University of Pittsburgh

Gurobi Workshop

PITT INFORMS Student Chapter

Luca Wrabetz(LUW28@pitt.edu)

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Introduction

- Gurobi is a *commercial optimizer* - a software to solve optimization models such as MIPs
- Gurobi interfaces with many languages such as Python
- First, we have to get everyone an Academic License and install Gurobi

Installation

Download Gurobi & Create License

- Go to gurobi's downloads page[1]
- Download and Install "Gurobi Optimizer"
- Select "Academic License" under "Request a License"
- Use grbgetkey command to activate license
- Test by typing gurobi.sh command in terminal (gurobi in windows)

[1] https://www.gurobi.com/downloads/

Installation

Install Anaconda

- Choose your OS and install anaconda to have access to packages and libraries[2]
- Follow instructions at link

[2] http://www.gurobi.com/downloads/get-anaconda

A Small Example

Model

- Let’s solve the following simple model using Gurobi:

A Small Example

Model

- Let's solve the following simple model using Gurobi:

$$\begin{aligned} \max \quad & 3x + 4y \\ \text{s.t.} \quad & 5x + 2y \leq 10 \\ & 3x + 5y \leq 12 \\ & x \geq 0, y \geq 0 \end{aligned}$$

A Small Example

How to Solve?

- Create instance of model class
  - Add variables
  - Add constraints
  - Set objective
  - Provide Data
  - Solve!

Sudoku

A 0-1 Model to solve Sudoku

- We want to solve an instance of the sudoku problem
- We receive a grid in a text file, with 0s for blanks
- How can we model this

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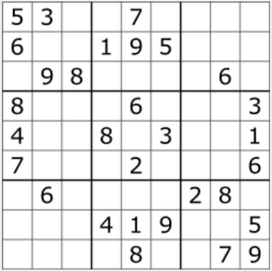

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

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$$x_{ijk} \in \{0, 1\} \quad \forall i, j, k \in \{1, \ldots, 9\}$$

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$$x_{ijk} = 1, \quad \forall \{i,j,k\} \quad \text{on original grid}$$

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$$x_{ijk} = 1, \quad \forall \{i,j,k\} \quad \text{on original grid}$$

$$\sum_{k=1}^{n} x_{ijk} = 1, \quad \forall i,j \in \{1,...,n\}$$

Sudoku

Continue Constraints

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$$\sum_{(j=(s-1)\sqrt{n}+1)}^{s\sqrt{n}} \sum_{(i=(t-1)\sqrt{n}+1)}^{t\sqrt{n}} x_{ijk} = 1, \qquad \forall t, s \in \{1, ..., \sqrt{n}\},$$

$$\forall k \in \{1, ..., n\}$$

Sudoku

Tuning

- Gurobi allows you to automatically ”tune” your model parameters
  - There are many parameters you can change in Gurobi
  - You are probably not an expert in all of them
  - Even if you are a super-genius, there are too many to keep track of

Travelling Salesman Problem (TSP)

Introduction

- Let $G = (V, E)$ be an undirected network, with cost weighted edges ($c_e > 0$)
- We want to find a minimum cost hamiltonian cycle (tour that visits all nodes)
- Edges are symmetric

Travelling Salesman Problem (TSP)

Formulation

$$\min \quad \sum_{e \in E} c_e x_e \tag{1}$$

$$s.t. \quad \sum_{e \in \delta(i)} x_e = 2 \quad i \in V \tag{2}$$

$$\sum_{e \in \delta(i)} x_e = 2 \quad S \subset V, |S| \geq 2 \tag{3}$$

$$x_e \in \{0, 1\}, \quad e \in E \tag{4}$$

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- How many constraints in set (3)?

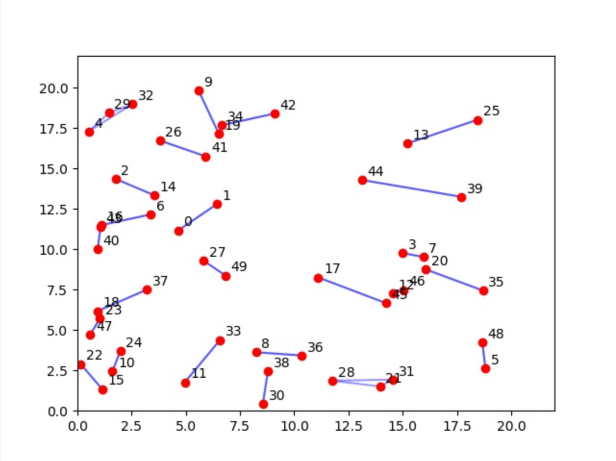
Travelling Salesman Problem (TSP)

Subtour Elimination

- Constraints in (3) are known as *subtour elimination* constraints
- There is an exponential number of them so we can't write out the whole formulation
- We will relax them and add them as lazy cuts in a Gurobi callback

Travelling Salesman Problem (TSP)

Example



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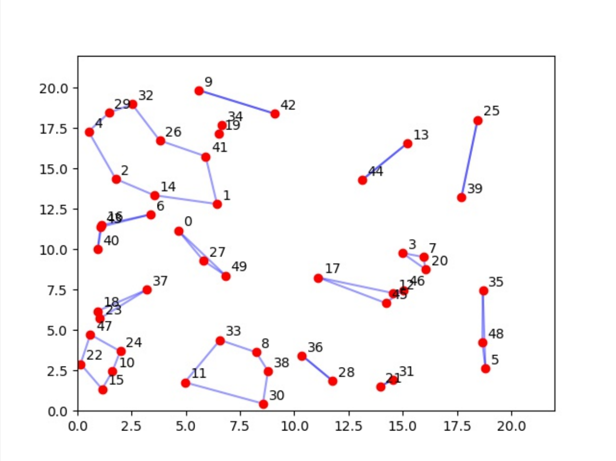

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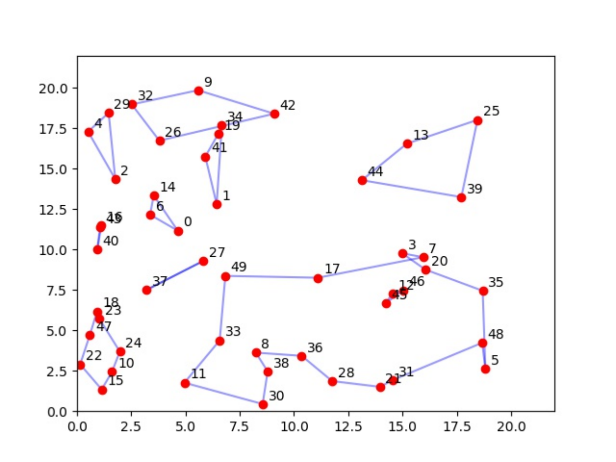

Travelling Salesman Problem (TSP)

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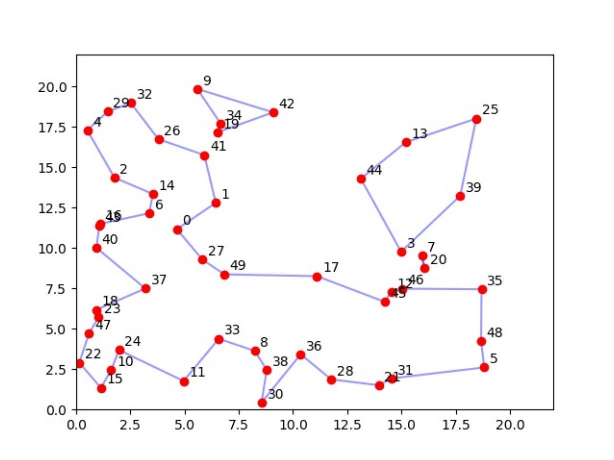

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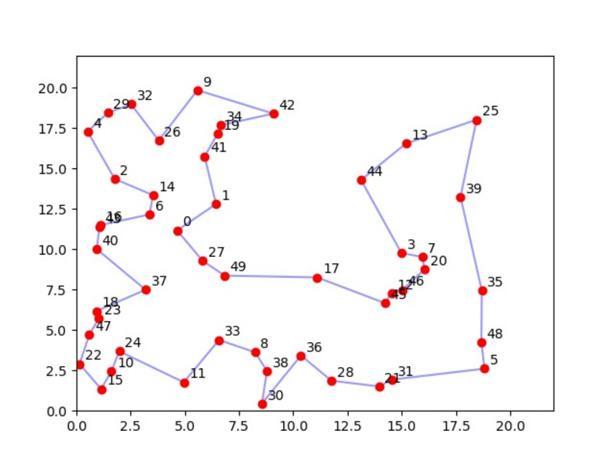

Thank You!

- Questions?