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Introduction
In this thesis we present the design and implementation of a distributed theorem prover for the non-monotonic description logic $\mathcal{ALC} + \mathbf{T}_{min}$.
Description Logics (DL) are a family of formal knowledge representation languages. They are a decidable fragment of the first-order logic formalism used to provide semantics to representation structures. DLs are used in artificial intelligence for formal reasoning on the concepts of an application domain. It is of particular importance in providing a logical formalism for ontologies and the Semantic Web. The most notable applications outside information science is in bioinformatics and in the codification of medical knowledge. A DL knowledge base (KB) comprises two components: the TBox, containing the definition of concepts (and possibly roles) and a specification of inclusion relations among them, and the ABox containing instances of concepts and roles. Since the very objective of the TBox is to build a taxonomy of concepts, the need of representing prototypical properties and of reasoning about defeasible inheritance of such properties naturally arise. The traditional approach is to handle defeasible inheritance by integrating some kind of nonmonotonic extension of DLs.

1. **Why**
2. **What**
3. **Logics**
4. **The problem**
5. **Automated reasoning**
6. **State of the art**

Description Logics
Research in the field of knowledge representation and reasoning is usually focused on methods for providing high-level descriptions of the world that can be effectively used to build intelligent applications. In this context, "intelligent" refers to the ability of a system to find implicit consequences of its explicitly represented knowledge. Such systems are therefore characterized as knowledge-based systems. Approaches to knowledge representation developed in the 1970's are sometimes divided roughly into two categories: logic-based formalisms, which evolved out of the intuition that predicate calculus could be used unambiguously to capture facts about the world; and other, non-logic-based representations. The latter were often developed by building on more cognitive notions for example network structures. Even though such approaches were often developed for specific representational chores, the resulting formalisms were usually expected to serve in general use. In other words, the non-logical systems created from very specific lines of thinking (e.g., early Production Systems) evolved to be treated as general purpose tools, expected to be applicable in different domains and on different types of problems. In a logic-based approach, the representation language is usually a variant of first-order predicate calculus, and reasoning amounts to verifying logical consequence. In the non-logical approaches, often based on the use of graphical interfaces, knowledge is represented by means of some ad hoc data structures, and reasoning is accomplished by similarly ad hoc procedures that manipulate the structures. Among these specialized representations we find semantic networks and frames. Owing to their more human-centered origins, the network-based systems were often considered more appealing and more effective from a practical viewpoint than the

logical systems. Unfortunately they were not fully satisfactory because of their usual lack of precise semantic characterization. The end result of this was that every system behaved differently from the others, in many cases despite virtually identical-looking components and even identical relationship names.
Description Logics for typicality

7. The logic $\mathcal{ALC} + \mathbf{T}$

In this section, we recall the original $\mathcal{ALC}+\mathbf{T}$, which is an extension of $\mathcal{ALC}$ by a typicality operator $\mathbf{T}$ introduced in [1]. Given an alphabet of concept names $\mathcal{C}$, of role names $\mathcal{R}$, and of individual constants $\mathcal{O}$, the language $\mathcal{L}$ of the logic $\mathcal{ALC}+\mathbf{T}$ is defined by distinguishing *concepts* and *extended concepts* as follows:

- (Concepts)
  - $A \in \mathcal{C}$, $\top$ and $\bot$ are *concepts* of $\mathcal{L}$;
  - if $C, D \in \mathcal{L}$ and $R \in \mathcal{R}$, then $C \sqcap D, C \sqcup D, \neg C, \forall R.C, \exists R.C$ are *concepts* of $\mathcal{L}$
- (Extended concepts)
  - if C is a concept of $\mathcal{L}$, then C and $\mathbf{T}(C)$ are *extended concepts* of $\mathcal{L}$
  - boolean combinations of extended concepts are extended concepts of $\mathcal{L}$.

A knowledge base is a pair (TBox, ABox). TBox contains subsumptions $C \sqsubseteq D$, where $C \in \mathcal{L}$ is an extended concept of the form either C' or $\mathbf{T}(C')$, and $C', D \in \mathcal{L}$ are concepts. ABox contains expressions of the form $C(a)$ and aRb where $C \in \mathcal{L}$ is an extended concept, $R \in \mathcal{R}$, and $a, b \in \mathcal{O}$.
In order to provide a semantics to the operator $\mathbf{T}$, we extend the definition of a model used in "standard"

Definition 1 (Semantics of T with selection function). A model is any structure

$$\langle \Delta, I, f_{\mathbf{T}} \rangle$$

where:

- Δ is the domain, whose elements are denoted with $x, y, z, \dots$;
- I is the extension function that maps each extended concept C to $C^I \subseteq \Delta$, and each role R to a $R^I \subseteq \Delta \times \Delta$. I assigns to each atomic concept $A \in \mathcal{C}$ a set $A^I \subseteq \Delta$ and it is extended to arbitrary extended concepts as follows:
  - $\top^I = \Delta$
  - $\bot^I = \emptyset$
  - $(\neg C)^I = \Delta \backslash C^I$
  - $(C \sqcap D)^I = C^I \cap D^I$
  - $(C \sqcup D)^I = C^I \cup D^I$
  - $(\forall R.C)^I = \{x \in \Delta \mid \forall y.(x, y) \in R^I \rightarrow y \in C^I\}$
  - $(\exists R.C)^I = \{x \in \Delta \mid \exists y.(x, y) \in R^I \text{ and } y \in C^I\}$
  - $(\mathbf{T}(C))^I = f_{\mathbf{T}}(C^I)$

- Given $S \subseteq \Delta$, $f_{\mathbf{T}}$ is a function $f_{\mathbf{T}} : Pow(\Delta) \rightarrow Pow(\Delta)$ satisfying the following properties:

  - $(f_{\mathbf{T}} - 1)$ $f_{\mathbf{T}}(S) \subseteq S$;
  - $(f_{\mathbf{T}} - 2)$ if $S \neq \emptyset$, then also $f_{\mathbf{T}}(S) \neq \emptyset$;
  - $(f_{\mathbf{T}} - 3)$ if $f_{\mathbf{T}}(S) \subseteq R$, then $f_{\mathbf{T}}(S) = f_{\mathbf{T}}(S \cap R)$;
  - $(f_{\mathbf{T}} - 4)$ $f_{\mathbf{T}}(\bigcup S_i) \subseteq \bigcup f_{\mathbf{T}}(S_i)$;
  - $(f_{\mathbf{T}} - 5)$ $\bigcap f_{\mathbf{T}}(S_i) \subseteq f_{\mathbf{T}}(\bigcup S_i)$.

Intuitively, given the extension of some concept C, the selection function $f_{\mathbf{T}}$ selects the *typical* instances of C. $(f_{\mathbf{T}} - 1)$ requests that typical elements of S belong to S. $(f_{\mathbf{T}} - 2)$ requests that if there are elements in S, then there are also *typical* such elements. The following properties constrain the behavior of $f_{\mathbf{T}}$ with respect to $\cap$ and $\cup$ in such a way that they do not entail monotonicity. According to $(f_{\mathbf{T}} - 3)$, if the typical elements of S are in R, then they coincide with the typical elements of $S \cap R$, thus expressing a weak form of monotonicity (namely, *cautious monotonicity*). $(f_{\mathbf{T}} - 4)$ corresponds to one direction of the equivalence $f_{\mathbf{T}}(\bigcup S_i) = \bigcup f_{\mathbf{T}}(S_i)$, so that it does not entail monotonicity. Similar considerations apply to the equation $f_{\mathbf{T}}(\bigcap S_i) = \bigcap f_{\mathbf{T}}(S_i)$, of which only the inclusion $\bigcap f_{\mathbf{T}}(S_i) \subseteq f_{\mathbf{T}}(\bigcap S_i)$ holds. $(f_{\mathbf{T}} - 5)$ is a further constraint on the behavior of $f_{\mathbf{T}}$ with respect to arbitrary unions and intersections; it would be derivable if $f_{\mathbf{T}}$ were monotonic. In [1], we have shown that one can give an equivalent, alternative semantics for $\mathbf{T}$ based on a *preference relation* semantics rather than on a selection function semantics. The idea is that there is a global, irreflexive and transitive relation among individuals and that the typical members of a concept C (i.e., those selected by $f_{\mathbf{T}}(C^I)$) are the minimal elements of C with respect to this relation. Observe that this notion is *global*, that is to say, it does not compare individuals with respect to a specific concept. For this reason, we cannot express the fact that y is more typical than x with respect to concept C, whereas x is more typical than y with respect to another concept D. All what we can say is that either x is incomparable with y or x is more typical than y or y is more typical than x. In this framework, an element $x \in \Delta$ is a *typical instance* of some concept C if $x \in C^I$ and there is no C-element in Δ *more typical* than x. The typicality preference relation is partial since it is not always possible to establish given two element which one of the two is more typical. Following KLM, the preference relation also satisfies a *Smoothness Condition*, which is related to the well known *Limit Assumption* in Conditional Logics [2] [1]; this condition ensures that, if the extension C^I of a concept C is not empty, then there is at least one *minimal* element of C^I. This is stated in a rigorous manner in the following definition:

Definition 2. Given an irreflexive and transitive relation $<$ over a domain Δ, called *preference relation*, for all $S \subseteq \Delta$, we define

$$Min_<(S) = \{x \in S \mid \nexists y \in S \text{ s.t. } y < x\}$$

We say that $<$ satisfies the *Smoothness Condition* if for all $S \subseteq \Delta$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$.

The following representation theorem is proved in [1]:

[1] More precisely, the Limit Assumption entails the Smoothness Condition (i.e. that there are no infinite $<$ descending chains). Both properties come for free in finite models.

Theorem 1 (Theorem 2.1 in [1]). *Given any model* $\langle \Delta, I, f_{\mathbf{T}} \rangle$*,* $f_{\mathbf{T}}$ *satisfies postulates* $(f_{\mathbf{T}} - 1)$ *to* $(f_{\mathbf{T}} - 5)$ *above iff there exists an irreflexive and transitive relation* $<$ *on* Δ*, satisfying the Smoothness Condition, such that for all* $S \subseteq \Delta$*,* $f_{\mathbf{T}}(S) = Min_<(S)$*.*

Having the above Representation Theorem, from now on, we will refer to the following semantics:

Definition 3 (Semantics of $\mathcal{ALC} + \mathbf{T}$). A model $\mathcal{M}$ of $\mathcal{ALC} + \mathbf{T}$ is any structure

$$\langle \Delta, I, < \rangle$$

where:

- Δ is the domain;
- $<$ is an irreflexive and transitive relation over Δ satisfying the Smoothness Condition (Definition 2)
- I is the extension function that maps each extended concept C to $C^I \subseteq \Delta$, and each role R to a $R^I \subseteq \Delta \times \Delta$. I assigns to each atomic concept $A \in \mathcal{C}$ a set $A^I \subseteq \Delta$. Furthermore, I is extended as in Definition 1 with the exception of $(\mathbf{T}(C))^I$, which is defined as
$$(\mathbf{T}(C))^I = Min_<(C^I).$$

Let us now introduce the notion of satisfiability of an $\mathcal{ALC}+\mathbf{T}$ knowledge base. In order to define the semantics of the assertions of the ABox, we extend the function I to individual constants; we assign to each individual constant $a \in \mathcal{O}$ a *distinct* domain element $a^I \in \Delta$, that is to say we enforce the *unique name assumption*. As usual, the adoption of the unique name assumption greatly simplifies reasoning about prototypical properties of individuals denoted by different individual constants. Considering the example of department staff having lunches, if (in addition to the TBox) the ABox only contains the following facts about Greg and Sara:

DepartmentMember(*greg*)
DepartmentMember(*sara*), *TemporaryWorker*(*sara*)

we would like to infer that Greg takes his lunches at the restaurant, whereas Sara does not; but without the unique name hypothesis, we cannot get this conclusion since Greg and Sara might be the same individual. To perform useful reasoning we would need to extend the language with equality and make a case analysis according to possible identities of individuals. While this is technically possible, we prefer to keep the things simple here by adopting the unique name assumption.

Definition 4 (Model satisfying a Knowledge Base). Consider a model $\mathcal{M}$, as defined in Definition 3. We extend I so that it assigns to each individual constant a of $\mathcal{O}$ an element $a^I \in \Delta$, and I satisfies the unique name assumption. Given a KB (TBox,ABox), we say that:

- $\mathcal{M}$ satisfies TBox iff for all inclusions $C \sqsubseteq D$ in TBox, $C^I \subseteq D^I$.
- $\mathcal{M}$ satisfies ABox iff: (i) for all $C(a)$ in ABox, we have that $a^I \in C^I$, (ii) for all aRb in ABox, we have that $(a^I, b^I) \in R^I$.

$\mathcal{M}$ satisfies a knowledge base if it satisfies both its TBox and its ABox. Last, a query F is entailed by KB in $\mathcal{ALC}+\mathbf{T}$ if it holds in all models satisfying KB. In this case we write $KB \models_{\mathcal{ALC}+\mathbf{T}} F$.

Notice that the meaning of $\mathbf{T}$ can be split into two parts: for any x of the domain Δ, $x \in (\mathbf{T}(C))^I$ just in case (i) $x \in C^I$, and (ii) there is no $y \in C^I$ such that $y < x$. As already mentioned in the Introduction, in order to isolate the second part of the meaning of $\mathbf{T}$ (for the purpose of the calculus that we will present in Section **??**), we introduce a new modality $\Box$. The basic idea is simply to interpret the preference relation $<$ as an accessibility relation. By the Smoothness Condition, it turns out that $\Box$ has the properties as in Gödel-Löb modal logic of provability G. The Smoothness Condition ensures that typical elements of C^I exist whenever $C^I \neq \emptyset$, by avoiding infinitely descending chains of elements. This condition therefore corresponds to the finite-chain condition on the accessibility relation (as in G). The interpretation of $\Box$ in $\mathcal{M}$ is as follows:

Definition 5. Given a model $\mathcal{M}$ as in Definition 3, we extend the definition of I with the following clause:

$$(\Box C)^I = \{x \in \Delta \mid \text{for every } y \in \Delta, \text{ if } y < x \text{ then } y \in C^I\}$$

It is easy to observe that x is a typical instance of C if and only if it is an instance of C and $\Box\neg C$, that is to say:

Proposition 1. *Given a model $\mathcal{M}$ as in Definition 3, given a concept C and an element $x \in \Delta$, we have that*

$$x \in (\mathbf{T}(C))^I \textit{ iff } x \in (C \sqcap \Box\neg C)^I$$

Since we only use $\Box$ to capture the meaning of $\mathbf{T}$, in the following we will always use the modality $\Box$ followed by a negated concept, as in $\Box\neg C$.
The Smoothness condition, together with the transitivity of $<$, ensures the following Lemma:

Lemma 1. *Given an $\mathcal{ALC}+\mathbf{T}$ model as in Definition 3, an extended concept C, and an element $x \in \Delta$, if there exists $y < x$ such that $y \in C^I$, then either $y \in Min_<(C^I)$ or there is $z < x$ such that $z \in Min_<(C^I)$.*

Proof. Since $y \in C^I$, by the Smoothness Condition we have that either (i) $y \in Min_<(C^I)$ or (ii) there is $z < y$ such that $z \in Min_<(C^I)$. In case (i) we are done. In case (ii), since $<$ is transitive, we have also that $z < x$ and we are done. $\Box$

Last, we state a theorem which will be used in the following:

Theorem 2 (Finite model property of $\mathcal{ALC}+\mathbf{T}$)**.** *The logic $\mathcal{ALC}+\mathbf{T}$ has the finite model property.*

Proof. The theorem is a consequence of Theorems 3.1 and 3.2 in [1], which prove the soundness, the completeness and the termination of a tableau calculus for $\mathcal{ALC}+\mathbf{T}$. Indeed, if a KB is satisfiable in an $\mathcal{ALC}+\mathbf{T}$ model, then there is a tableau with a finite open branch. With a construction similar to the one used in the proof of Theorem 3.1, from this branch we can build a finite model satisfying KB. $\Box$

8. The logic $\mathcal{ALC} + \mathbf{T}_{min}$

As mentioned in the Introduction, the logic $\mathcal{ALC}+\mathbf{T}$ presented in [1] allows to reason about typicality. As a difference with respect to standard $\mathcal{ALC}$, in $\mathcal{ALC}+\mathbf{T}$ we can consistently express, for instance, the fact that three different concepts, like *Department member*, *Temporary Department Member* and *Temporary Department member having restaurant tickets*, have a different status with respect to *Have lunch at a restaurant*. This can be consistently expressed by including in a knowledge base the three formulas:

$\mathbf{T}(\mathit{DepartmentMember}) \sqsubseteq \mathit{LunchAtRestaurant}$
$\mathbf{T}(\mathit{DepartmentMember} \sqcap \mathit{TemporaryResearcher}) \sqsubseteq \neg \mathit{LunchAtRestaurant}$
$\mathbf{T}(\mathit{DepartmentMember} \sqcap \mathit{TemporaryResearcher} \sqcap \exists \mathit{Owns}.\mathit{RestaurantTicket}) \sqsubseteq$
$\mathit{LunchAtRestaurant}$

Assume that *greg* is an instance of the concept $\mathit{DepartmentMember} \sqcap \mathit{TemporaryResearcher} \sqcap \exists \mathit{Owns}.\mathit{RestaurantTicket}$. What can we conclude about *greg*? We have already mentioned that if the ABox explicitly points out that *greg* is a *typical* instance of the concept, and it contains the assertion that:

$$(*)\ \mathbf{T}(\mathit{DepartmentMember} \sqcap \mathit{TemporaryResearcher} \sqcap \exists \mathit{Owns}.\mathit{RestaurantTicket})(\mathit{greg}),$$

then, in $\mathcal{ALC}+\mathbf{T}$, we can conclude that

$$\mathit{LunchAtRestaurant}(\mathit{greg}).$$

However, if (*) is replaced by the weaker

$$(**)\ (\mathit{DepartmentMember} \sqcap \mathit{TemporaryResearcher} \sqcap \exists \mathit{Owns}.\mathit{RestaurantTicket})(\mathit{greg}),$$

in which there is no information about the typicality of *greg*, in $\mathcal{ALC}+\mathbf{T}$ we can no longer draw this conclusion, and indeed we cannot make any inference about whether *greg* spends its lunch time at a restaurant or not. The limitation here lies in the fact that $\mathcal{ALC}+\mathbf{T}$ is *monotonic*, whereas we would like to make a non-monotonic inference. Indeed, we would like to non-monotonically assume, in the absence of information to the contrary, that *greg* is a typical instance of the concept. In general, we would like to infer that individuals are typical instances of the concepts they belong to, if this is consistent with the KB.
As a difference with respect to $\mathcal{ALC}+\mathbf{T}$, $\mathcal{ALC}+\mathbf{T}_{min}$ is *non-monotonic*, and it allows to make this kind of inference. Indeed, in $\mathcal{ALC}+\mathbf{T}_{min}$ if (**) is all the information about *greg* present in the ABox, we can derive that *greg* is a typical instance of the concept, and from the inclusions above we conclude that $\mathit{LunchAtRestaurant}(\mathit{greg})$. We have already mentioned that we obtain this non-monotonic behaviour by restricting our attention to the minimal $\mathcal{ALC}+\mathbf{T}$ models. As a difference with respect to $\mathcal{ALC}+\mathbf{T}$, in order to determine what is entailed by a given knowledge base KB, we do not consider *all* models of KB but only the *minimal* ones. These are the models that minimize the number of atypical instances of concepts.
Given a KB, we consider a finite set $\mathcal{L}_{\mathbf{T}}$ of concepts occurring in the KB: these are the concepts for which we want to minimize the atypical instances. The minimization of the set of atypical instances will apply to individuals explicitly occurring in the ABox as well as to implicit individuals. We assume that the set $\mathcal{L}_{\mathbf{T}}$ contains at least all concepts C such that $\mathbf{T}(C)$ occurs in the KB. Notice that in case $\mathcal{L}_{\mathbf{T}}$ contains more concepts than those occurring in the scope of $\mathbf{T}$ in KB, the atypical instances of these concepts will be

minimized but no extra properties will be inferred for the typical instances of the concepts, since the KB does not say anything about these instances.
We have seen that $(\mathbf{T}(C))^I = (C \sqcap \Box\neg C)^I$: x is a typical instance of a concept C ($x \in (\mathbf{T}(C))^I$) when it is an instance of C and there is no other instance of C preferred to x, i.e. $x \in (C \sqcap \Box\neg C)^I$. By contraposition an instance of C is atypical if $x \in (\neg\Box\neg C)^I$ therefore in order to minimize the atypical instances of C, we minimize the instances of $\neg\Box\neg C$. Notice that this is different from maximizing the instances of $\mathbf{T}(C)$. We have adopted this solution since it allows to maximize the set of typical instances of C without affecting the extension C^I of C (whereas maximizing the extension of $\mathbf{T}(C)$ would imply maximizing also the extension of C).
We define the set $\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}}$ of negated boxed formulas holding in a model, relative to the concepts in $\mathcal{L}_{\mathbf{T}}$:

Definition 6. Given a model $\mathcal{M} = \langle \Delta, I, < \rangle$ and a set of concepts $\mathcal{L}_{\mathbf{T}}$, we define

$$\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} = \{(x, \neg\Box\neg C) \mid x \in (\neg\Box\neg C)^I, \text{ with } x \in \Delta, C \in \mathcal{L}_{\mathbf{T}}\}$$

Let KB be a knowledge base and let $\mathcal{L}_{\mathbf{T}}$ be a set of concepts occurring in KB.

Definition 7 (Preferred and minimal models). Given a model $\mathcal{M} = \langle \Delta_{\mathcal{M}}, I_{\mathcal{M}}, <_{\mathcal{M}} \rangle$ of KB and a model $\mathcal{N} = \langle \Delta_{\mathcal{N}}, I_{\mathcal{N}}, <_{\mathcal{N}} \rangle$ of KB, we say that $\mathcal{M}$ is preferred to $\mathcal{N}$ with respect to $\mathcal{L}_{\mathbf{T}}$, and we write $\mathcal{M} <_{\mathcal{L}_{\mathbf{T}}} \mathcal{N}$, if the following conditions hold:

- $\Delta_{\mathcal{M}} = \Delta_{\mathcal{N}}$
- $a^{I_{\mathcal{M}}} = a^{I_{\mathcal{N}}}$ for all individual constants $a \in \mathcal{O}$
- $\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} \subset \mathcal{N}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}}$.

A model $\mathcal{M}$ is a *minimal model* for KB (with respect to $\mathcal{L}_{\mathbf{T}}$) if it is a model of KB and there is no a model $\mathcal{M}'$ of KB such that $\mathcal{M}' <_{\mathcal{L}_{\mathbf{T}}} \mathcal{M}$.

Given the notion of preferred and minimal models above, we introduce a notion of *minimal entailment*, that is to say we restrict our consideration to minimal models only. First of all, we introduce the notion of *query*, which can be minimally entailed from a given KB. A query F is a formula of the form $C(a)$ where C is an extended concept and $a \in \mathcal{O}$. We assume that, for all $\mathbf{T}(C')$ occurring in F, $C' \in \mathcal{L}_{\mathbf{T}}$. Given a KB and a model $\mathcal{M} = \langle \Delta, I, < \rangle$ satisfying it, we say that a query $C(a)$ holds in $\mathcal{M}$ if $a^I \in C^I$.
Let us now define minimal entailment of a query in $\mathcal{ALC} + \mathbf{T}_{min}$. In Section **??** we will reduce the other standard reasoning tasks to minimal entailment.

Definition 8 (Minimal Entailment in $\mathcal{ALC} + \mathbf{T}_{min}$). A query F is minimally entailed from a knowledge base *KB* with respect to $\mathcal{L}_{\mathbf{T}}$ if it holds in all models of *KB* that are minimal with respect to $\mathcal{L}_{\mathbf{T}}$. We write $KB \models^{\mathcal{L}_{\mathbf{T}}}_{min} F$.

The non-monotonic character of $\mathcal{ALC} + \mathbf{T}_{min}$ also allows to deal with the following examples.

Example 1. Consider the following KB:

$$\text{KB } = \{\mathbf{T}(\mathit{Athlet}) \sqsubseteq \mathit{Confident}, \mathit{Athlet}(\mathit{john}), \mathit{Finnish}(\mathit{john})\}$$

and $\mathcal{L}_{\mathbf{T}} = \{Athlet, Finnish\}$. We have

$$\text{KB } \models^{\mathcal{L}_{\mathbf{T}}}_{min} Confident(john)$$

Indeed, there is no minimal model of KB that contains a non typical instance of some concept (indeed in all minimal models of KB the relation $<$ is empty). Hence *john* is an instance of $\mathbf{T}(Athlet)$ (it can be easily verified that any model in which *john* is not an instance of $\mathbf{T}(Athlet)$ is not minimal). By KB, in all these models, *john* is an instance of *Confident*. Observe that *Confident*(*john*) is obtained, in spite of the presence of the irrelevant assertion *Finnish*(*john*).

Example 2. Consider now the knowledge base KB' obtained by adding to KB the formula $\mathbf{T}(Athlet \sqcap Finnish) \sqsubseteq \neg Confident$, that is to say:

$$\text{KB'} = \{\mathbf{T}(Athlet) \sqsubseteq Confident, \mathbf{T}(Athlet \sqcap Finnish) \sqsubseteq \neg Confident, Athlet(john), Finnish(john)\}$$

and to $\mathcal{L}_{\mathbf{T}}$ concept $Athlet \sqcap Finnish$. From KB', *Confident*(*john*) is no longer derivable. Instead, we have that

$$\text{KB}' \models^{\mathcal{L}_{\mathbf{T}}}_{min} \neg Confident(john).$$

Indeed, by reasoning as above it can be shown that in all the minimal models of KB', *john* is an instance of $\mathbf{T}(Athlet \sqcap Finnish)$, and it is no longer an instance of $\mathbf{T}(Athlet)$. This example shows that, in case of conflict (here, *john* cannot be both a typical instance of *Athlet* and of $Athlet \sqcap Finnish$), typicality in the more specific concept is preferred.

In general, a knowledge base KB may have no minimal model or more than one minimal model, with respect to a given $\mathcal{L}_{\mathbf{T}}$. The following properties hold.

Proposition 2. *If* KB *has a model, then* KB *has a minimal model with respect to any* $\mathcal{L}_{\mathbf{T}}$.

The above fact is a consequence of the *finite model property* of the logic $\mathcal{ALC}+\mathbf{T}$ (Theorem 2).

Proposition 3. *Given a knowledge base* KB *and a query* F*, let us replace all occurrences of* $\mathbf{T}(C)$ *in* KB *and in* F *with* C*. We call* KB' *the resulting knowledge base and* F' *the resulting query. If* KB $\models^{\mathcal{L}_{\mathbf{T}}}_{min} F$ *then* KB' $\models_{\mathcal{ALC}+\mathbf{T}} F'$.

Proof. We show the contrapositive that if KB' $\not\models_{\mathcal{ALC}+\mathbf{T}} F'$ then KB $\not\models^{\mathcal{L}_{\mathbf{T}}}_{min} F$. Let $\mathcal{M}$ be an $\mathcal{ALC}+\mathbf{T}$ model satisfying KB' and not satisfying F'. Since neither KB' nor F' contain any occurrence of $\mathbf{T}$, the relation $<$ does not play any role in $\mathcal{M}$ and we can assume that $<$ is empty. Notice that in $\mathcal{M}$, for all C, we have that $\mathbf{T}(C)^I = C^I$. Therefore it can be shown by induction on the complexity of formulas in KB and in F that $\mathcal{M}$ is also a model of KB that does not satisfy F.
Furthermore, by Definition 5, for all C: $(\neg\Box\neg C)^I = \emptyset$, hence $\mathcal{M}$ is a minimal model of KB. We therefore conclude that KB $\not\models^{\mathcal{L}_{\mathbf{T}}}_{min} F$, and the proposition follows by contraposition. $\Box$

The above proposition shows that the inferences allowed by $\mathcal{ALC}+\mathbf{T}_{min}$ have as upper approximation the consequences that can be drawn classically from the knowledge base *KB'* obtained by transforming $\mathbf{T}(C) \sqsubseteq C'$ into the trivial $C \sqsubseteq C'$, what corresponds to assume that all individuals are typical. Obviously the *KB'* may be inconsistent or degenerated (all concepts are empty), whereas the original *KB* is not. For this reason the inverse of the proposition obviously does not hold.
Background

9. Logics

10. Calculus

11. Implementation

Architecture
Conclusion

References

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