

HW 20: Taylor polynomials

Due: Monday, November 25th in SQRC by 9pm

Learning Goals:

- Find Taylor polynomial approximations of functions.

Questions:

1. Find the fifth degree Taylor polynomial centered at $x = 0$ for $f(x) = \frac{1}{1-x}$ by taking derivatives.
2. Find the fifth degree Taylor polynomial centered at $x = \pi/2$ for $\cos(x)$ by taking derivatives.
3. Find a seventh degree Taylor polynomial centered at $x = 0$ for the indicated antiderivatives.
 - a) $\int \frac{\sin(x)}{x} dx$ (answer: $x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!}$)
 - b) $\int e^{x^2} dx$
 - c) $\int \sin(x^2) dx$
4. Plot the 7th degree polynomial you found in part (a) above over the interval $[0, 5]$. Now plot the 9th degree approximation on the same graph. When do the two polynomials begin to differ visibly?
5. Find the fifth degree Taylor polynomial centered at $x = 0$ for $f(x) = \ln(1 - x)$ by
 - (a) computing derivatives $f^{(n)}(0)$;
 - (b) taking the integral of your answer you found in question 1.
6. Calculate the values of $\sin(.4)$ and $\sin(\pi/12)$ using the seventh degree Taylor polynomial centered at $x = 0$

$$\sin(x) \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}.$$

Compare your answers with what a calculator gives you.

7. Find the 7th degree Taylor polynomial centered at $x = 0$ for $f(x) = 3 \sin(x^2)$ by manipulating the Taylor polynomial for $\sin(x)$.
8. In this problem you will compare computations using Taylor polynomials centered at $x = \pi$ with computations using Taylor polynomials centered at $x = 0$.
 - (a) Calculate the value of $\sin(3)$ using a seventh degree Taylor polynomial centered at $x = 0$. How many decimal places of your estimate appear to be fixed?
 - (b) Now calculate the value of $\sin(3)$ using a seventh degree Taylor polynomial centered at $x = \pi$. Now how many decimal places of your estimate appear to be fixed?