# Problems

Task: Below is the type of puzzle that mathematics loves to solve (notice the anagram). To demonstrate how such a problem is solved we will walk through the steps together as a group. First, spend five minutes thinking about the problem on your own. Try examples, play around. Then spend five to ten minutes talking with your neighbor about the problem. Finally we will discuss as an entire group and see where we are. The end goal is to write a proof.

Fault lines The squares on an 6×6 chessboard are created by two families of evenly spaced parallel lines: 5 horizontal lines and 5 vertical lines. A domino piece that covers exactly two squares crosses one such line. Assuming the whole board is covered with dominoes, some of the lines will be crossed, but others may not be. Lets call the lines that have not be crossed fault lines. Prove that every covering of a 6×6 board with dominoes leaves at least one fault line. Give an example of a covering with exactly 1 fault.

Here is an example tiling with many faults.
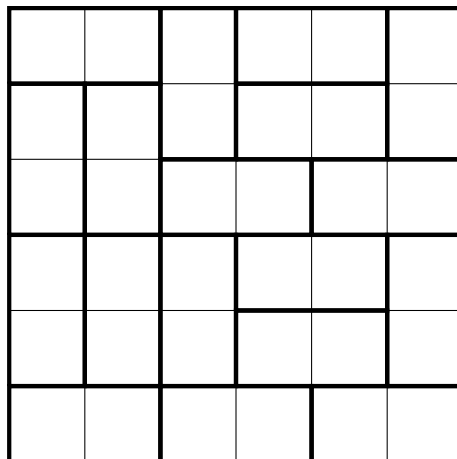
The Mad Veterinarian A mad veterinarian (that is, a mad scientist who studies animals) has invented a series of animal transmogrifying machines. In all the puzzles below, the goal is to use the machine to end up with 1 cat and no other animals. When can the problems be solved
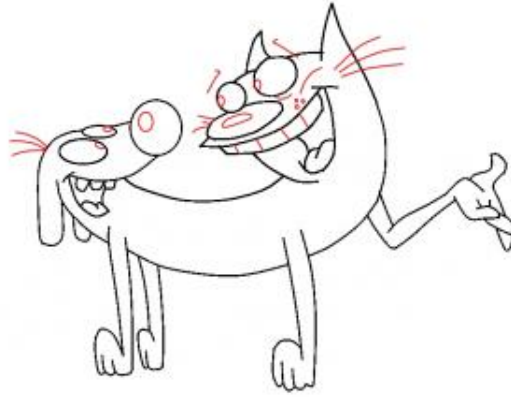
1. Machine one: Cats and Dogs

   - Put in two cats or two dogs and one dog comes out of the machine.
   - Put in one cat and one dog and one cat comes out

   Can you end up with 1 cat and 0 dogs if you start with:

   - 3 cats and 1 dog?
   - 3 cats and 5 dogs?
   - 4 cats and 2 dogs?
   - 11 cats and 15 dogs?
   - 20 cats and 101 dogs?

   Find a pattern that describes when it is possible to end with 1 cat and 0 dogs.

2. Machine two: Cats, Dogs, and Mice

   - Put in 1 dog + 1 cat to get 1 mouse
   - Put in 1 dog + 1 mouse to get 1 cat
   - Put in 1 cat + 1 mouse to get 1 dog

   Can the vet end with 1 cat, 0 dogs, 0 mice starting with:

   - 3 cats and 1 dog?
   - 3 cats, 3 dogs, 3 mice?
   - 4 cats, 4 dogs, 4 mice?

   Find a pattern that describes when it is possible to end with 1 cat and 0 dogs.