# Fourth Homework, DC Motor, Cart Pendulum, Decoupling

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## Abstract

The homework is structured in three problems.
First problem is about the comparison of different metho controller performances of a DC motor dynamical system, under uncertainties in the model and in the load.
Second problem is about the stabilization of an inverted cart pendulum system and comparing its performances with the SISO case.
Third problem is concerning about the reproduction of the Garrido’s paper, that deals about time delayed system and decoupling.

**Keywords**: DC Motor, Loop Shaping, Mu-synthesis, Inverted Cart Pendulum, Decoupling, MIMO

## 1. First Problem – DC Motor

In the previous homework I have considered a motor model taking the parameters from Faulhaber motor. There was an error in the conversions and all the previous plots were in rad/min instead of rad/s. I have changed the parameters; new source is in the Matlab file.

The motor is now modelled in Matlab using a manual interconnection, specifying each time the inputs and outputs of the systems.
Once the open loop nominal plant is built, I have built manually the augmented plant and applied a Hinfsyn on it, by considering differently control inputs and exogenous inputs, as well as measurements signals and performance outputs. After controller has been founded, I closed the loop and defined some fake blocks to explicit the output signals I would like to observe.

The nominal plant system response is plotted with respect to each input. I have considered a reference of 15 rad/s, a constant load torque and a sinusoidal noise input with an angular frequency of 1000 rad/s. Below are shown the output of the system (in rad/s with respect to the seconds) and the control signal (in Volt) with respect to each input during time (in seconds).

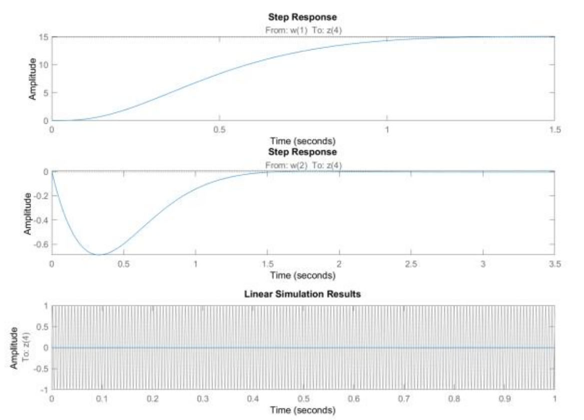


*Fig. 1 System Output Response (r-y) (Tl-y) (n-y)*

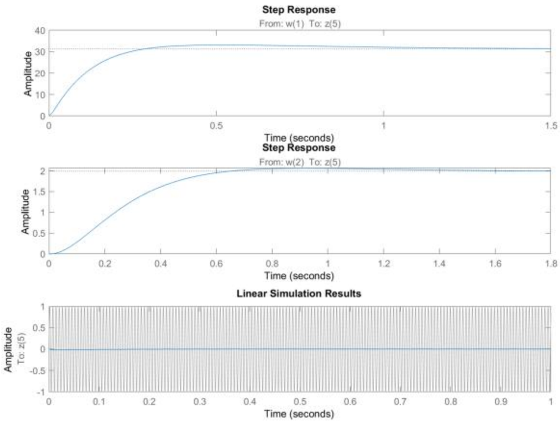


*Fig. 2 Control Signal*
*(r-u) (Tl-u) (n-u)*

Then I have built in the same way the uncertain system, considering perturbations in the parameters and in the load torque; then I applied the same “nominal plant-based” controller on the uncertain plant to found that it still robustly stabilized by it.

Then I synthetized a new controller using Mu synthesis, closed the control loop and analysed the robust performance of the closed loop system against parameter variations. Finally, I have found the worst case and found the corresponding Hinfinity norm of the closed loop system.
Having set a performance gain of 3, the system performances appear to be robust against the variations.

Below are shown the controlled system’s output with respect to each input. A comparison has been done between the controller designed with *hinfsyn (in blue) the musyn controller (in red) and the nominal plant with musyn controller (in green). All the y-axis values are in rad/s and the time unit is the seconds.*

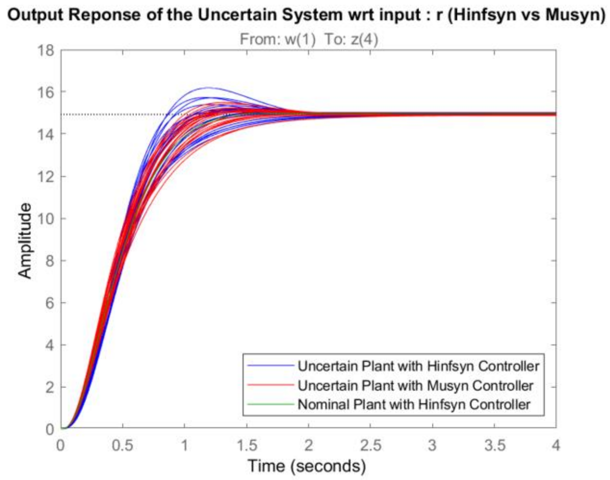


*Fig. 3 Closed Loop Uncertain System Output (r-y) Comparison*

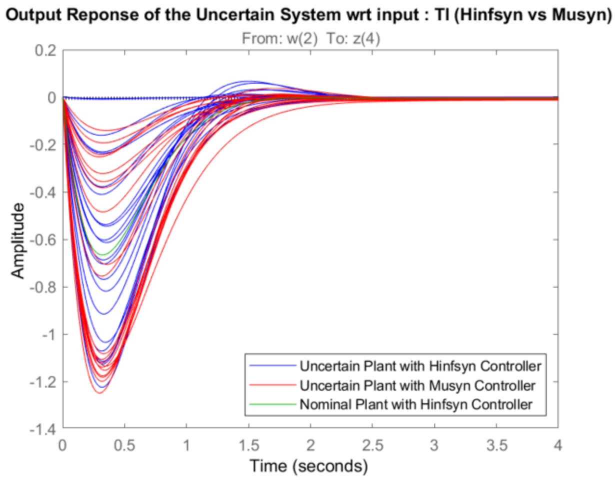


*Fig. 4 Closed Loop Uncertain System Output (Tl-y) Comparison*

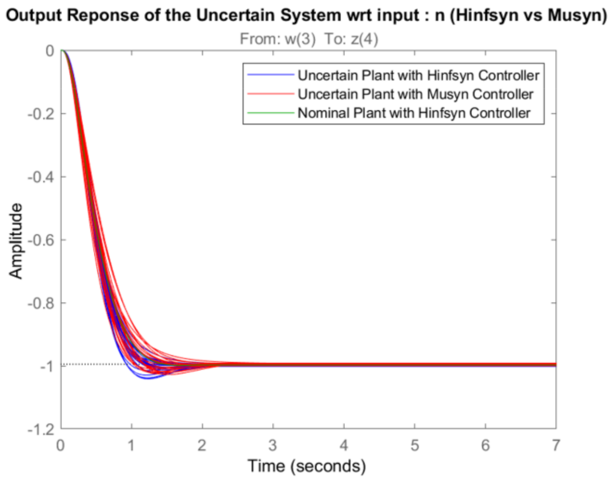


*Fig. 5 Closed Loop Uncertain System Output (n-y) Comparison (constant n, not sinusoidal)*

Morover, having set a parameter percentage variations up to 10% and a load torque variation up to 100%, system can tolerate up to 509% of the modeled uncertainties, and the destabilizing values for the parameters are Listed in Matlab.

The worst perturbation reference-output behavior are compared with the nominal one in the following plot.

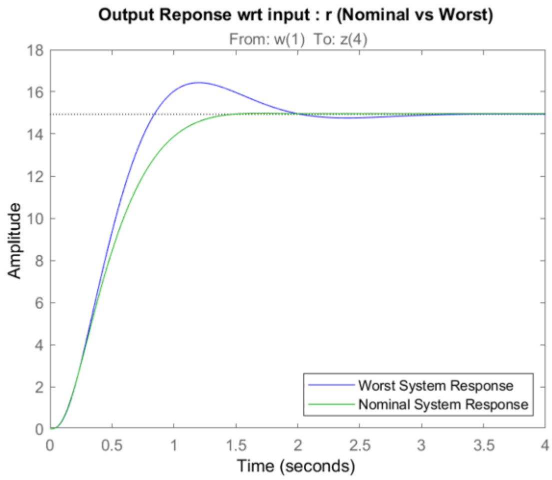


*Fig. 6 Closed Loop System Output (r-y) Comparison*

## 2. Second Problem – Inverted Cart Pendulum

The second problem deals with the stabilization of a S.I.M.O inverted cart pendulum system, and the comparison with the S.I.S.O case.

The plant required in the homework aim has been created in Matlab, after having built the S.I.M.O system with two outputs: the pendulum angle with respect to vertical position and position of the cart on the ground.
This is really differenct then considering the SISO problem, cause the transfer function to the control input to the position has a non minimum phase zero and it is unstable, with some values really near each other. In the SIMO case instead, the zero of the system disappear, cancelling the limitation in the sensitivity desired bandwidth.

The interconnected unstable system has been built and its zeroes and poles analyzed in Matlab. Then I've chosen the weights, built the augmented plant and founded a stabilizing (1x2) controller using hinfsyn.

The response of the system in term of the two outputs and control effort are shown below.

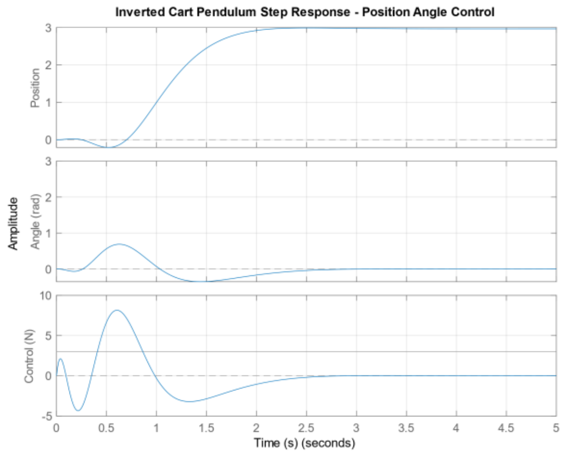


*Fig. 7 Closed Loop System Output and Control Effort Step Responses*

In term of performances, what we should analyze are the shapes of the Sensitivity and Complementary Sensitivity and their peaks, the lower it is and the more the positive area is spread, the better the performances.

In the SIMO case, from the second waterbed effect, since we have not anymore any transmission zeroes, the trade off between the positive area and the negative area of the sensitivity function is not anymore restricted to a limited range of frequencies (this is what happened instead in the SISO case).

In the Matlab file, I tried to found a controller that solve the problem with the same specifications, but mixsyn can't found that, so I lowered the performances and compared the shapes of these functions in the plots below.

In red we have the SISO system, in blue the singular values of the MIMO system.
From the plot below we can clearly see that the upper singular value of the MIMO has a smaller peak then in the SISO case.

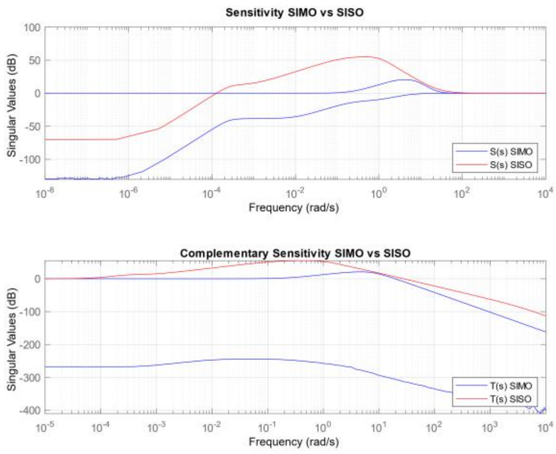


*Fig. 8 Sensitivity and Complementary Sensitivity ( SISO vs MIMO)*

### 3. Third Problem

The aim of this chapter is to analyze the example 4.1 of paper [1].

The model represented deals about an industrial-scale polymerization reactor. The time unit is the hour and the two control variables are the setpoints of the two reactor feed flow loops.

In our system we have no RHP zeroes, but there are some delays that has a bad effect on the gain margin of the system. Unfortunately, decoupling tecnique is not directly applicable case the stronger NMP terms act along the second input channel.
In the state space formulation of the problem this may be due to the fact that the decoupling matrix of the system is not invertible, or we don't have a strong relative degree condition. This kind of situations are solved in the nonlinear context as well, by some algorithmic tecniques like dynamic extensions. The idea is to add a delay along one or more input channels in order to achieve a strong relative degree, by retarding some input that affect the system too fast with respect to the others.

In order to achieve realizability, the first input-outputs channel is delayed by a factor of 0.2.
Here is shown the effect of this extra delay in the process response:

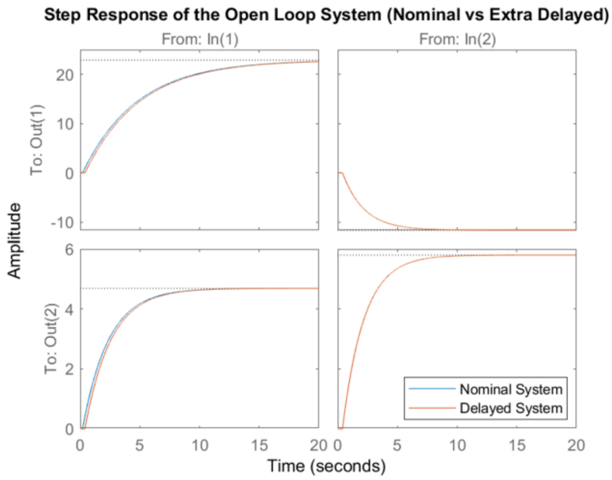


*Fig. 9 Open Loop System Step Response*

After the determinant of the new matrix is checked to be different from zero and after that we have noticed that we don't have any non minimum phase zero, the decoupling configuration (1-2) is chosen.
A diagonal identity structure is given to the matrix Dd(s) and the correspective matrix Do(s) is the following:

$$Do = \begin{pmatrix} 0 & -g12/g11 \\ -g21/g22 & 0 \end{pmatrix}$$

Once the decoupling matrix is found and system is decoupled, we can find two different controllers to the two input outputs links, avoiding coupling effects.
In the papers some controllers and control strategies are compared as well as their performances.
This comparison is shown in the plot below.
In green we have the process decoupled using inverted decoupling vs the non decoupled (dashed green). In blue and red we have the control schemes proposed by Xiaong and Lee.

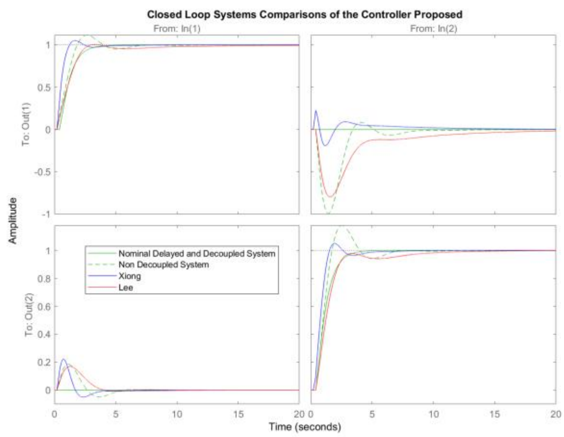


*Fig. 10 Proposed Control Schemes in the Paper*

I would like to highlight how the input-output step response behavior of the nominal decoupled and delayed closed loop process is better in term of coupling effects, with respect to the same nominal process, but not decoupled and without the extra delay block.

In the figure below the two plots in position (1,2) and (2,1) highlight how in the decoupled process (in green) is free from the coupling effects.

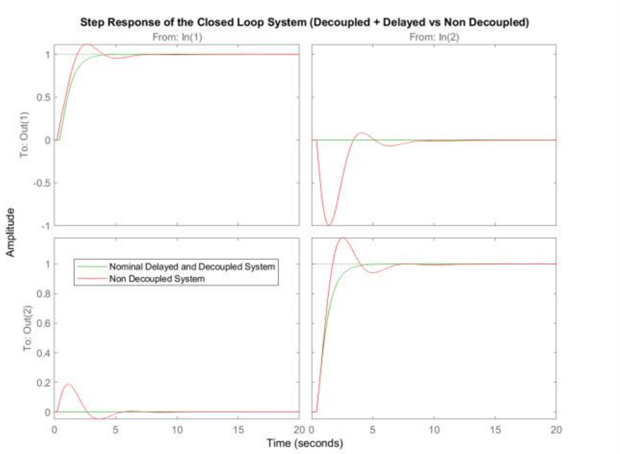


*Fig. 11 Closed Loop System Step Response (Decouped vs Nondecoupled)*

After that, I have constructed a uncertain augmented plant using two weights matrices, one to weight performances of the system and the other to represent an input multiplicative uncertainty.
After I have compared the performances of the closed loop decoupled system with the nominal one and analyzed robust stabilization and performances of both in the Matlab script.
The non decoupled plant appears to have about half of the stability margins, and its performance level is not robust to the model uncertainties.

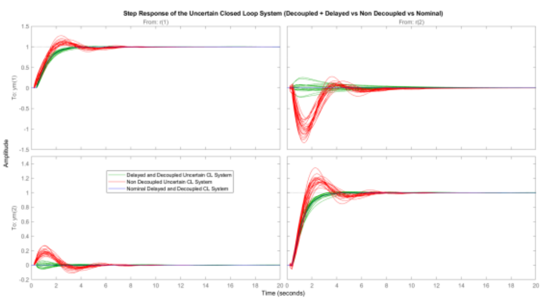

*Fig. 12 Closed Loop Uncertain System Step Response*

From the previous plot appears clearly evident the importance of this topic in the feedback control strategies for mimo systems.

## References


[1] J. Garrido et al. / Journal of Process Control 21 (2011) 55–68