Exam 2 Prep

The second exam will cover all material we have discussed since the first: GCD, Euclidean algorithm, logic (truth tables, deductions, functional completeness, quantifiers), and some set thoery (subsets, proper subsets, union, intersection, set minus, power sets, proving two sets are equal). The in-class portion of the exam will be 50 minutes.

I will send out the other portion of the exam on Thursday afternoon and it will be due on Monday by class time. For that portion you can use your notes and the book but no other sources. You may not discuss that portion of the exam with anyone but me.

Euclidean algorithm

- Review the Euclidean Algorithm for finding the greatest common divisor of two numbers.
- Make sure you know the exact statements of the Division Algorithm, the GCD theorem from Section 2.4 of your textbook and the Fundamental Theorem of Arithmetic.
- Review the process for finding some solutions of the Diophantine equations $mx + ny = d$.

---

1. (a) Find $\text{GCD}(2730, 3150)$ using the Euclidean Algorithm.

   (b) Express the above GCD as an integer linear combination of 2730 and 3150.

   (c) Find the GCD of 2730 and 3150 using the Fundamental Theorem of Arithmetic.

2. Find at least one pair (x, y) of integer solutions of the following equations

   (a) $53x + 42y = 1$.

   (b) $53x + 42y = 4$.

Simplifying Logical Operations

Express the following using only the logical operations $\wedge, \vee$ and $\neg$, and simplify as far as possible. The variables p, q, r are some statements.

1. $\neg(p \longrightarrow q)$
2. $\neg(p \longleftrightarrow q)$
3. $(p \wedge q) \longrightarrow (p \wedge r)$
4. $q \longrightarrow \neg(q \longrightarrow p)$
5. $(p \longrightarrow q) \longrightarrow (q \longrightarrow p)$

Working with Statements Involving Quantifiers

1. For each of the following, write the statement as an English sentence. Then negate the statement and write in in logical symbols.

   (a) $\exists (m,n) \in \mathbb{N}^2, \quad m^2 = 2n^2$

   (b) $\forall n \in \mathbb{N}, \quad n \big| 31 \longrightarrow n \in \{1, 31\}$

   (c) $\forall n \in \mathbb{N}, \quad 3 \big| n^2 \longrightarrow 3 \big| n$

   (d) $\forall x > 0, \ \exists y > 0, \quad y < x$

   (e) $\exists C > 0, \ \forall n \in \mathbb{N}, \quad e^n < C\, n!$

   (f) $\forall \varepsilon > 0, \ \exists N \in \mathbb{N}, \ \forall n \in \mathbb{N}, \quad n > N \longrightarrow \frac{1}{n} < \epsilon$

   (g) $\forall \varepsilon > 0, \ \exists \delta > 0, \ \forall x \in \mathbb{R}, \quad |x-1| < \delta \longrightarrow |x^2 - 1| < \epsilon$

2. Please prove or disprove the following:

   (a) $\forall x \in \mathbb{R}, \ x \neq -1 \longrightarrow \frac{x}{x+1} < 1$;

   (b) $\exists C \in \mathbb{N}, \ \forall n \in \mathbb{N}, \quad n + 1 \leq Cn$;

   (c) $\forall \varepsilon > 0, \ \exists n \in \mathbb{N}, \quad \frac{1}{n} < \varepsilon$;

   (d) $\exists n \in \mathbb{N}, \ \forall \varepsilon > 0, \quad \frac{1}{n} < \varepsilon$;

   (e) $\forall \varepsilon > 0, \ \exists n \in \mathbb{N}, \quad \frac{n-1}{n^2} < \varepsilon$.

Set Theory

1. Prove (or disprove) each of the following statements of set theory:

   (a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ for all sets A, B, C;

   (b) $(A \cup B) \cap (C \cup D) = (A \cap C) \cup (B \cap D)$ for all sets A, B, C, D;

   (c) $A \smallsetminus (B \cup C) = (A \smallsetminus B) \cap (A \smallsetminus C)$ for all sets A, B, C;

   (d) $(A \smallsetminus B) \smallsetminus C = A \smallsetminus (B \cup C)$ for all A, B, C;

   (e) $A \smallsetminus (A \smallsetminus B) = B$;

   (f) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ for all A, B;

   (g) $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$ for all A, B;