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# ***SYSC 4101 / 5105***

## **Graph Criteria (Part I)**

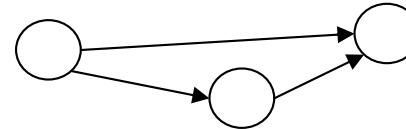
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## ***Test Criteria Based on Structure [Offutt]***

- **Graphs**

Method body  
Methods and calls  
Components interactions  
State and transitions  
...

- Logical Expressions



[not X or not Y] and A and B

A: {0,1,>1}

B: {600,700,800}

C: {swe,cs,isa,ifs}

if [x>y]

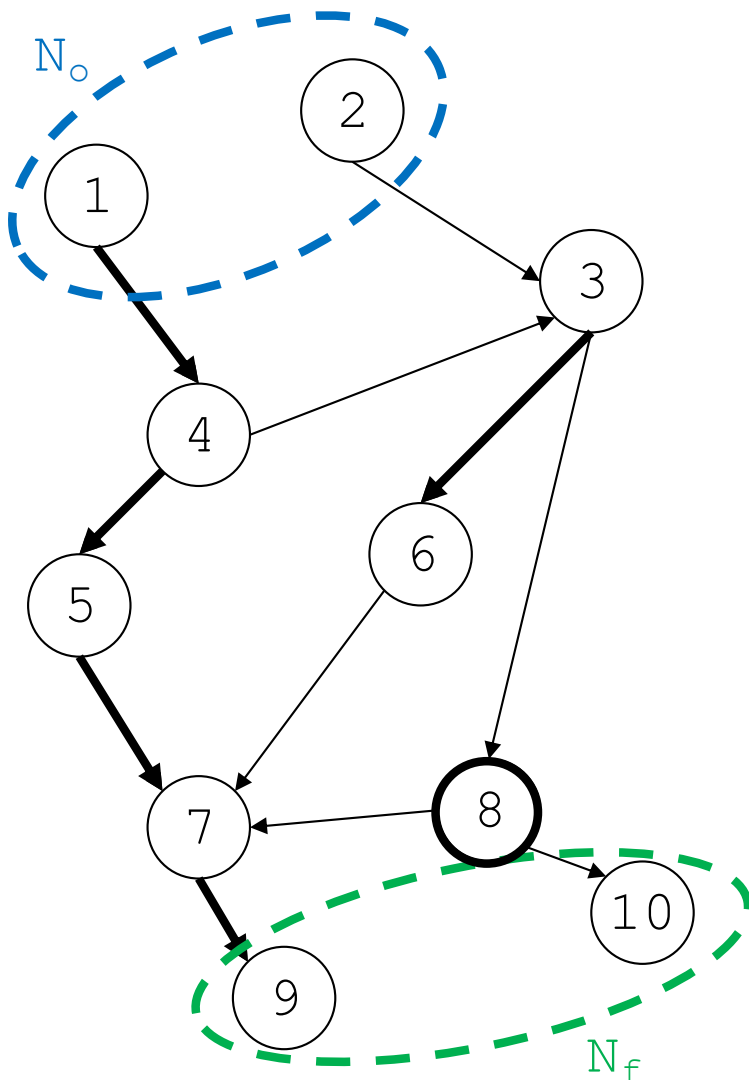
z = x - y;

else

z = 2 \* x

- Syntactic Structures

# Graph Principles



- A **directed graph** is defined by:
  - a set  $N$  of **nodes**
  - a set  $N_o$  of **initial nodes**, where  $N_o \subseteq N$ :  $|N_o| \geq 1$
  - a set  $N_f$  of **final nodes**, where  $N_f \subseteq N$ :  $|N_f| \geq 1$
  - a set  $E$  of **edges**, where  $E \subseteq N \times N$
- **Path** = a sequence  $[n_1, n_2, \dots, n_M]$  of nodes, where each pair of adjacent nodes  $[n_i, n_{i+1}]$ ,  $1 \leq i < M-1$ , is in  $E$ .
  - $[n]$  is a path of length zero
  - Two nodes may share several edges, in which case a path definition needs to indicate edges and nodes
- The **length** of path  $p$  is the number of its edges.
- A **sub-path** of path  $p$  is a subsequence of  $p$  (possibly  $p$  itself).
  - $[n_2, n_3]$  is a sub-path of  $[n_2, n_3, n_6, n_7]$
  - $[n_1, n_4, n_5]$  is a sub-path of  $[n_1, n_4, n_5, n_7, n_9]$
  - $[n_4]$  is a sub-path of  $[n_1, n_4, n_5]$

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## ***Graph Testing—Criteria***

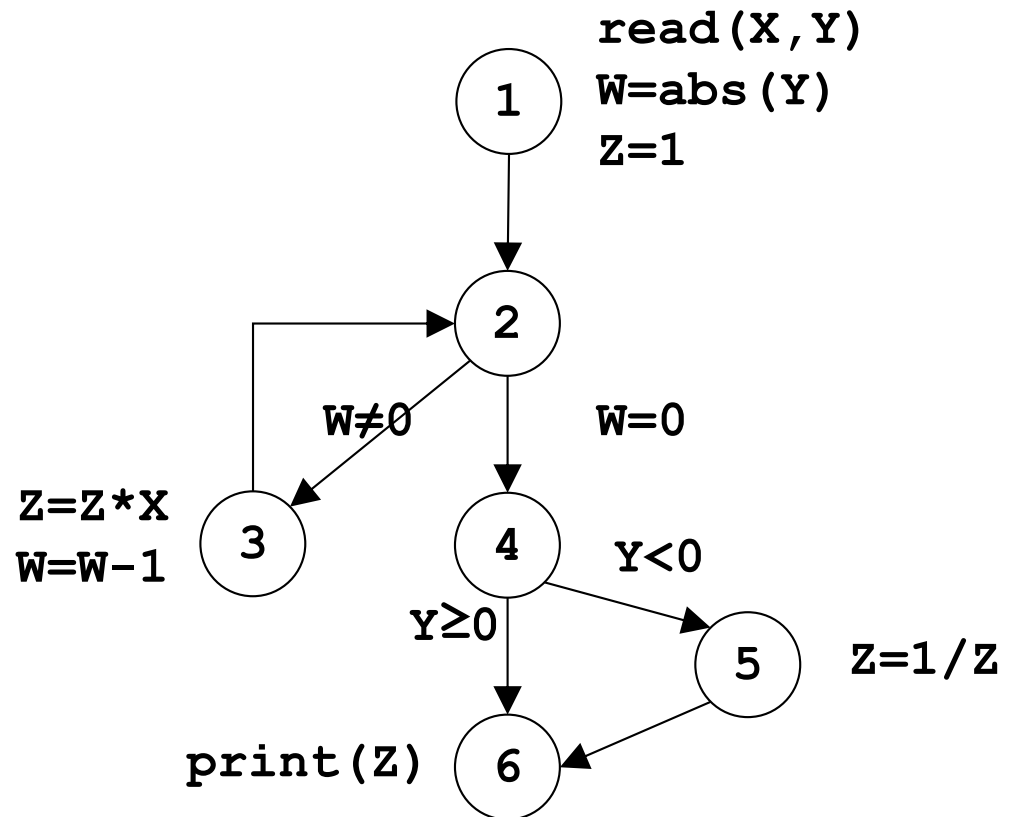
- Graph criteria define a set of test requirements to be achieved
  - These requirements are stated in terms of elements of the graph to be exercised.
  - The requirements then translate into paths from an initial to a final node [to exercise the required elements]: a.k.a. **test paths**.
  - The paths then become test cases: one identifies (inputs) values to execute the path.
- Graph criteria, therefore, consider the syntax of the graph
  - Not its semantics!
- A criterion may result in infeasible objectives (and therefore paths)
  - See example next.
- Reachability:
  - Node  $n$  [or edge  $e$ ] is syntactically reachable from node  $n_i$  if there exists a path from node  $n_i$  to  $n$  [or  $e$ ]
  - Node  $n$  [or edge  $e$ ] is also semantically reachable if it is possible to execute at least one of the paths with some input.

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## Example: Power Function

Program computing  $Z=X^Y$

```
BEGIN
  read (X, Y) ;
  W = abs (Y) ;
  Z = 1 ;
  WHILE (W <> 0) DO
    Z = Z * X ;
    W = W - 1 ;
  END
  IF (Y < 0) THEN
    Z = 1 / Z ;
  END
  print (Z) ;
END
```



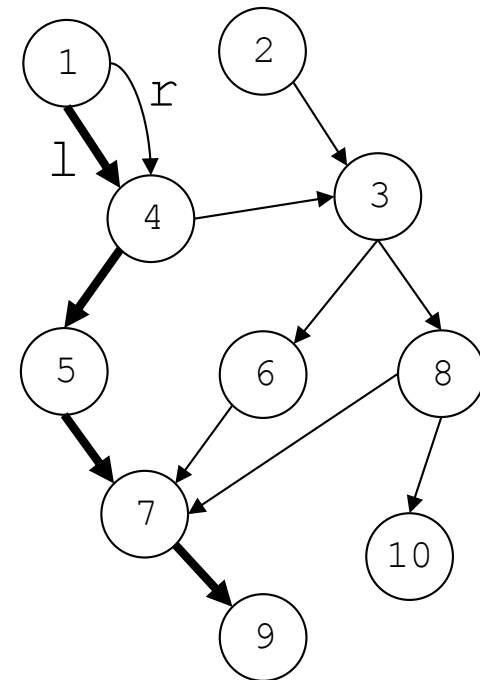
Test requirement [1,2,4,5] is:

- Syntactically feasible/reachable: path [1,2,4,5,6] is possible in the graph.
- Semantically infeasible/unreachable: need to satisfy  $y=0$  and  $y<0$ .

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# Graph Testing—Terminology

- Test Path
  - A path  $p$ , possibly of length zero, that starts at some node in  $N_0$  and ends at some node in  $N_f$ .
    - A complete traversal of the graph, from an initial node to a final node.
  - A test path  $p$  **visits** node  $n$  [resp. edge  $e$ ] if  $n$  [resp.  $e$ ] is in  $p$ .
  - A test path  $p$  **tours** sub-path  $q$  if  $q$  is a sub-path of  $p$ .
- Unless the system is non-deterministic, a test case executes one test path.
  - $\text{path}[T]$  is the set of test paths executed by test set  $T$ .



Path [1, l, 4, 5, 7, 9]:

- Visits node 5
- Visits edge (7,9)
- Tours path [5, 7, 9]

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## ***Graph Testing—Two Families of Criteria***

- **Control Flow Criteria**

- Only consider the flow of nodes and edges
- Seven criteria (many more criteria exist)

- **Data Flow Criteria**

- Considers the definitions and usages of data along paths
- Three criteria (many more criteria exist)

Reminder: a criterion can be an selection criterion or a coverage criterion.

- One can use selection control flow criterion A and coverage control flow criterion B
- One can use selection control flow criterion A and coverage data flow criterion B
- One can use selection data flow criterion A and coverage control flow criterion B
- ...

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## ***Graph Testing—Control Flow Criteria [textbook]***

- Graph Criteria (generic/template definition)
  - Given a set TR of test requirements for a graph criterion C, a test set T satisfies C on graph G if and only if for every test requirement tr in TR, there is at least one test path p in path[T] such that p meets tr.
    - Overall, all the tr in TR are exercised by paths in path[T], the paths exercised by T.
  - **Recall the procedure:**  
**Criterion → Test Requirements (paths) → Test Paths → Test inputs**
- All-Nodes Criterion [NC]:
  - TR contains each reachable node in G.
- All-Edges Criterion [EC]:
  - TR contains each reachable [sub-]path of length up to 1, inclusive, in G.
    - Length of 0 or 1, i.e., nodes and edges. (All-edges subsumes All-Nodes by construction.)
- All-Edge-Pairs Criterion [EPC]:
  - TR contains each reachable path of length up to 2, inclusive, in G.
    - Length of 0, 1, or 2, i.e., nodes, edges and pairs of consecutive edges. (All-Edge-Pairs subsumes All-Edges by construction.)



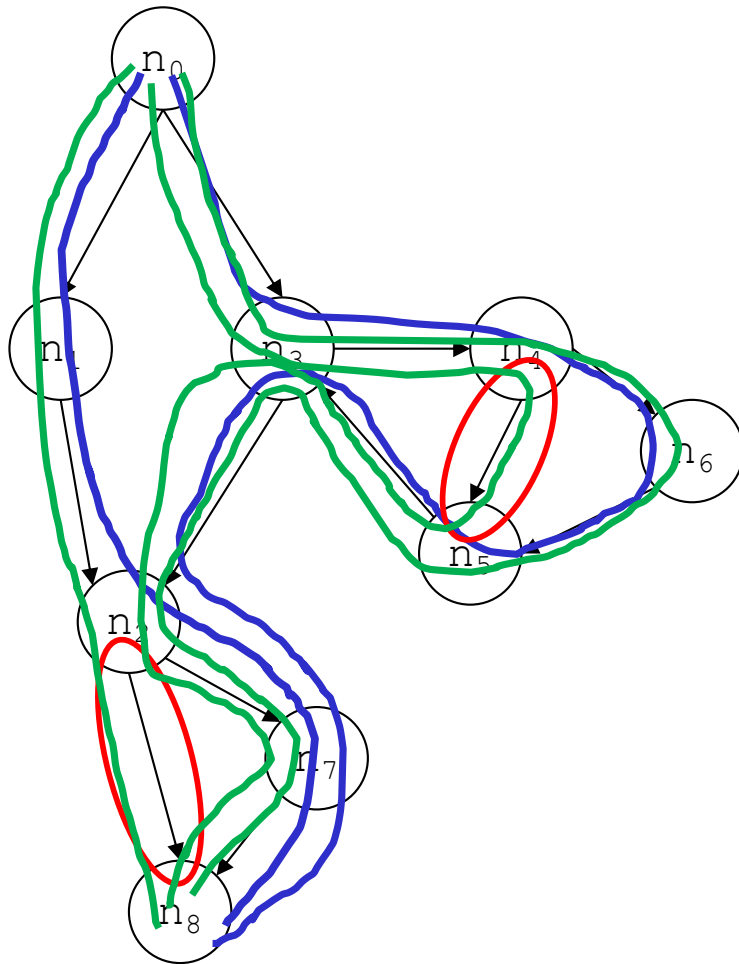
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## ***Graph Testing—Control Flow Criteria [2]***

- **Prime-Path Criterion [PPC]:** (textbook illustrates an algorithm to find them)
  - TR contains each prime path in  $G$ .
  - A path from  $n_i$  to  $n_j$  is a prime path if it is a simple path and it does not appear as a proper sub-path of any other simple path
  - A path from  $n_i$  to  $n_j$  is simple if no node appears more than once in the path.
    - Exception: the first and last nodes of the path may be identical (the sub-path is a loop).
  - A prime path is a simple path of maximum length.
- **Simple-Round-Trip Criterion [SRTC]:**
  - TR contains at least one round-trip path for each reachable node in  $G$  that begins and ends a round-trip path.
  - A round-trip path is a nonzero length prime path that starts and ends at the same node, i.e., a loop with at least one edge.
- **Complete-Round-Trip Criterion [CRTC]:**
  - TR contains all round-trip paths for each reachable node in  $G$ .
- **Complete-Path Criterion [CPC]:**
  - TR contains all paths in  $G$ .

## Example

Each time, we list the set of test requirements and then test paths that are adequate for the criterion.



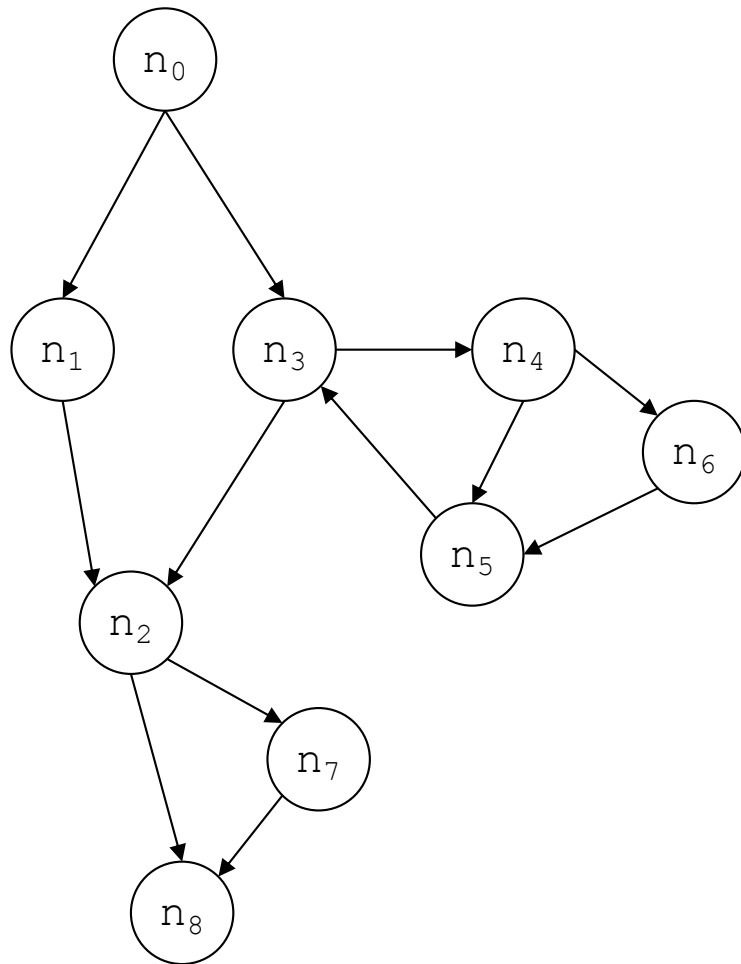
- All-Nodes Criterion

- Test requirements:  
[n<sub>0</sub>], [n<sub>1</sub>], [n<sub>2</sub>], [n<sub>3</sub>], [n<sub>4</sub>], [n<sub>5</sub>], [n<sub>6</sub>], [n<sub>7</sub>], [n<sub>8</sub>]
- path[t<sub>1</sub>]=[n<sub>0</sub>,n<sub>1</sub>,n<sub>2</sub>,n<sub>7</sub>,n<sub>8</sub>]
- path[t<sub>2</sub>]=[n<sub>0</sub>,n<sub>3</sub>,n<sub>4</sub>,n<sub>6</sub>,n<sub>5</sub>,n<sub>3</sub>,n<sub>2</sub>,n<sub>7</sub>,n<sub>8</sub>]
- T<sub>NC</sub>={t<sub>1</sub>,t<sub>2</sub>} satisfies the All-Nodes criterion
  - Many other possibilities...

- All-Edges Criterion

- Test requirements  
[n<sub>0</sub>], [n<sub>1</sub>], [n<sub>2</sub>], [n<sub>3</sub>], [n<sub>4</sub>], [n<sub>5</sub>], [n<sub>6</sub>], [n<sub>7</sub>], [n<sub>8</sub>]  
[n<sub>0</sub>,n<sub>1</sub>] [n<sub>0</sub>,n<sub>3</sub>] [n<sub>1</sub>,n<sub>2</sub>] [n<sub>2</sub>,n<sub>7</sub>] [n<sub>2</sub>,n<sub>8</sub>]  
[n<sub>3</sub>,n<sub>4</sub>] [n<sub>3</sub>,n<sub>2</sub>] [n<sub>4</sub>,n<sub>5</sub>] [n<sub>4</sub>,n<sub>6</sub>] [n<sub>5</sub>,n<sub>3</sub>]  
[n<sub>6</sub>,n<sub>5</sub>] [n<sub>7</sub>,n<sub>8</sub>]
- path[t<sub>3</sub>]=[n<sub>0</sub>,n<sub>1</sub>,n<sub>2</sub>,n<sub>8</sub>]
- path[t<sub>2</sub>]=[n<sub>0</sub>,n<sub>3</sub>,n<sub>4</sub>,n<sub>6</sub>,n<sub>5</sub>,n<sub>3</sub>,n<sub>2</sub>,n<sub>7</sub>,n<sub>8</sub>]
- path[t<sub>4</sub>]=[n<sub>0</sub>,n<sub>3</sub>,n<sub>4</sub>,n<sub>5</sub>,n<sub>3</sub>,n<sub>2</sub>,n<sub>7</sub>,n<sub>8</sub>]
- T<sub>EC</sub>={t<sub>2</sub>,t<sub>3</sub>,t<sub>4</sub>} satisfies the All-Edges criterion

## Example [cont.]



- All-Edge-Pairs Criterion

- Test requirements

$[n_0], [n_1], [n_2], [n_3], [n_4], [n_5], [n_6], [n_7], [n_8]$

$[n_0, n_1] \quad [n_0, n_3] \quad [n_1, n_2] \quad [n_2, n_7] \quad [n_2, n_8]$

$[n_3, n_4] \quad [n_3, n_2] \quad [n_4, n_5] \quad [n_4, n_6] \quad [n_5, n_3]$

$[n_6, n_5] \quad [n_7, n_8]$

$[n_0, n_1, n_2] \quad [n_0, n_3, n_2] \quad [n_0, n_3, n_4]$

$[n_1, n_2, n_7] \quad [n_1, n_2, n_8] \quad [n_2, n_7, n_8]$

$[n_3, n_4, n_6] \quad [n_3, n_4, n_5] \quad [n_3, n_2, n_7] \quad [n_3, n_2, n_8]$

$[n_4, n_5, n_3] \quad [n_4, n_6, n_5] \quad [n_5, n_3, n_2] \quad [n_5, n_3, n_4]$

$[n_6, n_5, n_3]$

- $\text{path}[t_1] = [n_0, n_1, n_2, n_7, n_8]$

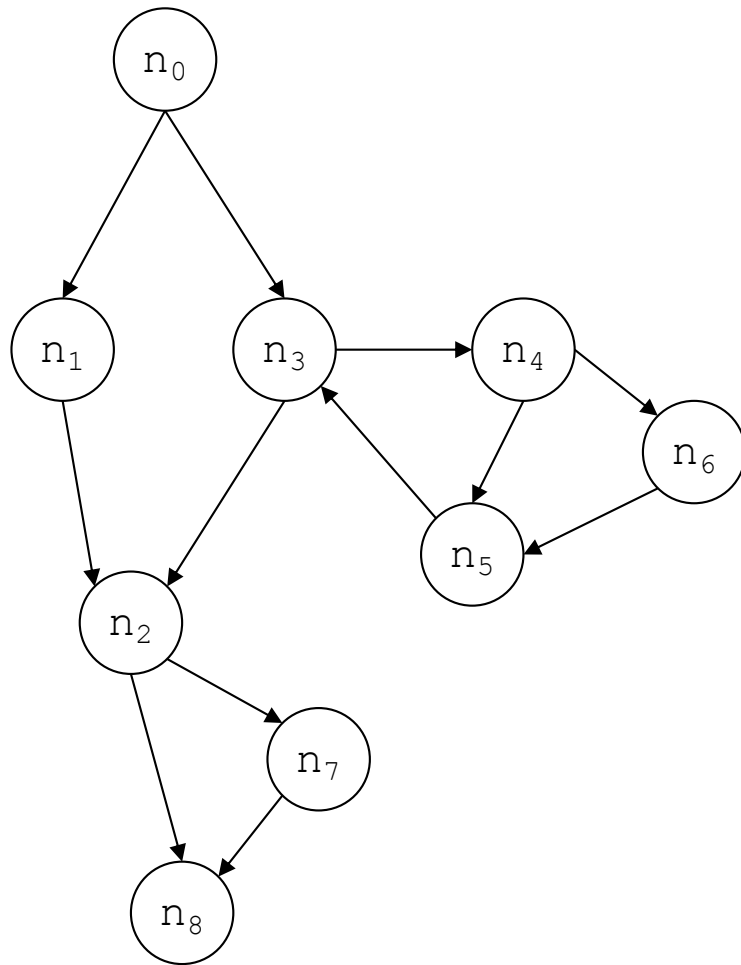
- $\text{path}[t_3] = [n_0, n_1, n_2, n_8]$

- $\text{path}[t_5] = [n_0, n_3, n_2, n_7, n_8]$

- $\text{path}[t_6] = [n_0, n_3, n_4, n_6, n_5, n_3, n_4, n_5, n_3, n_2, n_8]$

- $T_{\text{EPC}} = \{t_1, t_3, t_5, t_6\}$  satisfies the All-Edge-Pairs criterion

## Example [cont.]



- Prime-Path Criterion

- Test requirements

$[n_0, n_1, n_2, n_8]$	$[n_0, n_1, n_2, n_7, n_8]$	$[n_0, n_3, n_2, n_8]$
$[n_0, n_3, n_2, n_7, n_8]$	$[n_0, n_3, n_4, n_5]$	$[n_0, n_3, n_4, n_6, n_5]$
$[n_3, n_4, n_5, n_3]$	$[n_3, n_4, n_6, n_5, n_3]$	
$[n_4, n_5, n_3, n_2, n_8]$	$[n_4, n_5, n_3, n_2, n_7, n_8]$	$[n_4, n_6, n_5, n_3, n_4]$
$[n_4, n_6, n_5, n_3, n_2, n_8]$	$[n_4, n_6, n_5, n_3, n_2, n_7, n_8]$	$[n_4, n_5, n_3, n_4]$
$[n_5, n_3, n_4, n_5]$	$[n_6, n_5, n_3, n_4, n_6]$	$[n_5, n_3, n_4, n_6, n_5]$

- $\text{path}[t_1] = [n_0, n_1, n_2, n_7, n_8]$

- $\text{path}[t_3] = [n_0, n_1, n_2, n_8]$

- $\text{path}[t_5] = [n_0, n_3, n_2, n_7, n_8]$

- $\text{path}[t_6] = [n_0, n_3, n_4, n_6, n_5, n_3, n_4, n_5, n_3, n_2, n_8]$

- $\text{path}[t_7] = [n_0, n_3, n_2, n_8]$

- $\text{path}[t_8] = [n_0, n_3, n_4, n_5, n_3, n_2, n_7, n_8]$

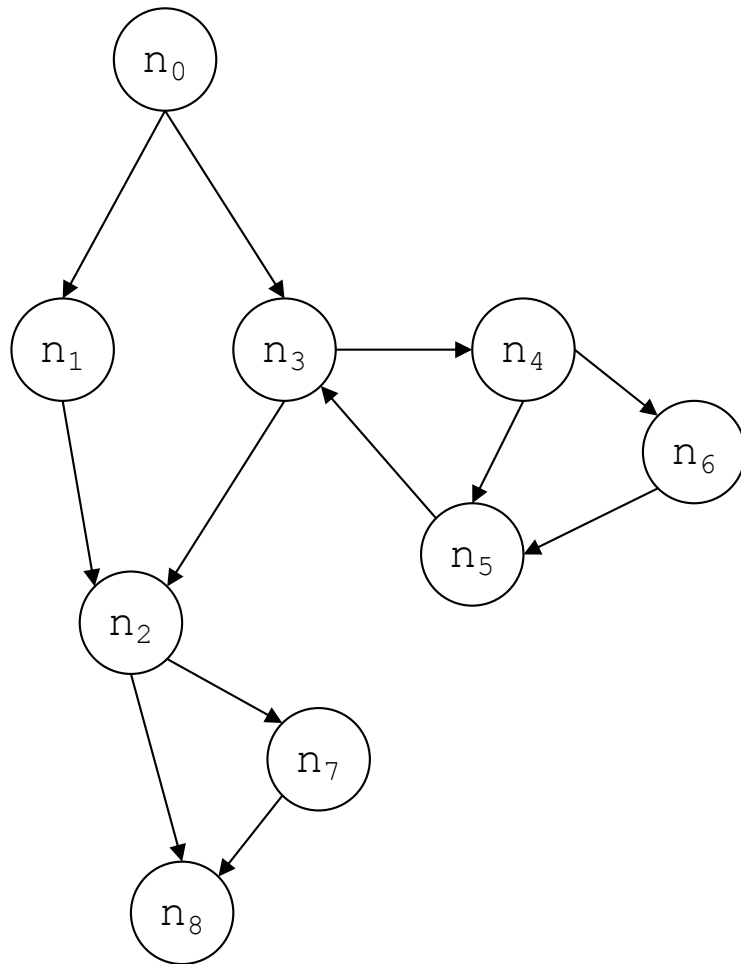
- $\text{path}[t_9] = [n_0, n_3, n_4, n_6, n_5, n_3, n_2, n_7, n_8]$

- $\text{path}[t_{10}] = [n_0, n_3, n_4, n_6, n_5, n_3, n_2, n_8]$

- $\text{path}[t_{11}] = [n_0, n_3, n_4, n_5, n_3, n_4, n_6, n_5, n_3, n_4, n_6, n_5, n_3, n_2, n_8]$

- $T_{\text{PPC}} = \{t_1, t_3, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}\}$  satisfies the Prime-Path criterion

## Example [cont.]



- Simple-Round-Trip Criterion

- Test requirements

$[n_3, n_4, n_5, n_3]$        $[n_3, n_4, n_6, n_5, n_3]$  (one of these 2)

$[n_4, n_5, n_3, n_4]$        $[n_4, n_6, n_5, n_3, n_4]$  (one of these 2)

$[n_5, n_3, n_4, n_5]$        $[n_5, n_3, n_4, n_6, n_5]$  (one of these 2)

$[n_6, n_5, n_3, n_4, n_6]$

- $\text{path}[t_{12}] = [n_0, n_3, n_4, n_5, n_3, n_4, n_6, n_5, n_3, n_4, n_6, n_5, n_3, n_2, n_7, n_8]$

- Covered requirements are underlined.

- $T_{\text{SRTC}} = \{t_{12}\}$  satisfies the Simple-Round-Trip criterion

- Complete-Round-Trip Criterion

- Test requirements

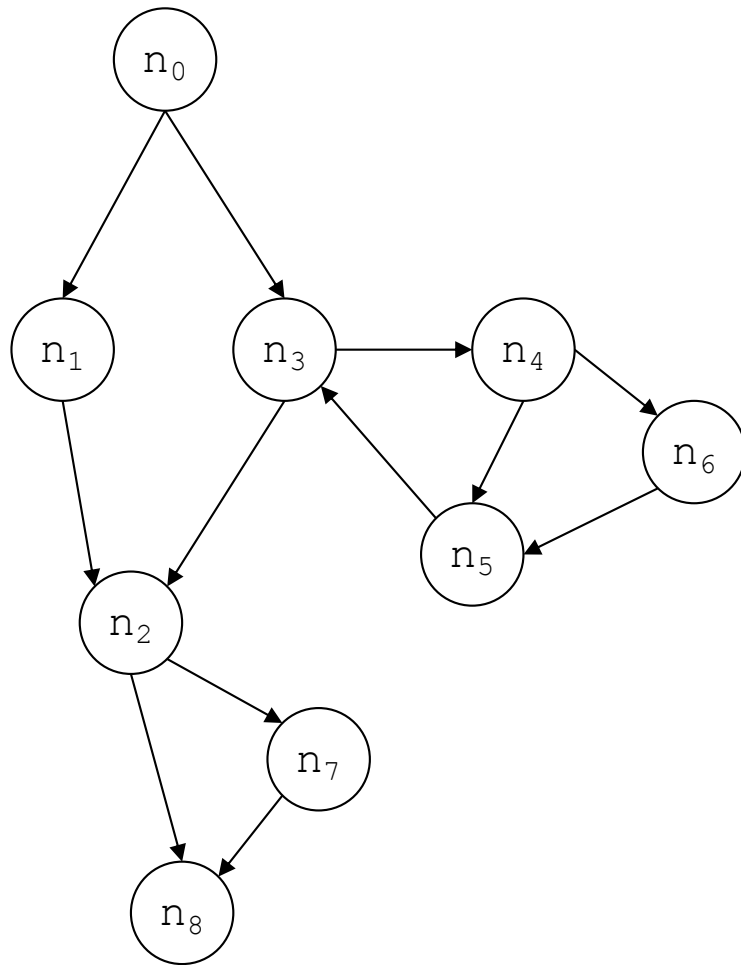
- See above: all the seven round trips must be exercises

- $\text{path}[t_{13}] = [n_0, n_3, n_4, n_5, n_3, n_4, n_6, n_5, n_3, n_4, n_6, n_5, n_3, n_4, n_5, n_3, n_2, n_7, n_8]$

- $T_{\text{SRTC}} = \{t_{13}\}$  satisfies the Complete-Round-Trip criterion

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## Example [cont.]



- Complete Path Coverage

$[n_0, n_1, n_2, n_8]$

$[n_0, n_1, n_2, n_7, n_8]$

$[n_0, n_3, n_2, n_8]$

$[n_0, n_3, n_2, n_7, n_8]$

infinite number of paths involving loops!

- Alternative to Complete Path Coverage when there are loops!

- Specified Path Coverage [SPC]

- TR contains a set S of test paths, where S is supplied as a parameter (judgement call by test engineer).
- For instance, for each loop, S contains
  - a path that bypasses the loop,
  - a path that takes the loop once,
  - a path that takes the loop a reasonable number of times (say, 5).(if these are feasible)

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## ***Example [cont.]***

- $T_{NC}$  does not satisfy the All-Edges Criterion.
  - It misses edges  $[n_2, n_8]$  and  $[n_4, n_5]$ .
- $T_{EC}$  does not satisfy the Edge-Pair Criterion.
  - It misses  $[n_0, n_3, n_2]$ ,  $[n_1, n_2, n_7]$ ,  $[n_3, n_2, n_8]$ ,  $[n_5, n_3, n_4]$ .
- $T_{EPC}$  does not satisfy the Prime-Path Criterion.
  - It misses  $[n_0, n_3, n_2, n_8]$ ,  $[n_0, n_3, n_4, n_5]$ ,  $[n_4, n_5, n_3, n_2, n_7, n_8]$ ,  $[n_4, n_6, n_5, n_3, n_2, n_8]$ ,  $[n_4, n_6, n_5, n_3, n_2, n_7, n_8]$ .
- $T_{PPC}$  does not satisfy the Complete-Path Criterion.
  - It misses many loops involving nodes  $n_3$ ,  $n_4$ ,  $n_5$ , and  $n_6$ .
- The Simple-Round-Trip Criterion and the Complete-Round-Trip Criterion do not even exercise all the transitions
  - But they are useful when testing state machines, which are often connected.
  - Exercising round-trips (SRTC, CRTC) must be combined with another criterion that exercises at least all the edges.
- The Complete-Path Criterion is often not achievable when there are loops.
  - An alternative exists.