# Problem Set 14

Due: Monday, April 6th

**Instructions:** Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1. Let $f : A \to B$ and $g : B \to C$ be functions. Assume that $f$ and $g$ are both injective functions. Either prove that $g \circ f : A \to C$ is always injective, or provide a counter-example.

2. Let $f : A \to B$ and $g : B \to C$ be functions. Assume that $f$ and $g$ are both surjective functions. Either prove that $g \circ f : A \to C$ is always surjective, or provide a counter-example.

3. Let $f : A \to B$ and $g : B \to C$ be functions. Assume that $f$ is a surjective function and $g$ is an injective function.

   (a) Either prove that $g \circ f : A \to C$ is always injective, or provide a counter-example.

   (b) Either prove that $g \circ f : A \to C$ is always surjective, or provide a counter-example.

4. Create functions $f : A \to B$ and $g : B \to C$ such that $g$ is **not** injective, but $g \circ f$ **is** injective.

5. Describe a bijection between the set of natural numbers $\mathbb{N}$ and the set of square numbers. Make sure you prove that you are actually describing a function (so for any given element you need to be able to describe where it is sent) and that you argue why it is injective and surjective.

6. Suppose that $|\mathbb{N}| = |A| = |B|$. Prove that $|\mathbb{N}| = |A \cup B|$.

7. Describe a bijection between the set of natural numbers $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N}$. Make sure you prove that you are actually describing a function (so for any given element you need to be able to describe where it is sent) and that you argue why it is injective and surjective.