HW 22: Taylor polynomials

Due: Mon Thursday, December 5th in SQRC by 9pm

Learning Goals:

- Investigate the concept of interval of convergence.
- Use the ratio test to compute intervals of convergence.

Questions:

1. (a) Suppose we wanted to calculate $\ln(5)$ to 7 decimal places. An obvious place to start is with the Taylor series centered at $x = 0$ for $\ln(1 - x)$:

$$\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots\right)$$

What happens when you set $x = -4$? Why is that happening? Try a few more values for x and see if you can guess what the interval of convergence for this Taylor series is.

   (b) Explain how you could use the fact that $\ln(1/A) = -\ln(A)$ for any real number $A > 0$ to approximate $\ln(x)$ for any $x > 2$. Use this to compute $\ln(5)$ to 7 decimals. How far out in the Taylor series did you have to go?

2. (a) Use to Taylor series for e^x based at $x = 0$ to approximate e to 3 decimal places.

   (b) Use to Taylor series for e^x based at $x = 0$ to approximate $\sqrt{e}$ to 3 decimal places.

3. Since $\tan(\pi/4) = 1$ we have $\pi/4 = \arctan(1)$.

   (a) Recall that the Taylor series for $\arctan(x)$ is $\frac{(-1)^n}{2n+1}x^{2n+1}$. Use the ratio test to ratio test to determine the interval of convergence. Remember to check the two endpoints of your interval by hand.

   (b) Use the Taylor series for $\arctan(x)$ based at $x = 0$ to approximate π to 3 decimal places. How far out in the series did you have to go? Why would you expect the series to converge slowly based on your answer to part (a)?

4. (a) Compute the Taylor series for $\frac{1}{2-x}$ based at $x = 0$.

   (b) Use the ratio test to compute the interval of convergence for this Taylor series. Use the graph of $\frac{1}{2-x}$ to explain why your answer should be expected.

5. Use Taylor's Theorem to compute a bound on the maximum error that could occur for the 4th degree Taylor polynomial of $\cos(x)$ centered at $x = 0.5$ on the interval $|x - 0.5| < 0.1$.