A curve on the sphere

Warning: this material is entirely optional and only for your enjoyment. Proceed only if you want to learn something really cool.

This problem investigates parameterizing a curve traveling due NE on a sphere.

To warm up consider the following question. If you are on the equator of a sphere and you travel a mile east and then a mile north will you end up in the same place as if you traveled a mile north and then a mile east?

Now imagine you are on the equator and you are holding a compass. You begin to walk due NE and you keep turning so that you are always walking exactly NE as you go. Eventually you will reach the North pole.

1. Find a parameterization for this curve.
2. Compute the length of the curve from the equator to the north pole (you might need to use numerical approximation if you can't compute the integral exactly.
3. How many times do you spiral around the north pole on your way there?