4.5 Worksheet: The Fundamental Theorem of Calculus

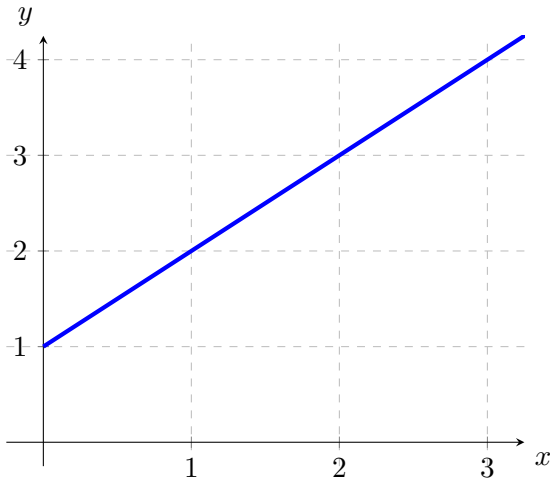
Purpose: The Fundamental Theorem of Calculus can look like it comes out of nowhere. In this worksheet we will try to explain why it is true through examples.

1. **Area functions:**

   a) Let $f(x)$ be the graph to the right and let $A(x) = \int_0^x f(x)\,dx$ be its *area function*. For this problem, use geometry to calculate the areas under the curve to compute the following.

   $A(1) = 1.5$ $\qquad$ $A(3) = 7.5$

   $A(2) = 4$ $\qquad$ $A(4) = 12$

   and, in general, $A(x) = x + x^2/2$
   (hint: think triangle plus rectangle)

   Take the derivative to compute
   $A'(x) = x + 1$

   How does $A'(x)$ compare to $f(x)$? They are the same!

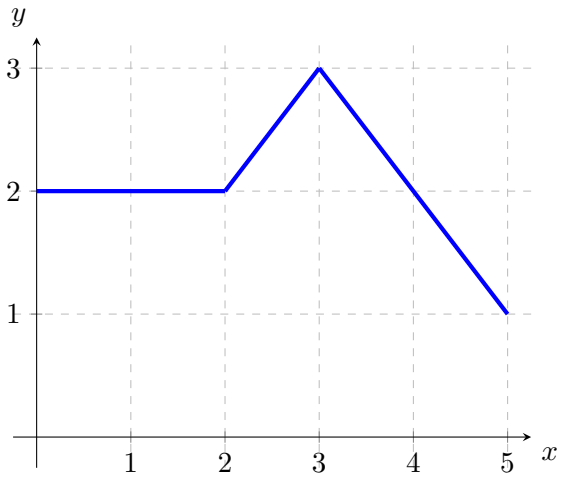
   b) Let $f(x)$ be the function with the given graph to the right and let $A(x) = \int_0^x f(x)\,dx$ be its area function. Compute the following.

   $A(1) = 2$ $\qquad$ $A'(1) = 2$

   $A(2) = 4$ $\qquad$ $A'(2) = 2$

   $A(3) = 6.5$ $\qquad$ $A'(3) = 2.5$

   $A(4) = 9$ $\qquad$ $A'(4) = 2$

   The maximum value of $A(x)$ on the interval $[0, 5]$ is 10.5.

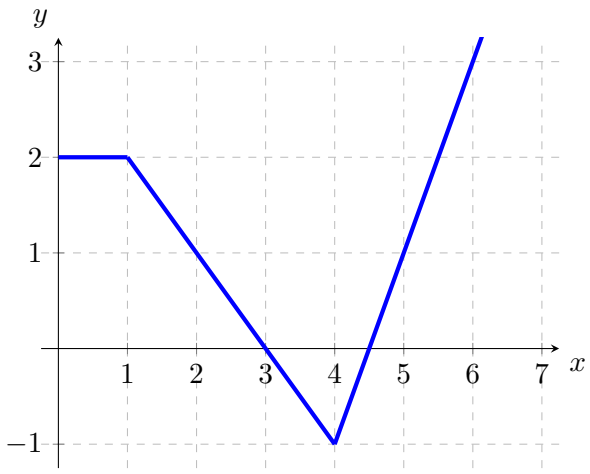
   The maximum value of $A'(x)$ on the interval $[0, 5]$ is 2.5.

2. **Velocity and Position:** A toy car is travelling on a stright track. Its velocity $v(t)$, in m/s, is given by the graph to the right. Define $s(t)$ to be the position of the car in meters. Choose coordinates so that $s(0) = 0$. Compute the following.

$s(2) = 3.5$ $\quad$ $v(2) = 1$

$s(4) = 3.5$ $\quad$ $v(4) = -1$

$s(6) = 5.5$ $\quad$ $v(6) = 3$



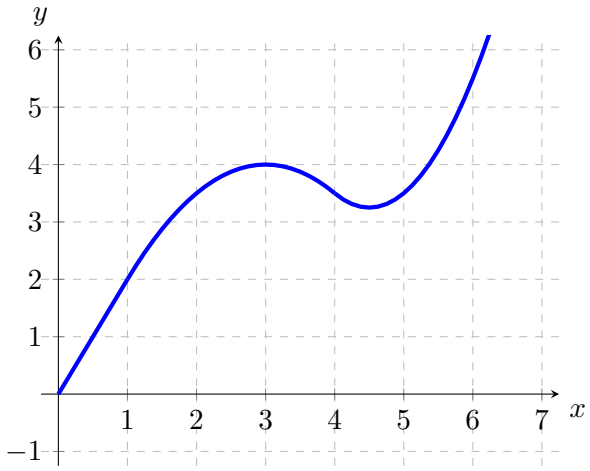
(a) What is the maximum value of $s(t)$ on the interval $[0, 6]$? 5.5

(b) What is the minimum value of $s(t)$ on the interval $[0, 6]$? 0

(c) What is the maximum value of $v(t)$ on the interval $[0, 6]$? 3

(d) What is the minimum value of $v(t)$ on the interval $[0, 6]$? -1

Sketch a graph of $s(t)$ on the empty graph to the right.



(a) What is the net change in position from $t = 1$ to $t = 3$? 2

(b) What is area under the velocity graph from $t = 1$ to $t = 3$? 2

(c) What is the net change in position from $t = 4$ to $t = 6$? 2

(d) Use the Fundamental theorem of calculus to compute the integral $\int_0^6 v(t)\,dt$. 2

3. **Extra Problems:** Use the Fundamental Theorem of Calculus to compute the definite integrals:

a) $\int_0^2 2x - 3\,dx = -2$

b) $\int_0^1 6e^{-3x} + 4\,dx = 6 - 2e^{-3}$

c) $\int_0^{\pi/4} \sec(t)\tan(t)\,dt = \sqrt{2} - 1$

d) $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx = \pi/2$