Daily Prep Assignment for March 11th

Overview

In section 10.7 we will learn how to find the maximum and minimum values of a function of several variables. The main ideas are the same as they were in single variable calculus but we will need to be more careful. We will define *critical* points and discuss the higher dimension version of the second derivative test.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Understand that at a maximum or minimum the tangent plane is horizontal so all partial derivatives are zero.
- Know the definition of a *critical point* and how they relate to maxima and minima values.
- Understand how concavity can tell us if a critical point is a max or min.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with practice**.

- Identify all critical points on a function of several variables.
- Give examples of critical points that are not maxima or minima.
- Use the Hessian matrix and the second derivative test to identify critical points.

To prepare for class

Preview activities: Read the example preview activity solution on the course website then,

- Preview activity 10.7.1

Reading:

- read section 10.7
- Also, if you do not watch the section overview, watch this 7 min video explaining the second derivative test: `https://youtu.be/8jHvMyooxSQ`

Watching: Watch these additional resources if you need support reading the text.

1. Overview of extra 10.7: `https://youtu.be/xpN4QnQwItU`

During and after class

- Activity 10.7.2
- Activity 10.7.3
- Activity 10.7.4
- Activity 10.7.5
- Activity 10.7.6
- This activity is about understanding the second derivative test in higher dimensions.

  a) The function $f(x, y, z) = xy + xz + 2yz - 1/x$ has a critical point at $(1, -1/2, -1/2)$. Explain why graphing this function is not a very successful way of determining what type or critical point this is.

  b) The Hessian matrix for a function of three variables is a 3×3 matrix of all the second order partials

  $$\begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix}$$

  Compute the Hessian for the function $f(x, y, z) = xy + xz + 2yz - 1/x$.

  c) Find the three eigenvalues of this matrix at our critical point.

  d) If our intuition from the two variable case carries over, what kind of critical point do you think $(1, -1/2, -1/2)$ is?

- This activity is about understanding the importance of conditions in the extreme value theorem.

  Extreme Value Theorem: Let $f = f(x, y)$ be a continuous function on a closed and bounded region R. Then f has an absolute maximum and an absolute minimum in R.

  1. Give an example of a function $f(x, y)$ and a region R where the region is closed but not bounded and no absolute maximum is achieved on R.
  2. Give an example of a function $g(x, y)$ and a region that is bounded but not closed and no absolute maximum is achieved on R.