HW 21: Section 4.3 and 4.4

Due: Monday, December 2nd in SQRC by 9pm

Learning Goals:

- Translate sums in to sigma notation.
- Evaluate expressions written in sigma notation.
- Approximate areas under a curve using a Riemann sum.
- Explain how a definite integral computes area under a curve.
- Use graphs and signed area to understand the value of a definite integral.

Questions:

1. Problem 4.3.16 Use technology (use the example worksheet on Desmos) to construct a table of Riemann sums as in example 3.4 (page 320) to show that sums with right-endpoint, midpoint, and left-endpoint evaluations all converge to the same value as $n \to \infty$:

   $$f(x) = \sin(x), [0, \pi/2].$$

   Write down the formula for the Riemann sums you are evaluating.

2. Problem 4.3.17 Use technology to construct a table of Riemann sums to show that sums with right-endpoint, midpoint, and left-endpoint evaluations all converge to the same value as $n \to \infty$:

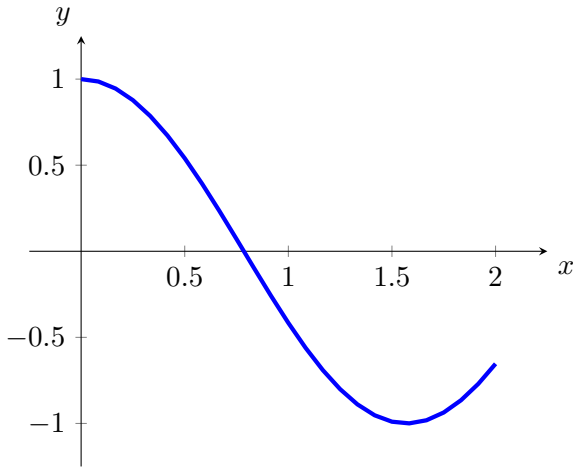
   $$f(x) = x^3 - 1, [-1, 1].$$

   Write down the formula for the Riemann sums you are evaluating.

3. Problem 4.4.24. Compute $\int_0^4 f(x)\,dx$ by drawing a picture, where

   $$f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}.$$

4. Problem 4.4.47. Use the graph to determine whether $\int_0^2 f(x)\,dx$ is positive or negative.



5. For the following Riemann sums determine what Δx, x_i, and $f(x)$ are. Also, state whether the Riemann sum is using left endpoints, right endpoints, or midpoints.

   (a)
   $$\sum_{i=0}^{n-1} \cos\left(\frac{3}{n}i\right)\frac{3}{n}$$

   (b)
   $$\sum_{i=0}^{n-1} \cos\left(\frac{2}{n}i+1+\frac{1}{n}\right)\frac{2}{n}$$

6. Write the following Riemann sum as an integral.
   $$\lim_{n\to\infty}\sum_{i=0}^{n-1} \ln\left(\frac{\pi}{n}i+\frac{\pi}{2n}\right)\frac{\pi}{n}$$