Third Homework, MIMO and SVD, Distillation Process

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Abstract

The homework is structured in two problems. The first one deals with the modelling of a SISO system that, considering some exogenous inputs, like input and ouput disutrbances and noise , became a MIMO system.

Keywords: MIMO, Singular Value Decompositio, Unstructured Uncetainties, Robust Stabilization, Loop Shaping

1. First Problem

Given the following plant transfer function:

$$G(s) = \frac{10}{(s+2)*(s+10)}$$

We need:

- Zero steady state error with respect to a constant reference
- Good disturbance rejection for both input and output sinusoidal disturbances d1 and d2 with frequencies up to 5 rad/s
- Sinusoidal measurement noise in the range [100, 1000] rad/s is sufficiently attenuated
- Satisfactory transient behaviour with respect to maximal overshoot allowed and settling time

In general, for the first point, we could obtain zero steady state error with respect to a constant reference by using an integral action.

Anyway to simplify the controller synthesis and obtain better performances, we will synthetize a controller using loop shaping.

We will use two weights on the sensitivity and complementary sensitivity functions let the loop function has a desired shape that mimic the inverse of the sensitivity at low frequencies and the complementary sensitivity at high frequency, doing so we will reject disturbances and noise. We will not consider control input minimization in the following scheme.
The weight parameters has been specified in Matlab.
To simulate the behavior of the system a Simulink model has been constructed.
Is important to say that in this example we are not considering a weight on the control action, that could imply more strict constraints and worst performances. This consideration could imply a big peak in the control.

The chosen specifications with respect to the transient behaviour are the following:

Bandwidth T(s)	**Crossover f. L(s)**	**Rise Time**	**Settling Time**
> 40 rad/s	> 35 rad/s	< 0.06 s	< 0.1 s
41.7961 rad/s	39.9912 rad/s	0.0524 s	0.0944 s

Steady State Disturbances Attenuation	**Steady State Error**	**Steady State Noise Attenuation**
< 1/10 of Amplitude	|e| < |0.3|	< 1/10 of Amplitude
MET	About |0.20|	MET

Some of them has been checked in Matlab, others by direct inspections.

Below we will show the response of the internally stable open loop system (in yellow), the PID controlled system (in red) and our system controlled with mixsyn synthetized controller (in blue).

In the first plot we will consider the closed loop system output without disturbances, in the second plot we will add the input and output disturbances and show the results, and in the third one a deterministic sinusoidal noise will be considered without considering disturbances.
Finally in the 4th plot we will consider all the combined case of study, talking about specifications.

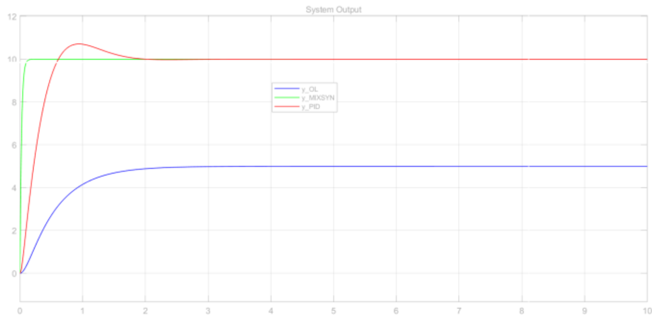


Fig. 1 System output without noise and disturbances

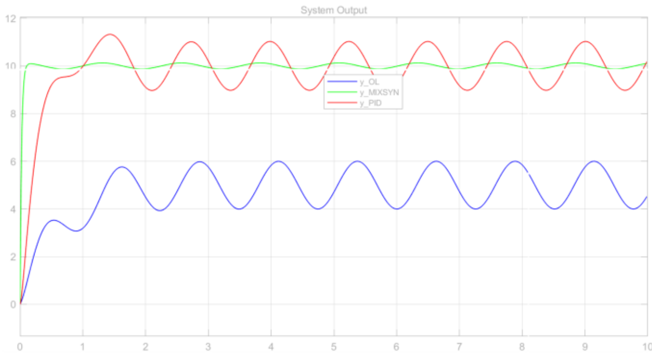


Fig. 2 System output without noise

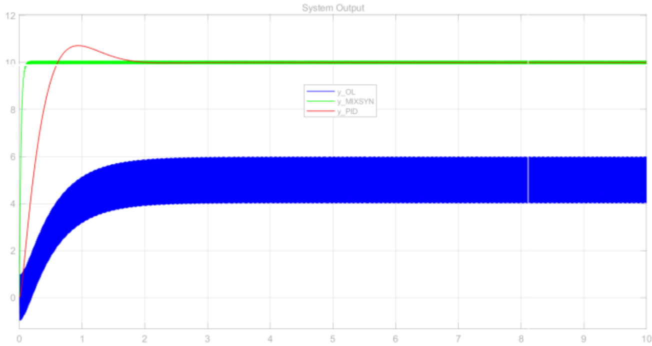


Fig. 3 System output without disturbances

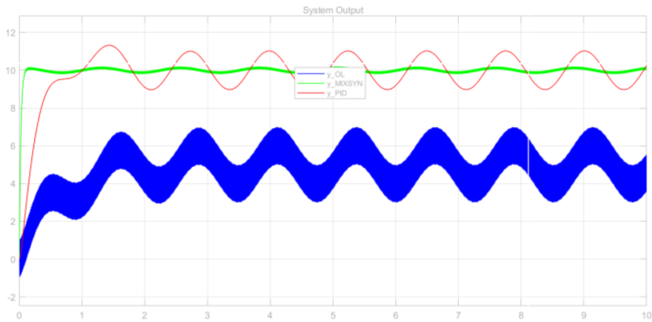
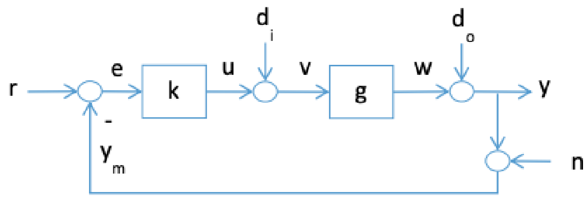


Fig. 4 System output with noise and disturbances

Now we will consider the MIMO system that follows:



This kind of structure can be considered as system with four inputs and three outputs, the output, the error and the control action.
The following (4x3) transfer function relates the inputs with the outputs, using the definitions of input and output sensitivity function and input and output complementary sensitivity function.

$$W(s) = \begin{pmatrix} To & G*So & So & -To \\ So & -G*So & -So & -So \\ C*So & -Ti & -C*So & -C*So \end{pmatrix}$$

The matrix for the specific controller founded can be obtained in Matlab.

Is important to notice that we are not yet considering a model of the disturbances and respective transfer functions.

The static gain of the matrix W(s) is the following:

$$W(0) = \begin{pmatrix} 1 & 4.995e-07 & 9.99e-07 & -1 \\ 9.99e-07 & -4.995e-07 & -9.99e-07 & -9.99e-07 \\ 2 & -1 & -2 & -2 \end{pmatrix}$$

We can perform a singular value decomposition on the static gains matrix of the system to obtain the singular values ordered on the diagonal in the second output matrix S, and their respective directions for the input-output links in the matrices U and V.

Since we are dealing with a nonsquare system we will have a redundant column in the S matrix.

The first output of the Matlab command will be U, a matrix in which the columns indicate output directions of the respective gains in S.

The third output will be V, a matrix in which each j-th column represent the input directions that can guarantee the gain on the j-th column of S. In our case the fourth column will represent a direction for the input in which the system will produce zero output.

Our condition number is very high, this means that the gain of the system is strongly dependent on the input direction.

Since we are not modelling yet the disturbances transfer functions, our steady state gain of the “fake” MIMO system will be consistent with the SISO system specifications.
In fact from the element in position (1,1) in the W(0) matrix, we could see that the gain at steady state from reference to output is equal to one, that is consistent with the fact that the steady state gain of the complementary sensitivity function of the SISO system is one at steady state.

2. Second Problem

In this problem we have modeled a four-tanks system with two water pumps input and two output. Our MIMO system is square and internally stable, but it is a nonminimum phase system.

First of all we have created an augmented plant with two tunable PI, in parallel between themselves, and in series with a decoupler. Then we’ve set the desired closed loop system bandwidth and obtained two PI controllers using looptune Matlab command and inserted all these components in series with the MIMO plant.

A Simulink model has been built to compare the results obtained with the two parallel PI, and a controller synthetized using loop shaping design method.

The weights relative to the Sensitivity and Complementary Sensitivity are chosen such that the crossover frequency of the loop function lies between 0.002 and 1 rad/s. This choice leads to faster settling time of the plant compared to PI.

After that a nonlinear model of the plant has been built and the controller has been tested on that, achieving the desired tracking of the nominal values also in nonlinear case.

The two plots below shows the output of the linearized system controlled with PI with respect to the system controlled with loop shaping synthetized controller, and the output of the linearized system controlled with the second controlled compared with the outputs of the nonlinear plant with the same controller.

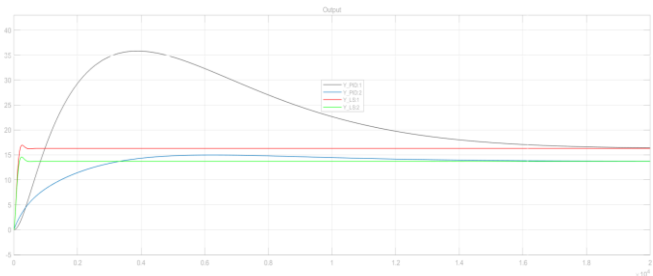


Fig. 5 System output, PID vs Loop Shaping

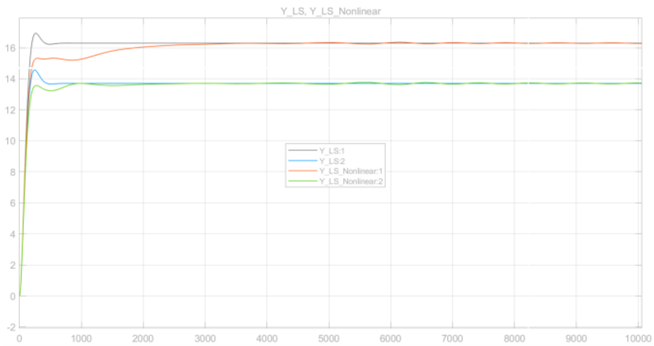


Fig. 6 System output with Loop Shaping, Linearize vs Nonlinear

Now we are going to define an uncertainty in the inputs of the system, in particular we will assume a 10% percentage variation in the parameters concerning the valve constants and pump proportionality constants.

After that we will consider our new uncertain plant and assume an unstructured case for the uncertainties as did in paper [3].

The plot below show the variations of the singular values of the system when he add uncertainties.

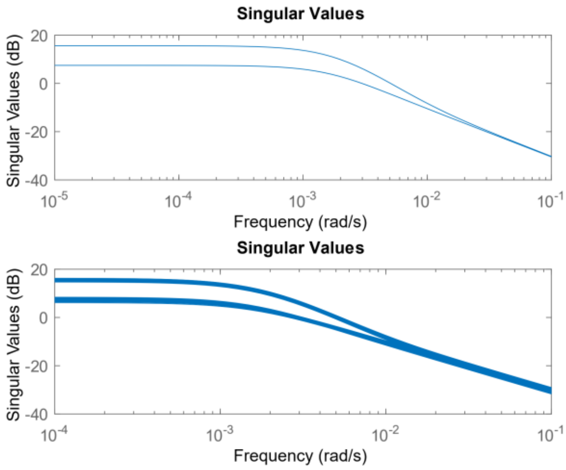


Fig. 7 Singual Values, Nominal vs Perturbed

From the following plot we can have a confirmations that the weight chosen to represent the uncertainties is consistent, since it must be a sup of the maximum singular value plot of the inverse of the plant, multiplied by the difference between the family of perturbed plant minus the nominal one.

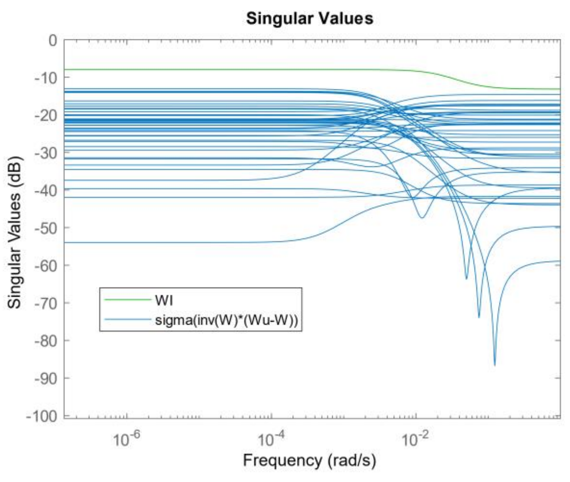


Fig. 8 Choice of the Weight for Multiplicative Uncertainty

Now that we have this information we could use that to build the augmented plant using linear fractional transformations.

Since we don’t need to build that explicitly to check the robust stabilization of the plant we will use the Nyquist plot of the perturbed loop function to investigate the stability.

Another condition required is that the hinfinity norm of the Delta block is less or equal then one, and this result is checked in Matlab.

Since the nominal plant is internally stable and stabilized by loop shaping controller, we should check that the perturbed cascade loop function do not encirle the point (-1,0) in the Nyquist Plot.

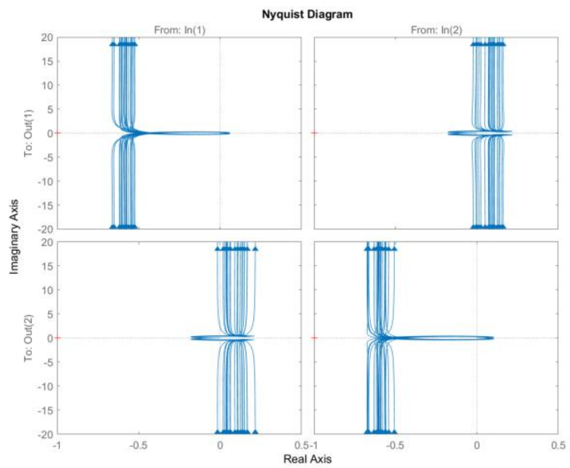


Fig. 9 Nyquist Plot of Perturbed Cascade Loop Function

The plot above shows that in the worst case we have no encirlements, so the robust stability condition is satisfied.
Morover we can have a look at the step response of the nomial and perturbed system and see that we have stability for all the perturbed response.

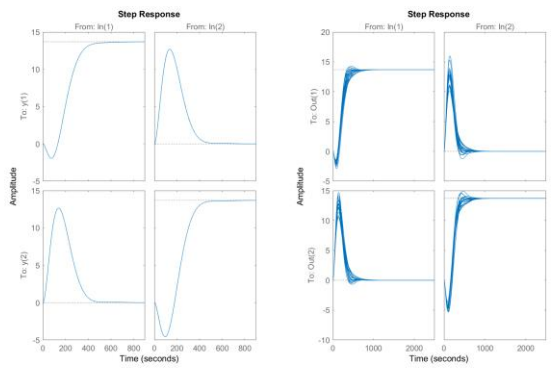


Fig. 10 Step Response, Nominal vs Perturbed Plant

Now we will try to obtain Robust Performances.

References

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[3] R. Vadigepalli, E. P. Gatzke, and F. J. Doyle. “Robust control of a multivariable experimental four-tank system.” Industrial & engineering chemistry research, vol. 40.8, 2001, pp. 1916–1927.