HW 20: Taylor polynomials

Due: Monday, November 25th in SQRC by 9pm

Learning Goals:

- Find Taylor polynomial approximations of functions.

Questions:

1. Find the fifth degree Taylor polynomial centered at $x = 0$ for $f(x) = \frac{1}{1-x}$ by computing $f^(k)(0)$.

2. Find the fifth degree Taylor polynomial centered at $x = \pi/2$ for $\cos(x)$ by computing $f^(k)(\pi/2)$.

3. Find the fifth degree Taylor polynomial centered at $x = 0$ for $f(x) = \ln(1 - x)$ by

   (a) computing derivatives $f^{(k)}(0)$;

   (b) taking the integral of your answer you found in question 1.

4. Find the 7th degree Taylor polynomial centered at $x = 0$ for $f(x) = 3\sin(x^2)$ by manipulating the Taylor polynomial for $\sin(x)$.

5. Find a seventh degree Taylor polynomial centered at $x = 0$ for the indicated antiderivatives.

   a) $\int \frac{\sin(x)}{x}\,dx$ (answer: $x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!}$)

   b) $\int e^{x^2}\,dx$

   c) $\int \sin(x^2)\,dx$

6. Plot the 7th degree polynomial you found in part (a) above over the interval $[0, 5]$. Now plot the 9th degree approximation on the same graph. When do the two polynomials begin to differ visibly?

7. Calculate the values of $\sin(.4)$ and $\sin(\pi/12)$ using the seventh degree Taylor polynomial centered at $x = 0$

   $$\sin(x) \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}.$$

   Compare your answers with what a calculator gives you.

8. In this problem you will compare computations using Taylor polynomials centered at $x = \pi$ with computations using Taylor polynomials centered at $x = 0$.

   (a) Calculate the value of $\sin(3)$ using a seventh degree Taylor polynomial centered at $x = 0$. How many decimal places of your estimate appear to be fixed?(you will need to compute the ninth degree Taylor polynomials to do this, do you see why?)

   (b) ow calculate the value of $\sin(3)$ using a seventh degree Taylor polynomial centered at $x = \pi$. Now how many decimal places of your estimate appear to be fixed?