Problem Set 3

Due: Monday, February 3rd

Instructions: Answer each of the following questions and provide a justification for your answer. In addition to the points assigned below, you will receive 0-2 writing points for the entire problem set.

1. A grocery store in a small town has 600 lottery tickets for sale. Since the town only has a population of 1200 people, the store has stated that no person may get more than 1 ticket.

   (a) How may ways are there to distribute all the tickets among the town residents? Explain your reasoning.

   (b) Suppose the store changes its policy so that no person may get more then 2 tickets. How many ways are there to distribute all the tickets. (hint: you might want to use summation notation in your answer).

2. Using a counting argument prove that

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$

   Please remember the three parts of writing up a counting argument: (a) a clear statement of the counting task being undertaken, (b) a counting argument that obtains the expression on the left hand side of the equation, and (c) a counting argument that obtains the expression on the right hand side of the equation.

3. (a) Prove that the sum of two even numbers is even.

   (b) Prove that the sum of two odd numbers is even.

   (c) What can you say about the sum of an even and an odd number? Prove it!

   (d) What can you say about the product of an even and odd number? Prove it!

   (e) What can you say about the product of two even numbers? Prove it!

   (f) What can you say about the product of two odd numbers? Prove it!

4. What is wrong with the following proof?

   Proposition: Let a, b, c be integers. If $a|b$ and $c|b$ then $a|b-c$.

   Proof: If $a|b$ then there is an integer q such that $b = aq$ and if $a|c$ then there is an integer q such that $c = aq$. Then $b - c = aq - aq$, and thus $a|b-c$.

5. (a) Let a and b be positive integers. Prove that if $a|b$ and $b|a$ then $a = b$.

   (b) If a, b and c are integers where $a|b$, $b|c$, and $c|a$, what can you conclude about a, b and c? Justify your answer.