Effective Coverage Control using Dynamic Sensor Networks

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***Abstract*— This paper studies the problem of dynamically covering a given region $\mathcal{D}$ in $\mathbb{R}^2$ using a set of N mobile sensor agents. First, a novel problem formulation is proposed that addresses a number of important multi-agent missions. The coverage goal, which is to cover a given search domain using mobile sensors such that each point is surveyed for a certain preset level, is then stated in a mathematically precise problem statement. Control laws are then developed that are guaranteed to meet this goal. The control laws are local in the sense that each agent makes use of information available from a subset of the other agents with which it can communicate. Several numerical examples are provided to illustrate the main ideas.**

I. Introduction

Recent natural disasters such as hurricane Katrina, the earthquake in the Indian subcontinent and the tsunami in Southeastern Asia have indicated that a speedy and efficient humanitarian and search and rescue responses are crucial in saving human lives. Such humanitarian as well as numerous military operations often involve tasks in adversarial, highly dynamic environments that are hazardous to human operators. Hence, there is a pressing need to develop autonomous multi-agent systems that seek to collect and process data from some domain $\mathcal{D} \subset \mathbb{R}^2$ under constrained resources such as time restrictions to mission accomplishment, fuel limitations, and dynamic and/or limited communication structures.

For some special geometries for $\mathcal{D}$, such as rectangular and circular ones, one may attempt to redeploy the agents at particular starting points and systematically sweep the domain. For example, a single agent may spiral from the center of a circular domain until it reaches the boundary, as in [1]. However, for arbitrary geometries, even if convex, such solutions may not be obvious or even exist. Even for circular or rectangular geometries, the application, such as in time-sensitive operations, may not afford a redeployment from an initial arbitrary configuration to an optimal one. In this case, the agents have to immediately respond to a certain application need without redeployment. This is the main goal of the control strategies developed in this paper.

The literature on mobile sensor coverage networks is relatively new due to novel sensor and wireless technologies that only emerged recently. In [2], the authors present a survey of the most recent activities in the control and design of both static and dynamic sensor networks. In their design criteria, they consider issues such as maximum coverage, detection of events and minimum communication energy expenditure. Of particular relevance in this paper is section 4.2 in [2], where the authors focus on coverage control missions. By means of illustrative examples, the authors in [3] discuss some challenges in modeling of robotic networks, motion coordination algorithms, sensing and estimation tasks, and complexity of distributed algorithms.

In [4], the authors consider a probabilistic network model and a density function to represent the frequency of random events taking place over the mission space. The authors develop an optimization problem that aims at maximizing coverage using sensors with limited ranges, while minimizing communication cost. Starting with initial sensor positions, the authors develop a gradient algorithm to converge to a (local) solution to the optimization problem. The sequence of sensor distributions along the solution is seen as a discrete time trajectory of the mobile sensor network until it converges to the local minimum. A similar goal is the focus of the paper [5], where the authors utilize the notion of "virtual forces" to enhance the coverage of a sensor network given an arbitrary initial deployment, and the paper [6], where the goal is to maximize target exposure in surveillance problems.

In [7], the authors address the same question, but instead of converging to a local solution of some optimization problem, the trajectory converges to the centroid of a cell in a Voronoi partition of the search domain. The authors propose stable control laws, in both continuous and discrete time, that converge to the centroids. In [8], the authors use a Voronoi-based polygonal path approach and aim at minimizing exposure of a UAV fleet to radar. Voronoi-based approaches, however, require exhaustive computational effort to compute the Voronoi cells continuously during a real-time implementation of the controllers. In the work presented herein, domain partitioning is not required and, hence, the computational complexity is reduced.

The fundamental difference between our approach and the approaches presented in the above literature is in the definition of *mobility*. In the above, researchers ask: Given an initial, random deployment of the sensor network, what is its final (optimal or suboptimal) configuration and how to control the network to get there? In other words they address the redeployment problem to improve network performance, while assuming mobility of the network in achieving redeployment. In this paper, a different question is addressed: Given a sensor network and mission (or, search) domain $\mathcal{D}$, how should the motion of each sensor agent be controlled such that the entire network surveys $\mathcal{D}$ by sensing each point in $\mathcal{D}$ by an amount of effective coverage equal to C^*? Though our work is currently purely deterministic, the quantity C^* is statistically related to the probability of event detection at each point in $\mathcal{D}$. The precise definition of "effective coverage" will be made in Section II. Hence, the aim is to actively sense the mission domain while the agents are moving in the space, which is applicable to search and rescue, and dynamic surveillance problems. This question has been studied in [9] for (optimal and suboptimal) motion planning of multiple spacecraft interferometric imaging systems (MSIIS). While MSIIS involves a slightly modified version of the coverage problem (in that a "sensor" corresponds to measurements made by a spacecraft *pair* and with inter-spacecraft relative positions and velocities as control variables instead of absolute positions and velocities), it is fundamentally a coverage problem of the type addressed in this paper. In [10], the author proves that the coverage optimization problem (with fuel expenditure as cost) is computationally intractable by mapping it into a traveling salesman problem and, thus, it is necessary to resort to heuristics. Time optimality was proven to be tractable and solved in [10] for a four spacecraft MSIIS.

Methodologies developed in [9], however, inspire the present approach. In this paper, a convergent feedback control law is proposed that achieves the coverage goal. Note that the proposed methodology is specifically developed for scenarios where there is a uniform probability distribution to event occurrence and, hence, there is no preferred fixed sensor configuration that maximizes detection of an event, and each point in the search domain has to be continuously and equally covered. Examples where this occurs include

- Human- or autonomous vehicle-based search and rescue missions, where each point in the search domain has to be equally surveyed.
- Aerial wildfire control in inaccessible and rugged country, where each point in the wildfire region has to be "suppressed" using fixed-wing aircraft or helicopters.
- Surveillance and aerial mapping.
- MSIIS, where each point in the domain (in this case the two-dimensional u-v frequency domain of an image, see [9], [10]) has to be surveyed by the spacecraft fleet.

These applications motivate the approach which is described in the next section.

II. Problem Formulation

In this paper an agent is denoted by $\mathcal{A}$. Let $\mathbb{R}^+ = \{a \in \mathbb{R} : a \geq 0\}$, $\mathsf{Q} = \mathbb{R}^2$ the configuration space of each agent and $\mathcal{D}$ be a compact subset of $\mathbb{R}^2$ which represents a region in $\mathbb{R}^2$ that the network is required to cover. Future research will consider other configuration spaces, in particular the group of planar rigid body motions with $\mathsf{Q} = \mathsf{SE}(2)$. Let the map $\phi : \mathcal{D} \to \mathbb{R}^+$, called a *distribution density function*, represent a measure of information or probability that some event takes place over $\mathcal{D}$. The function ϕ represents all prior information known by the fleet. A large value of ϕ indicates high likelihood of event detection and a smaller value indicates low likelihood. Let N be the number of agents in the fleet and let $\mathbf{q}_i \in \mathsf{Q}$ denote the position of agent $\mathcal{A}_i$, $i \in \mathcal{S} = \{1, 2, 3, \ldots, N\}$. Each agent $\mathcal{A}_i$, $i \in \mathcal{S}$, satisfies the following simple kinematic equations of motion

$$\dot{\mathbf{q}}_i = \mathbf{u}_i, \ i \in \mathcal{S} \tag{1}$$

where $\mathbf{u}_i \in \mathbb{R}^2$ is the control velocity of agent $\mathcal{A}_i$.

Define the *instantaneous coverage function* $\bar{A}_i : \mathcal{D} \times \mathsf{Q} \to \mathbb{R}^+$ as a $\mathcal{C}^1$-continuous map that describes how effective an agent $\mathcal{A}_i$ senses a point $\tilde{\mathbf{q}} \in \mathcal{D}$. Without loss of generality, we consider the following sensor model **SM**. We emphasize, however, that *this model is not an assumption for the ensuing theoretical results to be valid*. In the following sections, via straightforward modifications, similar results can easily be obtained for other sensor models. The most important property of any sensor model allowed by the theory is the fact that the sensors can have a finite sensor range.

Sensor Model SM. In this paper we consider sensors with the following properties:

SM1 Each agent has a peak *sensing capacity* of M_i exactly at the position $\mathbf{q}_i$ of $\mathcal{A}_i$. That is, we have $\bar{A}_i(\mathbf{q}_i, \mathbf{q}_i) = M_i > \bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i), \ \forall \tilde{\mathbf{q}} \neq \mathbf{q}_i$.

SM2 Each agent's sensor has a circular sensing symmetry about the position $\mathbf{q}_i$, $i \in \mathcal{S}$, in the sense that all points in $\mathcal{D}$ that are on the same circle centered at $\mathbf{q}_i$ are sensed with the same intensity. That is, $\bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i)$ is a constant $\forall \tilde{\mathbf{q}} \in \mathcal{D}$ such that $\|\mathbf{q}_i - \tilde{\mathbf{q}}\| = c$ for all constant c, $0 \leq c \leq r_i$, where r_i is the range of the sensor of agent $\mathcal{A}_i$. Hence, we introduce the new function $A_i : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i) = A_i\left(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2\right)$.

SM3 Each agent has a limited *sensory domain* $\mathcal{W}_i(t)$ with a *sensory range* r_i. The sensory domain of each agent is given by

$$\mathcal{W}_i(t) = \left\{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\| \leq r_i\right\}. \tag{2}$$

Mathematically, under assumption **SM2**, this requires that $A_i(\|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|^2) = 0 \ \forall \tilde{\mathbf{q}} \in \mathcal{D} \setminus \mathcal{W}_i(t) = \left\{\bar{\tilde{\mathbf{q}}} : \ \|\mathbf{q}_i(t) - \bar{\tilde{\mathbf{q}}}\| > r_i\right\}$. Let the union of all sensor domains be denoted by

$$\mathcal{W}(t) = \cup_{i \in \mathcal{S}} \mathcal{W}_i(t).$$

An example of such a sensor function is a second order polynomial function of $s = \|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2$ within the sensor range and zero otherwise. In particular, consider the function

$$A_i(s) = \begin{cases} \frac{M_i}{r_i^4}\left(s - r_i^2\right)^2 & \text{if } s \leq r_i \\ 0 & \text{if } s > r_i \end{cases}. \tag{3}$$

All simulations conducted in this paper employ the coverage function given by $A_i\left(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2\right)$, with A_i as given in equation (3). One can check that this sensor coverage function satisfies the model properties **SM1-SM3**. An example for the instantaneous coverage function (3) is given in Figure 1.

Fixing a point $\tilde{\mathbf{q}}$, the *effective coverage* achieved by an agent $\mathcal{A}_i$ surveying $\tilde{\mathbf{q}}$ from the initial time $t_0 = 0$ to time t

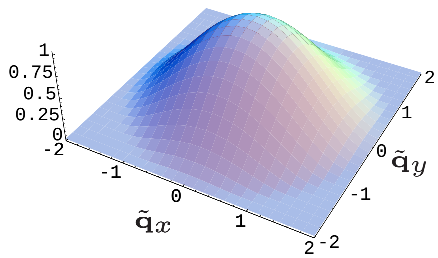


Fig. 1. The coverage function A_i with $\mathbf{q}_i = 0$, $M_i = 1$ and $r_i = 2$.

is *defined* to be

$$\mathcal{T}_i(\tilde{\mathbf{q}}, t) := \int_0^t A_i(\|\mathbf{q}_i(\tau) - \tilde{\mathbf{q}}\|^2)\mathrm{d}\tau$$

and the effective coverage by a subset of agents $\mathcal{A}_\mathcal{K} = \{\mathcal{A}_j | j \in \mathcal{K} \subseteq \mathcal{S}\}$ in surveying $\tilde{\mathbf{q}}$ is then given by

$$\mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, t) := \sum_{i\in\mathcal{K}} \mathcal{T}_i(\tilde{\mathbf{q}}, t) = \int_0^t \sum_{i\in\mathcal{K}} A_i(\|\mathbf{q}_i(\tau) - \tilde{\mathbf{q}}\|^2)\mathrm{d}\tau.$$

It can easily be checked that $\mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, \mathcal{K}, t)$ is a non-decreasing function of time t. In fact, note that

$$\frac{\partial}{\partial t}\mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, t) = \sum_{i\in\mathcal{K}} A_i\left(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2\right) \geq 0.$$

Let C^* be the *desired attained effective coverage* at all points $\tilde{\mathbf{q}} \in \mathcal{D}$. The goal is to attain a *network* coverage of $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) = C^*$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$ at some time t. The quantity C^* guarantees that, when $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) = C^*$, one can judge, with some level of confidence, whether an event happened at $\tilde{\mathbf{q}} \in \mathcal{D}$ or not. Consider the following *error function*

$$e(t) = \int_\mathcal{D} h\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right)\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}, \tag{4}$$

where $h(x)$ is a *penalty function* that is positive definite, twice differentiable, strictly convex on $(0, C^*]$ and that satisfies $h(x) = h'(x) = h''(x) = 0$ for all $x \leq 0$. Positivity and strict convexity in our case mean that $h(x), h'(x), h''(x) > 0$ for all $x \in (0, C^*]$. The penalty function penalizes lack of coverage of points in $\mathcal{D}$. It incurs a penalty whenever $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) < C^*$. Once $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) \geq C^*$ at a point in $\mathcal{D}$, the error at this point is zero no matter how much additional time agents spend surveying that point. The extra time spent there is beneficial, since it results in increasing the probability of event detection, and is hence not penalized. As will be seen, accumulated error will generate an attractive "force" on an agent, while excessive coverage has no effect on the motion. An example for the function $h(x)$ is

$$h(x) = \left(\max(0, x)\right)^n, \tag{5}$$

where $n > 1,\ n \in \mathbb{R}^+$. The total error is an average over the entire domain $\mathcal{D}$ weighted by the density function $\phi(\tilde{\mathbf{q}})$. When $e(t) = 0$, one says that the *mission is accomplished*.

III. Control Strategy for Dynamic Coverage

In this section, the error function in equation (4) is used to derive control laws that guarantee, under appropriate assumptions, coverage of the entire domain $\mathcal{D}$ with an effective coverage of C^*. Let $\hat{\mathcal{S}}_i \subseteq \mathcal{S}$ be the set of indexes representing all agents in the fleet with which agent $\mathcal{A}_i$ can *mutually* communicate measurement and state histories. Clearly, $i \in \hat{\mathcal{S}}_i$ holds. If $\hat{\mathcal{S}}_i = \mathcal{S}$, the multi-agent network is said to be *fully connected*. On the other hand, if $\hat{\mathcal{S}}_i \subset \mathcal{S}$ (strictly), the multi-agent network is said to be *partially-connected*. Due to communication bidirectionality we have

$$j \in \hat{\mathcal{S}}_i \ \Leftrightarrow\ i \in \hat{\mathcal{S}}_j. \tag{6}$$

In the next paragraphs, we first consider a control law for the fully connected case. Inspired by this control law, we propose a similar strategy for the partially connected case.

We will make, without any loss of generality, the following assumption, whose utility will become obvious later.

IC1 The initial coverage is identically zero:

$$\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, 0) = 0,\ \forall \tilde{\mathbf{q}} \in \mathcal{D}.$$

1) Fully-Connected Fleets: Let us first consider the following control law

$$\begin{aligned}\bar{\mathbf{u}}_i(t) \quad &= \quad \bar{k}_i \int_\mathcal{D} h'\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right) \left.\frac{\partial A_i(s)}{\partial s}\right|_{s=\|\mathbf{q}_i(t)-\tilde{\mathbf{q}}\|^2} \\ &\qquad \cdot \left(\mathbf{q}_i(t) - \tilde{\mathbf{q}}\right)\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}.,\end{aligned} \tag{7}$$

where $\bar{k}_i > 0$ are fixed feedback gains. Consider the function $\bar{V} = -e_t(t)$, where $e_t = \frac{\mathrm{d}e}{\mathrm{d}t}$, and note that $\dot{\bar{V}} = -e_{tt}$ where

$$\begin{aligned}e_t(t) \quad &= \quad -\int_\mathcal{D} h'\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right) \\ &\qquad \times \left(\textstyle\sum_{j\in\mathcal{S}} A_j(\|\mathbf{q}_j(t) - \tilde{\mathbf{q}}\|^2)\right)\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \\ e_{tt} \quad &= \quad \int_\mathcal{D} h''\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right) \\ &\qquad \times \left(\textstyle\sum_{j\in\mathcal{S}} A_j(\|\mathbf{q}_j(t) - \tilde{\mathbf{q}}\|^2)\right)^2\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \\ &\quad +2\textstyle\sum_{i\in\mathcal{S}} \bar{k}_i\Big[\int_\mathcal{D} h'\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right) \\ &\qquad \times \left.\frac{\partial A_j}{\partial s}\right|_{s=\|\mathbf{q}_j(t)-\tilde{\mathbf{q}}\|^2} \left(\mathbf{q}_j(t) - \tilde{\mathbf{q}}\right)\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}\Big]^2\end{aligned}$$

are the first and second time derivatives of $e(t)$ along the trajectory generated by the control law $\bar{\mathbf{u}}_i$ in equation (7).

Condition C1. $\quad \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) = C^*,\ \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t), \forall i \in \mathcal{S}.$

Lemma III.1. If for some $t \geq 0$ Condition **C1** holds, then $e_t(t) = 0$. Conversely, if $e_t(t) = 0$ for some time $t \geq 0$, then Condition **C1** holds.

Proof. By the property **SM3**, Condition **C1** implies that the h' term in the integrand in the expression for e_t is nonzero *possibly* only outside $\mathcal{W} = \cup_{i\in\mathcal{S}}\mathcal{W}_i$ where all coverage functions A_i are zero. Condition **C1**, by construction of the penalty function h, implies that

$$h'\left(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)\right) = 0$$

precisely inside $\mathcal{W}(t)$. Hence, under Condition **C1** $e_t = 0$.

The converse is easily verified by noting that the integrand in the expression for e_t is greater than or equal to zero everywhere in $\mathcal{D}$. Hence the integral over $\mathcal{D}$ has to be greater than or equal to zero. For e_t to be zero, the integrand has to be identically equal to zero everywhere on $\mathcal{D}$, which holds true only if Condition **C1** holds. This completes the proof.■

From the lemma, $\bar{V} = 0$ if and only if Condition **C1** holds since $\bar{V} = -e_t$. Clearly, e_t is negative semi-definite since the integrand in e_t is positive semi-definite. Hence, $\bar{V} = -e_t \geq 0$ (equality holding if and only if **C1** holds).

Next, consider the time derivative of $\bar{V}$:

$$\dot{\bar{V}} = -e_{tt}.$$

Under the control law (7) and the above expression for e_{tt}, note, by similar arguments as those given above, that $\dot{\bar{V}} \leq 0$ with equality holding if and only if Condition **C1** holds. The proof of this statement also relies on the fact that h is a strictly convex function on $(0, C^*]$ and that $h''(0) = 0$, which is true by construction of h. This implies that the function $\bar{V}$ is a Liapunov-type function that guarantees that the system always converges to the state described in Condition **C1**. This proves the following result.

Lemma III.2. Under the control law (7), a fully connected multi-agent sensor network will converge to the state described in Condition **C1**.

By construction of the function h and similar arguments as above, under the control law given in equation (7), $e_t < 0$ away from the state described by Condition **C1**. We then have the following lemma.

Lemma III.3. The control law $\bar{\mathbf{u}}_i$ in equation (7) guarantees that $e_t < 0$ away from the state described by Condition **C1**. At the state **C1**, the control law $\bar{\mathbf{u}}_i$ in equation (7) is identically zero.

Note that in the worst-case performance with unactuated agents, the agents' positions are fixed. Say the control is set to zero for all $t \geq t_1$ for some $t_1 \geq 0$. The error inside all agents' sensory ranges, $\mathcal{W}(t)$, keeps decreasing for all $t \geq t_1$ until we have

$$\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) \geq C^*, \ \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t) = \mathcal{W}_i(t_1).$$

At that point, the term $\sum_{i \in \mathcal{S}} A_i(\|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|)$ is not zero for all $\tilde{\mathbf{q}} \in \mathcal{D}$ exactly where $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) \geq C^*$ and is zero exactly where $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) < C^*$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$. Hence $e_t = 0$ while $e \neq 0$. From this, note that the zero control, which sets the second term in the equation for e_{tt} equal to zero, is guaranteed to converge to the state described in Condition **C1** (see [11] for more details). *This is not necessarily true for the control law $\bar{\mathbf{u}}_i$ in equation (7).* An example where the control law (7) drives the error to zero without satisfying the Condition **C1** is given in Section IV.

Under a dynamic nonzero control, such as that described in equation (7), the agents are in constant motion with $e_t < 0$ (i.e., error is always decreasing) as long as the Condition **C1** is not satisfied. In fact, this control law applied to agent $\mathcal{A}_i$ utilizes the gradient of the error distribution inside $\mathcal{W}_i(t)$ to move in directions with maximum error. Hence it locally seeks to maximize coverage. The gradient nature of (7) is what motivates it in this work. This choice of controller is obviously better than the zero control strategy albeit it does not guarantee effective coverage of at least C^* where an agent has been. This is of no concern, since this lack of full effective coverage implies that $e \neq 0$, which will induce some agent, as discussed later in our control strategy, to return and recover these partially covered regions. Optimal controllers will be the focus of future research.

We now study the behavior of the control law (7). First consider the following class of initial agent locations.

IC2 All agents are initialized such that:

1) the interior of each agent's sensory range intersects with the boundary of $\mathcal{D}$, $\partial\mathcal{D}$:
$$\mathcal{W}(0) \cap \partial\mathcal{D} \neq \emptyset,$$
2) *or*, the interior of the sensory range of *at least two agents* interferes with at least one other agent (not allowing exact overlapping):
$$\|\mathbf{q}_i(0) - \mathbf{q}_j(0)\| < r_i + r_j, \ i, j \in \mathcal{S}.$$

This class of initial conditions includes natural cases where a fleet begins a mission outside the domain $\mathcal{D}$ or when the fleet deploys from a flocking configuration. Note that if **IC2** is violated, then the system with control law (7) is guaranteed to converge to the state of Condition **C1**. To see why this is true, note that the gradient of the coverage function $\frac{\partial A_i}{\partial s}$ is skew symmetric about the agent location $\mathbf{q}_i$. If $\mathcal{W}$ were completely inside $\mathcal{D}$ (i.e., $\mathcal{W} \cap \partial\mathcal{D} = \emptyset$) or no two agents intersect (i.e., $\mathcal{W}_i \cap \mathcal{W}_j = \emptyset$ for all $i \neq j \in \mathcal{S}$) then, under assumption **IC1**, the integral in equation (7) is zero since the integrand is skew-symmetric and is being integrated over a symmetric domain $\mathcal{W}_i$ (property **SM2**). Since the integral of a skew-symmetric function over a symmetric domain is zero, then the control (7) is zero.

On the other hand satisfying the class of initial conditions **IC2**, the control law (7) is guaranteed to be nonzero. The integration over $\mathcal{D}$ is performed over non-symmetric subsets of each $\mathcal{W}_i$. Moreover, the integrand in equation (7) is generally not skew-symmetric and therefore $\bar{\mathbf{u}}_i \neq 0$. Each agent, then, is set to motion and will retain breaking the symmetry of the error distribution for all future time until, at a later time, it satisfies the Condition **C1** again. Hence, by Lemma III.3, the control law guarantees that $e_t < 0$ (see [11] for more details). Lemma III.2 is important if, in addition to the Condition **C1**, the error $e(t) = 0$. At that point the goal of achieving a minimum effective coverage of C^* is attained everywhere in $\mathcal{D}$. However, the above result does not guarantee full coverage of $\mathcal{D}$ using the control law (7) alone. Hence, we propose the following control strategy.

The control strategy is as follows. Under the control law (7), all agents in the system are in continuous motion (see previous remarks) as long as the state described in Condition **C1** is avoided. Whenever the Condition **C1** holds with nonzero error $e(t) \neq 0$, the system has to be perturbed by switching to some other control law $\bar{\bar{\mathbf{u}}}_i$ that ensures violating the Condition **C1**. Once away from the condition **C1**, the controller is switched back to the nominal control $\bar{\mathbf{u}}_i$ in equation (7). Only when both **C1** and $e(t) = 0$ are satisfied is when there is no need to switch to $\bar{\bar{\mathbf{u}}}_i$. Thus, the goal is to propose a simple linear feedback controller that guarantees driving the system away from the Condition **C1**. We will often refer to any such control law by *symmetry-breaking* controller since it attempts to move the agents to positions where the skew-symmetry inside all $\mathcal{W}_i$ is broken and the control law $\bar{\mathbf{u}}_i$ is no longer zero.

We now consider a simple symmetry-breaking control law. Define the time varying open set:

$$\mathcal{D}_e(t) = \{\tilde{\mathbf{q}} \in \mathcal{D} : \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) < C^*\}. \quad (8)$$

Clearly, this set is open. Let $\overline{\mathcal{D}}_e(t)$ be the closure of $\mathcal{D}_e(t)$. For each agent $\mathcal{A}_i$, let $\widetilde{\mathcal{D}}_e^i(t)$ denote the set of points in $\overline{\mathcal{D}}_e(t)$ that minimize the distance between $\mathbf{q}_i(t)$ and $\bar{\mathcal{D}}_e(t)$:

$$\widetilde{\mathcal{D}}_e^i(t) = \left\{ \bar{\tilde{\mathbf{q}}} \in \overline{\mathcal{D}}_e(t) : \bar{\tilde{\mathbf{q}}} = \mathrm{argmin}_{\tilde{\mathbf{q}} \in \bar{\mathcal{D}}_e(t)} \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\| \right\}.$$

Let t_s be the time at which the Condition **C1** holds and $e(t_s) > 0$, while $e_t \neq 0$ for all $t < t_s$. That is, t_s is the time of entry into the state described in Condition **C1** with nonzero error. At t_s, for each agent $\mathcal{A}_i$, consider a point $\tilde{\mathbf{q}}_i^*(t_s) \in \widetilde{\mathcal{D}}_e^i(t_s)$. (Note that the set $\widetilde{\mathcal{D}}_e^i(t_s)$ may include more than a single point. An example of this is when the condition on initial positions **IC2** is violated. In that case, one may assign a fixed rule for picking up a point $\tilde{\mathbf{q}}_i^*(t_s)$ in $\widetilde{\mathcal{D}}_e^i(t_s)$. This choice is immaterial in the present discussion and we will assume there is a single such point.)

Consider the control law

$$\bar{\bar{\mathbf{u}}}_i(t) = -\bar{\bar{k}}_i \left(\mathbf{q}_i(t) - \tilde{\mathbf{q}}_i^*(t_s)\right) \quad (9)$$

We claim that $\tilde{\mathbf{q}}_i^*(t_s)$ is fixed for some time interval $t_s < t < \hat{t}_s$. This claim will be justified shortly. Under the regime when $e_t = 0$ and $e(t) > 0$, this control law is a simple linear feedback controller and will drive each agent in the fleet towards its associated $\tilde{\mathbf{q}}_i^*(t_s)$. Note that it is possible that $\tilde{\mathbf{q}}_i^*(t_s) = \tilde{\mathbf{q}}_j^*(t_s)$ for some pair $i \neq j \in \mathcal{S}$. By simple linear systems theory, the feedback control law (9) will result in having $\mathbf{q}_i(\hat{t}_s)$, for some $i \in \mathcal{S}$, be inside a ball of radius $\varepsilon < r_i$ at some time $\hat{t}_s > t_s$. Hence the point $\mathbf{q}_i^*$ is guaranteed to lie *strictly* inside the sensory range of agent $\mathcal{A}_i$.

We now justify that the point $\tilde{\mathbf{q}}_i^*$ by construction does not move as a function of time. Let $\hat{t}_s$ be the first instance of any one agent's sensory domain overlapping with $\tilde{\mathbf{q}}_i^*$ (i.e., $\tilde{\mathbf{q}}_i^* \in \mathcal{W}_i$ for some $i \in \mathcal{S}$). Once an agent $\mathcal{A}_i$ reaches a set $\{\tilde{\mathbf{q}} \in \mathcal{D} : \|\tilde{\mathbf{q}} - \tilde{\mathbf{q}}_i^*(t_s)\| \le r_i\}$, the Condition **C1** is violated due to the skew-symmetry in the integrand of e_t at $t = \hat{t}_s$ within $\mathcal{W}_i$. At that point we have regained $e_t(t) \neq 0$ for some time $\hat{t}_s + \epsilon > \hat{t}_s$ and the control is switched back to $\bar{\mathbf{u}}_i$ in equation (7). However, up to time $\hat{t}_s$, by definition, we are guaranteed that skew-symmetry is unbroken and the uncovered nearest-points set $\widetilde{\mathcal{D}}_e^i(t)$ is unaltered for time $t_s < t < t_s^*$. This proves that $\tilde{\mathbf{q}}_i^*(t_s)$ is fixed during the phase of operation of $\bar{\bar{\mathbf{u}}}_i$. To conclude, we have the following result.

Theorem III.1 (Fully Connected)**.** Under the sensor model **SM1**-**SM3** and **IC1**, the control law

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i & \text{if Condition } \mathbf{C1} \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}_i & \text{if Condition } \mathbf{C1} \text{ holds} \end{cases}, \quad (10)$$

drives the error $e(t) \to 0$ as $t \to \infty$.

2) Partially-Connected Networks: Now assume that agent $\mathcal{A}_i$ receives information from agents $\mathcal{A}_j$ only, $j \in \hat{\mathcal{S}}_i \subset \mathcal{S} \setminus \{i\}$. Inspired by the error function (4), consider the *estimated error function of agent* $\mathcal{A}_i$:

$$\hat{e}_i(t) = \int_{\mathcal{D}} h\left(C^* - \mathcal{T}_i(\tilde{\mathbf{q}}, t) - \mathcal{T}_{\hat{\mathcal{S}}_i}(\tilde{\mathbf{q}}, t)\right) \phi(\tilde{\mathbf{q}}) \mathrm{d}\tilde{\mathbf{q}}.$$

Note that the error associated with agent $\mathcal{A}_i$ depends only on its own state history and that of all other agents from which it receives information. The following two remarks are crucial for the next theorem.

Remark. Note that

$$\hat{e}_i(t) \ge e(t),$$

equality holding if and only if $\hat{\mathcal{S}}_i = \mathcal{S} \setminus \{i\}$. In the case of bidirectional communication (i.e., if $j \in \hat{\mathcal{S}}_i$ then $i \in \hat{\mathcal{S}}_j$) then $\hat{e}_i = \hat{e}_j$. •

Theorem III.2 (Partially Connected with Bidirectional Communication)**.** The control law

$$\mathbf{u}_l^*(t) = \begin{cases} \bar{\mathbf{u}}_l & \text{if } \mathbf{C1} \text{ does not hold } \forall\, j \in \hat{\mathcal{S}}_l \\ \bar{\bar{\mathbf{u}}}_l & \text{if } \mathbf{C1} \text{ holds } \forall\, j \in \hat{\mathcal{S}}_l \end{cases} \quad (11)$$

drives the error $\hat{e}_l(t) \to 0$ and, hence, $e_l(t) \to 0$ as $t \to \infty$.

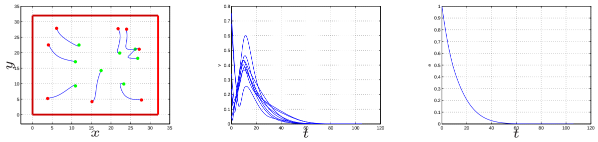
Proof. Define the function $\hat{V}_i = -\hat{e}_t$ and note that $\dot{\hat{V}} = -\hat{e}_{tt}$ where we get expressions similar to those obtained in the proof of the previous theorem. The rest of the proof follows exactly that of the fully-connected case to prove that $\hat{e}_l \to 0$ as $t \to \infty$. Since $\hat{e}_l \ge e_l$, the result follows immediately. See remark below for an intuitive explanation. ■

IV. Numerical Results

In this section we provide simulation results for the fully-connected network case. Assume that $\mathcal{D}$ is a square region of side length $d = 32$ units length. There are 8 agents ($N = 8$) with a randomly selected initial deployment as shown in Figure 3(a). Let the desired effective coverage C^* be 40. Here we use the control law in equation (7) with control gains $\bar{k}_i = 0.00001,\ i = 1, \ldots, 8$. In this example we do not employ switching control since the coverage regions of each agent are large enough for Condition **C1**. This example shows that switching may possibly not be required for applications with dense sensors. By "dense" we imply sensor ranges that are large relative to the size of the domain $\mathcal{D}$. Finally, we assume that $\phi(\tilde{\mathbf{q}}) = \text{constant},\ \forall \tilde{\mathbf{q}} \in \mathcal{D}$. For the sensor model, we have set $M_i = 1$, $r_i = 14$ for all $i = 1, \ldots, 8$.

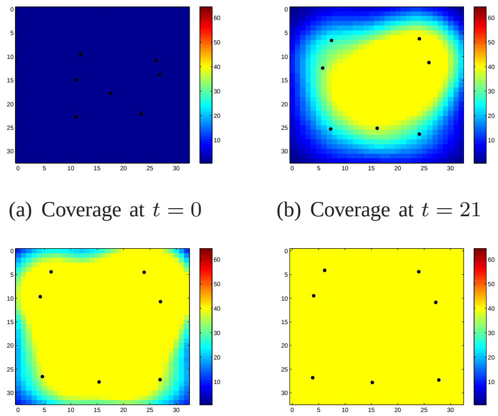
We used a simple trapezoidal method to compute integration over $\mathcal{D}$ and a simple first order Euler scheme to integrate with respect to time. The results are show in Figures 2 and 3. The total error $e(t)$ is shown in Figure 2(c). Note that the error converges to zero. Figure 3 shows the time evolution of the two dimensional function $h(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t))$. We note that coverage converges to C^* as desired. Hence, we see that we achieve satisfactory coverage using the control law (7) without switching control. Note that we have normalized error by dividing by $(C^*)^n d^2$ so that the initial error (which is precisely the volume under the graph of the function C^* for all $\tilde{\mathbf{q}} \in \mathcal{D}$), where d is the side length of $\mathcal{D}$ and $\beta = 2$ is defined in equation (5).

The second example is one with 8 agents, randomly deployment as shown in Figure 5(a), with $r_i = 8$, $i = 1, \ldots, 8$. In this example we employ the switching controller from Theorem III.1. An agent is set to switch to the linear feedback control law whenever Condition **C1** applies to it alone and not to the entire fleet. As mentioned above, this is a more efficient strategy than having the entire fleet satisfy the condition for switching to occur. The results are shown in Figures 4 and 5. Note that as the coverage error decreases,



(a) Motion in the plane (b) $\|\mathbf{u}_i\|$, $i \in \mathcal{S}$ (c) Error e(t)

Fig. 2. Motion in the plane (start at green dot and end at red dot), control effort and error function for a "dense" network with no control switching.

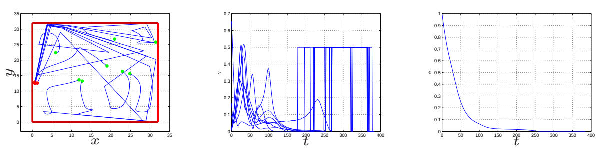


(a) Coverage at $t = 0$ (b) Coverage at $t = 21$

(c) Coverage at $t = 43$ (d) Coverage at $t = 107$

Fig. 3. Effective coverage (dark blue for low and yellow for full coverage) and fleet configuration at $t = 0, 21, 43, 107$ for a "dense" network with no control switching.

agents tend to flock since the agents' desired target points $\tilde{\mathbf{q}}_i^*$ tend to agree.

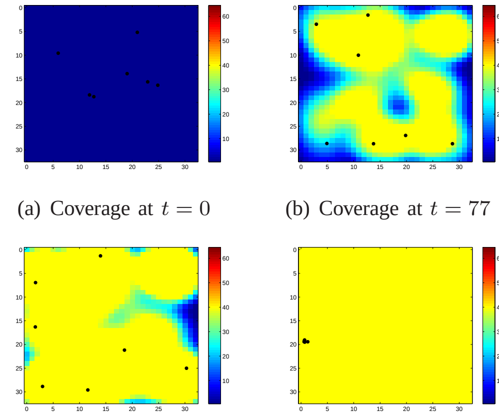


(a) Motion in the plane (b) $\|\mathbf{u}_i\|$, $i \in \mathcal{S}$ (c) Error e(t)

Fig. 4. Motion in the plane (start at green dot and end at red dot), control effort and error function with control switching.

V. Conclusion

In this paper we formulated a coverage control problem that addresses a wide variety of multi-agent system applications. The goal is to achieve coverage of each point in the search domain by an effective coverage of C^*. We explored the fully connected network, where all agents have access to the state histories of all other agents in the group. This result was specialized to a class of bidirectional partially connected network. We proposed a control law that ensures that the coverage error converges to zero for both communication structures. The control law for the fully connected fleet was validated numerically for various model parameters. Current work includes investigation of collision avoidance conditions, other (possibly dynamic) information structures, and, finally, the case when the missing person or target is mobile instead of being static.



(a) Coverage at $t = 0$ (b) Coverage at $t = 77$

(c) Coverage at $t = 154$ (d) Coverage at $t = 385$

Fig. 5. Effective coverage (dark blue for low and yellow for full coverage) and fleet configuration at $t = 0, 77, 154, 3858$ with control switching.

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