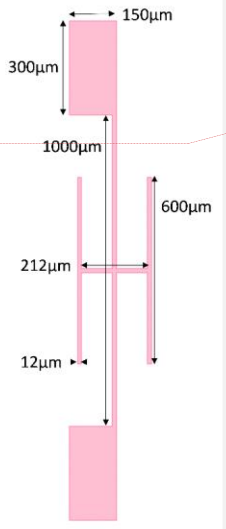
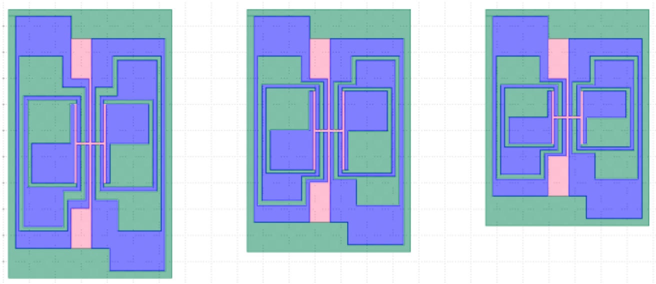
Device dimensions



Central beam length (L_{res}) = 1000/800/600 µm *
Lateral beams length ($L_{lateral}$) = 600/600/400 µm *
Transverse beams length (L_{trans}) = 100 µm
Gaps (G) = 5 µm
Width (W) = 12 µm
Thickness (T) = 50 µm
Anchors /connection pads are all 150 µm × 300 µm.

Commented [LRS1]: Where is the gap?

***** If the central beam is 1000 µm or 800 µm, the lateral beams are 600 µm. If the central beam is 600 µm, the lateral beams are 400 µm, as shown below.



The material is silicon, with Young's modulus $E = 160$ GPa and density $\rho = 2320$ kg/m^3.

The structure is surrounded with air, with permittivity $\epsilon_0 = 8.84 \times 10^{-12}$ F/m and viscosity $\mu = 1.8 \times 10^{-5}$ Pa. s.

Resonator model

Here we first derive the fully-nonlinear 1-DOF model of the resonator, then a linearized model and some example Butterworth-Van Dyke models for designing the electronics associated to the resonator.

1-DOF nonlinear model

Letting x be the deflection of the central beam midpoint, the 1-DOF model of the resonator is

$$K \times (1 + N/N_{crit} + x^2/x_D^2) \times x + B\dot{x} + M\ddot{x} = F_{elec} + F_{flu}$$

where K is the mechanical stiffness of the resonator, N is the axial force in the resonator ($N > 0$ means tensile stress), N_{crit} is the critical buckling force, x_D is the value of the static deflection for which stiffness is doubled because of mechanical stiffening, B is a linear damping coefficient, M is the effective mass of the resonator, F_{elec} is the electrostatic force acting on the resonator and F_{flu} is the viscous force due to squeezed-film damping in the air-gaps.

Commented [LRS2]: Because of the fixation points? Or, rather, forces in the longitudinal direction along the resonator beams

Commented [LRS3]: ?

Commented [LRS4]: Caused by a DC electric force, for example

Commented [LRS5]: Why effective mass?

Commented [LRS6R5]: Because we tend to use an approximated model for these complex resonator geometries

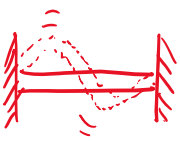
We may assimilate the central beam to a clamped-clamped beam with length L_{res}, so that we have

Formulas for

$$K \approx 16.56 \times E\frac{TW^3}{L_{res}^3}$$

$$N_{crit} \approx 0.205 \times KL_{res}$$

$$x_D \approx 1.18 \times W$$



(clamped-clamped beam)

$$M = \rho \times (0.397 L_{res} + 2 L_{lateral} + 2 L_{trans}) \times WT$$

This lets us define the natural frequency of the resonator in the absence of axial stress as

N = 0

$$\omega_0 = \sqrt{\frac{K}{M}}$$

In the presence of axial stress we have

$$\omega_N = \sqrt{\frac{K}{M} \times \left(1 + \frac{N}{N_{crit}}\right)} \approx \omega_0 \times \left(1 + \frac{1}{2}\frac{N}{N_{crit}}\right)$$

where the assumption holds if $|N| \ll N_{crit}$.

Supposing thermoelastic losses are the dominant cause of damping, one may use Zener's model to determine the linear damping coefficient

true for low air pressures

$$B = \sqrt{KM} \times \frac{\alpha^2 \Theta E}{c_v} \times \frac{\omega_0 \tau_R}{1 + \omega_0^2 \tau_R^2}$$

where

$$\tau_R = \frac{c_v}{k} \times \left(\frac{W}{\pi}\right)^2$$

in which $k = 156$ W/m/K is the thermal conductivity of silicon, $c_v = 713$ J/kg/K its specific heat at constant volume, $\alpha = 2.6$ ppm/K its coefficient of linear expansion and Θ is the temperature.

Assuming the flow has a low squeeze number, we may approximate fluidic forces as

Commented [LRS7]: Squeeze number = ?

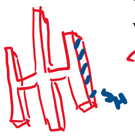
Commented [LRS8R7]: Low squeeze number is to say the air's compression is not as important as its viscosity effects

Commented [LRS9R7]: (During resonator movement)

$$F_{flu} = -\mu_{eff} \frac{T^2 S_H}{G^3} \times \left(\frac{2}{(1-\tilde{x})^3} + \frac{2}{(1+\tilde{x})^3} + 0.397 \frac{S_{res}}{S_H} \times \left(\frac{1}{(1-\tilde{x})^{5/2}} + \frac{1}{(1+\tilde{x})^{5/2}}\right)\right) \times \dot{\tilde{x}}$$

where $\tilde{x} = x/G$ is the normalized deflection with respect to the gap, $S_H = TL_{lateral}$ is the surface of the lateral H arms, $S_{res} = TL_{res}$ is the surface of the central beam, and μ_{eff} is the effective viscosity of air

%



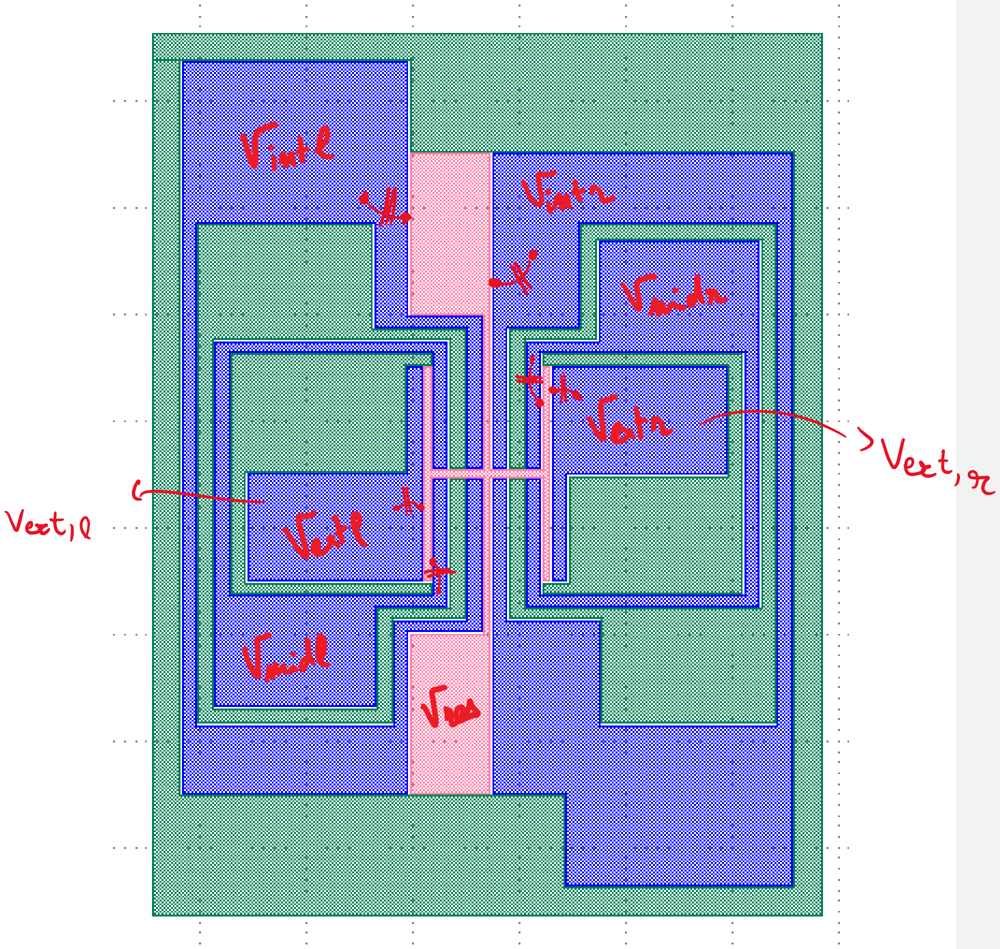
$$\mu_{eff} \approx \frac{\mu}{1 + 9.64 \times \left(\frac{\Lambda_0}{G} \times \frac{P_0}{P}\right)^{1.16}}$$

with $\Lambda_0 = 64$ nm the mean free path of air molecules at $P_0 = 10^5$ Pa and P is the ambient pressure.

Finally, electrostatic forces are given by

$$F_{elec} = \epsilon_0 \frac{S_H}{2G^2}\left(-\frac{(V_{res} - V_{extl})^2}{(1+\tilde{x})^2} + \frac{(V_{res} - V_{extr})^2}{(1-\tilde{x})^2} + \frac{(V_{res} - V_{midl})^2}{(1-\tilde{x})^2} - \frac{(V_{res} - V_{midr})^2}{(1+\tilde{x})^2} + 0.523 \frac{S_{res}}{S_H} \times \left(-\frac{(V_{res} - V_{intl})^2}{(1+\tilde{x})^{3/2}} + \frac{(V_{res} - V_{intr})^2}{(1-\tilde{x})^{3/2}}\right)\right)$$

where V_{res} is the voltage of the resonator, $V_{extl,r}$ are the voltages on the exterior (left and right) electrodes, $V_{midl,r}$ are the voltages on the middle (left and right) electrodes and $V_{intl,r}$ are the voltages on the interior (left and right) electrodes, as shown below. The electrodes in green are connected to the ground.



1-DOF linear model

One may linearize the previous expressions with respect to $x \ll G$ for the purpose of simplification. In this section, we also make the following assumptions:

small movements/vibrations

$$V_{intl} = -V_{intr} \equiv V_{int} \ll V_{res}$$

$$V_{midr} = V_{midl} \equiv V_{mid}$$

$$V_{extl} = V_{extr} = 0$$

This corresponds to a configuration where the interior electrodes are used for actuating the resonator, the middle electrodes are used for “fine tuning” of the electrostatic softening, and the exterior electrodes are used for detecting the motion of the resonator (and are connected to a transimpedance amplifier with ideal characteristics).

The electrostatic force then simplifies to

$$F_{elec} = \overbrace{\epsilon_0 \frac{2S_H}{G^3} V_{res}^2 \left(1 + \left(1 - \frac{V_{mid}}{V_{res}}\right)^2 + \frac{3}{4} \times 0.523 \frac{S_{res}}{S_H}\right) x}^{\text{positive restoring force } (K_{elec})} + \overbrace{0.523 \times \epsilon_0 \frac{2S_{res}}{G^2} \times V_{res} V_{int}}^{\text{force driving the resonator}}$$

The first term on the right hand-side is a positive restoring force, which results in electrostatic softening. The second term on the right-hand side is the force driving the resonator.

Defining the electrostatic stiffness

$$K_{elec} = -\epsilon_0 \frac{2S_H}{G^3} V_{res}^2 \left(1 + \left(1 - \frac{V_{mid}}{V_{res}}\right)^2 + 0.392 \frac{S_{res}}{S_H}\right)$$

and the linearized squeeze-film damping coefficient

$$B_{squeeze} = 4\mu_{eff} \frac{T^2 S_H}{G^3} \times \left(1 + 0.199 \frac{S_{res}}{S_H}\right)$$

the linearized model of the resonator is then

$$K \times \left(1 + \frac{N}{N_{crit}} + \frac{K_{elec}}{K}\right) \times x + (B + B_{squeeze})\dot{x} + M\ddot{x} = \overbrace{0.523 \times \epsilon_0 \frac{2S_{res}}{G^2} \times V_{res} V_{int}}^{F_{drive}}$$

Butterworth-Van Dyke (BVD) model

The equivalent electrical model of the resonator can be derived by calculating the motional current at the left and right exterior terminals of the system. We have

$i = \frac{dQ}{dt}$

Capacitor: $Q = C\Delta V$

$$i_{motl} = \frac{d}{dt}\left(\frac{\epsilon_0 S_H}{G + x} \times (V_{extl} - V_{res})\right) \approx \frac{\epsilon_0 S_H}{G^2} V_{res} \times \dot{x} = \eta_H \dot{x}$$

$$i_{motr} = \frac{d}{dt}\left(\frac{\epsilon_0 S_H}{G - x} \times (V_{extr} - V_{res})\right) \approx -\frac{\epsilon_0 S_H}{G^2} V_{res} \times \dot{x} = -\eta_H \dot{x}$$

where

$\Rightarrow i = \underbrace{C\frac{dV}{dt}}_{0} + \left(V\frac{dC}{dt}\right)$

$$\eta_H = \epsilon_0 \frac{S_H}{G^2} \times V_{res}$$

Replacing $\dot{x}$ by i_{motl}/η_H and V_{int} by V_{intl} in the linear 1-DOF model yields the BVD model for the left branch

Butterworth-Van Dyke

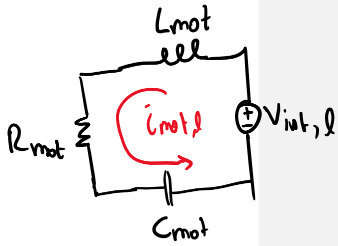
$$\frac{1}{C_{mot}} \int i_{motl} dt + R_{mot} i_{motl} + L_{mot} \frac{di_{motl}}{dt} = V_{intl}$$

with

$$R_{mot} = \frac{B + B_{squeeze}}{\eta_H \eta_{res}}$$

$$L_{mot} = \frac{M}{\eta_H \eta_{res}}$$

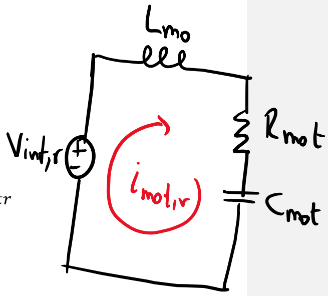
$$\frac{1}{C_{mot}} = \frac{K}{\eta_H \eta_{res}} \times \left(1 + \frac{N}{N_{crit}} + \frac{K_{elec}}{K}\right)$$



where

$$\eta_{res} = 0.523 \times \epsilon_0 \frac{2S_{res}}{G^2} \times V_{res}$$

For the right branch, the BVD model is



$$\frac{1}{C_{mot}}\int i_{motr}dt + R_{mot}i_{motr} + L_{mot}\frac{di_{motr}}{dt} = V_{intr}$$

with the same motional components as for the left branch.

First-step assignment

Make a Matlab function that computes the numerical values of the motional components for the three considered geometries.

The arguments of the Matlab function should be:

- the geometrical parameters of the resonators
- the value of the axial force N
- the value of the ambient pressure P and temperature Θ
- the (DC) values of V_{mid} and V_{res}

Calculate the numerical values of the motional components, the resonance frequency and the quality factor of the BVD model for the three considered geometries under the following conditions:

- $P = 10^5$ Pa, $P = 10^2$ Pa
- $V_{res} = 10$ V, $V_{res} = 50$ V

You will assume $V_{mid} = V_{res}$, $\Theta = 300$ K, $N = 0$ N for all the simulations. Assuming the amplitude of the AC driving voltage is $V_{int} = 1$ V, determine the value of the motional current at the resonance frequency.

Present the results in the form of a 24×6 table. Make a qualitative analysis of the dependence of the resonance frequency and of the quality factor to pressure P and to bias voltage V_{res}.

Second-step assignment

Dimension analog front-ends for this resonator. The AFEs will be based on two transimpedance amplifiers and one instrumentation amplifier. The values of the passive components need to be adapted to the different operating conditions above.

In terms of performance, this front-end should deliver signals of the order of 100s of mV (or more) and be as low noise as possible. Your design will be based on discrete active components from the Analog Devices catalog. To determine the best candidates, you will have to make an analytical model of the noise at the output of the amplifier and determine which parameters are critical for low noise operation. Possible candidates for the transimpedance amplifier are AD8065 (those I currently use), LTC-6268-10, but there may well be some smarter choices.

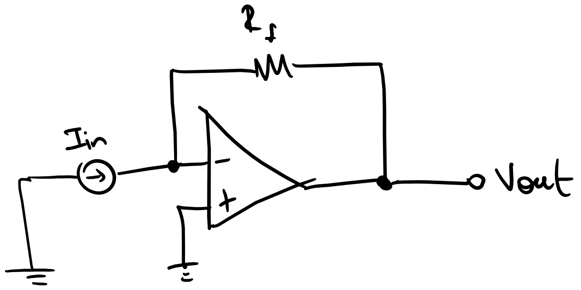
You will consider the following imperfections:

- existence of 10 pF parasitic capacitors at the inputs of the transimpedance amplifiers.
- 100 fF parasitic capacitors between the inputs and outputs of the transimpedance amplifiers.
- existence of offset voltages at the outputs of the transimpedance amplifiers.

Your design choices will be validated with LTSpice simulations.

TIA (Transimpedance Amplifier)

⇒ current to voltage converter



$V_{out} = -R_f I_{in}$

→ Read the datasheet of one amplifier and the chapter of an analog electronics book (or Analog Devices' website) to remember how to model imperfections (parasitics, offset voltages, gain-bandwidth product, noise etc.)

↳ Library