Weighted voting systems

1. **European Economic Community**

   Formed in 1957 by the Treaty of Rome the European Economic Community had six member nations: France, Germany, Italy, Belgium, Netherlands, Luxembourg.

Nation	Votes
France	4
Germany	4
Italy	4
Belgium	2
Netherlands	2
Luxembourg	1

   A resolution needs 12 votes to pass.

   This is called a *weighted voting system*: voters $v_1, v_2, \ldots, v_n$ have weights $w_1, w_2, \ldots, w_n$. A bill passes if the sum of the weights for yes votes $\geq q$, a quota.

   - Analyze this situation. What are the minimal winning coalitions? Do you notice anything strange? What questions does this example bring up?

2. **U.N. Security Council**

   The U.N. Security Council has fifteen members: five permanent members, and ten non-permanent members. (trivia: does anyone know the five permanent members?) A resolution needs 9/15 "yes" votes to pass, but any of the five permanent members can veto.

   This is a *yes-no voting system*. It can be defined by giving a set of voters, and also defining the set of "winning coalitions". A bill passes if and only if the set of voters is a winning coalition. In this example the winning coalitions are sets of 9 or more voters including all 5 permanent members.

   Proposition: Weighted voting systems are also yes-no voting systems.

   - The UN Security Council does not appear to be a weighted voting system but... is it possible to assign a certain number of votes to each member and a certain quota so that the winning coalitions are the same as the yes-no version? (so no more vetos, just weighted votes).

3. **Amending the Canadian constitution**

   To amend the Canadian constitution each province has a (2/3) majority vote. The amendment passes if it passes in at least 7/10 provinces *and* the sum of populations in those provinces is $> 50\%$ of population.

P.E.I.	0%
Newfoundland	2%
New Brunswick	2%
Nova Scotia	3%
Sasketchewan	3%
Manitoba	4%
Alberta	11%
British Columbia	13%
Quebec	23%
Ontario	39%

So, the winning coalitions are the sets of 7 or more provinces with populations summing to at least 50%. If populations are less than 50% or the number of provinces is less than 7, then it's a losing coalition.

- Analyze this voting system. What are some winning coalitions?
- Do you think this is a weighted voting system?

4. **Passing a bill in Congress**

To pass a bill in the United states Congress:

(a) The house must pass the bill with majority. There are 435 and we will assume no abstentions so no ties,

(b) the Senate (100 members) must pass the bill with a majority as well with the VP as tiebreaker,

(c) but President can veto unless both chambers exceed 2/3.

So, there are three kinds of winning coalitions:

(a) More than half the house, more than half the Senate, and President. (Possibly VP.)

(b) More than half the house, exactly half the Senate, President and VP.

(c) More than 2/3 the House, more than 2/3 the Senate. (Possibly VP).

- Is this a weighted voting system? Can you prove or disprove your claim?
- When is a yes-no voting system a weighted voting system? Can you put any conditions on the winning coalitions that must be true for a yes-no voting system to be a weighted voting system? Can you find sufficient conditions?

If you spend time analyzing the above material you can easily generate enough thoughts, questions, conjectures, and attempts at proving things for an lively hour (or many hours) of discussion. If for some reason that is not sparking joy you could instead consider the following question:

- In the first example you might have noticed an imbalance of power. How would you measure the power of each of the six nations? How would you measure the power of various voters in the other three examples?