Problem Set 10

Due: Friday, April 17th

Instructions: Do at least 4 of the following problems.

1. **Heads I Win, Tails You Lose** $\star$

   Consider a two-player game in which the players simultaneously show a penny, either heads up or tails up. If both players show heads, then both players lose their pennies to a lucky third party, and if both players show tails, each player keeps his or her own penny. If both players show different sides, then the player who shows heads gets both coins.

   (a) Write down the two-by-two matrix for this game.

   (b) Is Chicken or Prisoner's Dilemma or neither a model for this game?

   (c) Do the players have a dominant strategy?

   (d) Is there a Nash equilibrium?

2. **Do You Trust Me Now?** $\star$

   Iterated Prisoner's Dilemma is a two-player game in which two players play the Prisoner's Dilemma game a fixed finite number N of times. Their strategy for all games must be chosen before starting the game.

   (a) Determine each player's strategy when $N = 2$.

   (b) Determine each player's strategy when $N = 3$.

   (c) Explain why each player's strategy remains the same no matter how large N is.

3. **Math is a Game!** $\star$ **to** $\star\star\star$

   Here is a wonderful little game:

   <div align="center">

   https://ncase.me/trust/

   </div>

   It takes about 30 minutes to play. Play the game and write a reflection. In particular, I want you to critique the conclusions of the game. What insights does it have? What does it simplify or brush over? Do you think it gets everything right? Does it make claims without proving them? Are they true claims or false? How could you rephrase these ideas in the language of mathematics. What questions do you still have?

4. **Theory of Chicken** $\star\star$

   Do an analysis of the theory of moves version of Chicken that is analogous to what was done for Prisoner's Dilemma in the text.

5. **This is the Last Problem You Will _ever_ Do!** $\star\star$

   The ultimatum game is a two-player game, played as follows: Player 1 proposes a possible division of \$1 between the two players (for example, they might split the \$1 evenly between them). Only divisions requiring quarters (no dimes, nickels, or pennies) are allowed. Player 2 has two options: she can either accept the division and the dollar is split as proposed, or she can reject the division in which case neither player receives anything.

(a) Assuming each player just wants to maximize his profit, what is Player's 2 dominant strategy? What about Player 1?

(b) In practice, a large percentage of the people in Player 1's role offer a near 50-50 split. Compare this to your results in part (a). How might you explain this difference?

6. **Somewhat Sensical** ⋆⋆

A two-player game is said to be a somewhat finite game if every play of the game ends after finitely many moves. "Hypergame" was created by William Zwicker in the late 1970s. It is played by two players as follows: The first move consists of Player 1 naming a somewhat finite game of his or her choice. The second move in this play of hypergame consists of Player 2 making a legitimate first move in the somewhat finite game named in move 1. Player 1 now makes a second move in the game named, and they continue to alternate until this play of the game named is completed. (In some ways, hypergame is like dealer's choice poker.)

(a) Write down a compelling argument that hypergame is a some-what finite game.

(b) Write down a compelling argument that hypergame is not a somewhat finite game.