

Diversified Statistical Arbitrage: Dynamically combining mean reversion and momentum
investment strategies

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Abstract

The financial crisis has put into question the validity of commonly accepted investment strategies. Long only and static asset allocation methodologies proved to be incapable of enduring the 57% peak-to-trough decline of the S&P 500. Retail and institutional investors alike became wrought with fear as the capital markets continued to tumble. Nonetheless, these strategies are receiving significant capital inflows due to the 70% rally in the S&P 500 from March 2009 to March 2010, which has ignited the resurgence in risk appetite. This paper presents a strategy of investing that is capable of producing strong risk-adjusted returns in both up and down markets. The strategy utilizes dynamic portfolio optimization to rebalance exposures as the market fluctuates. It combines mean reversion and momentum investment strategies to construct a diversified statistical arbitrage approach. It was able to generate strong risk-adjusted returns in 2008 as the market declined, and in 2009 as the market rallied. The strategy has proven to be robust across two very different market environments in 2008 and 2009.

We created a mean reversion strategy that identifies short term mean reversion opportunities across stocks within the S&P 500. The strategy decomposed stock returns into market and idiosyncratic risk components. Through principal component analysis, stocks that are close substitutes are grouped together. The idiosyncratic return is calculated by comparing the return of each stock to the return of its entire group. This residual return often oscillates around a long-term mean.

The strategy identifies long opportunities when the residual return has been significantly lower than its mean, and it identifies short opportunities when the residual is significantly higher than its mean. The strategy was dollar neutral by construction, and had a high level of portfolio turnover. The strategy was able to produce a Sharpe ratio of 1.28 from 11/1/2005 – 10/31/2007 (in-sample) and a Sharpe ratio of 0.75 from 11/1/2007 – 10/31/2009 (out-of-sample). As a single strategy, these results were positive, but the combined strategy was able to produce far superior results by utilizing dynamic portfolio optimization.

The momentum strategies utilized technical trading rules to go long and short the nine S&P 500 sector exchange traded funds and the SPY. Medium-term trend following signals were created for each ETF. The momentum strategies were mixed during the in-sample period from 11/1/2005 – 10/31/2007, but they generated strong returns in the out-of-sample period from 11/1/2007 – 10/31/2009. The leading strategies during the out-of-sample period were the SPY with a Sharpe ratio of 1.06, XLF (0.90), XLI (0.88), and XLP (0.79). These were compelling results, but the trend following strategies had a heightened level of volatility. As such, combining these strategies with the mean reversion strategy was an effective way to maximize the returns while also minimizing the risk of the portfolio.

Dynamic portfolio optimization was used to maximize the risk-adjusted returns of the portfolio. By dynamically allocating to each strategy, the return of the portfolio was enhanced significantly. We created tactical strategy bounds for each of the 11 strategies,

and rebalanced the portfolio at the end of each month. The Sharpe ratio of the diversified portfolio was twice as high as the best performing stand alone strategy.

The out-of-sample results of the diversified portfolio were compelling. From 11/1/2007 – 10/31/2009, the diversified portfolio produced annualized returns of 39.14%, an annualized volatility of 16.84% and a Sharpe ratio of 2.27. Over this same time period, the S&P 500 had a Sharpe ratio of -0.74, the HFRI equity hedge index had a Sharpe of -0.61 and the HFRI Composite index had a Sharpe of -0.45. The diversified strategy clearly outperformed the benchmark indices and generated high risk-adjusted returns. The portfolio produced a significant amount of alpha as its beta exposure ranged from positive 30% to negative 35%. The strategy experienced one major drawdown of 16.9%, but recovered its net asset value within seven weeks. The drawdown resulted from the change in market direction starting on March 9, 2009.

The strategy was able to produce strong results during both bearish and bullish market environments. The short term trading focus of the strategy allowed it to be nimble and tactically change positioning. We are conducting future research that will allow the mean reversion and momentum models to have varying look back and investment time horizons. We feel the diversified model will be able to produce even better results with these additional enhancements.

I. Introduction

A true arbitrage opportunity is a zero-cost trading strategy that offers the possibility of a gain with no possibility of a loss. The mere existence of arbitrage opportunities directly contradicts the fundamental foundations of the efficient market hypothesis.¹ Since the efficient market hypothesis was developed by Eugene Fama, it has been under rather severe criticism from economists, financial professionals, and mathematicians alike. Yet, the word “arbitrage” is used very liberally throughout finance. It encompasses a number of strategies that take advantage of relative value opportunities, but with a varying level of risk. Statistical arbitrage focuses on quantitative rule based trading strategies that are often market neutral. The strategy seeks to minimize risk and provide near risk free return. Yet, as August 2007 indicated, even this strategy is far from an arbitrage opportunity.

Pairs trading is the simplest form of Statistical Arbitrage investing.² The strategy is made under the assumption that two stocks, A and C, will follow some process of tracking one another once accounting for market beta. The model for the system is expressed in the differential equation:

$$(1) \frac{dA_t}{A_t} = \alpha dt + \beta \frac{dC_t}{C_t} + dX_t,$$

where X_t is a mean reverting process. The process is commonly called the residual because it represents the residual return of the system from its mean.³ In the above

¹ Fama, E.F., "Efficient Capital Markets: II," *Journal of Finance* (December 1991).

² Pole, A., "Statistical Arbitrage: Algorithmic trading insights and techniques," Wiley Finance, 2007.

³ Pole (2007).

equation, α is defined as the drift function. In many systems, the α is small compared to X_t and can be ignored, meaning that the long and short positions oscillate around an equilibrium.⁴

Practitioners and academics expanded on the simple idea of pairs trading to trade groups of stocks versus a single stock or group.⁵ The focus of these studies is on setting up long/short portfolios that analyze several residuals, X_t . Signals can be generated by decomposing stock returns into systematic and residual components and statistically modeling the residuals. The general function for decomposing stock returns will look like the following:

$$(2) \frac{dA_t}{A_t} = \alpha dt + \sum_{j=1}^n \beta_j F_t + dX_t,^6$$

the terms F_t , $j = 1, 2, \dots, n$ represent the returns of systematic market risk factors. With this basic premise, we are able to construct a statistical equity mean reversion model. Yet, does equity mean reversion offer a diversified investment approach?

The benefits of a market neutral approach with limited factor risk create a compelling case. However, the quantitative equity meltdown in August 2007 has put this argument into question. Hence, a more diversified strategy is needed to help avoid the single model risk. Given the focus of mean reversion on stocks reverting to the mean of a system, a natural complement is momentum investing.

⁴ Avellaneda and Lee., "Statistical Arbitrage in the US Equities Markets," working paper version 3 drafted on June 15, 2009.

⁵ Pole (2007).

⁶ Avellaneda and Lee (2009).

The ground-breaking research conducted by Kenneth French and Eugene Fama on the Fama-French three factor model was carefully scrutinized because of the emergence of momentum as a legitimate equity factor. Jegadeesh and Titman first discovered the existence of momentum in stock returns in 1993.⁷ Since then, momentum has been widely analyzed and critiqued due to its lack of fundamental foundation. The theoretical explanation for the existence of momentum is based on behavior finance. The theory assumes that investors suffer from the disposition effect. It holds that investors will sell shares of a stock quickly after it increases, but hold shares longer when the stock has decreased because investors have different utility functions for losses than for gains.⁸ Other behavioral phenomena that help explain momentum are investor herding, confirmation bias, and anchoring.

Although, there is a significant amount of evidence indicating the existence of momentum, there is no study done that specifically examines the best momentum investment strategy. Moskowitz and Grinblatt (1999) confirmed that the existence of momentum is explained by industry sectors.⁹ The results of the statistical arbitrage study conducted by Avellaneda and Lee confirm these results as well. Therefore, a momentum strategy should be focused on the industry sectors, as opposed to single stocks. This paper will focus on momentum strategies that are implemented using sector exchange traded funds.

⁷ Jegadeesh, Narasimhan and Sheridan Titman, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, Volume 48 (1), 1993, pp. 65-92.

⁸ Jegadeesh and Titman (1993).

⁹ Moskowitz and Grinblatt, "Do Industries Explain Momentum?" *Journal of Finance*, Vol. 4, August 1999.

The first section of this paper will discuss the equity mean reversion model. The second section will discuss the momentum models. The third section will discuss the in-sample analysis. The fourth section will discuss portfolio optimization and out-of-sample performance. The final section will present conclusions and discuss ideas for future research.

II. Equity Mean Reversion Model

We use a statistical arbitrage model that is very similar to the one used by Avellaneda and Lee in Statistical Arbitrage in the US Equities Markets (2009) with certain distinct differences that will be discussed throughout this paper. The decomposition of stock returns as shown in formula (2): $\frac{dA_t}{A_t} = \alpha dt + \sum_{j=1}^n \beta_j F_t + dX_t$, illustrates that stock returns have a systematic market risk component and an idiosyncratic component. Thus, if we create a portfolio that remains market neutral, the returns will only be affected by the idiosyncratic component. The next step is to create a model that can easily identify the idiosyncratic component across a large number of stocks.

Principal Component Analysis

Principal Component Analysis is an eigenvector based multivariate analysis. The purpose of the analysis is to reveal the internal structure of the data to help best explain the variance. PCA is an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by any projection of the data comes to lie on the first principal component, with the second greatest variance on the second

principal component, and the nth greatest variance on the nth principal component.¹⁰ Data centering is an important element of PCA analysis because it helps minimize the error of mean squared deviation. As a result, we must standardize the sample returns using the following model:

$$(3) Y_{ik} = \frac{R_{ik} - \bar{R}_i}{\sigma_i} \text{ where } \bar{R}_i = \frac{1}{M} \sum_{k=1}^M R_{ik} \text{ and } \sigma_i^2 = \frac{1}{M-1} \sum_{k=1}^M (R_{ik} - \bar{R}_i)^2.$$

Given these equations, the correlation matrix for the data is defined as:

$$(4) P_{ij} = \frac{1}{M-1} \sum_{k=1}^M Y_{ik} Y_{jk},$$

The estimation window of the correlation matrix is always one year or 252 trading days. The correlation matrix is used to analyze the eigenvectors and eigenvalues of the data. These values help explain the variance in the data and exist for all n stocks in the universe. It is important to identify the eigenvectors that are important for the data and where a legitimate cut-off point should be. Avellaneda and Lee used the first 15 eigenvectors for the analysis, but our model uses the first 12 eigenvectors because we found that that these were sufficient to explain the systematic variance. For each index, j, we analyze the corresponding eigenportfolio which is defined by the equation below:

$$(5) Q_i^{(j)} = \frac{v_i^{(j)}}{\sigma_i}$$

with eigenportfolio returns of

$$(6) F_{jk} = \sum_{i=1}^n \frac{v_i^{(j)}}{\sigma_i} R_{ik} \text{ where } j = 1, 2, \dots, m.^{11}$$

¹⁰ Fukunaga, K., "Introduction to Statistical Pattern Recognition." (1990).

¹¹ Avellaneda and Lee (2009).

Chart 1: First 50 Eigenvalues for PCA Analysis on 4/25/2006

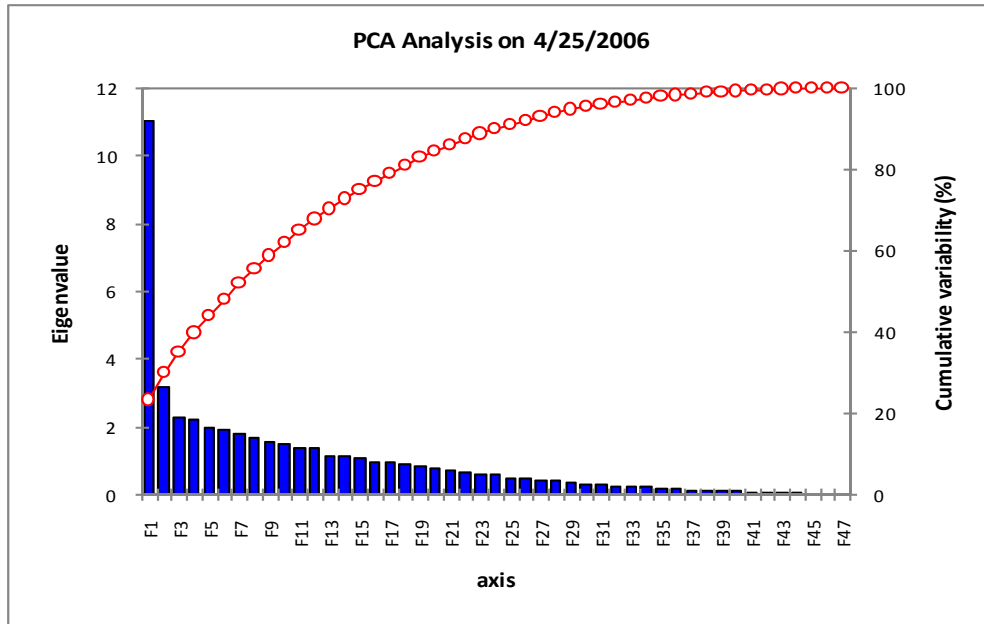
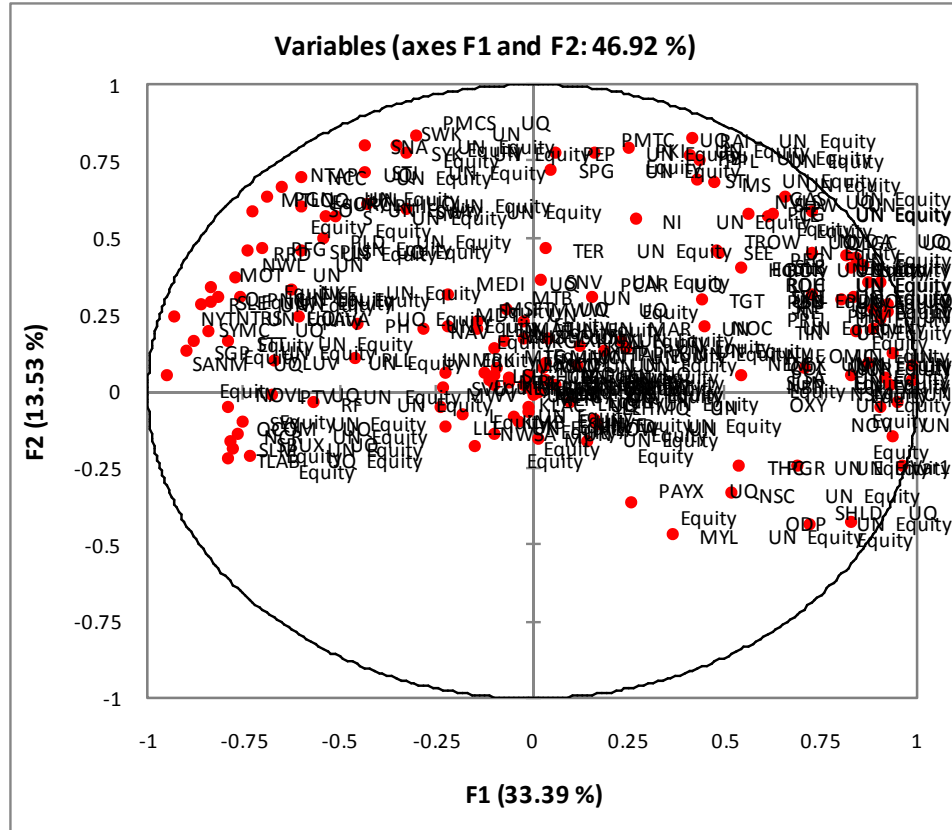


Chart 2: Stock Correlations to F1 and F2 (total variance explained = 46.92%) on 4/25/2006



Trading Signal

Building off the equation (2), (5) and (6), we know the idiosyncratic component of stock returns are defined by the following equation:

$$(7) \quad d\widetilde{X}_i(t) = \alpha_i dt + dX_i(t),$$

α_i represents the drift rate of the idiosyncratic part of the returns. For simplicity, we assume the drift rate is negligible and do not try to calculate it. Therefore, the simple parametric model can be created for $X_i(t)$:

$$(8) \quad dX_i(t) = k_i(m_i - X_i(t))dt + \sigma_i dW_i(t),$$

where $k_i > 0$. The expected one day return of the market neutral long short portfolio is:

$$(9) \quad \alpha_i dt + k_i(m_i - X_i(t))dt.$$

The model predicts a negative return if $X_i(t)$ is sufficiently high and a positive return in $X_i(t)$ is sufficiently low. Mean reversion is defined by the equation:

$$(10) \quad \tau_i = \frac{1}{k_i},$$

where k_i is the speed of mean reversion. We focus on stocks that have fast mean reversion such that k_i is less than the defined trading window, T_1 . We use the same estimation window as Avellaneda and Lee, but include stocks that will mean revert in 20 days as opposed to 30 days because we are seeking to have a shorter investment time horizon. We define a dimensionless variable:

$$(11) \quad s_i = \frac{X_i(t) - m_i}{\sigma_{eq,i}},$$

where s_i measures the distance to equilibrium in standard deviations for each stock. The trading signal we use is similar to Avellaneda and Lee (2009):

(12) buy to open if $s_i < -\bar{sbo}$ and sell to open if $s_i > +\bar{sso}$,

(13) close short if $s_i < +\bar{sbc}$, close long if $s_i > -\bar{ssc}$.

We use slightly different cut-offs than Avellaneda and Lee: $\bar{sbo} = \bar{sso} = 1.25$ and $\bar{sbc} = \bar{ssc} = 0.75$, where we close both long and short trades sooner than Avellaneda and Lee because we need to have a less volatile result. The model also is two-times levered per side or four-times levered gross, which is the industry standard for this type of statistical arbitrage model. The trade is made based on closing prices each day and continuous adjustments are not made to positions. There is 10 bps deducted for each trade to account for slippage and transaction costs.

III. Momentum Strategy

The momentum trading signal uses a simple approach to momentum investing because there has been no clear empirical evidence that gives credence to complex statistical strategies for momentum investing.¹² We focus on technical indicators to identify momentum opportunities across all of the S&P 500 industry sector ETFs and the S&P 500 ETF, SPY.

A number of different potential combinations were analyzed across the data. After data mining using data prior to the in-sample period, we found the most effective technical signal for these data was a 60 day and 5 day exponential moving average. An exponential moving average is defined by the following equation:

$$(14) S_t = \alpha \times Y_t + (1 - \alpha) \times S_{t-1}, \alpha = \frac{2}{N + 1},$$

¹² Asness, Moskowitz and Pedersen., “Value and Momentum Everywhere,” AFA 2010 Atlanta Meetings Paper, March 6, 2009.

Y_t = today's asset price. The signal is long the ETFs if the 5 day EMA is above the 60 day EMA for the previous 4 (or more) trading days. In all other scenarios, the signal is short. Similar to the equity mean reversion strategy, there is no rebalancing and the trade is made based on closing prices with a 10 bps transaction cost to account for fees and slippage.

IV. In-sample Analysis

Data

The equity mean reversion model used daily closing price data for the S&P 500 constituents as of 11/1/2005. The data was obtained from Bloomberg using the data toolbox in Matlab, and because the model is reconstructing the correlation matrix on a daily basis, there is no need to eliminate stocks without a full data history. The momentum strategy also used daily closing price data from Bloomberg and had a complete time series for the ten indices. The in-sample period is 11/1/2005 – 10/31/2007 and the out-of-sample period is 11/1/2007 – 10/30/2009.

Results

For the in-sample period no portfolio optimization was conducted. The portfolio was 50% mean reversion, and a 5% allocation was given to each of the ten momentum models. These allocations were used to create a return stream that was 50% mean reversion and 50% momentum. A table of the in-sample results is shown below:

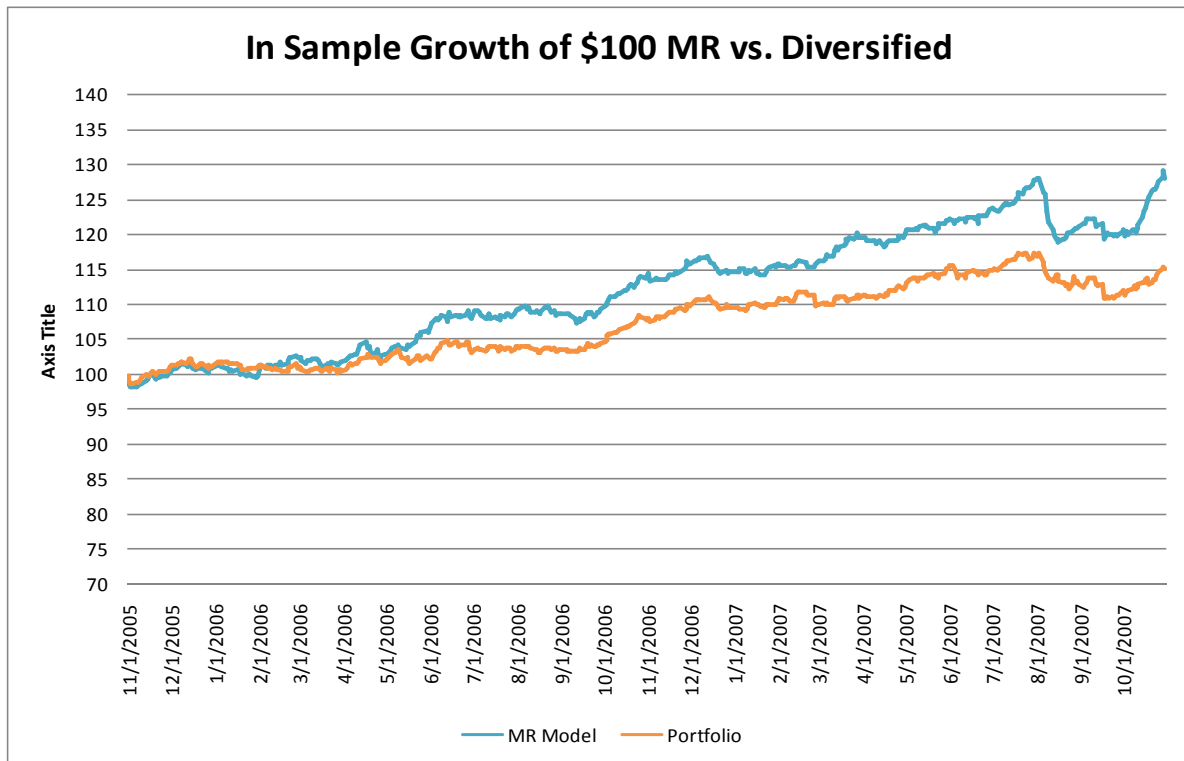
Table 1: In-sample Returns (11/1/2005 – 10/31/2007)

Strategy	Annualized Returns	Annualized SD	Sharpe	SPX Beta
Mean Reversion	13.14%	6.76%	1.28	-0.28
SPY	3.09%	11.93%	-0.12	0.94
XLF - Financials	-2.12%	15.70%	-0.42	0.22
XLE - Energy	-8.49%	24.03%	-0.54	1.40
XLK - Technology	7.87%	13.92%	0.24	0.27
XLP - Cons Staples	-2.54%	9.29%	-0.76	0.48
XLY - Cons Discr.	-5.19%	13.67%	-0.71	-0.15
XLB - Basic Materials	13.54%	18.48%	0.49	0.47
XLI - Industrials	8.48%	13.01%	0.31	0.46
XLV - Healthcare	-0.26%	10.56%	-0.45	-0.28
XLU - Utilities	-6.85%	14.34%	-0.79	0.37
Diversified Portfolio	6.96%	5.01%	0.49	0.07

Table 2: Index Returns (11/1/2005 – 10/31/2007)

Index	Annualized Returns	Annualized SD	Sharpe
S&P 500	15.44%	7.09%	1.54
HFRI Equity Hedge	14.96%	5.24%	1.99
HFRI Composite	14.32%	4.35%	2.25

Chart 3: In-sample Growth of \$100 MR vs. Diversified (11/1/2005 – 10/31/2007)



It is clear from the in-sample returns that the mean reversion strategy is far superior to the momentum strategies with a Sharpe Ratio of 1.28 (1 month T-bill is the risk free rate and note that the leverage of the strategy does not affect its Sharpe ratio). The diversified portfolio does have lower volatility than the stand alone mean reversion strategy, but the Sharpe ratio is much lower at 0.49. The SPX beta of the mean reversion strategy was negative for the in-sample time period. This may be due to our model not hedging out the beta risk as Avellaneda and Lee did, which is potentially tainting the results of the model. The betas of the momentum strategies vary greatly from -0.15 for Consumer Discretionary to 1.4 for Energy.

Over the same time period the benchmark indices outperformed all of these strategies. The SPX produced a Sharpe ratio of 1.54, the HFRI Equity Hedge index produced a Sharpe of 1.99, and the HFRI Composite produced a Sharpe of 2.25. Based on the in-sample results, the model needed to be enhanced significantly in order to be competitive with these benchmark indices.

Table 3: In-sample Strategy Correlation Matrix

Correlation Matrix	Mean Reversion	SPY	XLF	XLE	XLK	XLP	XLY	XLB	XLI	XLV	XLU
Mean Reversion	1.00										
SPY	0.04	1.00									
XLF - Financials	0.43	0.22	1.00								
XLE - Energy	0.13	-0.20	-0.07	1.00							
XLK - Technology	0.50	-0.16	0.11	0.32	1.00						
XLP - Cons Staples	-0.04	0.47	-0.02	0.17	0.15	1.00					
XLY - Cons Discr.	0.15	0.28	0.26	-0.09	0.22	0.14	1.00				
XLB - Basic Materials	0.36	0.08	0.22	0.39	0.46	0.38	0.26	1.00			
XLI - Industrials	-0.10	0.34	-0.02	-0.10	0.08	0.23	0.09	-0.20	1.00		
XLV - Healthcare	0.13	0.01	0.09	-0.15	0.30	0.27	0.45	0.29	-0.17	1.00	
XLU - Utilities	-0.33	-0.07	-0.04	0.35	-0.03	0.09	0.10	0.01	0.28	0.02	1.00

There still appear to be potential diversification benefits from utilizing momentum strategies. The Mean reversion was negatively correlated with three of the momentum strategies, XLP, XLI, and XLU and had no correlation with the SPY momentum strategy. We will examine these benefits in the out-of-sample analysis.

August 2007

The mean reversion strategy was not immune to the quant equity melt down in August of 2007. The strategy had a -7.12% drawdown from August 6th – August 17th. The daily performance of the strategy during the drawdown is shown below:

Table 4: Mean Reversion Daily Performance (8/6/2007 – 8/17/2007)

Date	MR Returns
8/6/2007	-1.50%
8/7/2007	-0.08%
8/8/2007	-0.11%
8/9/2007	-2.00%
8/10/2007	-1.24%
8/13/2007	-0.89%
8/14/2007	-0.75%
8/15/2007	-0.34%
8/16/2007	0.02%
8/17/2007	-0.45%
Total Drawdown	-7.12%

The events of 2007 were driven by a liquidity shock that was caused by funds unwinding their positions.¹³ The fundamentally driven quantitative equity managers were affected the most by the events of August 2007. Many large managers lost between 15% and 30% in a 5 day time span. These losses were magnified as fund managers manually overrode

¹³ Khandani and Lo, “What Happened to the Quants in August 2007?” SSRN, 2007.

their models and liquidated their portfolios. The events also affected statistical arbitrage strategies, but to a lesser extent. The effect was less pronounced because statistical arbitrage managers are less likely to hold a significant amount of common positions with other funds. The drawdown in the mean reversion strategy was large given the time period, but the strategy recovered in the second half of the month and was down only 4.9% during August.

V. Optimization and Out-of-sample Results

Dynamic Portfolio Optimization

The out-of-sample analysis utilizes dynamic portfolio optimization to help maximize the returns of the diversified portfolio. The in-sample results indicate that the mean reversion strategy as a stand alone is the best option. As a result, the goal of the portfolio optimization is to provide allocation timing benefits that can help provide better results for the diversified portfolio.

Dynamic portfolio optimization is a very important component of active investment strategies. Investment managers that keep allocations constant despite changing market conditions are eliminating a large portion of potential returns. Quadratic programming is one type of optimization that can provide a robust solution in an actively managed portfolio. It optimizes a quadratic function of several variables using linear constraints on these variables. The basic equation for QP optimization is the following:

$$(15) \quad \min_x \frac{1}{2} x^T H x + f^T x \rightarrow \begin{cases} A & x \leq b, \\ Aeq & x = beq, \\ lb & \leq x \leq ub \end{cases}$$

H is the symmetric matrix, f is the vector, lb/ub are the lower and upper bounds, Aineq is the matrix for linear inequality constraints, bineq is the vector for linear inequality constraints, Aeq is the matrix for linear equality constraints, and beq is the vector for linear equality constraints. An important input into the process is lower and upper bounds for each variable. Using expected return and allocation targets, we can customize the optimization process to best suit our portfolio specifications.

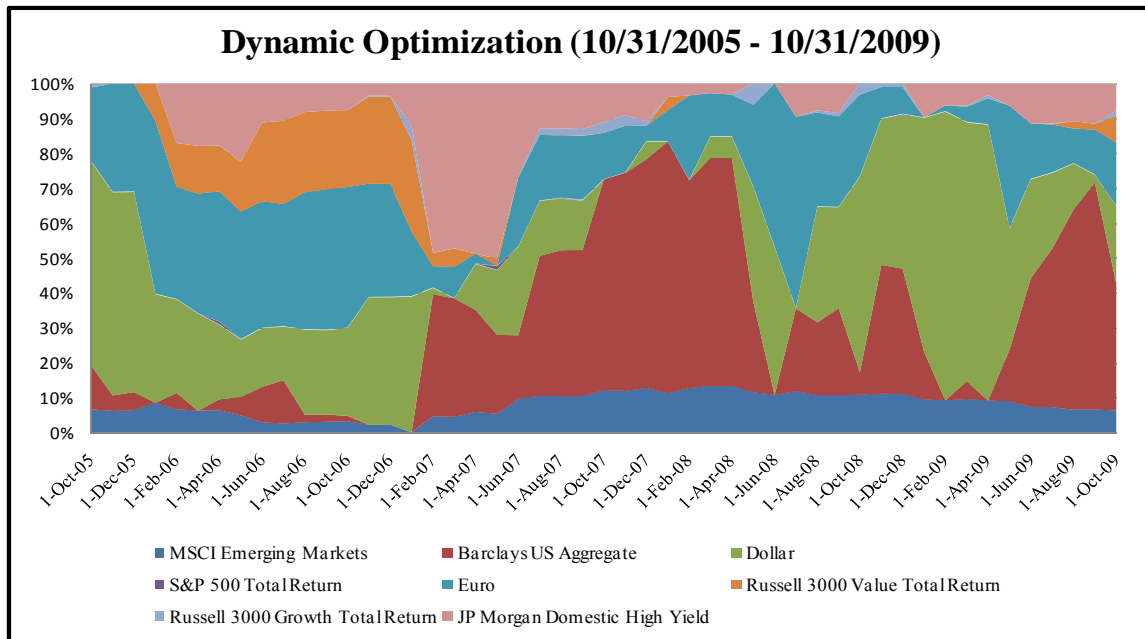
The best way to illustrate the significant impact this type of optimization can have on a portfolio is through the use of an example. We analyzed two portfolios that were invested in the MSCI Emerging Markets Index, Barclays US Aggregate Index, USD Index, EUR Index, Russell 3000 Value TR, Russell 3000 Growth TR, JPM Domestic HY, and the S&P 500 TR. The first portfolio uses similar weights to the universe of mutual and hedge fund managers with 25% in the Barclays US Aggregate, 19% in the S&P 500, 19% in the Russell Growth, 17% in the Russell Value, 8% in JPM HY, 5% in the Dollar Index, 5% in the Euro Index, and 2% in the MSCI EM Index.¹⁴ The second portfolio uses quadratic programming to dynamically allocate between these assets with a 1 year look back period, a minimum allocation of zero and the total weights equal to one.

Table 5: Market Index vs. Optimization Returns (10/31/2005 – 10/31/2009)

Index	Annualized Returns	Annualized SD	Sharpe
MSCI Emerging Markets	8.25%	29.87%	0.19
Barclays US Aggregate	5.72%	3.69%	0.82
USD Index	-3.84%	8.88%	-0.73
EUR Index	5.07%	11.23%	0.21
Russell 3000 Value Total Return	-3.40%	18.90%	-0.32
Russell 3000 Growth Total Return	-0.87%	17.60%	-0.20
JP Morgan Domestic High Yield	6.24%	13.81%	0.26
S&P 500 Total Return	-2.04%	17.21%	-0.27
Static Portfolio	1.02%	13.46%	-0.12
Dynamic Optimization	7.41%	3.85%	1.22

¹⁴ Fox, Kostin, Maasry and Sneider. "US Weekly Kickstart." February 26, 2010. Goldman Sachs Portfolio Strategy Research (portal.gs.com).

Chart 4: Portfolio Weights of Market Indices Using QP Optimization



The example illustrates the return potential of dynamic portfolio optimization. The best performing strategy over this time period was the Barclays US Aggregate Index which had a Sharpe ratio of 0.82. The static portfolio produced a Sharpe of -0.12, while the dynamic portfolio produced a Sharpe of 1.22. The Sharpe ratio of the portfolio was greatly increased by utilizing dynamic portfolio optimization. There were no bounds placed on each asset within the portfolio, however. This is not a prudent practice for dynamic optimization as the positions in a portfolio can become extremely concentrated, as shown by the portfolio weights above. As such, tactical strategy bounds should be utilized to eliminate the potential risk of portfolio concentration.

The out-of-sample optimization process identified qualitative strategy bounds to decrease the concentration risk of the portfolio. The goal of the optimization process was to maximize the Sharpe ratio of the diversified portfolio with a penalty for marginal risk contribution. Based on the in-sample performance of each strategy, the following upper and lower bound parameters were used:

Table 6: Upper and Lower Strategy Bounds for the QP Optimization Process

Strategy	Upper Bound	Lower Bound
Mean Reversion	100%	25%
SPY	25%	2.50%
XLF	25%	2.50%
XLE	25%	2.50%
XLK	25%	2.50%
XLP	25%	2.50%
XLY	25%	2.50%
XLB	25%	2.50%
XLI	25%	2.50%
XLV	25%	2.50%
XLU	25%	2.50%

The portfolio was optimized at the end of each month using the returns from the previous 252 trading days. There was no transaction cost penalty enforced in the optimizer, although the addition of transaction costs may have had a material impact on the results. As such, we assume portfolio changes are made for the following day with an immediate effect. There was no leverage or shorting strategies in the optimization, but the mean reversion returns already include the 4x gross leverage. The results of the portfolio optimization are shown below with monthly portfolio rebalancing.

Chart 5: Portfolio Weights Using QP Optimization

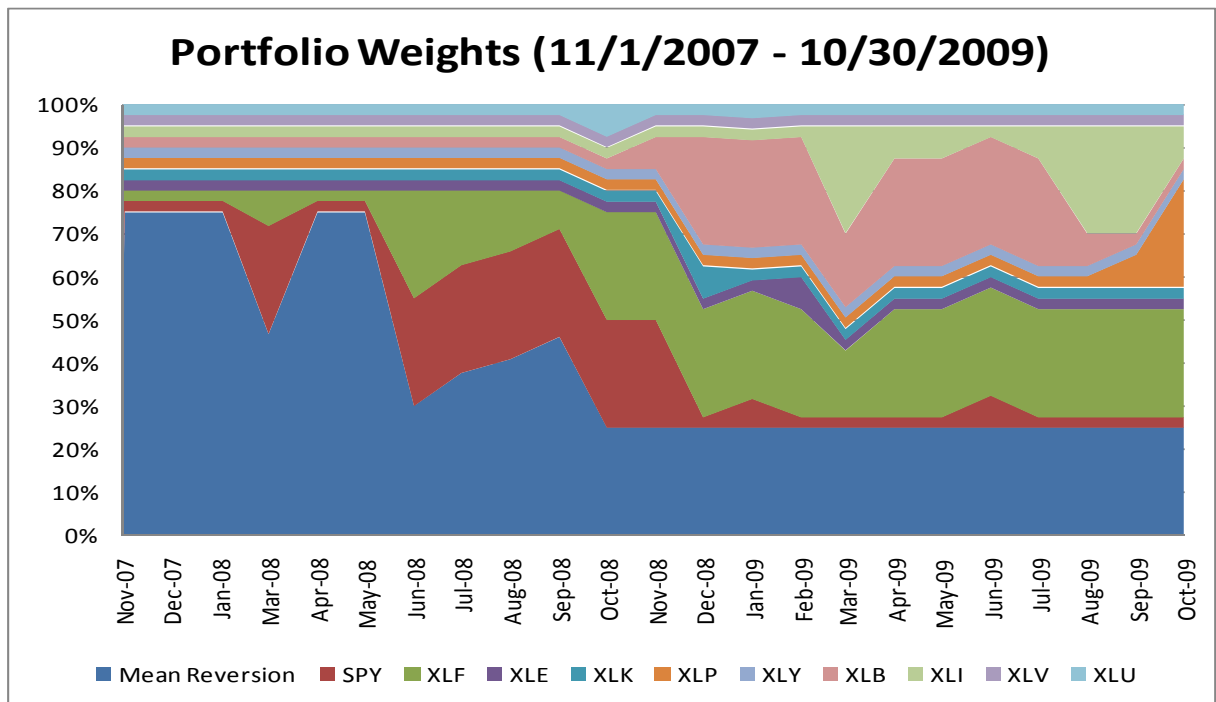


Table 7: Out-of-sample Returns (11/1/2007 – 10/30/2009)

Strategy	Annualized Returns	Annualized SD	Sharpe	SPX Beta
Mean Reversion	12.42%	15.42%	0.75	0.30
SPY	38.00%	35.18%	1.06	-0.67
XLF - Financials	62.20%	68.49%	0.90	-0.93
XLE - Energy	4.70%	50.88%	0.08	-0.53
XLK - Technology	10.12%	34.68%	0.27	-0.43
XLP - Cons Staples	17.70%	21.23%	0.79	-0.28
XLY - Cons Discr.	6.90%	38.55%	0.16	-0.63
XLB - Basic Materials	19.19%	41.70%	0.44	-0.57
XLI - Industrials	32.43%	35.86%	0.88	-0.82
XLV - Healthcare	4.11%	25.52%	0.13	-0.23
XLU - Utilities	6.71%	29.85%	0.20	-0.30
Diversified Portfolio	39.15%	16.84%	2.27	-0.37

Table 8: Index Returns (11/1/2007 – 10/30/2009)

Index	Annualized Returns	Annualized SD	Sharpe
S&P 500	-16.23%	22.98%	-0.74
HFRI Equity Hedge	-7.24%	13.29%	-0.61
HFRI Composite	-3.61%	9.80%	-0.45

Chart 6: Out-of-sample Growth of \$100 (11/1/2007 – 10/30/2009)

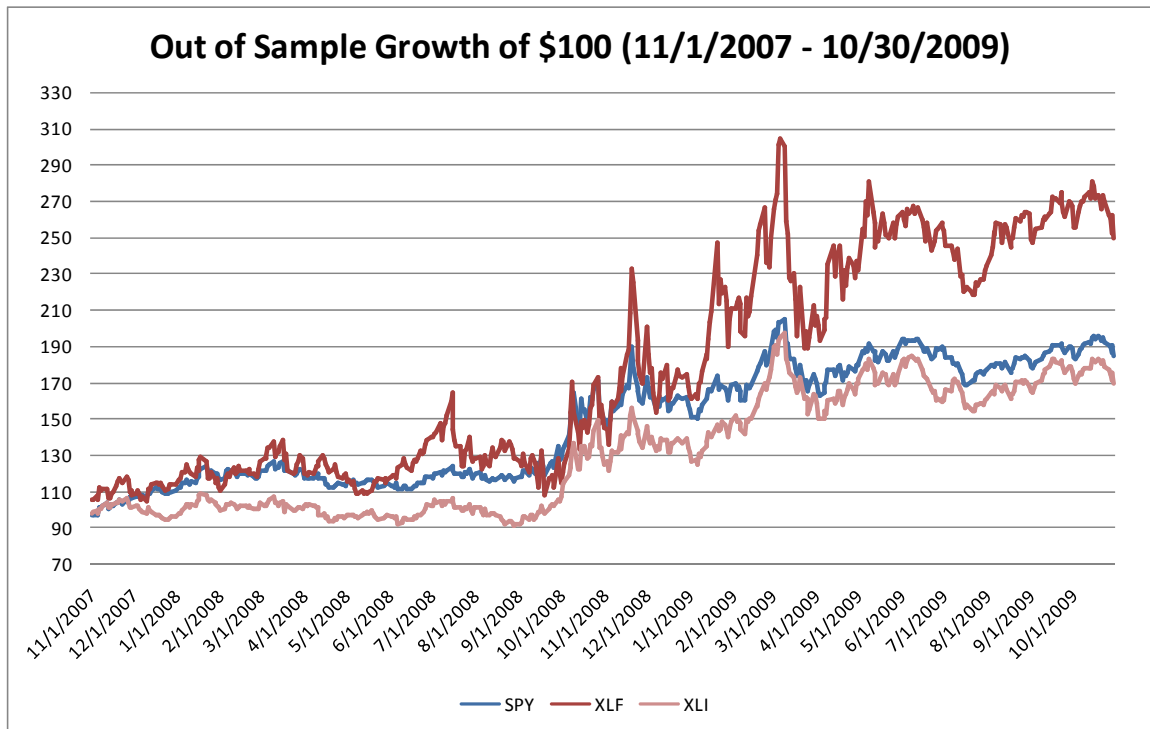
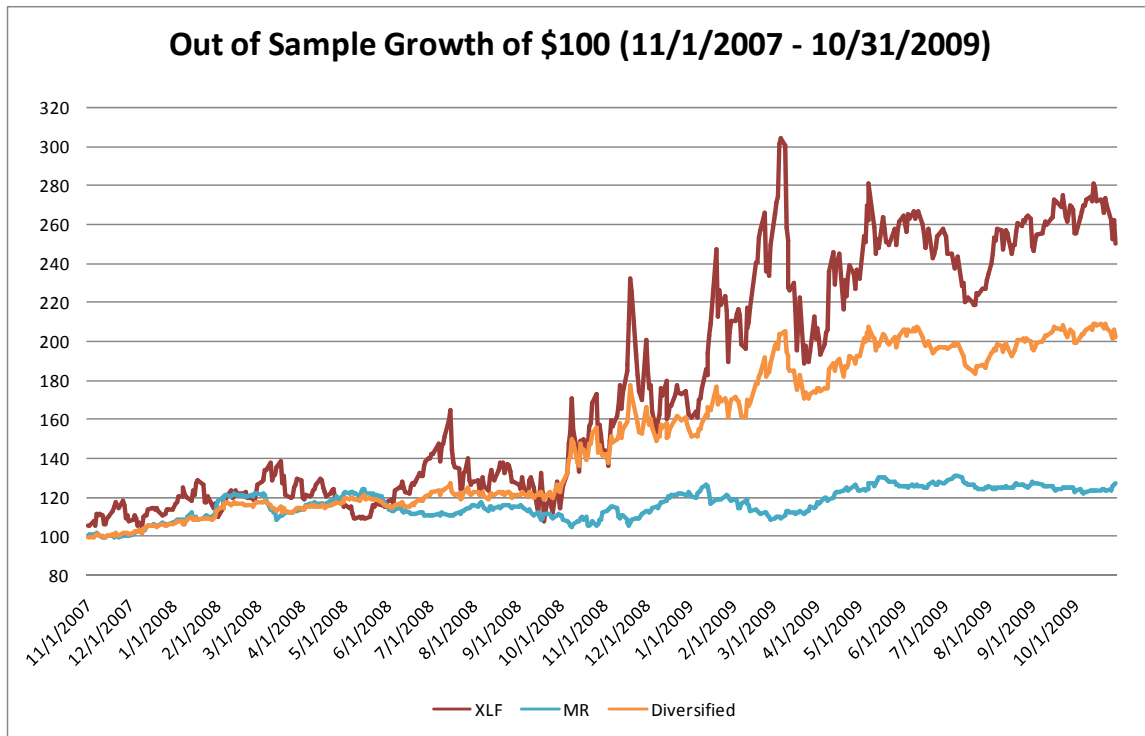


Chart 7: Out-of-sample Growth of \$100 (11/1/2007 – 10/30/2009)



The out-of-sample results of the diversified portfolio are much stronger than the in-sample results. This is largely due to strong performance by the momentum strategies, and the dynamic portfolio optimization. The SPY had a Sharpe ratio of 1.06, XLF was 0.90, XLI was 0.88, and XLP was 0.79. These momentum strategies outperformed the mean reversion strategy which had a Sharpe of 0.75 over the period. The strategies performed particularly well during the financial crisis in Q4 of 2008. Over the out-of-sample period the benchmark indices were all negative. The SPX had the worst performance with a Sharpe of -0.74, followed by the HFRI Equity Hedge Index (-0.61), and the HFRI Composite Index (-0.45). The flight from risk hurt these indices, but appears to have created a better environment for the momentum strategies.

The Sharpe ratio of the diversified program was very high at 2.27 (1 month T-bill is the risk free rate) indicating that the portfolio optimization helped immensely with the diversified returns. The SPY had the highest stand alone performance with a Sharpe ratio of 1.06. As such, the diversified portfolio more than doubled the Sharpe ratio of the best performing strategy. It also decreased the annualized standard deviation to 16.8%, which is lower than the stand alone momentum strategies.

If we had kept the portfolio weights at 50% mean reversion and 5% each of the momentum strategies, the portfolio would have had a Sharpe ratio of 1.56. The betas of each strategy, however, were extremely high. As mentioned previously, we did not use an overlay hedge for the portfolio and, as a result, we had a positive beta for the mean reversion strategy of 30% and a strongly negative beta for the diversified portfolio of -37%.

Table 9: Diversified Daily Performance (3/10/2009 – 3/26/2009)

Date	Diversified
3/10/2009	-5.17%
3/11/2009	-1.18%
3/12/2009	-3.23%
3/13/2009	-0.55%
3/16/2009	-0.35%
3/17/2009	-2.57%
3/18/2009	-2.51%
3/19/2009	1.42%
3/20/2009	2.78%
3/23/2009	-6.40%
3/24/2009	1.52%
3/25/2009	-0.81%
3/26/2009	-0.95%
Total Drawdown	-16.93%

The diversified portfolio had a significant drawdown starting on March 9, 2009. This was the market bottom for all of the major US equity indices, and the momentum strategies

were positioned short. As a result, the diversified fund was down 16.9% peak to trough over a 13 trading day time period. We also can view this as a positive because it only took the strategy 14 trading days to recalibrate based on the changed market conditions. The period immediately prior to the drawdown had strong performance, and as such, the total performance in March of 2009 was -8.2%. Additional risk constraints within the optimization process would have helped avoid this drawdown. If the diversified program had beta and volatility constraints, the drawdown would not have been so severe. Fortunately, the NAV was recovered by the first week in May.

IV. Conclusion

The results of the out-of-sample analysis indicate that there are potentially significant benefits to including both mean reversion and momentum models in quantitative trading platforms. Currently, there are a number of stand alone mean reversion hedge funds that do not include momentum trading strategies, but may be overlooking the potential diversification benefits.

Unlike Avellaneda and Lee, we did not hedge the beta risk using the SPY, and found that there was beta exposure for the mean reversion strategy in both the in-sample (-28%) and out-of-sample (30%) results. We also found that using a 12 PCA strategy had negligible effects on the performance of the model, but using a shorter mean reversion requirement appears to have decreased the average holding period for an investment. The analysis done by Avellaneda and Lee on August 2007 claims that the dislocations were the

greatest for the Technology and Consumer Discretionary sectors. We did not find enough evidence to confirm or refute these claims.

Threats to the Internal Validity

The momentum strategies used in this analysis were kept simple by design, but there is credence to using a more complex approach. We conducted preliminary analysis to estimate a momentum signal using the PCA eigenportfolios, but found that momentum was not apparent at the individual stock level. A more sophisticated model for momentum would likely produce stronger out-of-sample results. Neither the mean reversion nor the momentum strategy modeled transaction costs. We simply deducted a flat 10 bps transaction cost constant from the trading results, but this does not account for slippage, path dependency of stock returns, or the liquidity of each underlying stock. The liquidity element would not have had an impact on the ETF momentum strategy, but could have had an impact on the mean reversion strategy. The mean reversion model did not analyze volume data, and hence, could potentially end up suffering significant liquidity penalties. The drift value, α_i , in the mean reversion model was also ignored despite the possibility that it has a meaningful impact on stock returns. Avellaneda and Lee found that the drift rate did not have a meaningful impact and averaged 10-15 bps.¹⁵

Future Research

Although we have shown the potential benefits of combining mean reversion and momentum trading strategies, a more robust approach could be used to maximize the Sharpe ratio of the diversified portfolio. Trading momentum with the PCA

¹⁵ Avellaneda and Lee (2009).

eigenportfolios would potentially create less volatile and more consistent returns. The SPX beta of the diversified portfolio was close to zero for the in-sample analysis, but in the out-of-sample analysis the beta was sharply negative (-37%). As such, there may be additional risk parameters that can be implemented in the model specification. The dynamic optimization helped maximize the Sharpe ratio for the out-of-sample analysis, but could have included more thoughtful strategy bounds. We also find that there is potentially greater alpha on finer time scales, and we are working on future research that evaluates the effect of varying time scales with signal decay for both momentum and mean reversion strategies.

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