Module 10: Project - BayesNet Inference and Sampling

1. Select one of the attributes- on paper compute the probability of the attribute given an evidence of your choosing (example:Burglary=T)   using enumeration (show all steps) (10 points)

$$P(b|j,m) = \alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a).$$

- **P(Sepal_W_Top|Setosa=T)**
- = P(Sepal_W_Top, Setosa=T) / P(Setosa=T)
- = 1/ P(Setosa=T) * P(Sepal_W_Top, Setosa=T)
- = 1/ P(Setosa=T) * ∑(sepal_L_Top) * P(Sepal_W_Top, Sepal_L_Top, Setosa=T)
- = 1/ P(Setosa=T) * ∑(sepal_L_Top) * P(Sepal) * P(Sepal_L_Top|Setosa) * P(Sepal_W_Top|Sepal_L_Top)

$$
\begin{aligned}
&\text{if } b = t, a = t, e = t: P(b)P(e)P(a|b,e)P(j|a)P(m|a)\\
&\text{if } b = t, a = t, e = f: P(b)P(e = f)P(a|b,e = f)P(j|a)P(m|a)\\
&\text{if } b = t, a = f, e = t: P(b)P(e)P(a = f|b,e)P(j|a = f)P(m|a = f)\\
&\text{if } b = t, a = f, e = f: P(b)P(e = f)P(a = f|b,e = f)P(j|a = t)P(m|a = t)
\end{aligned}
$$

- **1. If Sepal_W_Top = T, Sepal_L_Top = T:** P(Setosa) * P(Sepal_L_Top|Setosa) * P(Sepal_W_Top|Sepal_L_Top)
- **2. If Sepal_W_Top = F, Sepal_L_Top = T:** P(Setosa) * P(Sepal_L_Top|Setosa) * P(Sepal_W_Top|Sepal_L_Top)
- **3. If Sepal_W_Top = T, Sepal_L_Top = F:** P(Setosa) * P(Sepal_L_Top|Setosa) * P(Sepal_W_Top|Sepal_L_Top)
- **4. If Sepal_W_Top = F, Sepal_L_Top = F:** P(Setosa) * P(Sepal_L_Top|Setosa) * P(Sepal_W_Top|Sepal_L_Top)

- 1. P(Setosa=T) = 0.33, P(Sepal_L_Top=T|Setosa) = 0.02, P(Sepal_W_top=T|Sepal_L_Top=T) = 0.545
- = 0.33 * 0.02 * 0.545 = **0.003**
- 2. P(Setosa=T) = 0.33, P(Sepal_L_Top=T|Setosa) = 0.02, P(Sepal_W_top=F|Sepal_L_Top=T) = 0.614
- = 0.33 * 0.02 * 0.614 = **0.004**
- 3. P(Setosa=T) = 0.33, P(Sepal_L_Top=F|Setosa) = 0.98, P(Sepal_W_top=T|Sepal_L_Top=F) = 0.698
- = 0.33 * 0.98 * 0.698= **0.225**
- 4. P(Setosa=T) = 0.33, P(Sepal_L_Top=F|Setosa) = 0.98, P(Sepal_W_top=F|Sepal_L_Top=F) = 0.385
- = 0.33 * 0.98 * 0.385 = **0.124**

- (1/P(Setosa=T)) * 0.003 + 0.004 + 0.225 + 0.124
- = (1/P(Setosa=T)) * 0.356
- = 0.356 / 0.33
- **= 1.07**

2. Select one of the attributes- on paper compute the probability of the attribute given evidence of your choosing (example:Burglary=T) using elimination (show all steps) (10 points)

$$\mathbf{P}(B|j,m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \sum_a \mathbf{f}_3(A,B,E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A).$$

- **P(Sepal_W_Top|Setosa=T)**
- = P(Sepal_W_Top, Setosa=T) / P(Setosa=T)
- = 1/ P(Setosa=T) * P(Sepal_W_Top) * Σ P(sepal_L_Top) * ΣP(Sepal_L_Top, Setosa=T) * P(Sepal_L_Top) * P(Sepal_L_Top)

$$\begin{aligned}
\mathbf{f}_6(B,E) &= \sum_a \mathbf{f}_3(A,B,E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A) \\
&= (\mathbf{f}_3(a,B,E) \times \mathbf{f}_4(a) \times \mathbf{f}_5(a)) + (\mathbf{f}_3(\neg a,B,E) \times \mathbf{f}_4(\neg a) \times \mathbf{f}_5(\neg a)).
\end{aligned}$$

- = (P(Sepal_L_Top=T, Setosa=T) * P(Sepal_L_Top=T)) + (P(Sepal_L_Top=F, Setosa=T) * P(Sepal_L_Top=F))
- = (0.02 * 0.51) + (0.98 + 0.38) = 0.0102 + 0.3724
- = **0.3826**

$$\mathbf{P}(B|j,m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \mathbf{f}_6(B,E).$$

- P(Sepal_W_Top=T) * (P(Sepal_L_Top=F) + P(Sepal_L_Top=T)) * P(Sepal_W_Top=T, Sepal_L_Top=T)
- 0.38 * (0.48 + 0.51) * 0.54
- 0.2052

- **Final integration of numbers:**
- = (1/ 0.33) * 0.62 * 0.2052 * 0.3826
- = **0.048 / 0.33 = 0.1454**