HW 19: Section 3.7

Due: Thursday, November 14th in SQRC by 9pm

Learning Goals:

- Write sums using Sigma notation.
- Evaluate expressions written in Sigma notation.
- Approximate the area under a curve using rectangles.
- Express the area under a curve in sigma notation.

Questions:

1. Problem 4.2.2 Translate into summation notation

$$\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \cdots + \sqrt{15-1}$$

2. Problem 4.2.8 Write out all the terms and compute the sum

$$\sum_{i=6}^{8} (i^2 + 2)$$

3. Problem 4.2.40 The table below shows the downwards velocity of an object. Estimate the distance fallen.

time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
velocity (m/s)	10	14.9	19.8	24.7	29.6	34.5	39.4	44.3	49.2

4. Problem 4.3.2/4 Sketch the function and the interval under which we want to measure the area. Then break the interval into the specified number of pieces. Label the midpoint of each interval. Sketch the rectangles that approximate the area using those midpoints to find their heights. Lastly, estimate the area using these rectangles.

   a) $f(x) = x^3 - 1$, on $[1, 2]$ with four rectangles.
   b) $f(x) = x^3 - 1$, on $[0, 3]$ with four rectangles.
   c) $f(x) = 4 - x^2$, on $[-1, 1]$ with four rectangles.
   d) $f(x) = 4 - x^2$, on $[-3, -1]$ with four rectangles.

5. Sketch the curve $y = x^2 + 1$ on the interval $[0, 2]$. Suppose we split this interval into 16 pieces

   a) List the left endpoint of these intervals. Write a Riemann sum expression in Sigma notation for the approximate area of under the curve using the left-hand rule with 16 rectangles. Evaluate this sum using Desmos or a calculator.
   b) Do the same using the right hand endpoints.
   c) Lastly, use the midpoints of each interval.

6. Consider the Riemann sum $\sum\limits_{i=1}^{6} \ln(0.5i + 1)(0.5)$. Notice that this has the form $f(x_i)\Delta x$, so that $\Delta x = 0.5$, $x_i = 0.5 + 1$ and $f(x) = \ln(x)$. Draw a sketch of this Riemann sum by first marking the points x_i, then sketching the function $f(x)$, and then draw 6 rectangles using the x_i as right endpoints of six rectangles that approximate the area under the curve.

7. Consider the Riemann sum $\sum\limits_{i=1}^{4} ((0.25i + 1)^2 + 1)(0.25)$. What is Δx? What are the x_i? What is $f(x)$? Draw a sketch of this Riemann sum by marking the points x_i, sketching the function, and then drawing the rectangles so that the x_i are the right endpoints of those rectangles.