Notes: Monday, February 3rd

Today, we continued working on definitions of criteria for voting methods for more than two candidates. We focused mainly on monotonicity.

We continued working with two potential definitions of monotonicity proposed last week: "**add monotonicity**," which has to do with adding an additional first place vote for a candidate, and "**up monotonicity**," which has to do with changing one voter's preference list to increase the ranking of a candidate.

We reviewed some of our proofs and counterexamples from last week concerning whether different methods satisfy either or both of these criteria. These results and those from the rest of the day are given below:

Method	**Add monotone?**	**Up monotone?**
Plurality	Yes	Yes
Borda	Yes	Yes
Copeland		Yes
Instant runoff		No
Coomb's	No	No
Sequential pairwise	Yes	
Dictatorship	Yes	Yes

We also continued to think about whether these definitions are equivalent, or whether there is some voting method which satisfies one but not the other. Eric gave the following example, which is up monotone, but not add monotone:

Example: We have two candidates: A and B. If the total number of voters is even, the outcome is A>B. If instead the number of voters is odd, the outcome is B>A. This method is up monotone: for a fixed number of voters, the outcome is determined, so it won't change if a voter's vote is changed. However, this method is not add monotone. If there are 4 voters, A wins. If we add a vote for A, there are 5 voters, so B wins.

We then worked on determining whether different methods satisfy either of these two definitions. Lucas gave this example, which shows that instant runoff is not up monotone:

Example: We have three candidates: A, B, and C.

11	8	8
A	B	C
B	C	A
C	A	B

Here, A wins.

12	7	8
A	B	C
B	C	A
C	A	B

A moved higher in one voter's vote, but now C wins.

Emily gave the following example, which shows that Coomb's method is not up monotone:

Example: We have three candidates: A, B, and C.

10	9	10
C	A	A
B	C	B
A	B	C

Here, B wins.

10	8	1	10
C	A	B	A
B	C	A	B
A	B	C	C

B moved higher in one voter's vote, but now A wins.

Eric gave this example, which shows that Coomb's method is also not add monotone:

Example: We have three candidates: A, B, and C.

10	11	11
C	B	C
B	A	A
A	C	B

Here, A wins

10	11	11	1
C	B	C	A
B	A	A	B
A	C	B	C

A received an additional first place vote, but now B wins.

Austin suggested that sequential pairwise is add monotone, and gave an idea of a proof. The idea was that gaining a first place vote can't cause a candidate to lose a previously won/tied pairwise race or to tie a previously won pairwise race. They then won't be eliminated any sooner from the race, and so their overall ranking won't decrease.