# A NE curve on the sphere

This problem investigates parameterizing a curve starting on the equator that travels due NE on the surface of the sphere. To be precise, we want to find a parameterization $r(t) = (x(t), y(t), z(t))$ so that $r(t)$ is always on the surface of the sphere and $r'(t)$ always points in the north east direction.

To begin, recall that the unit sphere is defined by $x^2 + y^2 + z^2 = 1$. So any paramaterization of a curve on the sphere must satisfy this equation. Next, let's investigate what traveling north east means.

a) Let $(0, 0, 1)$ be the north pole. If you are standing at $(1, 0, 0)$ which unit vector points in the north east direction?

b) What if you are standing at $(0, 1, 0)$?

c) What about $(1/\sqrt{2}, 0, 1/\sqrt{2})$?

d) What about $(1/\sqrt{2}, 1/\sqrt{2}, 0)$?

Now imagine you are on the equator and you are holding a compass. You begin to walk due NE and you keep turning so that you are always walking exactly NE as you go. Eventually you will reach the North pole.

1. Find a parameterization for this curve.
2. Compute the length of the curve from the equator to the north pole (you might need to use numerical approximation if you can't compute the integral exactly.
3. How many times do you spiral around the north pole on your way there?