistakes. Learning is my superpower.

Chapter 11: I can calculate, use, and interpret multiple integrals.

☐ I.1 I can define and interpret double integrals of functions of two variables over rectangles and numerically approximate them using double Riemann sums.

☐ I.2 I can set up and evaluate double integrals over general regions. I can interchange the order of integration.

☐ I.3 I can set up and evaluate double integrals in polar coordinates.

☐ I.4 I can set up and evaluate triple integrals over general regions. I can interchange the order of integration.

☐ I.5 ** I can set up and evaluate triple integrals in spherical and cylindrical coordinates.

☐ I.6 ** I can make change of coordinates to double and triple integrals by changing bounds and finding the Jacobian.

☐ I.7 I can define, evaluate, and interpret line integrals of scalar functions on parametrized lines.

☐ I.8 I can define, evaluate, and interpret surface integrals of scalar functions across parametrized surfaces.

I.1 Consider the integral $\int_0^2 \int_1^3 x^2 \, dy \, dx$.

(a) Sketch the region of integration.

(b) Partition the region into 2x3 boxes and use this partition to estimate the value of the integral using a Riemann sum. For each box use the top right corner to estimate the value of the function.

(c) In at least one full English sentence describe what the geometric meaning of $f(x_{ij}^*, y_{ij}^*)\Delta x \Delta y$ is in the Riemann sum $\sum_{i=1}^{n} \sum_{j=1}^{m} f(x_{ij}^*, y_{ij}^*)\Delta x \Delta y$.

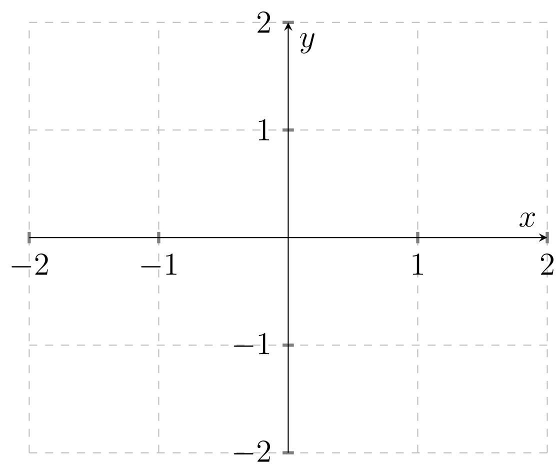
(d) Explain why the approximation you made in part b) would be better if you used more rectangles.

I.2 Exchange the order of the bounds of the integral.

$$\int_1^2 \int_0^{(y-1)^2} f(x,y) \, dx \, dy = \int_?^? \int_?^? f(x,y) \, dy \, dx.$$

I.3 Draw a sketch of the region of integration. Convert the integral to polar coordinates. You do not need to evaluate the integral.

$$\int_0^1 \int_{-y}^{y} x^2 + y^2 \, dx \, dy.$$



I.4 Let E be the tetrahedron defined bounded by the planes $y = 0, x = 0, z = 0$, and $x + y + z = 4$. In this problem we set up the integral to find the volume of E with the given order of integration. You may need to write your answer as the sum of two or more integrals. **Do not evaluate** any of the integrals.

a) Sketch the region of integration.

b) Set up the integral to find the volume of E with order of integration $dz \, dx \, dy$.

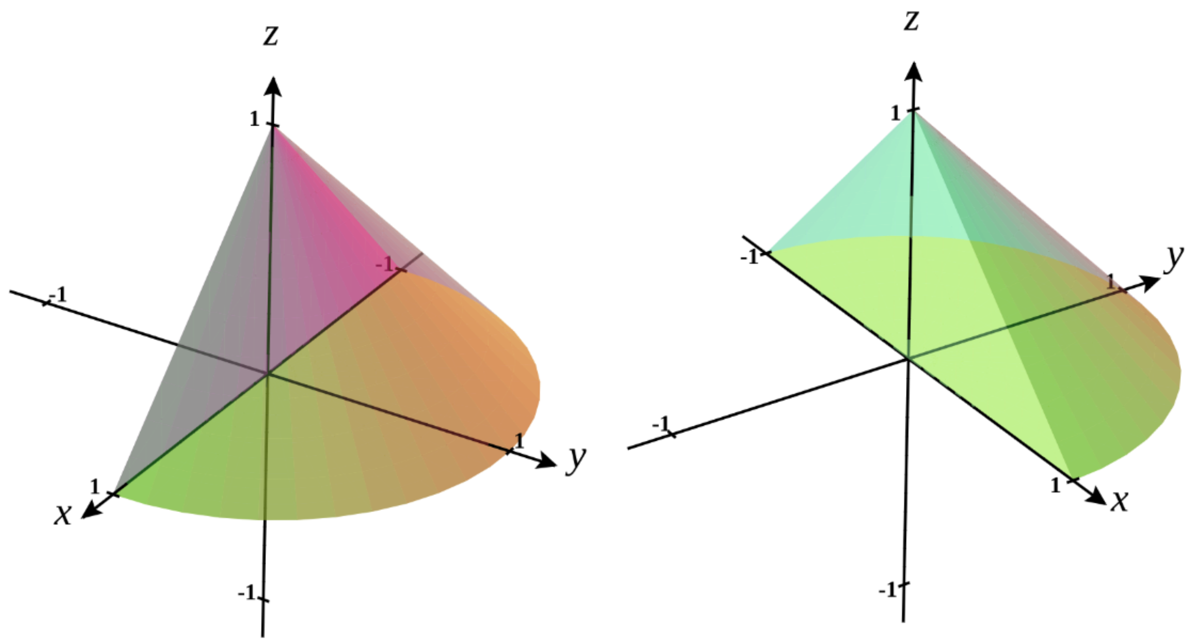
c) Set up the integral to find the volume of E with order of integration $dy \, dz \, dx$.

I.5 Let E be the cone depicted below. In this problem we will set up (but **do not evaluate**) an integral to find the volume of E using the specified coordinate system and order of integration.

a) Set up the integral to find the volume of E in cylindrical coordinates with order of integration $dz\, dr\, d\theta$.

b) Set up the integral to find the volume of E in cylindrical coordinates with order of integration $dr\, dz\, d\theta$.

c) Set up the integral to find the volume of E in spherical coordinates with order of integration $d\rho\, d\phi\, d\theta$.



I.6 (a) Let $T(x, y) = (s, t)$ be a change of variables that rotates the plane by $\pi/13$ radians around the point $(1, 1)$. Use a geometric argument to find the Jacobian of this change of variables.

(b) Let R be the region bounded by $x^{1/3} \leq y \leq x^{1/3} + 3$ where $0 \leq x \leq 8$, use the transformation $T : S \to R$ given by $x = u^3$ and $y = v$. Apply this change of coordinates to the integral

$$\iint_R x + y\, dA$$

I.7 A wire is parameterized by $r(t) = (t, 2t, 3t)$ between $y = 0$ and $y = 8$. The density (grams per meter) of the wire is given by $\delta(x, y, z) = xyz$. Set up but **do not evaluate** an integral to compute the total mass of the wire.

I.8 **Evaluate** the surface integral of the function $f(x, y) = y$ over the surface of the part of the parabaloid $y = x^2 + z^2$ such that $x^2 + z^2 \leq 4$ and $z \geq 0$. (*note: please make sure you read the equations above correctly since they are familiar looking but slightly non-standard.*)