Problem Set 9

Due: Monday, March 2nd

Instructions: Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1. For each of the following logical statements state if it is a tautology, contradiction, or neither. You do not need to give a truth table, just state your answer.

   (a)
   $$(A \to B) \leftrightarrow (\neg B \to \neg A).$$

   (b)
   $$(A \to B) \to (B \to A).$$

   (c)
   $$(Q \to (P \wedge \neg Q)) \wedge Q.$$

2. Create a truth table for every part of the following logical statements.

   (a)
   $$(A \vee B) \to (B \to A).$$

   (b)
   $$A \to (B \to (A \wedge B)).$$

3. The logical connective NOR is defined as the negation of "or". In other words $(p \,\mathrm{NOR}\, q)$ is equivalent to $\neg(p \vee q)$.

   (a) Write a truth table for NOR.

   (b) Prove that the logical connective NOR by itself is a functionally complete set (i.e. every truth table can be built using just NOR's). In your answer you may assume that $\{\neg, \vee, \wedge\}$ is a functionally complete set of logical connectives. I would suggest proving that you can get these three from NOR instead of showing you can get all truth tables from scratch.

4. For this problem you may use any of the logical deductive steps:

   - Modus Ponens: $((p \to q) \wedge p) \implies q$
   - Modus Tollens: $((p \to q) \wedge \neg q) \implies \neg p$
   - Addition: $p \implies (p \vee q)$
   - Simplification: $(p \wedge q) \implies p$
   - Modus Tollendo Ponens: $((p \vee q) \wedge \neg p) \implies q$
   - Hypothetical Syllogism: $((p \to q) \wedge (q \to r)) \implies (p \to r)$

   (a) Consider the set of hypotheses $\{(C \vee E) \to \neg M, R \to M, C\}$. Give a deduction which concludes with $\neg R$.

   (b) Consider the set of hypotheses $\{P \wedge Q, (P \vee Q) \to R\}$. Give a deduction which concludes with R.

(c) Consider the set of hypotheses $\{\neg J \vee S, \neg L \to \neg S, J \wedge \neg L\}$. Give a deduction which concludes with F.

(d) Consider the set of hypotheses $\{\neg J \vee S, \neg L \to \neg S, J \wedge \neg L\}$. Give a deduction which concludes with $\neg F$.

(e) (Optional) (0 points) In the previous two problems you started with the same set of hypotheses and were able to deduce both F and $\neg F$.

What does this tell you about the hypotheses?