Chapter 12 Checkpoint 1 a

Directions:

- You will have 2 hours to complete as many of the following questions as you can. When you begin the checkpoint, please write down the current time at the top of your cover page, and leave a space to write the time you finish. When you finish please immediately write the time.
- You may use your notes, the book, and any materials posted on the course website. Also, feel free to ask me clarifying questions or about typos. You may not use any other resource. In particular, you may not use any other resource on the internet, you may not use a computer to assist you with graphing or computations (unless the problem explicitly states otherwise) and you may not discuss the problems with anyone else.
- Each problem corresponds to a standard and specifically asks about that standard. You many complete as many or as few of the problems as you wish.
- If you have a question about any of the problems, or think there is an error please email me immediately. Also, if something occurs during your allotted time or some other special circumstance arises, please email me immediately.
- Write your own personal growth mindset statement. This really does help you do better on the checkpoint. If you have trouble thinking of a growth mindset statement you can use this one:

  I am a problem solver and my mind grows everyday. I improve with lots of practice. I learn from my mistakes. Learning is my superpower.

Chapter 12: I can calculate, use, and interpret vector calculus

☐ VC.1 I can identify, evaluate, sketch and interpret vector fields in the plane and in space.

☐ VC.2 I can define and interpret line integrals of vector fields along oriented curves. I can use parametrizations to evaluate line integrals of vector fields along oriented curves.

☐ VC.3 I can use the Fundamental Theorem of Calculus for Line Integrals to evaluate line integrals of gradient fields.

☐ VC.4 ** I can define, evaluate, and interpret the divergence of vector fields. I can define, evaluate, and interpret the curl of vector fields.

☐ VC.5 ** I can use Green's Theorem to evaluate circulations of smooth vector fields along simple closed curves in the plane.

☐ VC.6 ** I can define, evaluate, and interpret flux integrals of vector fields across parametrized surfaces.

☐ VC.7 ** I can use Stokes' Theorem to evaluate circulations of smooth vector fields along simple closed curves in space.

☐ VC.8 ** I can use The Divergence Theorem to evaluate flux of continuous vector fields through closed surfaces in space.

VC.1 a) Sketch the vector field $\langle y, x\rangle$ by hand. Make sure to include at least several vectors in all four quadrants of the plane.

b) Give three real life examples of vector fields.

c) Which of the following are vector fields?

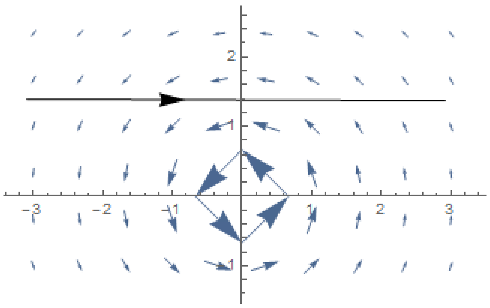
(i) $r(t) = \langle t^2 + 2t, t, \sin(t)\rangle$

(ii) $f(x, y) = \langle x^2 y, y\rangle$

(iii) $g(x, y) = e^{x^2+y}$

(iv) $h(x, y, z) = \langle 3x, x, x^2\rangle$

VC.2 (a) Let C be the horizontal line traveled from $(-2, 1.5)$ to $(2, 1.5)$. Determine if $\int_C \vec{F} \cdot d\vec{r}$ is positive, negative, or zero for the vector field $\vec{F}$ show below.



(b) Compute $\int_C F \cdot dr$ where $F = (-yx, 2x + 1)$ and C is the straight line starting at $(1, 2)$ and ending at $(-1, 2)$.

VC.3 (a) Give an example of a vector field that is not a gradient vector field. Justify your answer.

(b) Let $\mathbf{F}(x, y) = \langle 2xy, x^2 + 1\rangle$. Use the fundamental theorem of line integrals to find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve that starts at $(\sqrt{2}, 1)$ and ends at $(\sqrt{3}, 7)$.

VC.4 (a) Sketch a 2D vector field such that the divergence is clearly negative at every point.

(b) Sketch a 2D vector field so that the at $(1, 0)$ the circulating density is positive and at $(-1, 0)$ it is negative.