

$$\frac{\langle S, x : C, x : \neg C \mid U \mid K \rangle}{(\text{Clash})}$$

$$\frac{\langle S, x : \perp \mid U \mid K \rangle}{(\text{Clash})_{\perp}}$$

$$\frac{\langle S, x : \neg \top \mid U \mid K \rangle}{(\text{Clash})_{\top}}$$

$$\frac{\langle S \mid U \mid \emptyset \rangle}{(\text{Clash})_{\emptyset}}$$

$$\frac{\langle S, x : \neg \Box \neg C \mid U \mid K \rangle}{(\text{Clash})_{\Box \neg}} \text{ if } x : \neg \Box \neg C \notin \mathbf{B}^{\Box \neg}$$

$$\frac{\langle S, x : \neg(C \sqcap D) \mid U \mid K \rangle}{\langle S, x : \neg C \mid U \mid K \rangle \quad \langle S, x : \neg D \mid U \mid K \rangle} (\sqcap^-)$$

$$\frac{\langle S, x : C \sqcap D \mid U \mid K \rangle}{\langle S, x : C, x : D \mid U \mid K \rangle} (\sqcap^+)$$

$$\frac{\langle S, x : \neg \neg C \mid U \mid K \rangle}{\langle S, x : C \mid U \mid K \rangle} (\neg)$$

$$\frac{\langle S, x : \mathbf{T}(C) \mid U \mid K \rangle}{\langle S, x : C, x : \Box \neg C \mid U \mid K \rangle} (\mathbf{T}^+)$$

$$\frac{\langle S, x : \neg \mathbf{T}(C) \mid U \mid K \rangle}{\langle S, x : \neg C \mid U \mid K \rangle \quad \langle S, x : \neg \Box \neg C \mid U \mid K \rangle} (\mathbf{T}^-)$$

$$\frac{\langle S \mid U, C \sqsubseteq D^L \mid K \rangle}{\langle S, x : \neg C \sqcup D \mid U, C \sqsubseteq D^{L,x} \mid K \rangle} (\sqsubseteq) \text{ if } x \in \mathcal{D}(\mathbf{B}) \text{ and } x \notin L$$

$$\frac{\langle S, x : \forall R.C, x \xrightarrow{R} y \mid U \mid K \rangle}{\langle S, x : \forall R.C, x \xrightarrow{R} y, y : C \mid U \mid K \rangle} (\forall^+) \text{ if } y : C \notin S$$

$$\frac{\langle S, x : \neg \Box \neg C \mid U \mid K, x : \neg \Box \neg C \rangle}{\langle S, v_1 : C, v_1 : \Box \neg C, S_{x \rightarrow v_1}^M, x : \neg \Box \neg C \mid U \mid K \rangle \quad \langle S, v_2 : C, v_2 : \Box \neg C, S_{x \rightarrow v_2}^M, x : \neg \Box \neg C \mid U \mid K \rangle \quad \cdots \quad \langle S, v_n : C, v_n : \Box \neg C, S_{x \rightarrow v_n}^M, x : \neg \Box \neg C \mid U \mid K \rangle} (\Box^-) \text{ if } \nexists u \text{ s.t. } \{u : C, u : \Box \neg C, S_{x \rightarrow u}^M\} \subseteq S \text{ and } \forall v_i \in \mathcal{D}(\mathbf{B}), x \neq v_i$$

$$\frac{\langle S \mid U \mid K \rangle}{\langle S, x : \Box \neg C \mid U \mid K \rangle \quad \langle S, x : \neg \Box \neg C \mid U \mid K \rangle} (cut) \text{ if } x : \neg \Box \neg C \notin S \text{ and } x : \Box \neg C \notin S, C \in \mathcal{L}_{\mathbf{T}}, x \in \mathcal{D}(\mathbf{B})$$

$$\frac{\langle S, x : \exists R.C \mid U \mid K \rangle}{\langle S, x \xrightarrow{R} v_1, v_1 : C \mid U \mid K \rangle \quad \langle S, x \xrightarrow{R} v_2, v_2 : C \mid U \mid K \rangle \quad \cdots \quad \langle S, x \xrightarrow{R} v_n, v_n : C \mid U \mid K \rangle} (\exists^+) \text{ if } \forall v_i \in \mathcal{D}(\mathbf{B})$$