Control System for Speed and Position of an Armature Controlled DC Motor

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Abstract

The aim of this paper is to analize the linear S.I.S.O. dynamics of a DC motor and design two controllers for a velocity control problem and a position control problem. The control system will be based on performance weights on the sensitivity, complementary sensitivity and control sensitivity functions. The result obtained will be compared with a PID, tuned in Simulink via Pidtuner.

Keywords: DC Motor, Sensitivity Function, Performance Weights, PID, Control System

1. Model of the DC Motor

An armature controlled DC motor is a direct current motor that uses a permanent magnet driven by the armature coils only. It converts electrical energy in to rotational mechanical energy. The motor consists in a stator, a fixed part and a mobile rotor. The current flowing in their windings, and consequently the magnetic flux generated, results in the motion of the rotor.

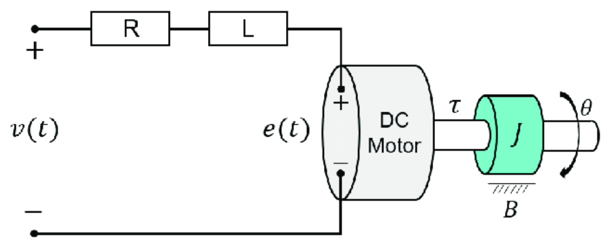

Fig. 1 D.C. Motor

The governing equations of the DC motor dynamics are the following:

$$J_m\ddot{\theta} + B_m\dot{\theta} = K_t i \tag{1}$$

$$L_a\frac{di}{dt} + R_a i = V - K_e\dot{\theta} \tag{2}$$

Parameters of the motor are taken from [1].

1.1 State Space Model

First, the motor dynamical system is designed with state space method. The following linear time invariant model is obtained:

$$\frac{d}{dt}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} = \begin{bmatrix}\frac{-B_m}{J_m} & \frac{K_t}{J_m}\\ \frac{-K_e}{L_a} & \frac{-R_a}{L_a}\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L_a}\end{bmatrix}V \tag{3}$$

1.2 Transfer Function Model

The model design in the frequency domain is more intuitive since it considers the Input-Output behavior. The transfer function of the open loop plant, having angular speed as output and without disturbances is the following:

$$W(s) = \frac{K}{((J_m s+B_m)\cdot(L_a s+R_a)+K^2)} \tag{4}$$

1.3 Open Loop Behavior

This DC motor has a particular kind of transfer function, since one of its denominator coefficient is almost zero, so it’s behavior is similar to that one of a transfer function with a single left half complex plane pole. The bode plot and the response of the output to a nominal voltage input of 6 Volt is the following :

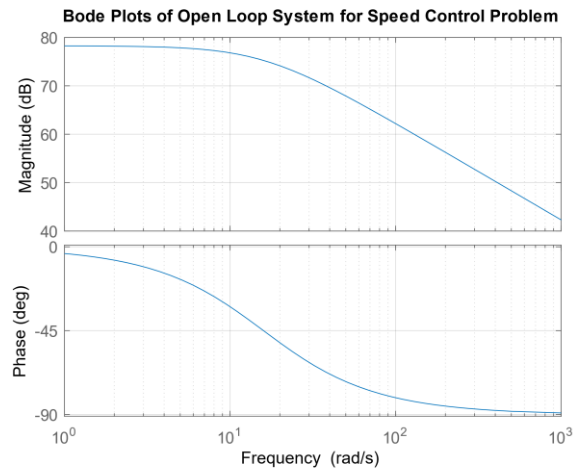

Fig.1 Bode diagram of Open Loop System (Speed Control)

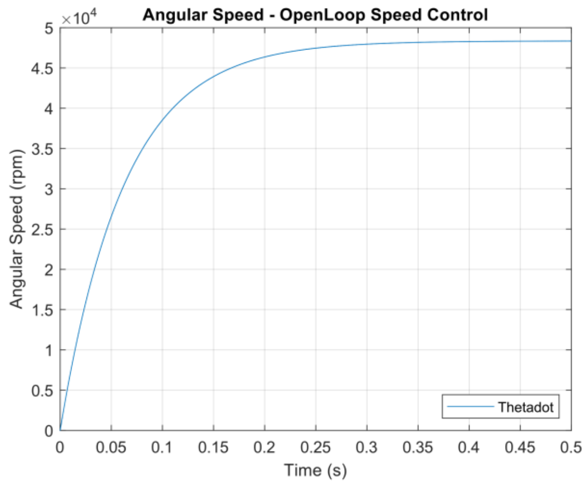

Fig.2 Output Step Response of the Open Loop System

From the last plot is clear that the motor speed with nominal input voltage is perfectly matching the no load speed indicated in the reference of 48600 rpm.

1.4 Performance Weights

The control technique described here is known as Mixed Sensitivity Loop Shaping and uses the Matlab command *mixsyn*, that, given a plant, computes a controller that minimize the H_∞ norm of the weighted closed loop transfer function. The role of the three weights is to give a hierarchy to the minimization priority of tracking error, disturbance rejection, robustness, noise attenuation and control effort, in some desired frequency ranges.

2. Velocity Control Problem

First, we are interested in rotating the motor at a fixed angular speed of 25000 rpm. In order have a smoother reference for the step input, in Simulink two exponential Matlab functions were created.

Then we considered in the model a constant load torque and a static friction as disturbances and also a white noise in the exit, to let the weights be effective.

The control action should be, at nominal values, around 6 Volt at steady state, while the current flowing in the armature of the stator should be around 0.3 Ampere.

2.1 Performance Weights Control System

The weights are created to reject DC disturbances and output noise, while limiting control effort to ensure a steady state current around 0.3 A.

The following shapes are obtained for the Loop Function, Sensitivity, Complementary Sensitivity, and respective weights functions.

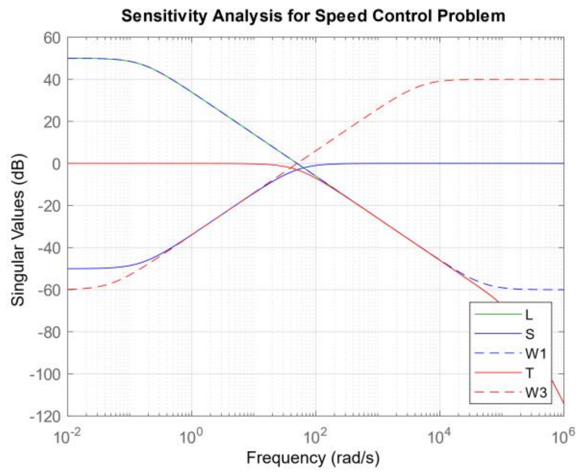

Fig.3 Mixed Sensitivity Analysis of Speed Control Problem

This shapes for the Sensitivity and Complementary Sensitivity highlight the role of the weighting functions and the fact that S(s) is required to be small outside the control bandwidth, while is the opposite for the T(s).

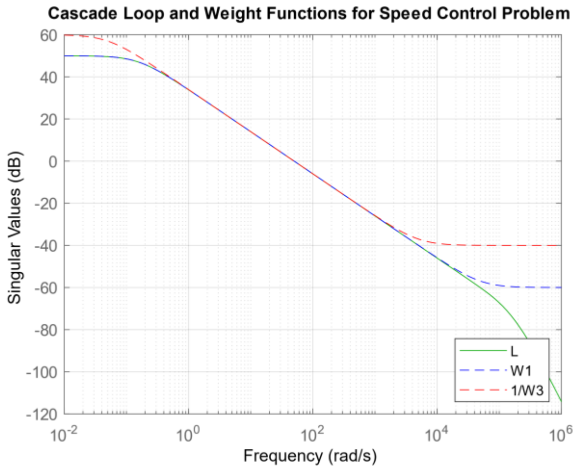

Fig.4 Closed Loop function

2.2 Comparison of the Results

The results obtained are compared with a PID controller tuned with the Simulink tool Pidtuner. In the following plots the Speed Output and Voltage Input of the two control systems are compared: (PID in red)

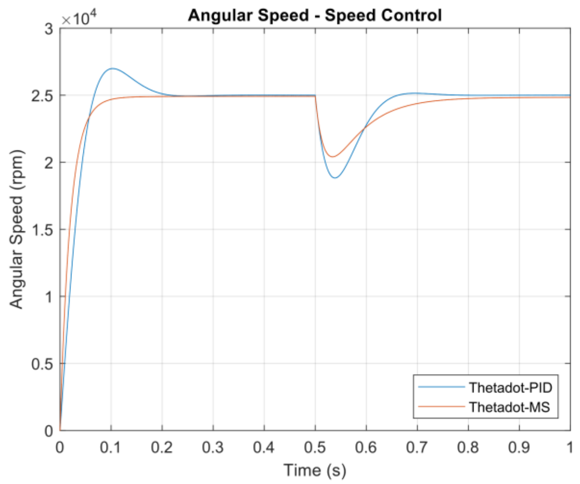

Fig.5 Angular Speed Output

Our steady state error will not be zero here because we included in the controller also a shape for complementary sensitivity, and a trade-off between performances will be present.

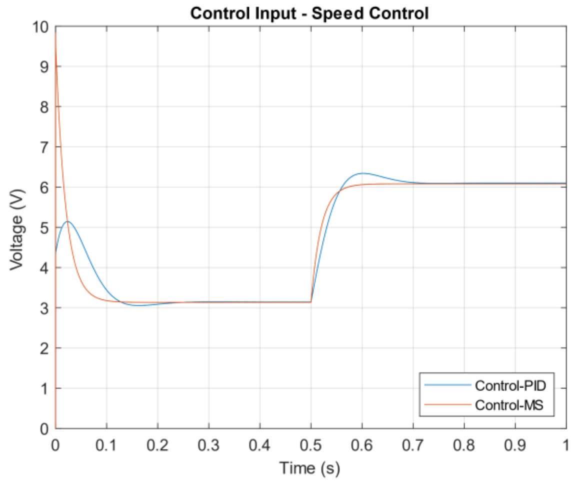

Fig.6 Voltage Input

The Voltage Input converge to the nominal value of 6 V, while the current inside the armature is around its thermal limit.

3. Position Control Problem

Now another control problem is considered by adding an integral action inside the plant, to obtain the rotor position instead its speed, and we would like to stop that at a given position. The same smoother reference is considered, as long as the disturbances and white noises. Is important to note that the integrator adds a pole in zero, decreasing the phase by 90 degrees.

3.1 Performance Weights Control System

After having choose the weights for the control system, the following results are obtained:

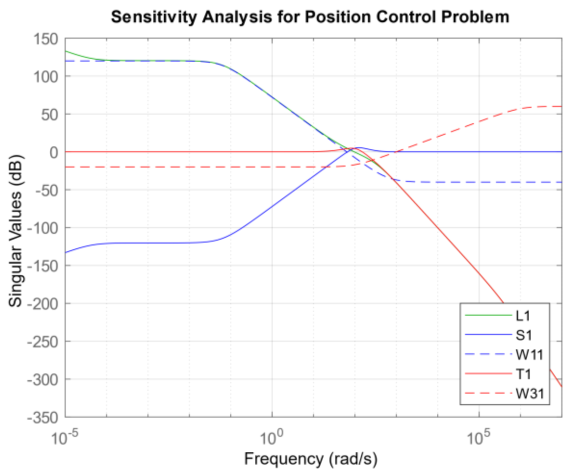

Fig.6 Mixed Sensitivity Analysis of Position Control Problem

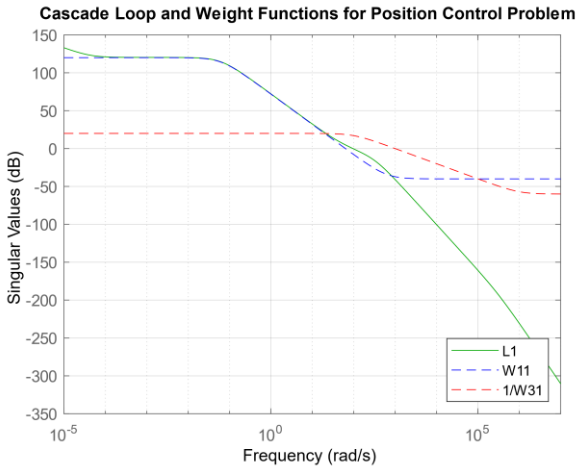

Fig.7 Loop function

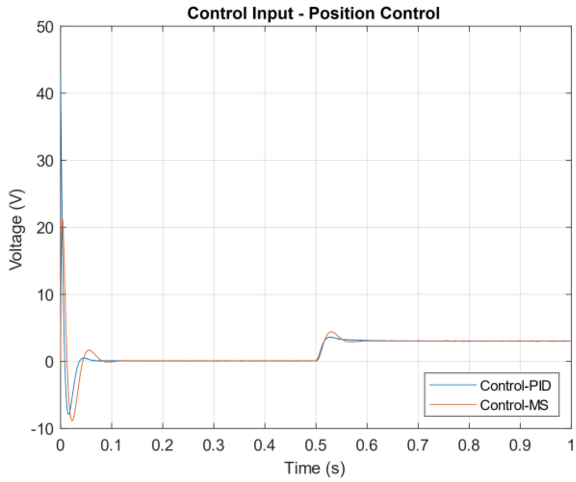

Fig.9 Voltage Input

3.2 Comparison of the Results

Results are compared with another PID controller, tuned with Pidtuner. The following results are obtained:

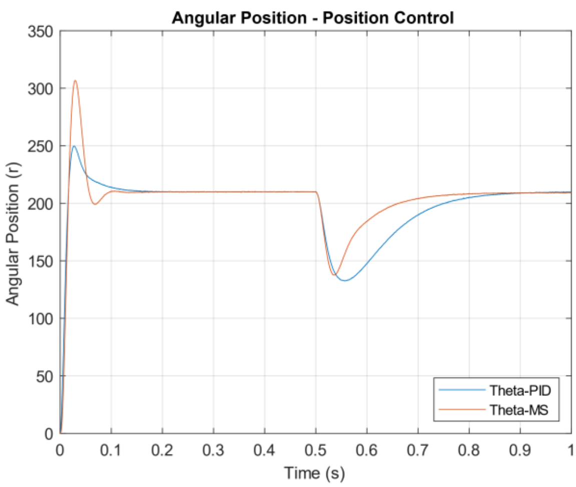

Fig.8 Angular Position Output

4. Simulink Control Schemes

In this section the simulink scheme adopted is briefly reported and discussed.
I considered both the noise (as a white noise) and the static friction and load torque constant disturbances.

These are the plant scheme for the position and the control scheme for the rotor speed:

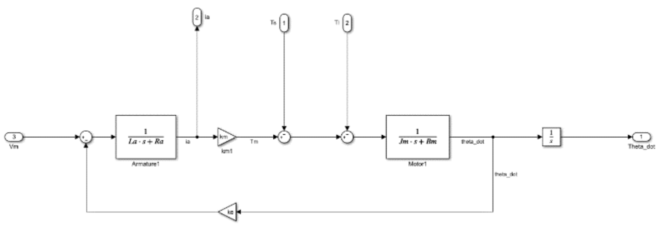

Fig.10 Plant Scheme for Position

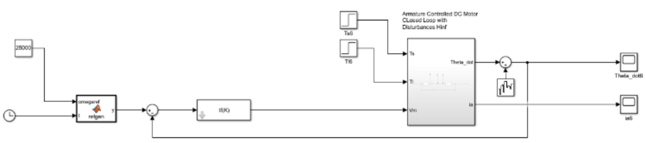

Fig.11 Speed Control Scheme

What is new:

After having understood better the topics, i choosed again the weights, and new results, similar to PID controller are obtained.

I removed smoother reference cause was out of the goals.

I added plot number 17 to highlight the effects of the sequential inclusion of the weights.
As you can see, without changing the closed loop specifications modifying the weights, in the nyquist plot, we reduce the distance from the critical point, when adding weight on control sensititivity. (we could obtained better performances for the first two by changing the weight specifications).

Logspace and time set definitions are not needed for the plots so I did not used them, also because by using the command sigma instead of bode we could obtain for siso certain system automatically the optimal frequency range.

I also founded two controllers for position and speed control by considering the plant without a knowledge of the disturbances. (not included Td and Ts in the nominal plant).

References

[1] https://www.faulhaber.com/fileadmin/Import/Media/EN_0620_B_FMM.pdf