Problem Set 10

Due: Monday, March 5th

Instructions: Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

Note: when we write $\mathbb{N}$ we mean the set of non-negative integers, $0, 1, 2, 3, ...$ When we write $\mathbb{Z}$ we mean the set of all integers $..., -2, -1, 0, 1, 2, ...$ The notation $\mathbb{Z}^+$ refers to the set of positive integers $1, 2, 3, ...$ And finally, $\mathbb{R}$ is the set of real numbers, things like $0, 1, \sqrt{2}, \pi, ...$ etc.

1. (a) Translate the following into a statement written in English (no math symbols!):

$$\forall a \in \mathbb{Z}^+, \forall b \in \mathbb{Z}^+, p|ab \rightarrow (p|a \vee p|b).$$

   Do you recognize this definition?

   (b) Translate the following into a statement written in English:

$$\forall a \in \mathbb{Z}^+, \forall b \in \mathbb{Z}^+, p = ab \rightarrow (a = 1 \vee b = 1).$$

   Do you recognize this definition?

2. Are the following statements true or false? If the statement is true please prove it; if it is false first negate the statement and then prove the negation. I imagine all the proofs are short except possibly part (c). The goal of this problem is to help you get sense for logical quantifiers and how we use them.

   (a) $\forall n \in \mathbb{N}, \frac{2n}{2n+1} < 1$
   (b) $\forall x \in \mathbb{Z}, \frac{2x}{2x+1} < 1$
   (c) $\forall n \in \mathbb{N}, (3|n \wedge 2|n) \rightarrow 6|n$
   (d) $\exists x \in \mathbb{R}, x^2 + x - 2 = 0$
   (e) $\exists x \in \mathbb{R}, (x+1)^2 + 1 = 0$
   (f) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N}, n|m$
   (g) $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}, n|m$
   (h) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N}, m|n$
   (i) $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}, m|n$
   (j) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x = e^y$
   (k) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + 1 = e^y$