

Mathematical Thinking, MATH 3

I want you . . .

1. *to be successful.* My top goal in teaching this course is to see you be successful. By “successful,” I mean that you will **grow** considerably as a learner of mathematics, **demonstrate** deep personal understanding of the main content in the course, **solve** challenging problems independently or with modest support, considerably **improve** your oral and written communication skills.
2. *to be an active, caring, and productive contributor* in our Math 003 learning community.
3. *to improve in your ability to learn independently* through readings, videos, activities, working with peers in class on group activities, and solving challenging problems.
4. *to be willing to struggle.* Struggle - temporary failure - is an important part of learning mathematics. Or anything. Thomas Edison famously said, “I haven’t failed. I’ve found 1000 ways that don’t work.” Part of answering hard questions involves trying and failing. Part of learning involves making mistakes and understanding why they are mistakes. The best kind of struggle is productive struggle: productive struggle pushes us further down the path of understanding and often provides new ideas that are key to making progress. Work to have a growth mind-set and challenge yourself to always get better through your work.
5. *to enjoy the course.* You may have had previous experiences with mathematics that were frustrating to you, ones that made you think that you don’t like mathematics. Especially if that is the case, I want you to have a good experience in this one and to find learning calculus interesting, satisfying, and yes, enjoyable.
6. *to build conceptual understanding* of the topics we study from several perspectives so that you gain greater ability to apply ideas in new settings, deepen your logical reasoning skills, and build an appreciation for the beautiful connections present in the mathematics. Throughout the course, I urge you to ask yourself questions like “What is the big idea here?,” “How does this connect to my life?,” “Do the concepts we are studying make sense to me?”
7. *to grow in your written and oral communication skills.* Through regularly assigned work throughout the course, you will be expected to communicate your thinking in writing. In addition, through opportunities to discuss material with your peers and contribute to whole-class discussions, you will have the chance to grow in your oral communication of mathematical ideas.
8. *to be successful in the course.* Again, this is my top goal for the course. By “successful” I mean that I want you to develop deep, personal understanding of fundamental concepts so that you can demonstrate this understanding on written assignments, verbally in class, and in your own projects. In order to help you reach this level of success, expect that I will consistently push you to work hard and challenge you with difficult ideas. Ultimately, I want you to be able to describe yourself - for your performance in this course - as a creative and independent problem solver, an effective communicator, and a mathematical thinker.

Course Website: <https://lucasvanmeter.github.io/classes/math3/index.html>

What is mathematical thinking?

“You probably already *are* doing math, even if you don’t call it that. Math is woven into the way we reason. And math makes you better at things. Knowing mathematics is like wearing a pair of X-ray specs that reveal hidden structures underneath the messy and chaotic surface of the world... With the tools of mathematics in hand, you can understand the world in a deeper, sounder, and more meaningful way.” -*How not to be wrong*, by Jordan Ellenberg

It is hard to say exactly what *math is*, but roughly speaking it is a collection of tools and a way of thinking. It is especially useful at detecting patterns and transforming questions that at first seem complex into ones that are understandable with definite answers. Many of these tools can be described as *problem solving techniques*:

- using experimentation and play to uncover patterns or structure;
- abstracting ideas and observations;
- exploring all possible cases/scenarios;
- generalizing patterns;
- breaking a difficult problem into one or more smaller, easier ones;
- Look at extreme examples (the largest number, the smallest number, etc).
- Work backwards.
- Draw a picture or figure.
- the power of a mathematical proof; In other words being very, very careful with your logic.
- Add an auxiliary element and/or find an invariant.

Course Learning Objectives

My goals in this course are for you:

1. to develop a deeper understanding of what we mean by mathematical thinking;
2. to enhance your knowledge and appreciation of a few beautiful and profound ideas in mathematics;
3. to develop your skills in mathematical thinking and writing;
4. to become a sharper, more creative and capable thinker when analyzing issues and problems both inside and outside of mathematics.

Course Structure

Each week we will focus on a different topic in mathematics. These topics will range from logic to mathematical art. Before each class I will provide a short prompt or inspiration to get you started thinking about the topic. In class we will delve deeper and practice our own mathematical thinking. See the course website for a calendar of topics.

Outside of class I will be assigning a variety of tasks, some will ask you to put some mathematical tools to use and others will ask you to reflect on math in our society or your own life. These assignments will be due on Fridays.

Over the course of the semester I will ask you to complete three projects. The two projects will be smaller and the last one will be larger with a presentation to the class. You can choose the topics of these projects from a list of ideas I provide, as a continuation of something we did in class, or something else we agree on.

Building a portfolio

The guiding goal for this course will be to build a portfolio that demonstrates evidence of meeting the course learning objectives. Your portfolio will contain a variety of things including:

- After discussing a topic in class a short 1-2 page essay reflecting on the topic or summarizing an important part of the topic.
- Work done during class investigating a topic. This will be a surprisingly varied set ranging from origami to thoughtful peer feedback.
- Solutions to mathematical puzzles and riddles that demonstrate the uses of mathematical thinking tools.
- Reflections on how you view yourself as a mathematician and how math fits into your life and society.
- Two small projects and a major project completed over the course of the semester on topics of your choosing.

Contract Grading

Instead of a usual grading system we will be using a *contract grading* system. At the start of the semester we will agree on a contract that describes the type and quality of work a student must do to earn certain grades. At the end of the semester you will evaluate how well you met these criterion and thus, what grade you earned in the course. You will use your portfolio, the assignments and feedback given throughout the course, as evidence in this final evaluation. I will probably agree with you, because you know yourself, your work, and your understanding. If I substantially disagree with your self evaluation we will talk about it.

Letter Grade	Qualitative Description	Measurable outcomes
A		
B		
C		
D		
E (okay, F)		

Collaboration and Academic Integrity

Collaboration is an important part of learning mathematics and I strongly encourage you collaborate with your classmate. In order to get the most out of each assignment the expectation for each group is that each member will

be involved with solving every question on an assignment and each person will write up their own submission. One good strategy for this is to do scratch work “solving” the problems while working with your group, but then write up the copy to hand in on your own. If you find yourself asking a group mate a question and then immediately writing down what they say verbatim you probably did not process the answer. In general, work that is done by copying another source does not show evidence of your own learning.

Academic Accommodations

If you believe you need accommodations for a disability or a chronic medical condition, please contact Student Disability Services via email at studentdisabilityservices@swarthmore.edu to arrange an appointment to discuss your needs. As appropriate, the office will issue students with documented disabilities or medical conditions a formal Accommodations Letter. Since accommodations require early planning and are not retroactive, please contact Student Disability Services as soon as possible. For details about the accommodations process, [visit the Student Disability Services website](#). You are also welcome to contact me privately to discuss your academic needs. However, all disability-related accommodations must be arranged, in advance, through Student Disability Services.