Problem Set 13

Due: Thursday, March 2nd

Instructions: Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1. Let A and B be sets. Let $\sim_A$ be an equivalence relation on A. Let $\sim_B$ be an equivalence relation on B. Let $\sim$ be a relation on $A \times B$ such that $(a_1, b_1) \sim (a_2, b_2)$ if and only if $a_1 \sim_A a_2$ **or** $b_1 \sim_B b_2$.

   (a) Is $\sim$ always reflexive? If so prove it, and if not give counter example.

   (b) Is $\sim$ always symmetric? If so prove it, and if not give counter example.

   (c) Is $\sim$ always transitive? If so prove it, and if not give counter example.

2. Let A be a non-empty set. Let $\sim^+$ be a arbitrary relation on A. Let $\sim^-$ be a relation on A such that $a_1 \sim^- a_2$ if and only if $a_1 \not\sim^+ a_2$. Define a relation $\sim$ on A such that $a_1 \sim a_2$ if and only if $a_1 \not\sim^+ a_2$ **or** $a_1 \not\sim^- a_2$.

   (a) Prove that $\sim$ is an equivalence relation. (hint: it might help to do part b) first).

   (b) State how many equivalence classes $\sim$ has.

3. Prove or disprove that each of the following functions is 1-1 and/or onto. (So two carefully written proofs must be given for each part.)

   a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 4x - 1$

   b) $f : \mathbb{N} \to \mathbb{N}$ given by $f(n) = 4n - 1$

   c) $f : \mathcal{P}(\{a, b, c, d\}) \to \mathcal{P}(\{a, b, c, d\})$ given by $f(S) = S \cap \{a, b\}$

   d) $f : \mathcal{P}(\{a, b, c, d\}) \to \mathcal{P}(\{a, b, c, d\})$ given by $f(S) = S \cup \{a\}$

   e) $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 + y^2$

   f) $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = xy$