Chapter 10 Checkpoint, version 2

Directions:

- You will have 30 minutes per question to complete whichever question you want. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- Write your own personal growth mindset statement. This really does help you do better on the checkpoint. If you have trouble thinking of a growth mindset statement you can use this one:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. I learn from my mistakes. Learning is my superpower.

Chapter 10: I can calculate, use, and interpret partial derivatives.

☐ D.1 ** I can evaluate limits of functions of two variables.

☐ D.2 I can evaluate and interpret first-order partial derivatives of functions of two variables using formulas, tables, graphs, and contour maps.

☐ D.3 I can evaluate and interpret second-order partial derivatives of functions of two variables using formulas, tables, graphs, and contour maps.

☐ D.4 I can find equations of tangent planes for functions of two variables and use them to approximate function values.

☐ D.5 ** I can compute and interpret derivatives using various chain rules.

☐ D.6 I can evaluate and interpret directional derivatives and gradients of functions of multiple variables.

☐ D.7 I can find and classify critical points of functions of two variables.

D.1 Consider the function $f(x,y) = \frac{x^3 y}{x^6+y^3}$.

a) Is the value of the function at $(0,0)$ defined? What does this tell you about the value of the limit of $f(x,y)$ as (x,y) approaches $(0,0)$?

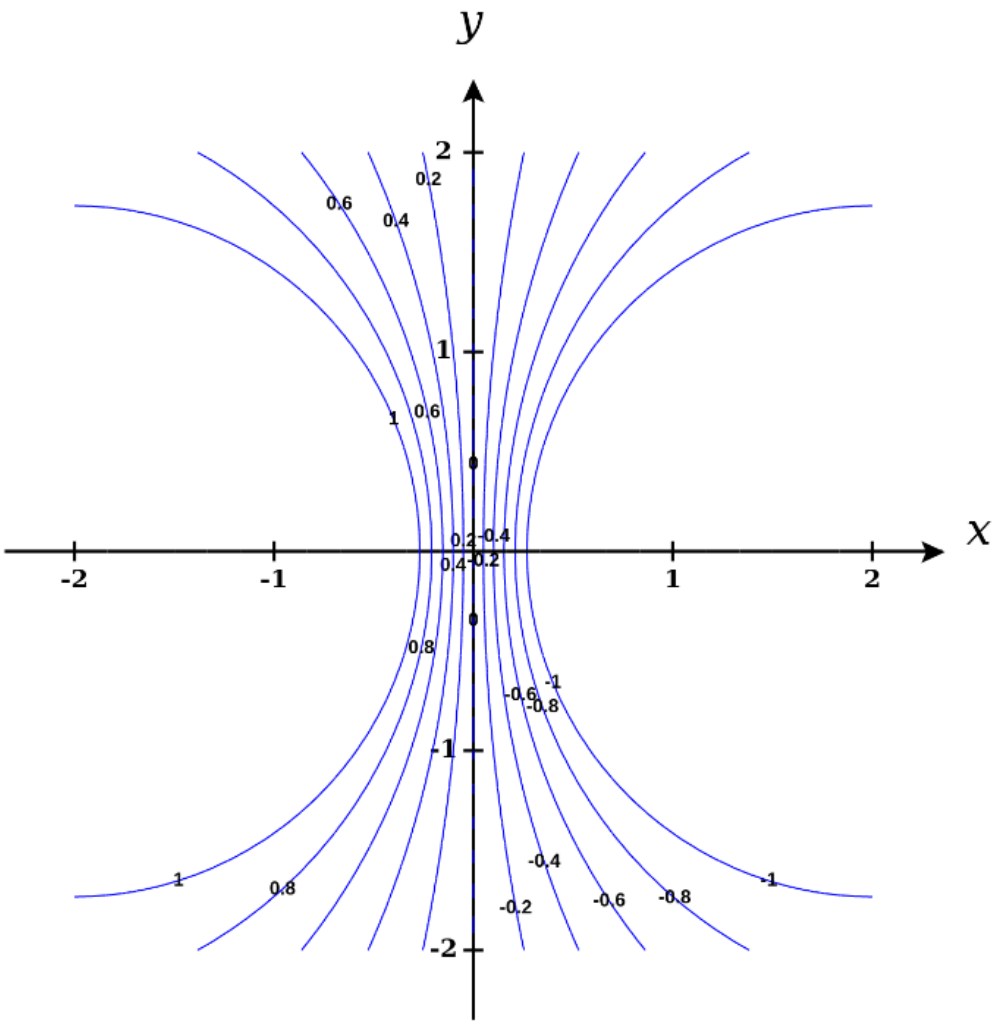
b) Compute the limit as (x,y) approaches $(0,0)$ along the x-axis.

c) Compute the limit as (x,y) approaches $(0,0)$ along the y-axis.

d) Compute the limit as (x,y) approaches $(0,0)$ along the line $y = x$.

e) Do you have enough information to conclude if the limit exists? Explain.

D.2 Consider the contour plot of $g(x,y)$ below.

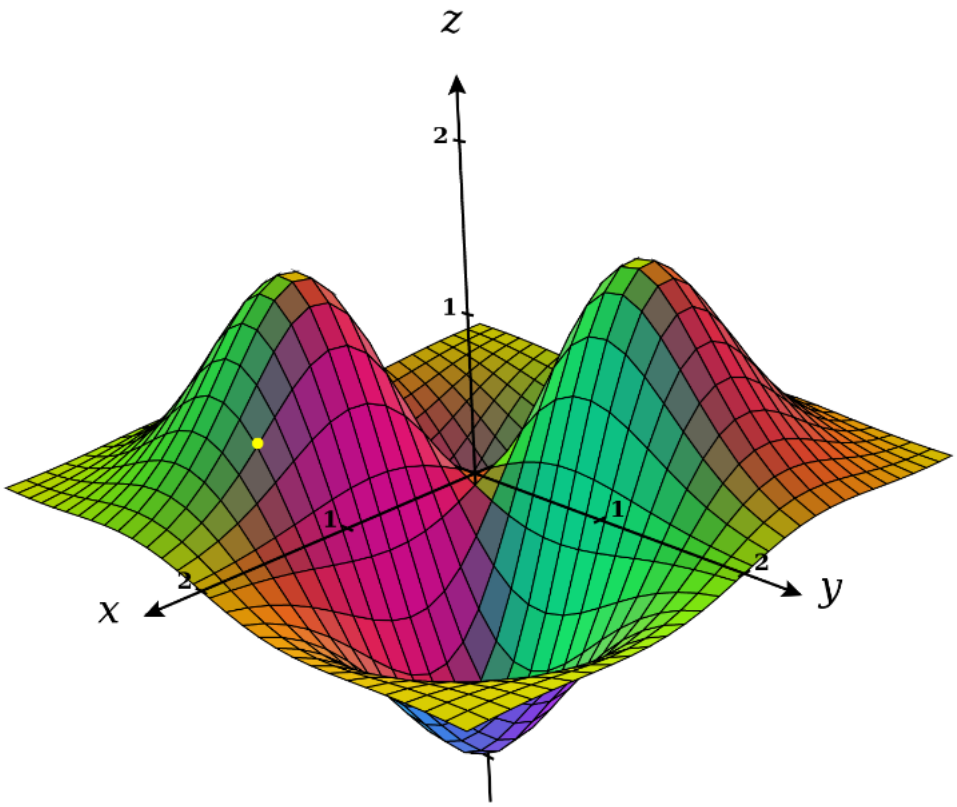


(a) Estimate a point at which $g_y(x,y)$ is positive. Write a sentence explaining your answer.

(b) Estimate a point at which $g_y(x,y)$ is zero. Write a sentence explaining your answer.

(c) Estimate a point at which $g_x(x,y)$ is zero. Write a sentence explaining your answer.

D.3 Below is an image of the graph of a function $f(x, y)$. The point $(1.25, -0.25)$ is shown in yellow.



(a) Is $f_{xx}(1.25, -0.25)$ positive? Negative? Zero? Justify your answer with a written explanation and possibly a picture. Do not justify your answer with algebra.

(b) Is $f_{yx}(1.25, -0.25)$ positive? Negative? Zero? Justify your answer with a written explanation and possibly a picture.

D.4 Use the tangent plane of $z = x^2 - e^y$ at $(3, 0, 8)$ to estimate $(2.9)^2 - e^{(0.1)}$ by hand.

D.5 Suppose $v = x^2 \sin(y) + ye^{xy}$, and $x = s + 2t$ and $y = st$. Use the chain rule to evaluate $\frac{\partial v}{\partial s}$ when $s = 0$ and $t = 1$.

D.6 a) Let $f(x, y) = x^2 + 5y + 1$. Find a direction $\vec{u}$ so that $D_{\vec{u}} f(e, 2) = 0$.

b) Is it possible for $D_{\mathbf{u}} f(a, b) > 0$ in all directions $\mathbf{u}$. If so give an example function $f(x, y)$ and point (a, b). If not explain why.

c) Is it possible for $D_{\mathbf{u}} f(a, b) = 0$ in all directions $\mathbf{u}$. If so give an example function $f(x, y)$ and point (a, b). If not explain why.

D.7 Consider the function $f(x, y) = x^2 y - y$ on the region defined by $y \geq x^2$ and $y \leq 8 - x^2$. Find the absolute maximum and absolute minimum this function attains. Please show all the work necessary. You may use a calculator to assist with evaluating the function at various points.