Daily Prep Assignment for April 15th

Overview

To prepare for class

Preview activities:

- Preview activity 12.3.1

Reading:

- Read section 12.3

Watching: Watch these additional resources if you need support reading the text.

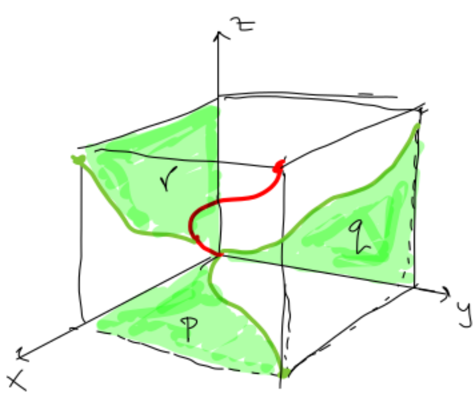
1. 12.3 overview: `https://youtu.be/A2GNoTSSelg`

During and after class

These problems are not in the book but I'm going to number them as if they were.

- Activity 12.3.2
- Activity 12.3.3
- Activity 12.3.4
- Extra problem: I want to share with you a problem that came up in my own research last week. The research question I'm asking is about exploring the possible probabilities that a set of three dice can beat each other with. I'm not going to explain how to get from there to the question I'm about to ask you but I thought you might find it interesting that multivariable calculus would show up in the study of dice.

  Consider the following picture:



  This is a picture of the cube $[0,1] \times [0,1] \times [0,1]$. The red curve is parameterized by $r(t) = \langle x(t), y(t), x(t) \rangle$ with $0 \leq t \leq 1$ such that $r(0) = (0,0,0)$ and $r(1) = (1,1,1)$. We will also assume that $x(t), y(t)$ and $z(t)$ are increasing functions so the curve can't double back on itself. On each of the three coordinate planes I have drawn the projection of this red curve. So for example, in the xy-plane, the green curve is give by $\alpha(t) = \langle x(t), y(t) \rangle$.

a) My first question is about finding the area labeled p in the picture. Draw a sketch in the xy-plane of the region p and the curve $\alpha(t)$. Now set up an integral that computes the area of p. (*hint: it would be nice to simply integrate y with respect to x, but we are in t-coordinates. How can you convert between dt and dx?*)

b) Double check your answer from a) by checking that it works if $\alpha(t) = (t, t^2)$, i.e. that it is the same as the integral $\int_0^1 x^2\, dx$.

c) Write similar integrals that compute the area of the regions labeled r and q.

d) Add these three integrals together to get a single integral that computes the sum of the three areas.

e) The integral you found in part d) is actually a line integral of a vector field over the red curve. Rewrite it in our standard line integral form.