Take-home Exam 1 Math 215 Spring 2020

Please print this cover sheet for your work. You do not need to print the problems, just this cover sheet.

Name: ______________________________

ID number: ______________________________

Directions:

- You are taking this exam under the honor system. Your signature at the bottom of this page is your promise to abide by the conditions described below. Breaking this promise violates Lewis and Clark's academic integrity policy.
- To answer these questions, use only the knowledge in your head, the course textbook, your notes and a calculator. You may not use any other text, or anything on a computer.
- The exam is due on Thursday at the start of class. If you do not believe you will have it ready at that time please email me immediately.
- Do not talk to any other person, whether enrolled in Math 215 or not, whether they have already taken the exam or not, about this exam until Friday, February 21st. That includes: discussing specific questions, its difficulty, how long you worked on it, how much you are enjoying it, as well as any other exam-related topic you might imagine. The only exception to this rule is that you can talk about it with me if you have any questions.
- Be sure to carefully read the directions for each problem. Please show your work. I expect the same level of explanation as on the homework.

Signature: ______________________________

Question	Points	Score
1	10	
2	20	
3	10	
4	10	
5	10	
Total:	60	

1. (10 points) Prove that for $n \geq 1$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

2. (20 points) Assume the standard deck of cards: 52 cards, 4 different suits and 13 different ranks.

   1. How many 5-card hands have at least two aces? Show your work.
   2. How many 5-card hands have three cards of one rank and two of a different rank (I.e. a full house). Show your work.
   3. How many 5-card hands have exactly one jack and at least one king? Show your work.
   4. How many 5-card hands have exactly one jack, queen, and king such that all the jacks, queens and kings, have different suits? Show your work.

3. (10 points) Let $S(n, k)$ denote the number of ways to place n labeled balls into k unlabeled boxes such that every box has at least one ball. (these are called Stirling numbers of the second kind). For example, if we had 3 balls and 2 boxes, there would be three ways to do this: three different ways to put two balls in one box and one in the other box depending on which ball we put by itself. In this problem we will prove

$$S(n+1, k) = kS(n, k) + S(n, k-1),$$

for $n \geq 1$ and $k \geq 1$.

1. Verify that the identity holds for $n = 3$ and $k = 2$ by computing all parts of the identity and showing that they are equal.
2. Prove the above identity by using a counting argument. (interesting fact: there is actually no formula for $S(n, k)$ so a counting argument is the only way to prove this.)

4. (10 points) Prove that if a does not divide c then a does not divide b or a does not divide $b + c$.

5. (10 points) Use induction to prove that a convex n-gon has $\frac{n(n-3)}{2}$ diagonals. For example, a pentagon has 5 diagonals and if you draw them all it looks like a star.