

## Problem Set 4

Due: Thursday, February 6th

**Instructions:** Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1. Let  $a, b$  and  $c$  be integers. Prove that if  $a|b + c$  and  $a|c$  then  $a|b$ .
2. Prove that for any integers  $a, b$ , if  $ab$  is even then either  $a$  is even or  $b$  is even.
3. Let  $a, b$  and  $c$  be integers. Show that  $11|(a - b + c)$  if and only if  $11|(100a + 10b + c)$ .
4. We define a positive integer  $p$  to be **prime** if whenever  $p|ab$  either  $p|a$  or  $p|b$ .  
We define a positive integer  $p$  to be **irreducible** if whenever  $p = ab$  either  $a = 1$  or  $b = 1$ .  
Prove that for any integer  $p$ , if  $p$  is prime, then it is irreducible.