HW 2: Sections 4.2, 4.3, and 4.4

Due: Monday, September 9th in SQRC by 9pm

Learning Goals:

- Translate sums in to sigma notation.
- Evaluate expressions written in sigma notation.
- Approximate areas under a curve using a Riemann sum.
- Explain how a definite integral computes area under a curve.
- Use graphs and signed area to understand the value of a definite integral.

A reminder about collaboration: Collaboration is an important part of learning mathematics and I strongly encourage you collaborate with your classmates on homework and studying for exams. With that said, there is a difference between working with someone else and copying down what they say or write without understanding it. I encourage you to write up final solutions on your own after you understand a problem, which might mean stepping away from your study group for 5 or 10 minutes.

Questions:

1. Problem 4.2.6. Write out all the terms and compute the sum:

$$\sum_{i=3}^{7}(i^2+1).$$

2. Problem 4.2.10. Use summation rules to compute the sum:

$$\sum_{i=1}^{45}(3i-4).$$

3. Problem 4.3.2.b. List the evaluation points corresponding to the midpoint of each sub-interval, sketch the function and approximating rectangles and evaluate the Riemann sum:

$$f(x) = x^3 - 1, [1,3], n = 4.$$

4. Problem 4.3.16 Use technology (try typing "sum" into `https://www.desmos.com/calculator`) to construct a table of Riemann sums as in example 3.4 to show that sums with right-endpoint, midpoint, and left-endpoint evaluations all converge to the same value as $n \to \infty$:

$$f(x) = \sin(x), [0, \pi/2].$$

5. Problem 4.4.24. Compute $\int_0^4 f(x)\,dx$ by drawing a picture, where

$$f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}.$$

6. Problem 4.4.47. Use the graph to determine whether $\int_0^2 f(x)\,dx$ is positive or negative.