Section 1.2 Challenge Problem

In this challenge problem we explore example 1.2.4 (b) from the book in greater detail.

Challenge Problem 1.2
Consider the function $f(x) = \sin(\frac{\pi}{x})$.

1. Find a sequence of x-values that approach 0 so that when you evaluate $f(x)$ on those values the output approaches $\frac{1}{2}$.

2. Find a sequence of x-values that approach 0 so that when you evaluate $f(x)$ on those values the output does not appear to converge to any number in particular.

3. Consider the function $g(x) = x\sin(\frac{\pi}{x})$. What can you say about $\lim_{x\to 0} g(x)$? Can you come up with an algebraic argument to justify your answer that does not depend on looking at the graph? (Hint: try bounding $g(x)$ between two other functions that you know more about).