Chapter 11 Checkpoint

Directions:

- You will have 2 hours to complete as many of the following questions as you can. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- Write your own personal growth mindset statement. This really does help you do better on the checkpoint. If you have trouble thinking of a growth mindset statement you can use this one:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. I learn from my mistakes. Learning is my superpower.

Chapter 11: I can calculate, use, and interpret multiple integrals.

☐ I.1 ** I can set up and evaluate triple integrals in spherical and cylindrical coordinates.

☐ I.2 ** I can make change of coordinates to double and triple integrals by changing bounds and finding the Jacobian.

☐ I.3 I can define, evaluate, and interpret line integrals of scalar functions on parametrized lines.

☐ I.4 I can define, evaluate, and interpret surface integrals of scalar functions across parametrized surfaces.

I.5 Let E be the volume above the cone $z = \sqrt{x^2 + y^2}$ and below the half sphere $z = \sqrt{1 - x^2 - y^2} + 1$. (this volume looks like an ice cream cone.) In this problem we will set up (but **do not evaluate**) an integral to find the volume of E using the specified coordinate system and order of integration.

a) Sketch the region of integration.

b) Set up the integral to find the volume of E in cylindrical coordinates with order of integration $dz\, dr\, d\theta$.

c) Set up the integral to find the volume of E in cylindrical coordinates with order of integration $dr\, dz\, d\theta$.

d) Set up the integral to find the volume of E in spherical coordinates with order of integration $d\rho\, d\phi\, d\theta$.

I.6 Let

$$u = x + y$$
$$v = y$$

be a change of coordinates.

(a) Explain using a picture, geometry and a sentences why

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 1.$$

In particular, please do not compute the Jacobian using a formula. instead I am asking about the geometry of what the Jacobian means.

(b) Apply this change of coordinates to the integral. **Do not evaluate** the integral.

$$\int_0^1 \int_0^x xy\, dy\, dx$$

I.7 A wire is in the shape of a spiral given by $r(t) = (2\cos(t), 2\sin(t), 2t)$ between $z = 0$ and $z = 4\pi$. The density (grams per meter) of the wire is given by $\delta(x, y, z) = x^2 + y$. Set up but **do not evaluate** an integral to compute the total mass of the wire.

I.8 Suppose you want to find the amount of glaze on your doughnut. The surface of your doughnut is parametrized by

$$\vec{r}(u, v) = \cos(u)[3 + \cos(v)]\mathbf{i} + \sin(u)[3 + \cos(v)]\mathbf{j} + \sin(v)\mathbf{k} \text{ for } 0 \le u \le 2\pi \text{ and } 0 \le v \le 2\pi.$$

The glaze is heavier on the top than on the bottom, so the glaze has a density function of $1 - z^2$ g/cm^2. Set up an integral to find the mass of the glaze on your doughnut and simplify to the point just before you would evaluate it. **do not evaluate**