Problem Set 16

Due: Thursday, April 16th

Instructions: Answer each of the following questions and provide a justification for your answer.

1. Take a look at the multiplication table for S_3 below.

	id	(12)	(23)	(13)	(123)	(132)
id	id	(12)	(23)	(13)	(123)	(132)
(12)	(12)	id	(123)	(132)	(23)	(13)
(23)	(23)	(132)	id	(123)	(13)	(12)
(13)	(13)	(123)	(132)	id	(12)	(23)
(123)	(123)	(13)	(12)	(23)	(132)	id
(132)	(132)	(23)	(13)	(12)	id	(123)

   (a) How many times does the identity element appear in the multiplication table?

   (b) How many times does it appear in each row?

   (c) In each column?

   (d) Pick another element of S_3 and answer the above questions for it.

   (e) What patterns do you notice? Do you have any conjectures for properties in the multiplication table for S_4?

2. If π_1 and π_2 are two permutations and $\pi_1\pi_2 = \text{id}$ then we say π_1 and π_2 are inverses. Prove that for any permutation there exists an inverse.

3. In class we saw that D_4 is a subset of S_4. From the previous question we know that every element of S_4 has an inverse in S_4. Does every element of D_4 have an inverse in D_4? Give two proofs, one using geometry and one using permutations.

4. Label the vertices of a regular pentagon $1, 2, 3, 4, 5$ clockwise with the one at the top. Consider all the possible rigid movements of the pentagon and the corresponding permutations they induce on the numbers. Write down all the permutations generated this way in cyclic notation. This set is called D_5.