

MODELING AND SIMULATION OF THE BOILING CRISIS  
WITHIN PWR AT CFD SCALE

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Improving the Modeling of Wall Boiling for Multiphase CFD Simulations

Paris, December 2022



test

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## ABSTRACT

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To write!

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## PUBLICATIONS

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Some ideas and figures have appeared previously in the following publications:

Put your publications from the thesis here. The packages `multibib` or `bibtopic` etc. can be used to handle multiple different bibliographies in your document.

We have seen that computer programming is an art,  
because it applies accumulated knowledge to the world,  
because it requires skill and ingenuity, and especially  
because it produces objects of beauty.

— knuth:1974 [knuth:1974]

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Put your acknowledgements here.

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<sup>1</sup> Members of GuIT (Gruppo Italiano Utilizzatori di T<sub>E</sub>X e L<sup>A</sup>T<sub>E</sub>X)

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## NOMENCLATURE

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### Acronyms

BC	Boiling Crisis
CHF	Critical Heat Flux
DNS	Direct Numerical Simulations
HFP	Heat Flux Partitioning
PWR	Pressurized Water Reactor
RPE	Rayleigh-Plesset Equation
SMR	Small Modular Reactor

### Greek symbols

$\eta$	Thermal diffusivity [m <sup>2</sup> /s]
$\gamma$	Shear rate [s <sup>-1</sup> ]
$\lambda$	Thermal conductivity [W/m/K]
$\mu$	Dynamic viscosity [J.s/m <sup>-3</sup> ]
$\nu$	Kinematic viscosity [m <sup>2</sup> /s]
$\phi$	Heat flux [J/m <sup>2</sup> /s]
$\rho$	Density [kg/m <sup>3</sup> ]
$\sigma$	Surface tension [J/m <sup>2</sup> ]
$\theta, d\theta$	Contact angle and half-hysteresis [° or rad]
$\theta_i$	Bubble inclination angle [° or rad]

### Latin symbols

$C$	Force coefficient [-]
$D$	Bubble diameter [m]
$D_h$	Hydraulic diameter [m]
$E$	Kinetic energy [J]
$F$	Force [N]
$G$	Mass flux [kg/m <sup>2</sup> /s]
$g$	Gravity acceleration [m <sup>2</sup> /s]
$h_{LV}$	Latent heat of vaporization [J/kg]
$K$	Bubble growth constant [-]
$L_c = \sqrt{\frac{\sigma}{(\rho_L - \rho_V)g}}$	Capillary length [m]
$l_{sl}$	Bubble sliding length [m]
$R$	Bubble radius [m]

$R_c$	Bubble curvature radius [m]
$r_w$	Bubble foot radius [m]
$T$	Temperature [K]
$U$	Velocity [m/s]
$U_\tau$	Wall friction velocity [m/s]
$U_{rel} = U_L - U_b$	Relative velocity [m/s]
$V$	Volume [ $\text{m}^3$ ]

### Non-dimensional numbers

$\text{Ca} = \frac{\mu_L U_L}{\sigma}$	Capillary number [-]
$\text{Eo} = \frac{(\rho_L - \rho_V) g R^2}{\sigma}$	Eotvos number [-]
$\text{Fr} = \frac{\rho_L U_L^2}{(\rho_L - \rho_V) g R}$	Froude number [-]
$\text{Ja} = \frac{\rho_L c_{P,L}  T - T_{sat} }{\rho_V h_{LV}}$	Jakob number [-]
$\text{Pr} = \frac{\nu}{\eta}$	Prandtl number [-]
$\text{Re}_b = \frac{U_{rel} D_b}{\nu_L}$	Bubble Reynolds number [-]
$\text{Re}_{D_h} = \frac{G_L D_h}{\mu_L}$	Liquid bulk Reynolds number [-]
$\text{Sr} = \frac{2\gamma R}{ U_{rel} }$	Non-dimensional shear rate [-]
$y^+ = \frac{y U_\tau}{\nu_L}$	Non-dimensional wall distance [-]

### Subscripts

$AM$	Added-Mass
$b$	Bubble
$D$	Drag
$d$	Departure or Downstream
$L$	Lift
$L$	Liquid
$lo$	Lift-off
$sat$	Saturation
$u$	upstream
$V$	Vapor
$w$	Wall

### ACRONYMS

CFD Computational Fluid Dynamics

CMFD Computational Multi-Fluid Dynamics

NCFD NEPTUNE\_CFD

DNS Direct Numerical Simulation

NSD Nucleation Site Density

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## NOMENCLATURE

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$N_{sit,0}$  Nucleation Site Density computed by correlation [m<sup>-2</sup>]

Part I  
INTRODUCTION

## Part II

### MODELING AND SIMULATION OF BOILING FLOWS USING CFD

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NEPTUNE\_CFD CODE

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## 1.1 THE NEPTUNE\_CFD CODE AND PHYSICAL MODELING

1.1.1 *Simulation framework*

NEPTUNE\_CFD is an eulerian multiphase CFD solver co-developed by EDF, CEA, IRSN and Framatome mostly for nuclear reactor applications. The code consists of a local three-dimensional modeling based on a two-fluid one pressure approach combined with mass, momentum and energy conservation equations for each phase[**Guelfi2007**].

The constitutive equations are solved using a pressure correction, and is based on a finite-volume discretization along with a collocated arrangement of the variables. Moreover, NEPTUNE\_CFD allows the use of all type of meshes (hexahedra, tetrahedra, pyramids, etc.), even non-conforming ones, thanks to its face-based data structure. Finally, the code is well-suited for parallel computing, widening its computing capacity to very large meshes.

1.1.2 *Governing equations for turbulent boiling bubbly flows*

To simulate two-phase dispersed boiling flows, NEPTUNE\_CFD solves the ensemble-averaged equations of mass conservation, momentum balance and energy conservation for each phase (total of 6 equations) :

Mass conservation :

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \bar{\nabla} \cdot (\alpha_k \rho_k \bar{U}_k) = \Gamma_k \quad (1.1)$$

Where  $\alpha_k$ ,  $\rho_k$ ,  $\bar{U}_k$  are the time fraction, average density and velocity of phase  $k$  ;  $\Gamma_k = \Gamma_{k,i} + \Gamma_{k,w}$  the interfacial mass transfer term per unit of volume and time splitted between bulk and wall contribution. Subscripts  $k = L$  or  $G$  denotes the liquid or gas phase,  $i$  the interfacial quantities and  $w$  the wall contribution.

Momentum balance :

$$\frac{\partial \alpha_k \rho_k \bar{U}_k}{\partial t} + \bar{\nabla} \cdot (\alpha_k \rho_k \bar{U}_k \otimes \bar{U}_k) = -\alpha_k \bar{\nabla} (P) + \bar{F}_{k,i} + \Gamma_k \bar{U}_{k,i} + \alpha_k \rho_k \bar{g} + \bar{\nabla} \cdot (\alpha_k (\bar{\tau}_{k,m} + \bar{\tau}_{k,T})) \quad (1.2)$$

Where  $P$  is the pressure,  $\bar{g}$  the gravity,  $\bar{F}_{k,i}$  the interfacial forces accounting for momentum transfer between phases per unit of volume and time,  $\bar{U}_{k,i}$  the interfacial velocity,  $\bar{\tau}_{k,m}$  and  $\bar{\tau}_{k,T}$  respectively the viscous and turbulent (or Reynolds) stress tensor. Subscript  $m$  and  $T$  respectively denote the molecular (or laminar) and turbulent terms.

Energy conservation :

$$\begin{aligned} \frac{\partial \alpha_k \rho_k H_k}{\partial t} + \bar{\nabla} \cdot (\alpha_k \rho_k H_k \bar{U}_k) &= \frac{\partial \alpha_k P}{\partial t} + \Gamma_k H_{k,i} + \bar{F}_{k,i} \cdot \bar{U}_k + Q_{k,I} + \bar{\nabla} \cdot (\alpha_k (\bar{\tau}_k + \bar{\tau}_{k,T}) \cdot \bar{U}_k) \\ &\quad + \bar{\nabla} \cdot (\alpha_k (-(\lambda_{k,m} + \lambda_{k,T}) \bar{\nabla} (T_k))) + \alpha_k \rho_k \bar{g} \cdot \bar{U}_k + Q_{k,w} \end{aligned} \quad (1.3)$$

Where  $H_k = e_k + \frac{U_k^2}{2} + \frac{P}{\rho_k} = h_k + \frac{U_k^2}{2}$  is the total enthalpy of phase  $k$ ,  $H_{k,i}$  the interfacial-averaged enthalpy,  $Q_{k,i}$  the interfacial heat flux per unit of volume and time,  $\lambda_{k,m}$  and  $\lambda_{k,T}$  respectively being the laminar and turbulent thermal conductivity,  $T_k$  the temperature,  $Q_{k,w}$  the heat flux from the wall to phase  $k$  per unit of volume and time.

However, this ensemble-average approach requires a given number of closure laws since this operation removes most of the information about smaller scales physics such as interfacial exchanges between phases

or wall-fluid interaction. Terms for which this modeling effort is needed are colored in orange in equations 1.1, 1.2 and 1.3. The chosen expressions for those terms are detailed in subsections 1.1.3, 1.1.4 and 1.1.5.

### 1.1.3 Interfacial transfers closure laws

The interfacial transfers of mass, momentum and energy are respectively noted in equations 1.1, 1.2 and 1.3 :  $\Gamma_k$ ,  $\overline{F}_{k,i}$  and  $Q_{k,i}$ .

#### Heat and mass transfers :

The mass transfer term, can be written as :  $\Gamma_{L,i} + \Gamma_{G,i} = 0$ ,  $\Gamma_{L,w} + \Gamma_{G,w} = 0$  with  $\Gamma_{G,w} \geq 0$  in the case of boiling flows. This finally gives  $\Gamma_L = -\Gamma_G$ .

The interfacial heat flux  $Q_{k,i}$  can be rewritten in terms of interfacial area concentration  $a_i$  :  $Q_{k,i} = q''_{k,i} a_i$ . Neglecting the mechanical contribution compared to the thermal terms, the energy jump condition can then be expressed as :

$$\sum_{k=L,G} \left( \Gamma_{k,i} h_{k,i} + q''_{k,i} a_i \right) = 0 \quad (1.4)$$

The estimation of  $h_{k,i}$  is not straightforward since it can either be supposed to be the saturation enthalpy of phase  $k$  at the system pressure (H1) or  $h_{k,i}$  can be identified to the phase-averaged enthalpy (H2). In NEPTUNE\_CFD, the assumption H2 is chosen, thus giving the bulk condensation rate :

$$\Gamma_{L,i} = \frac{a_i \left( q''_{L,i} + q''_{G,i} \right)}{h_{G,sat} - h_{L,sat}} \quad (1.5)$$

The interfacial heat flux densities  $q''_{k,i}$  and interfacial area concentration  $a_i$  are expressed as  $q''_{k,i} = C_{k,i} (T_{sat}(P) - T_k)$  and  $a_i = 6\alpha_G/d_G$ ,  $d_G$  being the gas phase Sauter mean bubble diameter. The interfacial area is computed using the transport equation of RUYER & SEILER [Ruyer2009].

For subcooled liquid, the following heat transfer coefficient is used [Manon2000]:

$$C_{L,i} = \frac{Nu_L \lambda_L}{d_G} \text{ and } Nu_L = 2 + 0.6 Re^{1/2} Pr_L^{1/3} \quad (1.6)$$

Where  $Re$  is the bubble Reynolds number  $Re = ||\overline{U}_G - \overline{U}_L|| d_G / \nu_L$  and  $Pr_L$  the liquid Prandtl number  $Pr_L = \nu_L / \eta_L$  with  $\nu_L$  and  $\eta_L$  respectively being the liquid kinematic viscosity and thermal diffusivity. On the other hand, if the liquid is overheated, the maximum of three heat transfer coefficients accounting for different heat tranfer mechanisms is taken [Berne1983]:

$$C_{L,i} = \max (C_{L,i,1}; C_{L,i,2}; C_{L,i,3}) \quad (1.7)$$

With  $C_{L,i,n} = \lambda_L Nu_{L,n} / d_G$  and :

$$Nu_1 = \sqrt{\frac{4}{\pi} Pe} ; Nu_2 = 2 ; Nu_3 = \frac{12}{\pi} Ja \quad (1.8)$$

where  $Pe = ||\overline{U}_G - \overline{U}_L|| d_G / \eta_L$  is the Peclet number and  $Ja = \rho_L c_{p,L} |T_{sat} - T_L| / (\rho_g h_{lg})$  the Jakob number, with  $h_{lg}$  the latent heat of vaporization. Those three Nusselt numbers respectively correspond to convection heat transfer, stationnary conduction and transient conduction.

For the gas phase, a simple law that ensures that the vapor temperature remains close to the saturation temperature is used (which is expected for small bubbles, e. g. in a PWR) :

$$C_{G,i} a_i = \frac{\alpha_G \rho_v c_{p,G}}{t_c} \quad (1.9)$$

where  $c_{p,G}$  is the gas heat capacity at constant pressure, and  $t_c$  a characteristic (relaxation) time given by the user (default value being  $t_c = 0.01s$ ) .

#### Interfacial forces :

The interfacial momentum transfer (excluding transfer associated to transfer of mass  $\Gamma_k$ ) is assumed to be composed of 4 different forces being the, drag  $D$ , the added mass  $AM$ , the lift  $L$  and the turbulent dispersion  $TD$  :

$$\overline{F}_{k,i} = \overline{F}_{k,D} + \overline{F}_{k,AM} + \overline{F}_{k,L} + \overline{F}_{k,TD} \quad (1.10)$$

The turbulent dispersion force  $\overline{F_{k,TD}}$  originates from the averaging operation conducted on the three other forces' expressions, detailed in equations 1.11, 1.12, 1.13 and 1.14.

$$\overline{F_{G,D}} = -\overline{F_{L,D}} = -\frac{1}{8}a_i\rho_L C_D \left| \left| \overline{U_G} - \overline{U_L} \right| \right| (\overline{U_G} - \overline{U_L}) \quad (1.11)$$

$$\begin{aligned} \overline{F_{G,AM}} = -\overline{F_{L,AM}} &= -C_{AM} \frac{1+2\alpha_G}{1-\alpha_G} \alpha_G \rho_L \\ &\times \left[ \left( \frac{\partial \overline{U_G}}{\partial t} + \bar{\nabla}(\overline{U_G}) \cdot \overline{U_G} \right) - \left( \frac{\partial \overline{U_L}}{\partial t} + \bar{\nabla}(\overline{U_L}) \cdot \overline{U_L} \right) \right] \end{aligned} \quad (1.12)$$

$$\overline{F_{G,L}} = -\overline{F_{L,L}} = -C_L \alpha_G \rho_L (\overline{U_G} - \overline{U_L}) \wedge (\bar{\nabla} \wedge \overline{U_L}) \quad (1.13)$$

$$\overline{F_{G,TD}} = -\overline{F_{L,TD}} = -\frac{2}{3}\alpha_L \alpha_G C_{TD} \bar{\nabla}(\alpha_G) \quad (1.14)$$

with  $C_D$ ,  $C_{AM}$ ,  $C_L$  and  $C_{TD}$  the associated forces coefficients, respectively taken from ISHII [Ishii1967], ZUBER [Zuber1964], TOMIYAMA [Tomiyama2002] and the Generalized Turbulent Dispersion model (GTD) from LAVIEVILLE *et al.* [Lavieville2017].

#### 1.1.4 Turbulence modeling

For bubbly flow simulations, only liquid phase turbulence is taken into account. The prescribed model is the Reynolds Stress Model (RSM)  $R_{ij} - \varepsilon SSG$  from SPEZIALE, SARKAR and GATSKI [Speziale1991] adapted to two-phase boiling flows by MIMOUNI *et al.* [Mimouni2011].

#### 1.1.5 Wall boiling model

The modeling of the heterogeneous boiling phenomenon at the wall is based on a Heat Flux Partitioning (HFP) model, inspired by KURUL & PODOWSKI original work [Kurul1990] who divided the wall heat flux density  $\phi_w$  in three terms :

- A single phase convective heat flux  $\phi_{c,L}$  heating the liquid through the fraction of the wall area unaffected by the vapor bubbles
- A vaporization heat flux  $\phi_e$  which accounts for the generation of vapor through heterogeneous nucleation
- A quenching heat flux  $\phi_q$  to represent the thermal impact of bubbles departing from the wall and being replaced by cool liquid

A fourth flux is added to this HFP in NEPTUNE\_CFD, following MIMOUNI *et al.* [Mimouni2016] who consider a convective heat flux heating the vapour  $\phi_{c,G}$  when the wall area is covered by a dense accumulation of bubbles.

The model thus gives Equation 7.1 :

$$\phi_w = \phi_{c,L} + \phi_e + \phi_q + \phi_{c,G} \quad (1.15)$$

The convective heat fluxes are expressed as  $\phi_{c,k} = A_k h_{k,log} (T_w - T_k)$  with  $h_{k,log} = \rho_k c_{p,k} u^* / T_L^+$ ; where  $A_k$  the fraction of the wall area facing phase  $k$ ,  $T_w$  the wall temperature and  $h_{k,log}$  the wall logarithmic convective heat transfer coefficient to phase  $k$  based on the wall functions for friction velocity  $u^*$  and non-dimensional liquid temperature  $T_L^+$  described in 1.1.6.

The vaporization heat flux is computed following  $\phi_e = N_{sit} f \rho_G h_{lg} \pi d_d^2 / 6$  with :

- $N_{sit}$  the nucleation site density modeled as [Lemmert1977] :  $N_{sit} = [210 (T_w - T_{sat})]^{1.8}$
- $f$  the bubble detachment frequency expressed as [Cole1960] :  $f = \sqrt{\frac{4}{3} \frac{g |\rho_v - \rho_l|}{\rho_l d_d}}$
- $d_d$  the bubble detachment diameter given by Unal correlation [Unal1976] corrected by Borée *et al.* [ref] (Equation 1.16)

$$d_d = 2.42 \times 10^{-5} P^{0.709} \frac{a}{\sqrt{b\varphi}} \text{ with } a = \frac{(T_w - T_{sat}) \lambda_w}{2\rho_G h_{lg} \sqrt{\pi \eta_w}} \text{ and } b = \begin{cases} \frac{T_{sat} - T_L}{2(1 - \rho_G/\rho_L)}, & \text{if } St \leq 0.0065 \\ \frac{1}{2(1 - \rho_G/\rho_L)} \frac{\phi_{c,L} + \phi_e + \phi_q}{0.0065 \rho_L c_{p,L} \|\overline{U}_L\|}, & \text{if } St > 0.0065 \end{cases} \quad (1.16)$$

where  $\lambda_w$  and  $\eta_w$  are the wall thermal conductivity and diffusivity,  $St = \frac{\phi_{c,L} + \phi_e + \phi_q}{\rho_L c_{p,L} \|\overline{U}_L\| (T_{sat} - T_L)}$  is the Stanton number and  $\varphi = \max \left( 1; \left( \frac{\|\overline{U}_L\|}{U_0} \right)^{0.47} \right)$  with  $U_0 = 0.61 \text{ m/s}$

Finally, the quenching heat flux follows the approach of DEL VALLE & KENNING [DelValle1985] supposing that it follows a semi-infinite transient conduction regime :  $\phi_q = A_G t_q f \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi \eta_L t_q}}$  where  $t_q$  is the quenching time, supposed to be equal to  $1/f$ .

### 1.1.6 Wall function for dispersed boiling flows

In boiling flows, the formation of bubbles at the wall may disturb the liquid velocity profile in the boundary layer. To take this phenomena into account, MIMOUNI *et al.* [Mimouni2016] proposed a wall function for boiling flows which tends to the single-phase formulation when  $\alpha_G \rightarrow 0$  and depends on the bubble diameter and density at the wall :

$$u^+ = \frac{1}{\kappa} \ln(y^+) + B - \Delta u^+ \text{ with } \Delta u^+ = \begin{cases} 0 & \text{if } k_r^+ \leq 11.3 \\ \frac{1}{\kappa} \ln(1 + C_{kr} k_r^+) & \text{if } k_r^+ > 11.3 \end{cases} \quad (1.17)$$

where  $\kappa=0.41$  is the Von Karman constant,  $B = 5.3$  the standard single-phase logarithmic law constant.  $\Delta u^+$  represents the offset of  $u^+$  due to the wall roughness induced by the presence of bubble. We have  $C_{kr} = 0.5$  and  $k_r = \frac{k_r \sqrt{u^* u_T}}{\nu_L}$  the "roughness Reynolds number" with  $u_T = C_\mu^{1/4} \sqrt{k_L}$  defined from the  $k - \varepsilon$  constant  $C_\mu = 0.09$  and  $k_L$  the liquid turbulent kinetic energy. Finally,  $k_r = \alpha_G d_G$ .

The non-dimensional wall liquid temperature  $T_L^+$  is modeled according to LEDUC [Leduc1995]:

$$T_L^+ = \begin{cases} Pr_L y^+, & \text{if } y^+ \leq 13.2 \\ 8.67 Pr_{L,T} \left( \frac{Pr_L}{Pr_{L,T}} - 1 \right) \left( \frac{Pr_{L,T}}{Pr_L} \right)^{0.25} + \frac{Pr_{L,T}}{\kappa} \ln(Ey^+) & \text{if } y^+ > 13.2 \end{cases} \quad (1.18)$$

With  $Pr_{L,T} = 0.9$  the turbulent liquid Prandtl number, and  $E = 7.76$  a constant for smooth walls.

# 2

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## DEBORA CASES

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### 2.1 BOILING FREON IN A SIMPLE TUBE : DEBORA EXPERIMENTS

In this section, we simulate upward boiling flows of R12 in a vertical tube and compare the NEPTUNE\_CFD (NCFD) results with experimental measurements conducted by CEA & EDF on the DEBORA test facility.

#### 2.1.1 *Description of the experiment*

In the end of 1990's, CEA and EDF built a test facility called DEBORA which goal was to conduct series of experiments and measurements to establish a database for boiling flows of freon R12. The choice of freon is justified because of its use as a simulating fluid for water in PWR conditions (very close phase density ratio, Weber number We, Boiling number Bo and thermodynamic flow quality  $x_{eq}$ ). Table 2.1 sums up the flow conditions scaling between R12 and water.

Table 2.1: Water/R12 scaling (from Garnier *et al.* [Garnier2001],  $D_h = 19.2\text{mm}$  for We)

Fluid	Water	Freon R12
Pressure $P$ (bar)	100 - 180	14 - 30
Mass velocity $G$ ( $\text{kg}/\text{m}^2/\text{s}$ )	1000 - 5000	1000 - 5000
Wall heat flux $\phi_w$ ( $\text{MW}/\text{m}^2$ )	0.5 - 6	0.05 - 0.65
Thermodynamic flow quality $x_{eq}$ (-)	(-0.4) - (+0.4)	(-0.4) - (+0.4)
Density ratio $\rho_G / \rho_L$ (-)	0.08 - 0.25	0.07 - 0.22
Weber number We (-)	2374 - 368 579	3319 - 438 966
Boiling number Bo (-)	$3.67 \times 10^{-5} - 2.39 \times 10^{-3}$	$2.65 \times 10^{-5} - 1.74 \times 10^{-3}$

The DEBORA experiment consists of an upward subcooled boiling flow of R12 in a 4m length pipe uniformly heated over 3.5m with an hydraulic diameter  $D_h = 19.2\text{mm}$ . Measurements of void fraction ( $\alpha$ ), interfacial velocity (*i.e.* axial gas velocity  $U_{G,z}$ ), bubble diameter ( $d_G$ ), liquid temperature ( $T_L$ ) and wall temperature ( $T_w$ ) at the end of the heating length were conducted through different series of tests. Experimental apparatus is detailed in Garnier *et al.* [Garnier2001] and Manon[Manon2000].

Different test campaigns were conducted on this experimental setup, in particular :

- Campaign 2900 : measurements of  $\alpha$ ,  $U_{g,z}$  and  $d_G$  using one optical probe
- Campaign 3000 : measurements of  $\alpha$ ,  $U_{g,z}$  and  $d_G$  using two optical probes
- Campaign 800 : measurements of  $T_L$  and  $T_w$  using thermocouples

Each experimental case is named following this nomenclature : CccGgPppWwwTtt with cc being the campaign number, g the inlet mass velocity ( $G$ ), pp the outlet pressure ( $P$ ), ww the total heat power applied ( $W$ ) and tt the inlet temperature ( $T_{in}$ ). For instance, C8G3P26W23Te69 refers to the case from the campaign 800 with  $G \approx 3000 \text{ kg/m}^2/\text{s}$ ,  $P \approx 26 \text{ bar}$ ,  $W \approx 23 \text{ kW}$  and  $T_{in} \approx 69^\circ\text{C}$ .

### 2.1.2 Analysis of the Experimental Measurements

### 2.1.3 Comparison with One Dimensional Correlations

### 2.1.4 Experimental Heat Flux Corrections

### 2.1.5 NEPTUNE\_CFD simulations of DEBORA cases

In this work, we present the simulations of the following cases :

- C8G2P26W16Te44.9 and C8G2P26W16Te49.6 (single-phase flow)
- C8G2P26W16Te66.6 and C8G2P26W16Te70.3 (two-phase flow)
- C30G2P26W16Te66.6 and C30G2P26W16Te70.6 (two-phase flow)

The pressure of 26 bar is chosen to match the pressure of the mixing vanes cases (DEBORA-Promoteur, Section 8.1). Mesh sensitivity is performed over two meshes : a large mesh (M1) with 460 356 cells = 338 radial  $\times$  1362 axial cells and a fine mesh (M2) with 3 157 952 cells = 1568 radial  $\times$  2014 axial cells. On Figure 2.1, we present the results regarding liquid temperature at the outlet and wall temperature. The liquid temperature profile seems to be correctly reproduced by the simulations, though we see a slight overestimation close to the wall. Looking closer at boiling cases shows a difference of  $\approx 0.5^\circ\text{C}$ , which is close to the uncertainty of the measurements [Garnier2001]. Concerning the wall temperature, it appears that it is underestimated before the **Onset of Nucleate Boiling** (ONB) ( $T_w < T_{sat}$ ) and overestimated after the ONB ( $\approx +5^\circ\text{C}$ ). Post-ONB wall temperature is characterized by a stabilization of its value above the saturation temperature (here  $T_{w,ONB} - T_{sat} \approx 2^\circ\text{C}$ ).

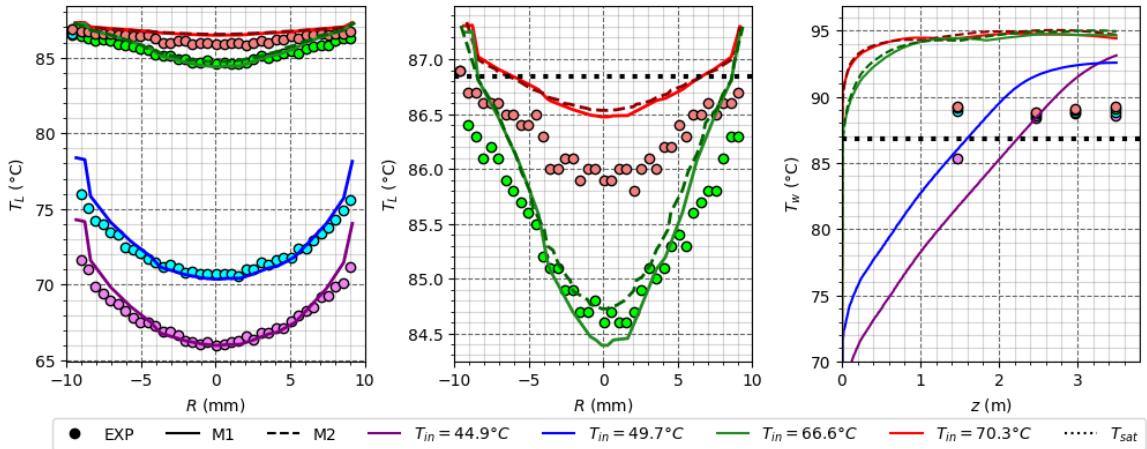


Figure 2.1: NCFD (lines) vs. Exp. (circles) -  $T_L$  and  $T_w$  - Cases C8G2P26W16Te44.9, Te49.6, Te66.6 and Te70.3 - Simulations using two meshes M1 (coarse) and M2 (fine).

On Figure 2.2, we compare the results of the simulations to the experiments regarding void fraction, bubble Sauter diameter and axial gas velocity. Void fraction profiles are quite correctly reproduced, though we observe a 10% higher peak at the wall for  $T_{in} = 66.6^\circ\text{C}$ . The order of magnitude of bubble diameter is correct ( $\sim 0.1\text{mm}$ ) and NEPTUNE\_CFD manages to detect coalescence (increase of bubble diameter when leaving the wall) and bulk condensation (decrease of bubble diameter when reaching the core of the flow), which is in qualitative agreement with the experiments. Quantitatively speaking, bubble diameter is globally underestimated. Finally, gas velocity profile is reasonably reproduced for  $T_{in} = 66.6^\circ\text{C}$ , but not for  $T_{in} = 70.6^\circ\text{C}$ . The latter experimental profile is flatter, which could be explained by a change of flow regime since uncondensed vapor is detected in the bulk.

Finally, the simulations reasonably agree with the experiments. The strongest discrepancies being mostly the wall temperature and bubble diameter. Potential ways of improving those results are investigated in next sub-section.

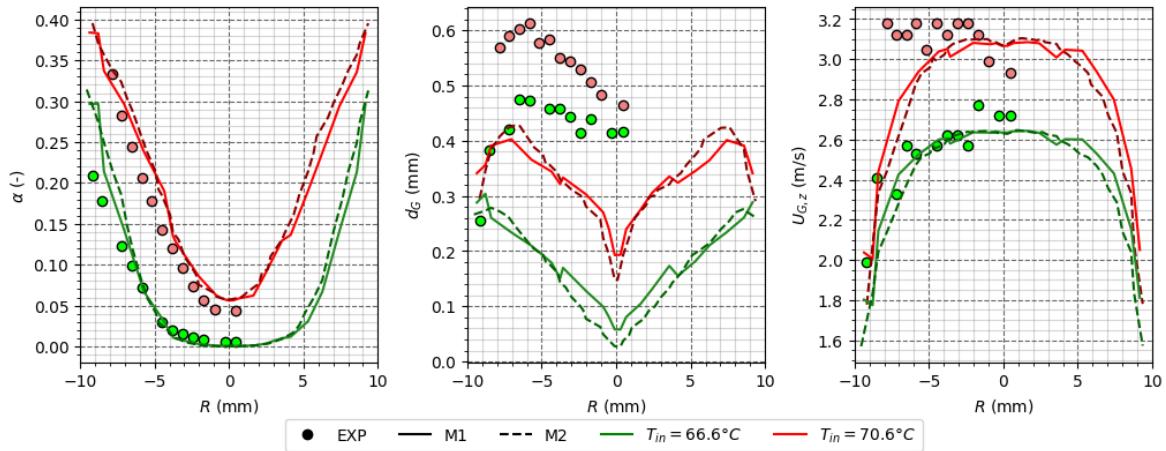


Figure 2.2: NCFD (lines) vs. Exp. (circles) -  $\alpha$ ,  $d_G$  and  $U_{G,z}$  - Cases C30G2P26W16Te66.6 and Te70.6 - Simulations using two meshes M1 (coarse) and M2 (fine).

### 2.1.6 Investigating the nucleation site density modeling $N_{sit}$

In NEPTUNE\_CFD, wall temperature is computed through the Heat Flux Partitioning model, which role is to find the appropriate  $T_w$  which balances Equation 7.1. However, some laws used to express parameters such as  $N_{sit}$ ,  $f$ , or  $d_d$  are quite old and simple. For instance, the Lemmert & Chawla [Lemmert1977] expression of  $N_{sit}$  only depends on the wall superheat (Sub-section 1.1.5).

A comparison of the Lemmert & Chawla law [Lemmert1977] with the Hibiki & Ishii [Hibiki2003] law for  $N_{sit}$  against 4 data sets from the literature is presentend on Figure 2.3. The Hibiki & Ishii correlation depends simultaneously on wall superheat, pressure and contact angle. Experimental measurements of Borishanskii *et al.* [Borishanskii1961], Richenderfer *et al.* [Richenderfer2018], Kossolapov *et al.* [Kossolapov2020] and Zhou *et al.* [Zhou2020] are used to assess the two nucleation site density correlations.

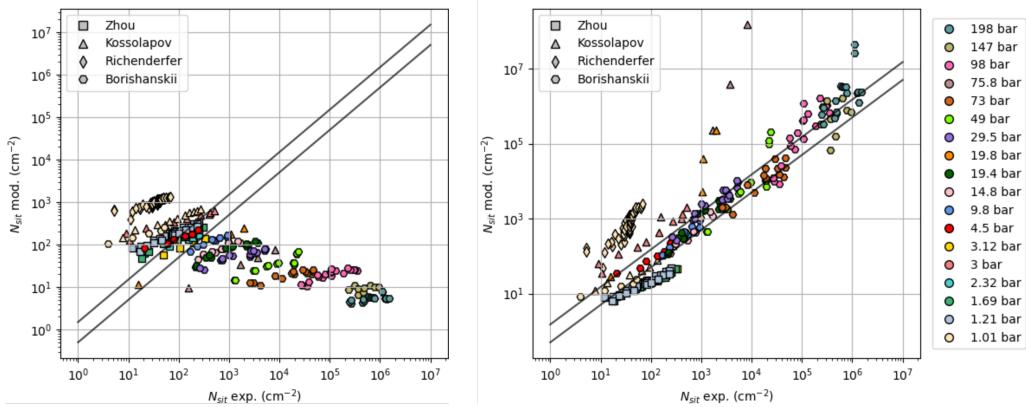


Figure 2.3:  $N_{sit}$  correlations of Lemmert & Chawla (left) and Hibiki & Ishii (right) vs. exp. data from literature. Operation pressures are displayed.  $\pm 50\%$  error bars are drawn in black.

Figure 2.3 clearly shows that the Lemmert & Chawla law lack of pressure dependence fails to reproduce high pressure measurements contrary to the Hibiki & Ishii one. Even though Hibiki & Ishii correlation shows significant discrepancies with measurements of Kossolapov *et al.* and Richenderfer *et al.*, its prediction capability is greater in average than Lemmert & Chawla correlation.

To assess the influence of nucleation site density law on NEPTUNE\_CFD computations, we compare results obtained with both correlations on Figure 2.4, which shows a remarkable impact of the modification of  $N_{sit}$  correlation. Using Hibiki & Ishii correlation reduces the error on  $T_w$  by approximately  $2^\circ\text{C}$  while  $\alpha$  and  $T_L$  remain unchanged. This implies that the same heat flux partitioning is found with the two models, but that the pressure dependence of Hibiki & Ishii law helped to balance Equation 7.1 using a lower  $T_w$ , thus closer to experimental measurements.

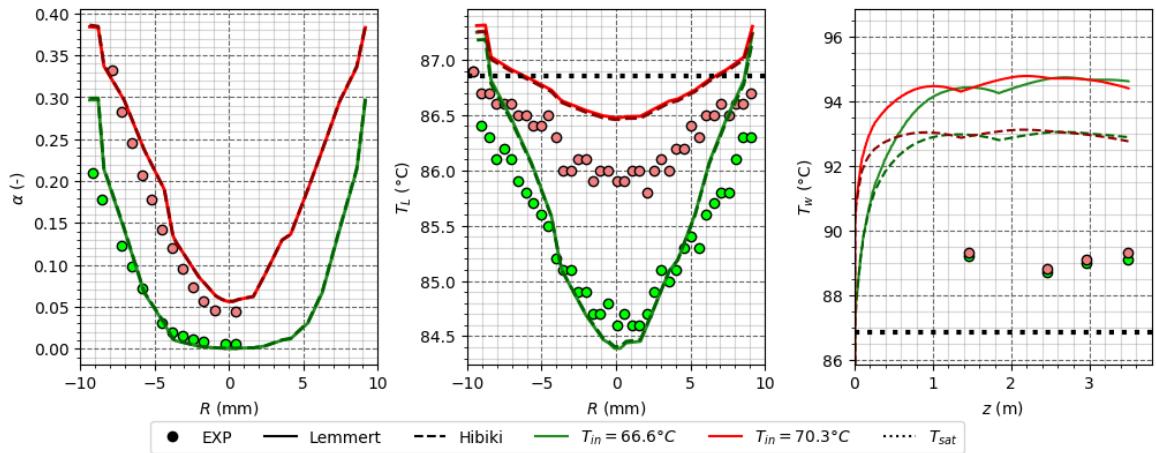


Figure 2.4: NCFD results for  $\alpha$ ,  $T_L$  and  $T_w$  using Lemmert & Chawla and Hibiki & Ishii correlation. Cases 8G2P26W23Te66.6 and Te70.3, 30G2P26W23Te66.6 and 70.6.

Such a result indicates that the HFP model could be improved through a systematic analysis of each parameter's impact and modeling (bubble departure diameter, detachment frequency, etc.). Assembling a more recent and consistent model could provide better results regarding wall temperature prediction. Models such as the one developed by Kommajosyula [**Kommajosyula2020**] could be interesting to apply for high-pressure flows.

Now that simple tube boiling flow has been assessed through the presented results, next section will focus on the simulation of boiling flow in a tube equipped with a mixing device.

### Part III

## DEVELOPMENT OF A NEW WALL HEAT FLUX PARTITIONING MODEL

# 3

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## INTRODUCTION

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### 3.1 FIRST APPROACHES : CORRELATIONS

# 4

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## EXISTING HEAT FLUX PARTITIONING MODELS

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### 4.1 KURUL & PODOWSKI (1990)

In their original work published in 1990, Kurul & Podowski [**kurul\_1990**] proposed a complete closure for the wall heat flux partitioning. They considered the applied heat flux to be divided between three mechanisms:

- A liquid single-phase heat flux  $\phi_{c,L}$  ;
- A boiling heat flux  $\phi_e$  ;
- A quenching heat flux  $\phi_q$  induced by bubbles leaving the surface.

The total wall heat flux being :

$$\phi_w = \phi_{c,L} + \phi_e + \phi_q \quad (4.1)$$

The convective heat flux is expressed as :

$$\phi_{c,l} = A_{c,L} \rho_L c_{p,L} U_{L,\delta} \text{St}_{L,\delta} (T_w - T_{L,\delta}) \quad (4.2)$$

with  $\delta$  a location in the buffer layer.

Assuming bubbles are spherical and leave the surface at diameter  $D_{lo}$ , they write:

$$\phi_e = \frac{1}{6} \pi D_{lo}^3 \rho_V h_{LV} f N_{sit} \quad (4.3)$$

$$(4.4)$$

The quenching heat flux occurring over the wait time  $t_w$  between two nucleated bubbles is computed as:

$$\phi_q = t_w f A_q \frac{2\lambda_L (T_w - T_{L,\delta})}{\sqrt{\pi \eta_L t_w}} \quad (4.5)$$

This expression corresponds to the average heat flux for semi-infinite conduction over a time  $t_w$ , as expressed by Del Valle and Kenning [**DeValle**].

They also estimate the portion of the surface affected by the bubbles as:

$$A_q = \min(1 ; F_A \pi R_{lo}^2 N_{sit}) = 1 - A_{c,L} \quad (4.6)$$

where  $F_A = 4$  accounts for the bubble influence area when leaving the surface.

**Needed closure relationships :**  $N_{sit}$ ,  $f$ ,  $t_w$ ,  $D_{lo}$

### 4.2 BASU (2000)

In 2005, Basu *et al.* [**Basu2005**, **Basu2005a**] proposed a new HFP model together with a series of experiments to further study the different needed closure relationships. This model was meant to account for finer descriptions of the multiple phenomena at stake in subcooled flow boiling. In particular, they account for bubble sliding and merging and thus distinguish bubble departure diameter  $D_d$  (leaving the nucleation site) and lift-off diameter  $D_{lo}$  (leaving the wall).

Their approach consist of separating the boiling flow in three regions (Figure 4.1):

- Pre-ONB zone, where only liquid forced convection occurs, yielding:

$$\phi_w = h_{c,L} (T_w - T_L) \quad (4.7)$$

- Zone between the ONB and the OSV, prior to observing a net amount of vapor with bubble lifting off the surface. The heat flux is then still totally transferred to the liquid, but the equivalent convective heat transfer coefficient is supposed enhanced by 30% due to the presence of bubbles on the wall:

$$\phi_w = \overline{h_{c,L}} (T_w - T_L) \approx 1.3 h_{c,L} (T_w - T_L) \quad (4.8)$$

Basu *et al.* define the ONB as:

$$T_{w,ONB} = T_{sat} + \frac{4\sigma T_{sat}}{D_c \rho_V h_{LV}} \quad (4.9)$$

$$D_c = \sqrt{\frac{8\sigma T_{sat} \lambda_L}{\rho_V h_{LV} \phi_w}} (1 - \exp(-\theta^3 - 0.5\theta)) \quad (4.10)$$

- Post-OSV zone, where bubbles now leave the surface towards the bulk flow. This is where the other parts of the HFP appear *i.e.* the boiling and quenching fluxes. The beginning of OSV is defined by Basu *et al.* as:

$$T_{L,OSV} = T_{sat} - 0.7 \exp \left( -0.065 \frac{D_d h_{c,L}}{\lambda_L} \right) \frac{\phi_w}{h_{c,L}} \quad (4.11)$$

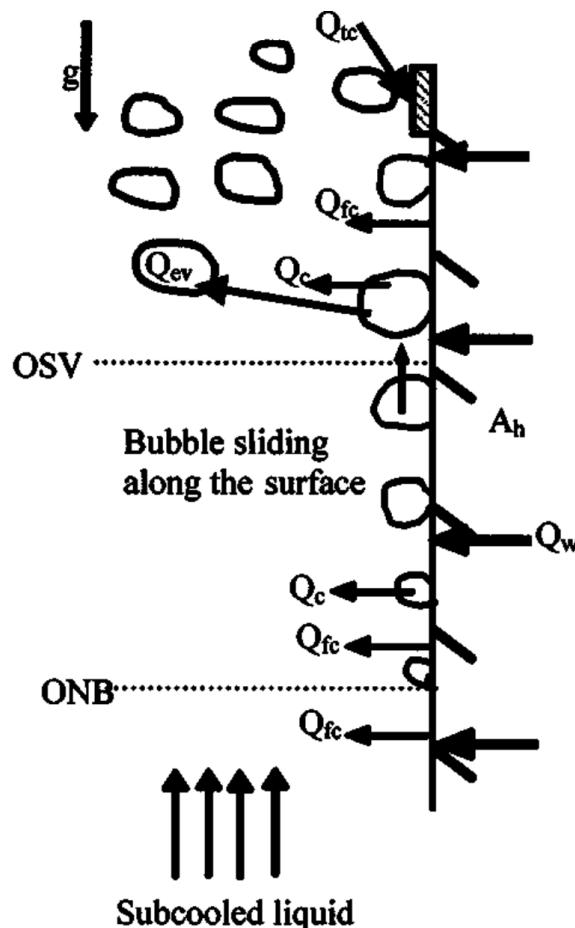


Figure 4.1: Sketch of the heat transfers zones considered by Basu *et al.*. (Adapted from [Basu2005])

The hypothesis of Basu *et al.* is that the heat flux is first transferred to the superheated liquid close to the wall (by convection and transient quenching), part of which contributing to the evaporation through

the liquid-vapor interface. The remaining heat is transferred to the bulk liquid ( $\phi_L$ ) either from the superheated liquid layer or bubble condensation. The whole heat transfer mechanism can thus be written as:

$$\phi_w = \phi_{c,L} + \phi_q = \phi_e + \phi_L \quad (4.12)$$

In order to estimate the quenching heat flux associated to bubble sliding and lift-off, Basu *et al.* consider two cases:

- 1) Bubble sliding from departure ( $D = D_d$ ) to lift-off ( $D = D_{lo}$ ) ;
- 2) Bubble coalescence with neighboring sites before departure.

Those two cases are distinguished using the average distance between nucleation sites  $s$ , which they suppose equal to  $1/\sqrt{N_{sit}}$ .

#### 4.2.1 Case 1 : Bubble sliding, $D_d < s$

In this situation, the bubble will grow up to its departure diameter  $D_d$  and slide over a length  $l_{sl,0}$  before lifting-off. If  $l_{sl,0} < s$ , the bubble will slide up to its lift-off diameter  $D_l$  and leave the wall without colliding with other bubbles. On the contrary, if  $l_{sl,0} \geq s$  the sliding bubble will merge with bubbles growing on their nucleation site, inducing a sudden growth of the bubble diameter that can exceed  $D_{lo}$  and thus lift-off after sliding over a reduced length  $l_{sl} < l_{sl,0}$ . Those assumptions are summarized on Figure 4.2.

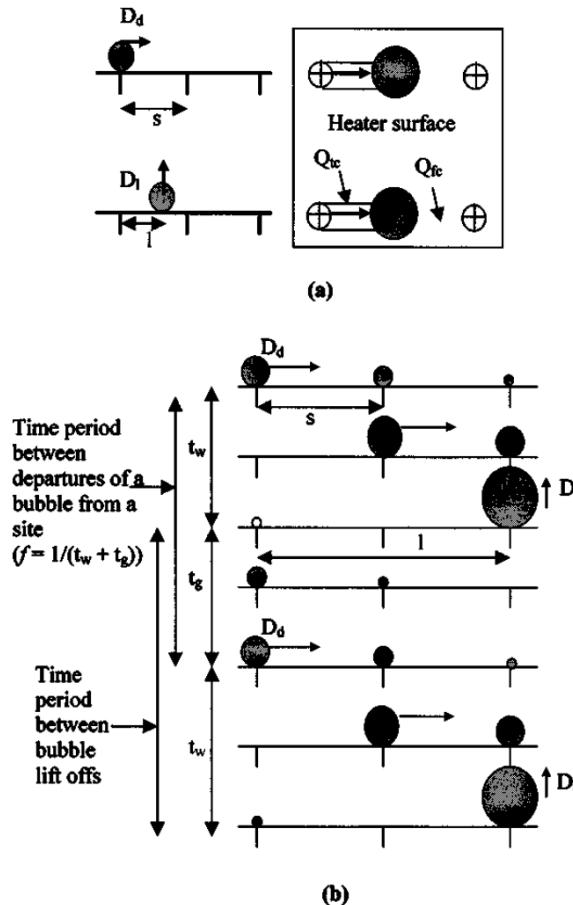


Figure 4.2: Sliding bubble behavior considered by Basu *et al.* (Adapted from [Basu2005])

If bubble coalescence occurs, the number of bubbles lifting-off the surface is lower than the actual number of nucleating sites. Basu *et al.* thus define a reduction factor:

$$R_f = \begin{cases} \frac{s}{l_{sl}} = \frac{1}{l\sqrt{N_{sit}}} & \text{if } l_{sl,0} \geq s \\ 1 & \text{if } l_{sl,0} < s \end{cases} \quad (4.13)$$

Regarding bubble sizes, they suppose that bubbles coalesced by a sliding bubble while growing have a diameter  $D_d$  *i.e.* they were close to departure (in reality, the coalesced bubble would have a diameter  $D < D_d$ ). This results in a bubble of diameter  $D = (D_{sl}^3 + D_d^3)^{1/3}$  which will lift-off if  $D > D_{lo}$ . Consequently, a sliding bubble can merge with numerous bubbles before lifting off. Noting  $N_{merg}$  the number of coalesced bubble and  $D_N$  the resulting bubble diameter, the sliding distance is:

$$l_{sl} = N_{merg}s + l_{D_N \rightarrow D_{lo}} \quad (4.14)$$

where  $l_{D_N \rightarrow D_{lo}}$  is the remaining distance to slide if  $D_N < D_{lo}$ , being 0 if  $D_N > D_{lo}$ .

The surface swiped by the sliding bubble is then expressed as  $A_{sl} = C\bar{D}l_{sl}$  with  $\bar{D}$  the average bubble diameter during sliding and  $C$  the ration between the bubble diameter and its foot, expressed correlating measurements from Maity [Maity2000] as :

$$C = 1 - \exp(2 - \theta^{0.6}) \quad (4.15)$$

After observing in their experiments that  $D_d \approx 0.5D_{lo}$ , Basu *et al.* choose:

$$\bar{D} = \frac{D_{lo} + D_d}{2} \approx 0.75D_{lo} \quad (4.16)$$

Noting  $t^* = \left(\frac{\lambda_L}{h_{c,L}}\right)^2 \frac{1}{\pi\eta_L}$  the time at which transient conduction heat transfer becomes equal to forced liquid convection, the quenching heat flux is expressed as:

$$\phi_q = \frac{1}{t_w + t_g} \int_0^T \frac{\lambda_L}{\sqrt{\pi\eta_L t}} (T_w - T_L) A_{sl} R_f N_{sit} dt \quad (4.17)$$

where  $T = t^*$  if  $t^* < t_w + t_g$  (forced convection dominates at some point during a nucleation cycle) or  $T = t_w + t_g$  if  $t^* \geq t_w + t_g$  (transient conduction dominates over the whole nucleation cycle).

The liquid convective heat transfer is then:

$$\phi_{c,L} = \overline{h_{c,L}} (T_w - T_L) A_{c,L} + \overline{h_{c,L}} (T_w - T_L) A_{sl} R_f N_{sit} \left(1 - \min\left(1 ; \frac{t^*}{t_w + t_g}\right)\right) \quad (4.18)$$

with  $A_{c,L} = 1 - A_{sl} R_f N_{sit}$ .

And the boiling heat flux:

$$\phi_e = \rho_V h_{LV} \frac{\pi}{6} D_{lo}^3 R_f N_{sit} \frac{1}{t_w + t_g} \quad (4.19)$$

#### 4.2.2 Case 2 : Bubble coalescence without sliding, $D_d \geq s$

Under higher wall superheats, the subsequent rise in the nucleation site density  $N_{sit}$  can lead to boiling regimes where bubbles coalesce with each other at early stages of their lifetime *i.e.* while still attached to their nucleation site. This situation is accounted for by Basu *et al.* in the case when  $D_d \geq s$  by considering immediate lift-off of coalesced bubble at radius  $D > D_{lo}$ . In this case, the total number of bubbles leaving the surface is lower than  $N_{sit}$  and is thus reduced using:

$$R_f = \frac{s^3}{D_{lo}^3} \quad (4.20)$$

Under this massive coalescing regime, the entire surface will experience quenching due to bubble lift-off all over the heater. Depending on the values of  $t^*$ , we have:

$$\phi_q = \begin{cases} \frac{1}{t_w + t_g} \int_0^{t^*} \frac{\lambda_L}{\sqrt{\pi \eta_L t}} (T_w - T_L) dt & \text{if } t^* < t_w \\ \frac{1}{t_w + t_g} \left[ \int_0^{t_w} \frac{\lambda_L}{\sqrt{\pi \eta_L t}} (T_w - T_L) dt + \int_0^T \frac{\lambda_L}{\sqrt{\pi \eta_L t}} (T_w - T_L) [1 - S_b N_{sit}] dt \right] & \text{if } t^* \geq t_w \end{cases} \quad (4.21)$$

$$\phi_{c,L} = \begin{cases} \overline{h_{c,L}} (T_w - T_L) \frac{t_w - t^*}{t_w + t_g} + \overline{h_{c,L}} (T_w - T_L) [1 - A_b N_{sit}] \frac{t_g}{t_w + t_g} & \text{if } t^* < t_w \\ \overline{h_{c,L}} (T_w - T_L) [1 - A_b N_{sit}] \frac{t_w + t_g - t^*}{t_w + t_g} & \text{if } t^* \geq t_w \end{cases} \quad (4.22)$$

with  $A_b = \frac{\pi (Cs) 2}{4}$ .

And the boiling heat flux still expressed as Eq. 4.19.

Needed closure relationships :  $N_{sit}$ ,  $t_w$ ,  $t_g$ ,  $D_d$ ,  $D_{lo}$ ,  $l_{sl,0}$ ,  $h_{c,L}$ .

### 4.3 GILMAN (2017)

A more recent HFP model dedicated to CFD simulations has been proposed by Gilman & Baglietto in 2017 [gilman2017]. Among the different advances proposed in their work, we can mention :

- A probabilistic law to account for static interaction between nucleation sites ;
- A force-balance approach to compute the bubble departure and lift-off diameters ;
- A generic law for the enhanced forced convection coefficient accounting for bubble presence ;
- The presence of a modified quenching term accounting for local wall superheat beneath a bubble dry spot.

The total heat flux is partitioned between the liquid forced convection  $\phi_{c,L}$ , the solid quenching  $\phi_{q,s}$ , the quenching due to bubble sliding  $\phi_{q,sl}$  and the evaporation flux  $\phi_e$ . Yielding:

$$\phi_w = \phi_{c,L} + \phi_{q,s} + \phi_{q,sl} + \phi_e \quad (4.23)$$

The convective term is computed in a way similar to Basu *et al.* [Basu2005] in Eq. 4.18:

$$\phi_{c,L} = \phi_{c1,L} + \phi_{c2,L} \quad (4.24)$$

$$= h_{c,L} (1 - A_{sl} N_{sit,a}^*) (T_w - T_L) + \overline{h_{c,L}} A_{sl} N_{sit,a}^* \left( 1 - \frac{t^*}{t_w + t_g} \right) (T_w - T_L) \quad (4.25)$$

where  $N_{sit,a}^*$  is the active nucleation site density that will generate sliding bubbles, that can differ from the empirical value of available sites  $N_{sit}$  usually computed by a correlation.

The active nucleation site density is actually smaller than  $N_{sit}$  since Gilman considers an static interaction between the available sites *i.e.* the fact that a bubble laying on a site may be blocking nucleation from sites laying beneath its foot. Following a Complete Spatial Randomness (CSR) approach, they express the probability to find a site under a growing bubble of radius  $R_d$  as:

$$\mathcal{P} = 1 - e^{-N_b \pi R_d^2} \quad (4.26)$$

where  $N_b = \frac{t_g}{t_w + t_g} N_{sit}$  is the density of bubbles covering the heater.

The number of active sites is then computed as:

$$N_{sit,a} = (1 - \mathcal{P}) N_{sit} \quad (4.27)$$

$$= \exp \left( -\frac{t_g}{t_w + t_g} N_{sit} \pi R_d^2 \right) N_{sit} \quad (4.28)$$

This value is then reduced by Gilman to obtain  $N_{sit,a}^*$  using a reduction factor representing sliding bubble coalescence (similar to Basu in Eq. 4.13):

$$N_{sit,a}^* = R_f N_{sit,a} = \frac{s}{l_{sl,0} + s} N_{sit,a} \quad (4.29)$$

The sliding quenching term is also computed in a similar way to Basu as:

$$\phi_{q,sl} = \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi\eta_L t^*}} A_{sl} N_{sit,a}^* \quad (4.30)$$

$$A_{sl} = \overline{D} l_{sl} = \frac{D_d + D_{lo}}{2} (N_{merg}s + l_{D_N \rightarrow D_{lo}}) \quad (4.31)$$

Regarding the boiling heat flux, Gilman splits it in two contributions respectively associated with the inception of nucleation and liquid microlayer evaporation :

$$\phi_e = \phi_{e,init} + \phi_{e,ML} \quad (4.32)$$

$$= \frac{4}{3}\pi R_d^3 \rho_V h_{LV} \frac{1}{t_w + t_g} N_{sit,a} + V_{ML} \rho_L h_{LV} \frac{1}{t_w + t_g} N_{sit,a} \quad (4.33)$$

$$\text{with } V_{ML} = \frac{2}{3}\pi \left(\frac{R_d}{2}\right)^3 \delta_{max} \quad (4.34)$$

where  $\delta_{max} = 2 \mu\text{m}$  based on experiments from Gerardi [Gerardi].

Finally, the solid quenching term is written as:

$$\phi_{q,s} = \rho_w c_{p,w} V_q \delta T_q \frac{1}{t_g + t_w} N_{sit,a} \quad (4.35)$$

$$V_q = \frac{2}{3}\pi r_w^2 \quad (4.36)$$

with  $\Delta T_q = 2 K$  as suggested by Gerardi *et al.* [Gerardi\_eta].

**Needed closure relationships :**  $N_{sit}$ ,  $t_w$ ,  $t_g$ ,  $D_d$ ,  $D_{lo}$ ,  $l_{sl,0}$ ,  $h_{c,L}$ ,  $\overline{h_{c,L}}$ .

#### 4.4 ZHOU (2020)

The last HFP model we will look through in this Chapter was proposed by Zhou *et al.* [zhou]. It is one of the most recent available in the literature and was built along with associated experiments for validation. In particular they compute separate heat flux contributions for static (*st*) or sliding bubbles (*sl*), yielding a total heat flux:

$$\phi_w = \phi_{c,L} + (\phi_{e,st} + \phi_{e,sl}) + (\phi_{e,sl} + \phi_{q,sl}) \quad (4.37)$$

An interesting aspect of Zhou *et al.* work is the presence of a condensation term in the evaporation heat fluxes, written as:

$$\phi_{e,st} = \rho_V h_{LV} \frac{4}{3}\pi R_d^3 N_{sit,a} f + h_{cond} (T_{sat} - T_L) A_{cond} N_{sit,a} f (t_g - t_s) \quad (4.38)$$

$$\phi_{e,sl} = \frac{\pi}{6} \rho_V h_{LV} (D_{lo}^3 - D_d^3) f N_{sit}^* + h_{cond} (T_{sat} - T_L) A_{cond} N_{sit}^* t_{sl} f \quad (4.39)$$

$$A_{cond} = \pi D_d \max(R_d (1 + \cos(\theta)) - y_{sat}; 0) \quad (4.40)$$

where  $y_{sat}$  is the wall distance where the liquid is at saturation temperature,  $t_s$  the moment when  $D_b = y_{sat}$  and  $N_{sit}^*$  the number of sliding bubbles.

The condensation heat transfer coefficient is computed using the correlation from Ranz & Marshall:

$$h_{cond} = \frac{\lambda_L}{D_d} (2 + 0.6 \text{Re}_b^{0.5}) \text{Pr}_L^{0.3} \quad (4.41)$$

Following similar approaches to HFP models from Basu or Gilman, the quenching heat flux for static bubbles is expressed as:

$$\phi_{q,st} = \begin{cases} t_w f \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi\eta_L t_w}} A_b N_{sit} & \text{if } t^* \geq t_w \\ t^* f \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi\eta_L t^*}} A_b N_{sit} + h_{c,L} (T_w - T_L) A_b N_{sit}^* f(t_w - t^*) & \text{if } t^* < t_w \end{cases} \quad (4.42)$$

with  $A_b = \underbrace{K_b}_{=2} \pi R_d^2$ .

And for the sliding bubbles:

$$\phi_{q,sl} = \begin{cases} \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi\eta_L (t_g + t_w)}} A_{sl} N_{sit}^* & \text{if } t^* \geq t_g + t_w \\ t^* f \frac{2\lambda_L (T_w - T_L)}{\sqrt{\pi\eta_L t^*}} A_{sl} N_{sit}^* + h_{c,L} (T_w - T_L) A_{sl} N_{sit}^* f(t_g + t_w - t^*) & \text{if } t^* < t_w \end{cases} \quad (4.43)$$

with  $A_{sl} = \underbrace{K_{sl}}_{=2} \frac{D_d + D_{lo}}{2} l_{sl}$  and  $l_{sl} = \min \left( l_{sl,0} ; \frac{1}{\sqrt{t_g f N_{sit}}} \right)$ .

Finally, the forced liquid convective heat flux is computed:

$$\phi_{c,L} = h_{c,L} (1 - A_b N_{sit} - A_{sl} N_{sit}^*) (T_w - T_L) \quad (4.44)$$

Needed closure relationships :  $N_{sit}$ ,  $t_w$ ,  $t_g$ ,  $D_d$ ,  $D_{lo}$ ,  $l_{sl,0}$ ,  $h_{c,L}$ .

## 4.5 CLOSURE LAWS FOR REMAINING PARAMETERS

In each of the 4 presented HFP models, there is a number of parameters that still need to be computed in order for the model to be fully expressed. Those parameters are often the nucleation site density, bubble diameters, etc. To do so, closure relationships are used and differ from one model to another. Here we want to sum up the different choices of the authors to point the variety of possibilities that exist.

### 4.5.1 Nucleation Site Density : $N_{sit}$

The value of the nucleation site density is a very sensitive parameter that controls the intensity of the boiling heat transfer. Unfortunately, its value can vary over a very large range of value depending on the operating conditions and heater material and is thus expressed using experimental correlations.

In their work, Kurul & Podowski used the law of Lemmert and Chawla [**Lemmert**]:

$$N_{sit} = [210 (T_w - T_{sat})]^{1.8} \quad (4.45)$$

Later, Basu [**Basu**] correlated her own experimental results to obtain:

$$N_{sit} = \begin{cases} 0.34 [1 - \cos(\theta)] \Delta T_w^2 & \text{if } \Delta T_{w,ONB} < \Delta T_w < 15 K \\ 3.4 \times 10^{-5} [1 - \cos(\theta)] \Delta T_w^{5.3} & \text{if } \Delta T_w > 15 K \end{cases} \quad (4.46)$$

Finally, Zhou *et al.* followed a approach similar to Basu *et al.* by correlating their own experimental data:

$$N_{sit} = N_0 (1 - \cos(\theta)) [\exp(f(P) \Delta T_w) - 1] f(P) = 0.218 \ln \left( \frac{P}{P_0} \right) + 0.1907 \quad (4.47)$$

with  $N_0 = 55\ 395.26\ m^{-2}$  and  $P_0 = 1.01\ bar$ .

### 4.5.2 Bubble Departure Frequency $f$ , Growth Time $t_g$ and Wait Time $t_w$

Historically, Kurul & Podowski modeled the BDF according to Cole [Cole] which derived an expression using photographic observations of bubble nucleation, further verified by Ceumern & Lindenstjerna for pool boiling of water at pressures up to 8 bar. :

$$f = \sqrt{\frac{4}{3} \frac{g(\rho_L - \rho_V)}{\rho_L D_d}} \quad (4.48)$$

Then, assuming that the growth time of the bubble before departure is small compared to the wait time before a new bubble nucleates ( $t_{g,d} \ll t_w$ ) gives :

$$t_w \approx \frac{1}{f} \quad (4.49)$$

Although the assumption of a negligible growth time is true in different cases, notably when bubble departure diameter is small *i.e.* they leave their nucleation site nearly instantly, this is not generally true and the value of the ratio  $\frac{t_{g,d}}{t_w}$  can vary by decades depending on the thermal-hydraulics conditions.

Thus, later models considered separate modeling of the growth and wait time in order to compute the BDF as :

$$f = \frac{1}{t_{g,d} + t_w} \quad (4.50)$$

In this scope, Basu *et al.* [Basu] correlated their own experimental results to obtain :

$$t_g = \frac{D_d^2}{45 e^{-0.02} \text{Ja}_L \eta L \text{Ja}_w} \quad (4.51)$$

$$t_w = 139.1 \Delta T_w^{-4.1} \quad (4.52)$$

Gilman & Baglietto [Gilman] chose use the growth law of Zuber [Zuber1961] and the total BDF of Cole (Eq. 4.48) to close the relation 4.50.

$$t_{g,d} = \frac{\pi R_d^2}{4b^2 \text{Ja}_w \eta L}, \quad b = 1.56 \quad (4.53)$$

# 5

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## BOILING BUBBLE DYNAMICS

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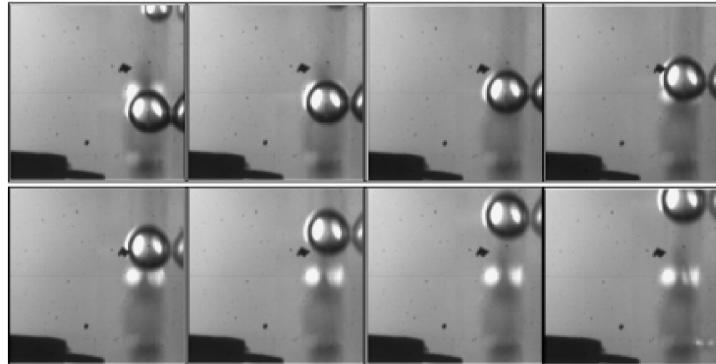
### 5.1 INTRODUCTION

Boiling bubble parameters are playing an important role in the Heat Flux Partitioning models. For instance, the evaporation heat flux  $\phi_e$  is directly proportional to the bubble lift-off radius  $R_{lo}$  ?? while the quenching heat flux  $\phi_q$  depends on the wall area visited by a bubble  $A_{q,lb}$  (Eq. ??) which depends on the bubble sliding length  $l_{sl}$ , departure radius  $R_d$  and lift-off radius  $R_{lo}$ .

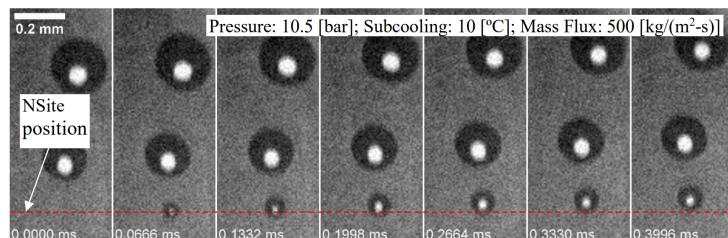
#### 5.1.1 Experimental Insights

Consequently, many experimental investigations have been conducted to further understand the behavior of nucleated bubbles on a wall while facing a liquid flow. In the case of vertical flow boiling, a typical bubble life cycle can be described as follows:

- Beginning of nucleation, growth while attached to the nucleation site ;
- Detachment occurring at radius  $R_d$ , from which the bubble will start to slide and accelerate along the wall ;
- Lift-off from the wall at radius  $R_{lo}$  after sliding over a length  $l_{sl}$ .



(a) Bubble sliding visualized and adapted from Maity [21] at atmospheric pressure.



(b) Bubble sliding visualized and adapted from Kossolapov [15] at higher pressure.

Figure 5.1: Visualization of bubble sliding at various pressures.

This behavior has been supported by many experimental observations who clearly observed three stages (departure, sliding, lift-off) both at low pressure (Maity [21], Situ [32], Thorncroft [37], Prodanovic [prodanovic\_2006], Chen [3], Ren [28], etc.) and high pressure (March [march\_1990], Kossolapov [15]). Altogether, those works cover various flow conditions and operating fluids which insist on the generality of this bubble behavior in vertical flow boiling. Examples from the literature of visualizations of bubble sliding at atmospheric and high pressure are reproduced on Figure 5.1.

The bubble sliding process has also been thermally studied to quantify its impact over the wall heat transfer. Estrada-Perez *et al.* [6] observed the significant thermal impact of sliding bubbles footprints. Kossolapov [15] also investigated the sliding of boiling bubbles and measured the magnitude of the transient heat transfer induced by the disruption of the liquid thermal boundary layer in the bubble's wake. Typical experimental observations from those works are reproduced on Figure 5.2

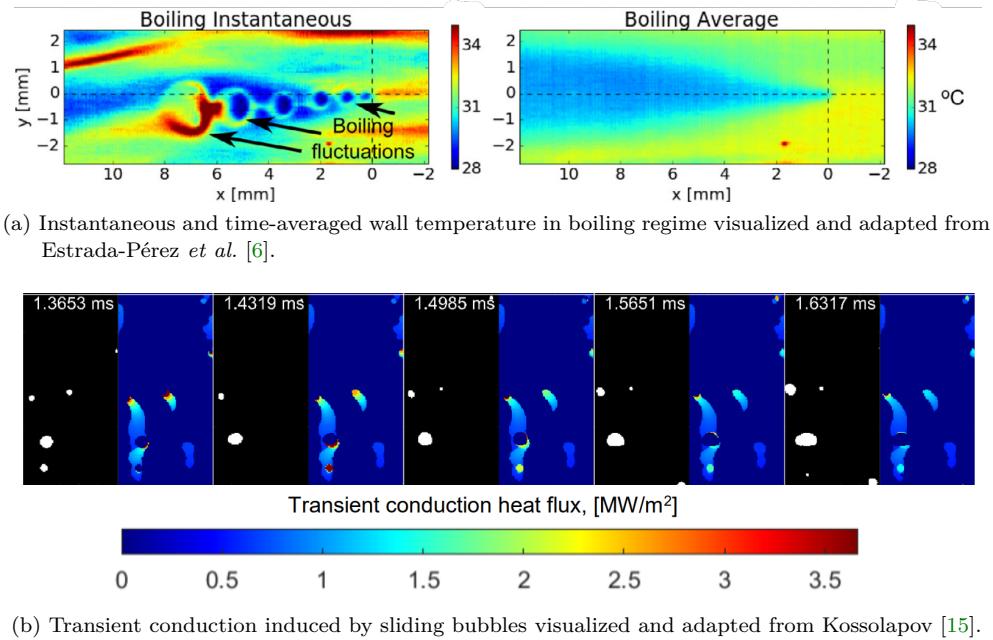


Figure 5.2: Visualization of bubble sliding thermal impact.

Those experimental observations highlight the significant magnitude of the transient heat transfer triggered by bubble movement on the wall that can represent up to 40% of the total wall heat flux [15]. All the aforementioned observations are summed-up on Figure ??.

Predicting the HFP in vertical flow boiling thus requires a descriptions of single bubble dynamics that includes accurate estimations of bubble departure and lift-off radiiuses  $R_d$  and  $R_{lo}$  as well as bubble sliding velocity  $\bar{U}_b$  to predict the sliding length  $l_{sl}$ .

### 5.1.2 Existing Approaches

#### 5.1.2.1 Departure / Lift-Off Diameters

Historically, first approaches to estimate the bubble diameter consisted of experimental-based correlations for pool boiling of horizontal surfaces through photographic studies. In those cases, departure from the nucleation site coincides with the bubble lift-off. Among the mainly used in HFP models and CFD, we can mention the law of Tolubinsky & Kostanchuk (1970)[tolubinsky\_diam] which depends only on the local liquid subcooling:

$$D_{lo} = D_0 e^{-\Delta T_L/45}, \quad D_0 = 15\text{mm} \quad (5.1)$$

On the other hand, authors such as Cole & Rohsenow (1968) proposed relationships including the influence of pressure through the capillary length  $L_c = \sqrt{\frac{\sigma}{g(\rho_L - \rho_V)}}$ :

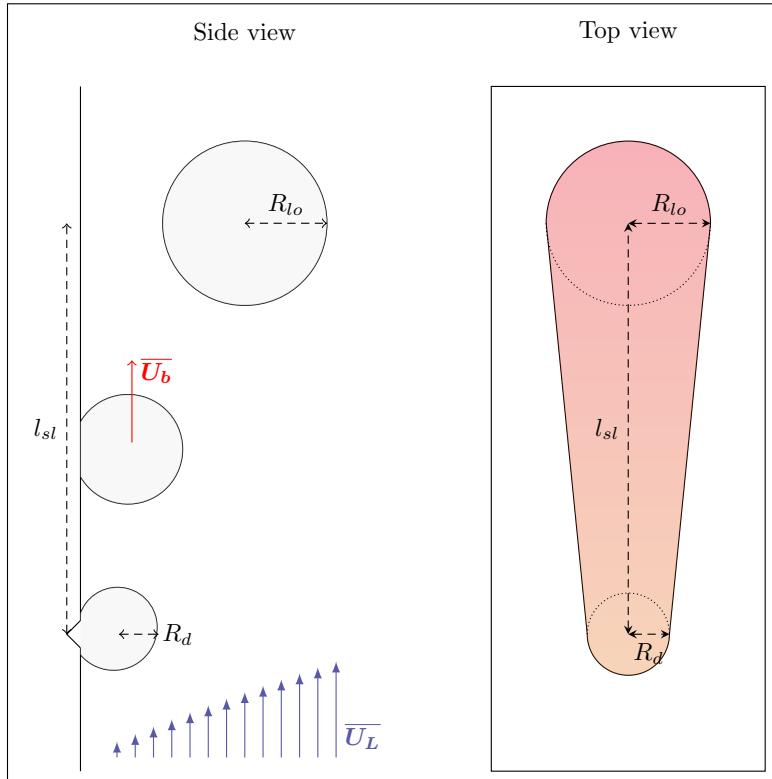


Figure 5.3: Sketch of a typical bubble lifetime in vertical flow boiling. Left depicts a typical side view of the heater with identification of departure, sliding and lift-off. Right depicts a top view of the heater, exhibiting the area that will undergo transient heat transfer.

$$D_{lo} = CL_c \left( \frac{\rho_L c_{p,L} T_{sat}}{\rho_V h_{LV}} \right)^{5/4} \quad (5.2)$$

$C = 1.5 \times 10^{-4}$  for water and  $4.65 \times 10^{-4}$  otherwise.

This equations provides a good trend for the evolution of bubble departure diameter with pressure as shown by Kossolapov [15].

Later, Ünal (1976)[49] derived a correlation based on semi-analytical approach of the heat transfer mechanisms around a bubble to estimate its maximum diameter, including simultaneous influences of pressure, heater material, liquid velocity and subcooling:

$$D_{lo} = 2.42 \times 10^{-5} P^{0.709} \frac{a}{\sqrt{b\varphi}} \quad (5.3)$$

$$a = \frac{\Delta T_w \lambda_w}{2\rho_V h_{LV} \sqrt{\pi \eta_w}}$$

$$b = \frac{\Delta T_L}{2(1 - \rho_V / \rho_L)}$$

$$\varphi = \max \left( 1 ; \left( \frac{U_L}{U_0} \right)^{0.47} \right), \quad U_0 = 0.61 \text{ m/s}$$

Ünal validated his law against several measurements from the literature covering pressures from 1 to 177 bars, liquid velocities from 0.08 to 9.15 m/s, subcoolings from 3 to 86K and heat fluxes from 0.47 to 10.64 MW/m<sup>2</sup>.

**Remark :** The law of Ünal is used in the HFP model of Kurul & Podowski. It as also implemented in NEPTUNE\_CFD and includes a correction of Borée *et al.* (Eq. ?? to avoid divergence in bubble diameter when reaching saturated conditions.

More recently, the several developments around HFP models has lead many researchers to propose dedicated correlations for bubble departure or lift-off diameter. For instance, Basu *et al.* fitted expressions for  $D_d$  and  $D_{lo}$  on their own measurements:

$$\frac{D_d}{L_c} = 1.3 \sin(\theta_s)^{0.4} \left[ 0.13 e^{-1.75 \times 10^{-4} \text{Re}_{L,D_h}} + 0.005 \right] \text{Ja}_w^{0.45} e^{-0.0065 \text{Ja}_L} \quad (5.4)$$

$$\frac{D_{lo}}{L_c} = 1.3 \sin(\theta_s)^{0.4} \left[ 0.2 e^{-1.28 \times 10^{-4} \text{Re}_{L,D_h}} + 0.005 \right] \text{Ja}_w^{0.45} e^{-0.0065 \text{Ja}_L} \quad (5.5)$$

They were validated for  $14 \leq \text{Ja}_w \leq 56$ ,  $1 \leq \text{Ja}_L \leq 138$ ,  $0 \leq \text{Re}_{L,D_h} \leq 7980$  and  $30^\circ \leq \theta_s \leq 90^\circ$ .

**Remark :** Basu *et al.* use these own-developed laws in their HFP formulation to estimate bubble diameters.

Similarly, Kommajosyula gathered several bubble departure and lift-off diameter measurements from the literature and proposed the following reduced correlation:

$$D_d = 18.9 \times 10^{-6} \left( \frac{\rho_L - \rho_V}{\rho_V} \right)^{0.27} \text{Ja}_w^{0.75} (1 + \text{Ja}_L)^{-0.3} U_{L,bulk}^{-0.26} \quad (5.6)$$

$$D_{lo} = 1.2 D_d \quad (5.7)$$

**Remark :** Although this law (used in Kommajosyula's HFP model) present coherent trends with flow conditions, the raw presence of  $U_{L,bulk}$  in the expression is questionable because:

- The relationship is not dimensionless and the constant  $18.9 \times 10^{-6}$  must be in  $\text{m}^{1.26} \cdot \text{s}^{-0.26}$  ;
- The negative exponent will yield diverging values when reaching pool boiling conditions, which is physically inconsistent.

The same type of approach is conducted by Zhou *et al.* [46] by correlating low pressure measurements:

$$\frac{D_d}{L_o} = 10^{2.4086} \left( \frac{\rho_V}{\rho_L} \right)^{-0.6613} \text{Ja}_w^{* 0.1557} \text{Ja}_L^{* -0.01592} \text{Re}_{L_o}^{-0.6647} \text{Pr}_L^{-1.8477} \sin(\theta_s)^{0.4} \quad (5.8)$$

$$\frac{D_{lo}}{L_c} = 10^{-1.1990} \left( \frac{\rho_V}{\rho_L} \right)^{-0.9785} \text{Ja}_w^{* 0.1435} \text{Ja}_L^{* -0.0119} \text{Re}_{L_c}^{-0.5129} \text{Pr}_L^{-1.8784} \quad (5.9)$$

with the Reynolds numbers based on  $L_o = \frac{\rho_L \nu_L^2}{\sigma}$  and  $L_c$  the capillary length, and  $\text{Ja}^* = \frac{\rho_L c_{p,L}}{h_{LV}}$  reduced Jakob numbers that do not include the density ratio.

### 5.1.2.2 Sliding Length and Velocity

Regarding bubble sliding phase, one of the most used correlations to predict bubble diameter evolution has been developed by Maity [21]. Based on atmospheric pressure visualization of boiling single bubbles in water, it predicts the resulting sliding diameter  $D_{sl}$  provided a sliding time  $t_{sl}$  and initial diameter  $D_{in}$  through:

$$\frac{(D_{sl}^2 - D_{in}^2)}{t_{sl} \eta_L \text{Ja}_w} = \frac{1}{15 (0.015 + 0.023 \text{Re}_b^{0.5}) (0.04 + 0.023 \text{Ja}_L^{0.5})} \quad (5.10)$$

where  $\text{Re}_b = \frac{U_L D_b}{\nu_L}$

**Remark :** This correlation is used in Basu *et al.* and Gilman & Baglietto HFP model.

Using Maity measurements of bubble sliding velocity, Basu *et al.* proposed an estimation of the sliding distance for a single bubble  $l_{sl,0}$ :

$$l_{sl,0} = \int_0^{t_{sl}} U_b \, dt = \int_0^{t_{sl}} C_U \sqrt{t} \, dt = \frac{2}{3} C_U t_{sl}^{3/2} \quad (5.11)$$

$$C_U = 3.2 U_L + 1 \quad (5.12)$$

where  $C_U$  represents a correlated acceleration coefficient.

**Remark :** Basu *et al.* also use this correlation in their model to estimate the bubble sliding length. The estimation of the bubble sliding velocity through an explicit correlation since it varies over the bubble lifetime. Therefore, some authors simply suppose that  $U_b = U_L$  such as Gilman & Baglietto.

Other assumptions regarding the sliding length relies on the value of the bubble density on the heater  $N_{bub}$ . By supposing that bubbles usually lift-off after coalescing with an other, thus traveling the average distance between two bubbles:

$$l_{sl} = \frac{1}{\sqrt{N_{bub}}} \quad (5.13)$$

**Remark :** This modeling choice is made by Kommajosyula. Zhou *et al.* choose the minimum value between the  $l_{sl,0}$  of Basu (Eq. 5.11) and this average distance between bubbles.

### 5.1.2.3 Conclusion on Correlations

Albeit proposing coherent trend with the flow boiling conditions along with good estimations of the desired parameters on given experimental datasets, explicit correlations inherently include a limited range of application. Moreover, the constant increase of the number of works proposing data-fitted laws makes the selection of a proper relationship a complicated matter due to their potential lack of generality.

To try to overcome this drawback and come up with more generalized models, researchers have explored an alternative approach by developing Mechanistic Models based on a force-balance to precisely depict the external efforts experienced by the growing bubble. The goal is to compute the sum of the forces applied to the bubble over its growing time and to detect departure and lift-off events using associated criteria such as a change in the force balance sign. This will be the subject of the next section.

As a summary, we gather the presented correlations on Table 5.1.

Bubble Departure Diameter	
Author (Year)	Correlation
Basu <i>et al.</i> (2005)	$\frac{D_d}{L_c} = 1.3 \sin(\theta_s)^{0.4} \left[ 0.13 e^{-1.75 \times 10^{-4} \text{Re}_{L,D_h}} + 0.005 \right] \text{Ja}_w^{0.45} e^{-0.0065 \text{Ja}_L}$ $L_c = \sqrt{\frac{\sigma}{g(\rho_L - \rho_V)}}$
Kommajosyula (2020)	$D_d = 18.9 \times 10^{-6} \left( \frac{\rho_L - \rho_V}{\rho_V} \right)^{0.27} \text{Ja}_w^{0.75} (1 + \text{Ja}_L)^{-0.3} U_{L,bulk}^{-0.26}$
Zhou (2021)	$\frac{D_d}{L_o} = 10^{2.4086} \left( \frac{\rho_V}{\rho_L} \right)^{-0.6613} \text{Ja}_w^{*0.1557} \text{Ja}_L^{*-0.01592} \text{Re}_{L_o}^{-0.6647} \text{Pr}_L^{-1.8477} \sin(\theta_s)^{0.4}$ $L_o = \frac{\rho_L \nu_L^2}{\sigma}$
Bubble Lift-Off Diameter	
Author (Year)	Correlation
Tolubinsky & Kostanchuk (1970)	$D_{lo} = D_0 e^{-\Delta T_L / 45}, D_0 = 15 \text{mm}$
Cole & Rohsenow (1968)	$D_{lo} = CL_c \left( \frac{\rho_L c_{p,L} T_{sat}}{\rho_V h_{LV}} \right)^{5/4}$ $C = 1.5 \times 10^{-4} \text{ (water)} \text{ or } 4.65 \times 10^{-4} \text{ (other)}, L_c = \sqrt{\frac{\sigma}{g(\rho_L - \rho_V)}}$
Ünal (1976)	$D_{lo} = 2.42 \times 10^{-5} P^{0.709} \frac{a}{\sqrt{b\varphi}}, a = \frac{\Delta T_w \lambda_w}{2\rho_V h_{LV} \sqrt{\pi \eta_w}}$ $b = \frac{\Delta T_L}{2(1 - \rho_V/\rho_L)}, \varphi = \max \left( 1 ; \left( \frac{U_L}{U_0} \right)^{0.47} \right), U_0 = 0.61 \text{ m/s}$
Basu <i>et al.</i> (2005)	$\frac{D_{lo}}{L_c} = 1.3 \sin(\theta_s)^{0.4} \left[ 0.2 e^{-1.28 \times 10^{-4} \text{Re}_{L,D_h}} + 0.005 \right] \text{Ja}_w^{0.45} e^{-0.0065 \text{Ja}_L}$ $L_c = \sqrt{\frac{\sigma}{g(\rho_L - \rho_V)}}$
Kommajosyula (2020)	$D_{lo} = 1.2 D_d$
Zhou (2021)	$\frac{D_{lo}}{L_c} = 10^{-1.1990} \left( \frac{\rho_V}{\rho_L} \right)^{-0.9785} \text{Ja}_w^{*0.1435} \text{Ja}_L^{*-0.0119} \text{Re}_{L_c}^{-0.5129} \text{Pr}_L^{-1.8784}$ $L_c = \sqrt{\frac{\sigma}{g(\rho_L - \rho_V)}}$
Sliding Length, Diameter and Velocity	
Author (Year)	Correlation
Maity (2000)	$\frac{(D_{sl}^2 - D_{in}^2)}{t_{sl} \eta_L \text{Ja}_w} = \left[ 15 \left( 0.015 + 0.023 \text{Re}_b^{0.5} \right) \left( 0.04 + 0.023 \text{Ja}_L^{0.5} \right) \right]^{-1}$ $\text{Re}_b = \frac{U_L D_b}{\nu_L}$
Basu <i>et al.</i> (2005)	$l_{sl,0} = \frac{2}{3} C_U t_{sl}^{3/2}, C_U = 3.2 U_L + 1$
Bubble Density Average Distance	$l_{sl} = \frac{1}{\sqrt{N_{bub}}}$

Table 5.1: Summary of the presented correlations

## 5.2 BUBBLE FORCE BALANCE IN VERTICAL FLOW BOILING

### 5.2.1 Introduction

The derivation of the force balance over a growing bubble on a wall in a liquid flow is a very complicated problem that many researchers have tried to tackle over the past decades. Many theoretical and numerical approaches have been conducted to estimate the forces at stake in bubble dynamics and sometimes compared to experimental visualization of bubbles in movement.

Among the first propositions of the whole force-balance closure, the work of Klausner *et al.* in 1993 [13] is probably among the most referred to. They proposed a tentatively complete force-balance for a growing bubble in a boiling flow and supposed that departure from the nucleation site is reached when the force balance becomes positive either in the direction of the flow or perpendicular to the wall. They validated their approach against measurements for horizontal flow boiling of refrigerant R113.

In the same framework, many subsequent works were published such as:

- Van Helden *et al.* [40] (1995) who assessed forces coefficients using injected air bubbles in a vertical flow ;
- Thorncroft *et al.* [37, 38] (1998, 2001) who conducted experiments on horizontal and vertical flow boiling of R113 while proposing more general formulations of the force balance that were used to predict bubble diameter measurements ;
- Duhar & Colin [5] (2006) who validated a force balance on bubbles created by air injection in a shear flow. They extended their work with boiling N-pentane experiments and studied the growth and detachment of single bubbles [**duhar\_npentane**] ;
- Van Der Geld (2009) [10] used potential flow theory to analytically derive the force balance for deforming bubbles near a plane ;
- Sugrue *et al.* (2014) [35] conducted measurements on boiling bubble for water at atmospheric pressure and various surface orientations. Their measurements were then used to validate a force-balance approach predicting bubble departure by sliding [34] ;
- Mazzocco *et al.* (2018) [22] gathered several measurements of bubble departure and lift-off diameters and proposed a reassessed force-balance approach including new drag coefficient and growth law to achieve predictions with a reasonable accuracy over the database ;
- Ren *et al.* (2020) [28] measured bubble departure diameter for vertical flow boiling of water up to 5 bars which they used to validate a force-balance model.

While not exhaustive, this list aims to show that force-balance modeling has become an increasingly interesting approach for authors. It is though not exempted of limitations because each force requires a proper modeling which needs sometimes to go through empirical choices as we will later discuss. This drawback is particularly noted by Bucci *et al.* [1] who points out that traditional force balances are not equal to zero when the bubble is immobile. On the other hand, they show that this is not due to the absence of unknown forces in the balance but rather associated to the computation of well-known forces such as capillary forces. Moreover, Duhar & Colin [5] managed to reach a zero total balance for their air-injected bubbles, and emphasized the interest of force modeling to deeper understand the physical phenomena behind bubble dynamics.

Each of the previously listed models proposed different upgrades and modifications to the force balance over the bubble. Unfortunately, they were all validated using low pressure experiments due to the lack of pressurized measurements in the literature. In addition, the mentioned common use of empirical parameters makes it difficult to reach a general validation of those models as we will see.

In this section, we aim to propose an update of the bubble force balance for vertical flow boiling with a reduced empiricism and to cover the whole bubble lifetime (departure, sliding, lift-off) while achieving a larger generality by including pressurized measurements up to 40 bar conducted by Kossolapov [15].

### 5.2.2 General Considerations

When trying to derive the force balance over a bubble, the first step consists of splitting the whole effort experienced by the bubble between different contributions depending on their nature. In our case, we focus on a bubble growing on a vertical wall and facing an upward flow as depicted in Figure 5.4.

Static forces :

- The buoyancy force  $\overline{F}_B$ , including Archimedes force and the weight of the bubble ;
- The capillary or surface tension force  $\overline{F}_C$  ;
- The contact pressure force  $\overline{F}_{CP}$ .

Hydrodynamic forces :

- The drag and lift forces  $\overline{F}_D$  and  $\overline{F}_L$  ;
- The inertia force  $\overline{F}_I$ , including added-mass and Tchen force.

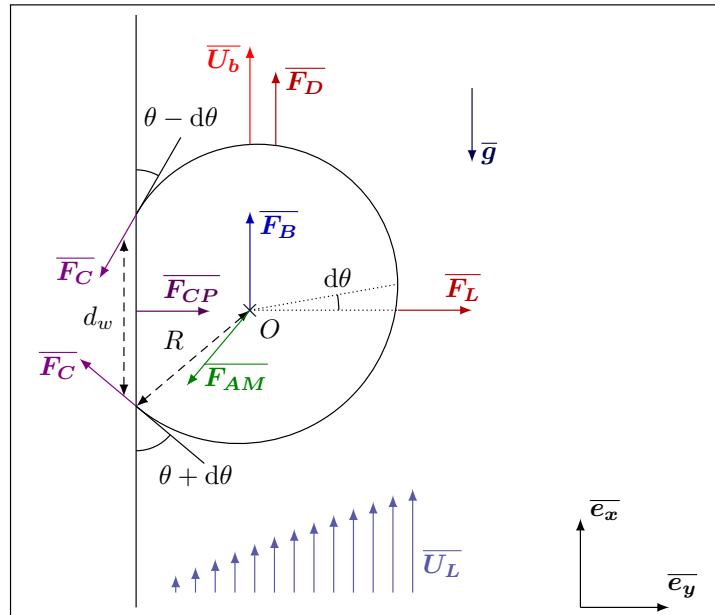


Figure 5.4: Sketch of the forces applied to the bubble facing an upward flow  $\overline{U}_L$  and sliding at velocity  $\overline{U}_b$

Regarding the bubble shape, we consider a quasi-spherical bubble of radius  $R$  with a circular contact area with the wall of radius  $r_w$ . It has a static contact angle  $\theta$  and is tilted under the influence of the flow by an inclination angle  $d\theta$  (half the total angle hysteresis). The resulting downstream and upstream contact angles are therefore  $\theta_d = \theta - d\theta$  and  $\theta_u = \theta + d\theta$ . If the bubble has a shape close to a truncated sphere, we can approximate the bubble foot radius as:

$$r_w \approx R \sin \left( \frac{\theta_u + \theta_d}{2} \right) = R \sin (\theta) \quad (5.14)$$

Some authors rather take  $r_w \approx \frac{1}{2}R (\sin (\theta_u) + \sin (\theta_d)) = R \sin (\theta) \cos (d\theta)$ , however this expression tends to zero when reaching  $d\theta \rightarrow 90^\circ$  which is undesirable regarding the expression of forces such as Contact Pressure and Surface Tension.

We suppose  $V_b \approx \frac{4}{3}\pi R^3$  for the bubble volume.

### 5.2.3 Buoyancy Force

The buoyancy force results from both the weight of the bubble and the integration of the hydrostatic liquid pressure over its surface which naturally yields:

$$\overline{F}_B = \rho_V V_b \bar{g} + \left( \iint_{S_b} \rho_L g z \, d^2 S \right) \bar{e}_x = V_b (\rho_V - \rho_L) \bar{g} = \frac{4}{3}\pi R^3 (\rho_L - \rho_V) g \bar{e}_x \quad (5.15)$$

### 5.2.4 Contact Pressure Force

The contact pressure force arises due to the pressure difference between the center of the bubble and the surrounding liquid. Combined with the Archimedes force, it can be expressed versus the difference of liquid and vapor pressure at the bubble foot using Laplace's equation  $\Delta P = 2\sigma/R_c$  as:

$$\overline{F_{CP}} \approx \frac{2\sigma}{R_c} \pi r_w^2 \overline{e_y} \approx \pi R \sigma 2 \sin(\theta)^2 \overline{e_y} \quad (5.16)$$

Here,  $R_c$  is the curvature radius of the bubble which is often assumed to be equal to  $5R$  [13, 22, 34] without other explanation than avoiding an overestimation of the contact pressure force. To avoid this arbitrary choice, following the hypothesis of a nearly spherical bubble shape gives  $R_c = R$ .

### 5.2.5 Capillary Force

The capillary force acts at the triple contact line at the bubble's foot and is an important adhesive force maintaining the bubble attached to the wall. Its derivation can be done by integration of the effort exerted over the triple contact line. Noting  $\Phi$  the polar angle around the bubble foot, we have :

$$\overline{F_C} = 2 \int_0^\pi \sigma r_w \bar{\tau}(\Phi) d\Phi \quad (5.17)$$

where  $\bar{\tau}$  is the unit vector tangent to the interface.

To compute the resulting components parallel and tangent to the wall, Klausner *et al.* [13] account for a contact angle difference between the upstream (receding) contact angle  $\theta_u$  and downstream (advancing) contact angle  $\theta_d$ . If the local contact angle is noted  $\gamma$ , then:

$$\bar{\tau}(\Phi) = \cos(\gamma) \cos(\Phi) \overline{e_x} + \sin(\gamma) \overline{e_y} \quad (5.18)$$

Then representing the evolution of the local contact angle  $\gamma$  from  $\theta_u$  to  $\theta_d$  using a polynomial expression of degree 3:

$$\gamma(\Phi) = \theta_d + (\theta_u - \theta_d) \left[ 3 \left( \frac{\Phi}{\pi} \right)^2 - 2 \left( \frac{\Phi}{\pi} \right)^3 \right], \quad 0 \leq \Phi \leq \pi \quad (5.19)$$

which verifies symmetry conditions:

$$\gamma'(0) = \theta_d, \quad \gamma(\pi) = \theta_u, \quad \gamma'(0) = \gamma'(\pi) = 0 \quad (5.20)$$

To obtain analytic expression, Klausner *et al.* also consider a first order linear interpolation:

$$\gamma(\Phi) = \theta_d + (\theta_u - \theta_d) \frac{\Phi}{\pi} \quad (5.21)$$

This yields:

$$\overline{F_C} = -2r_w \sigma \frac{\pi(\theta_u - \theta_d)}{\pi^2 - (\theta_u - \theta_d)^2} (\sin(\theta_u) + \sin(\theta_d)) \overline{e_x} - 2r_w \sigma \frac{\pi}{\theta_u - \theta_d} (\cos(\theta_d) - \cos(\theta_u)) \overline{e_y} \quad (5.22)$$

By comparing the analytic expression of Eq. 5.22 with the values obtained by numerical integration of Eq. 5.19, Klausner *et al.* introduce a correction factor of 1.25 over the  $x$  component, finally giving :

$$\overline{F_C} = -2.5r_w \sigma \frac{\pi(\theta_u - \theta_d)}{\pi^2 - (\theta_u - \theta_d)^2} (\sin(\theta_u) + \sin(\theta_d)) \overline{e_x} - 2r_w \sigma \frac{\pi}{\theta_u - \theta_d} (\cos(\theta_d) - \cos(\theta_u)) \overline{e_y} \quad (5.23)$$

$$= -\pi R \sigma \underbrace{\left[ 2.5 \frac{r_w}{R} \frac{d\theta}{(\frac{\pi}{2})^2 - d\theta^2} \sin(\theta) \cos(d\theta) \right]}_{f_{C,x}} \overline{e_x} - \pi R \sigma \underbrace{\left[ 2 \frac{r_w}{R} \sin(\theta) \frac{\sin(d\theta)}{d\theta} \right]}_{f_{C,y}} \overline{e_y} \quad (5.24)$$

**Remark :** We can see that  $f_{C,x} \rightarrow 0$  and  $f_{C,y} \rightarrow 2 \frac{r_w}{R} \sin(\theta)$  when  $d\theta \rightarrow 0$ . In that case,  $\overline{F_C} = -\overline{F_{CP}}$ .

### 5.2.6 Drag and Lift Forces

The external liquid flow over the bubble induces the well-known drag and lift forces, acting respectively in the flow direction and perpendicular to the flow. They are usually expressed using associated coefficients  $C_D$  and  $C_L$  defined by:

$$\overline{F_D} = \frac{1}{2} C_D \rho_L S_p \|\overline{U_L} - \overline{U_b}\| (\overline{U_L} - \overline{U_b}) \quad (5.25)$$

$$\overline{F_L} = \frac{1}{2} C_L \rho_L S_p \|\overline{U_L} - \overline{U_b}\|^2 \overline{e_y} \quad (5.26)$$

with  $S_p = \pi R^2$  the projected area of the bubble in the direction of the flow.

#### 5.2.6.1 Drag Coefficient

Derivations of analytic expressions for the drag coefficient in an infinite fluid medium exist for more than a century, starting with Hadamard-Rybinski (1911) [hadamard\_1911]:

$$C_D = \frac{16}{Re_b}, \text{ if } Re_b < 1 \quad (5.27)$$

where  $Re_b = \frac{|U_{rel}| D_B}{\nu_L}$  is the bubble Reynolds number and  $U_{rel}$  the relative velocity between the bubble and the surrounding fluid.

For  $Re_b \gg 1$ , Levich (1962) [levich\_1962] found for a potential flow:

$$C_D = \frac{48}{Re_b} \quad (5.28)$$

For intermediate values of  $Re_b$ , traditional approaches rely on expressions of the drag force for a bubble in an infinite medium based on numerical correlations as proposed by Mei & Klausner [24], used in many different mechanistic approaches [3, 28, 34, 38, 44]:

$$C_{D,U} = \frac{16}{Re_b} \left[ 1 + \left( \frac{8}{Re_b} + \frac{1}{2} \left( 1 + \frac{3.315}{\sqrt{Re_b}} \right) \right)^{-1} \right] \quad (5.29)$$

Results from DNS conducted by Legendre *et al.* [18] proposed expressions of the drag and lift forces for a hemispherical bubble on a wall facing a viscous shear flow. Earlier, Legendre & Magnaudet [18] analytically derived coefficients to transpose drag and lift expressions for a particle to the case of a bubble. This was applied by Mazzocco *et al.* [22] to the Drag for a solid particle near a wall in a shear flow proposed by Zeng *et al.* [45].

In this work, we propose to rely on the recent work of Shi *et al.* [31] who conducted DNS of a shear flow over a spherical bubble of constant radius close to a wall for bubble Reynolds number between  $10^{-1}$  and  $10^3$  and non-dimensional shear rates between -0.5 and 0.5. A sketch of the situation simulated by Shi *et al.* is depicted on Figure 5.5.

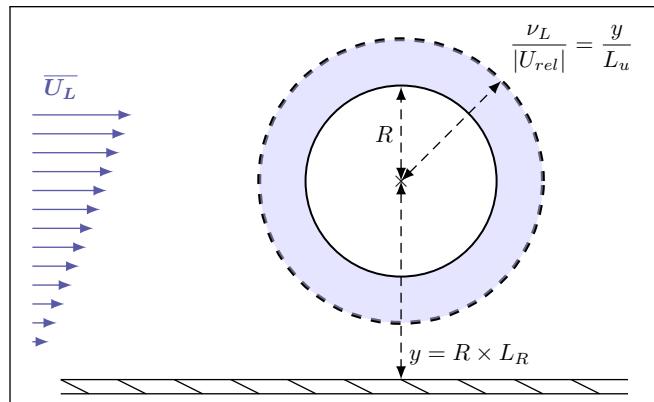


Figure 5.5: Physical situation considered by Shi *et al.* [31].

They computed the resulting drag and lift coefficients for each simulations and proposed correlations fitting their numerical results. The total Drag coefficient is expressed as a correction of the Drag coefficient for a bubble in an unbounded uniform flow  $C_{D,U}$ . The total drag is given by:

$$C_D = (1 + \Delta C_D) C_{D,U} \quad (5.30)$$

where  $\Delta C_D$  accounts for both the effect of the shear flow and the wall vicinity.

To cover the whole range of bubble Reynolds numbers, correlations at low and high  $Re_b$  are smoothly connected using an exponential term.

$$\Delta C_D = \Delta C_{D,Re_b=O(1)} + \left(1 - e^{-0.07Re_b}\right) \Delta C_{D,Re_b \gg 1} \quad (5.31)$$

Each of those corrections is computed depending on  $Re_b$ , the non-dimensional shear rate  $Sr = \frac{2\gamma R}{|U_{rel}|}$  where  $\gamma = \frac{\partial U_{L,x}}{\partial y}$ , the non dimensional wall distance  $L_R = \frac{y}{R}$  ( $L_R = 1$  being a spherical bubble laying on a wall) and non-dimensional viscous (or Stokes) length  $L_u = \frac{y}{\nu_L / |U_{rel}|}$ .

$$\begin{aligned} \Delta C_{D,Re_b=O(1)} &= \frac{1 + \tanh(0.012Re_b^{0.8}) + \tanh(0.07Re_b^{0.8})^2}{1 + 0.16L_u(L_u + 4)} \\ &\times \left[ \left( \frac{3}{8}L_R^{-1} + \frac{3}{64}L_R^{-4} \right) \left( 1 - \frac{3}{8}L_R^{-1} - \frac{3}{64}L_R^{-4} \right)^{-1} - \frac{1}{16} \left( L_R^{-2} + \frac{3}{8}L_R^{-3} \right) Sr \right] \end{aligned} \quad (5.32)$$

$$\Delta C_{D,Re_b \gg 1} = 0.47L_R^{-4} + 0.0055L_R^{-6}Re_b^{3/4} + 0.002|Sr|^{1.9}Re_b + 0.05L_R^{-7/2}SrRe_b^{1/3} \quad (5.33)$$

Figure 5.6 shows the evolution of the drag correction  $\Delta C_D$  against the bubble Reynolds number for different distances to the wall  $L_R$  and two values of  $Sr$ . We can see that as the distance between the wall and the bubble increases the drag correction logically approaches zero and that increasing the shear rate  $Sr$  increases  $\Delta C_D$  for higher values of  $Re_b$ .

Shi *et al.* [31] conducted DNS for wall distances down to  $L_R = 1.5$ . However, Scheiff *et al.* [29] compared the values obtained for  $L_R = 1$  with measured drag coefficients of bubbles sliding on a wall and observed a good agreement, which legitimates the use of this new drag correlation by extending its application to the case of a bubble laying on a wall and using the uniform drag coefficient of Eq. 5.29.

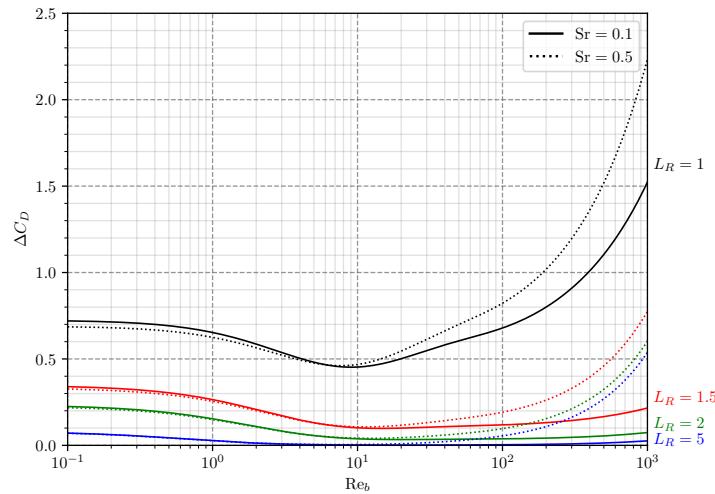


Figure 5.6: Drag correction from Shi *et al.* [31].

**Remark :** In PWR conditions, a static bubble of radius 0.01 mm on a wall with a bulk liquid velocity of 5 m/s leads to a non-dimensional shear rate  $Sr \approx 0.7$  with  $Re_b \approx 500$ . In this case, the drag correction can reach 180% compared to the unbounded uniform flow formulation.

### 5.2.6.2 Lift Coefficient

In 1987, Auton *et al.* [auton\_1998] analytically derived the lift force for an inviscid fluid in unstationary motion in a weak velocity gradient and found  $C_L = 0.5$ , with the lift force defined as:

$$\overline{F}_L = -\rho_L C_L V_b (\overline{U}_L - \overline{U}_b) \wedge \overline{\omega} \quad (5.34)$$

where  $\overline{\omega}$  is the flow vorticity.

This result was enriched by Legendre & Magnaudet (1998) [legendre\_lift\_1998bis] who used numerical results to propose a dependency of  $C_L$  on the bubble Reynolds number for a sphere in an infinite medium facing a weakly sheared flows as:

$$C_L = (C_{L,\text{Re}_b \sim 1}^2 + C_{L,\text{Re}_b \gg 1}^2)^{1/2} \quad (5.35)$$

$$= \left( \left[ \frac{6}{\pi^2} \frac{2.255 (\text{Re}_b \omega^*)^{-1/2}}{\left( 1 + 0.2 \frac{\text{Re}_b}{\omega^*} \right)^{3/2}} \right]^2 + \left[ \frac{1}{2} \frac{1 + 16 \text{Re}_b^{-1}}{1 + 29 \text{Re}_b^{-1}} \right]^2 \right)^{1/2} \quad (5.36)$$

where  $\omega^* = \frac{2R |\overline{\omega}|}{|U_{\text{rel}}|}$  is the non-dimensional vorticity of the flow.

**Remark :** In the case of a steady linear shear flow near a wall, the vorticity is  $\overline{\omega} = \nabla \wedge \overline{U}_L = -\gamma \overline{e}_y$ .

In that case, the non-dimensional vorticity becomes the non-dimensional shear rate :  $\omega^* = \frac{2R\gamma}{|U_{\text{rel}}|} = \text{Sr}$ .

Later, Mei & Klausner (1994) [mei\_klausner\_lift] derived the lift force induced by the shear for a spherical bubble in an unbounded flow for low Reynolds numbers, based on the expression of Saffmann [saffman\_lift\_particle]. By interpolating this result with the solution of Auton [auton\_lift\_1987], they obtained a formulation for a large range of  $\text{Re}_b$  :

$$C_L = 2.74 \sqrt{\text{Sr}} \times \left[ \text{Re}_b^{-2} + \left( 0.24 \sqrt{\text{Sr}} \right)^4 \right]^{1/4} \quad (5.37)$$

This expression is actually used in many mechanistic force balance [3, 13, 28, 34] [3] [28]

In his force-balance approach, Mazzocco *et al.* [22] used a constant lift coefficient by using the upper bound for the lift of a solid particle touching a wall in a Stokes flow, multiplied by  $\frac{4}{9}$  to transpose this value to the bubble case as suggested by Legendre & Magnaudet [legendre\_lift\_1997\_coeff]. This resulted in:

$$C_L = 2.61 \quad (5.38)$$

In accordance with the computation of the drag coefficient, our model will rely on the expression of the lift coefficient proposed by Shi *et al.* [31]. Their formulation includes extra parameters compared to the drag coefficient :

- The non-dimensional Saffman length  $L_\omega = \frac{y}{\sqrt{\nu_L/\omega}}$  ;
- The Stokes (or Oseen) length to Saffman length ratio  $\varepsilon = \frac{\nu_L/|U_{\text{rel}}|}{\sqrt{\nu_L/\omega}}$ , which quantifies the origin of inertial effects being either shear ( $\varepsilon > 1$ ) or the relative slip of the bubble ( $\varepsilon < 1$ ).

The resulting formulation of  $C_L$  corresponds to the superpositions of two contributions respectively associated to the uniform flow and the shear rate, both coupled with the wall presence.

$$C_L^W = C_{Lu}^W + C_{L\omega}^W \quad (5.39)$$

The lift associated to the uniform flow near a wall is computed as follows:

$$\begin{aligned} C_{Lu}^W &= e^{-0.22 \varepsilon^{0.8} L_{\omega}^{2.5}} \frac{\left[1 + \tanh(0.012 \text{Re}_b^{0.8}) + \tanh(0.07 \text{Re}_b^{0.8})\right]^2}{1 + 0.13 L_u (L_u + 0.53)} \\ &\times \left(\frac{L_R}{3}\right)^{-2.0 \tanh(0.01 \text{Re}_b)} C_{Lu}^{\text{W-in}} \\ &+ \left(1 - e^{-0.22 \text{Re}_b^{0.6}}\right) \left[C_{Lu, \text{Re}_b \rightarrow \infty}^W + 15 \tanh(0.01 \text{Re}_b) \text{Re}_b^{-1} L_R^{-4}\right] \end{aligned} \quad (5.40)$$

Where:

$$C_{Lu}^{\text{W-in}} = \frac{1}{2} \left(1 + \frac{1}{8} L_R^{-1} - \frac{33}{64} L_R^{-2}\right) \quad (5.41)$$

$$C_{Lu, \text{Re}_b \rightarrow \infty}^W = -\frac{3}{8} L_R^{-4} \left[1 + \frac{1}{8} L_R^{-3} + \frac{1}{6} L_R^{-5}\right] + O(L_R^{-10}) \quad (5.42)$$

The lift associated to the vorticity near a wall is computed as follows:

$$C_{L\omega}^W = \left[1 - \exp\left(-\frac{11}{96} \pi^2 \frac{L_{\omega}}{J_L(\varepsilon)} \left(1 + \frac{9}{8} L_R^{-1} - \frac{1271}{3520} L_R^{-2}\right)\right)\right] C_{L\omega, \text{Re}_b \ll 1}^U \quad (5.43)$$

$$+ \left(1 - e^{-0.3 \text{Re}_b}\right) \left[1 + 0.23 L_R^{-7/2} \left(1 + 13 \text{Re}_b^{-1/2}\right)\right] C_{L\omega, \text{Re}_b \gg 1}^U \quad (5.44)$$

Where:

$$J_L(\varepsilon) = 2.254 (1 + 0.2\varepsilon^{-2})^{-3/2} \quad (5.45)$$

$$C_{L\omega, \text{Re}_b \ll 1}^U = \frac{8}{\pi^2} \frac{\text{Sr}}{|\text{Sr}|} \varepsilon J_L(\varepsilon) \quad (5.46)$$

$$C_{L\omega, \text{Re}_b \gg 1}^U = \frac{2}{3} \text{Sr} (1 - 0.07 |\text{Sr}|) \frac{1 + 16 \text{Re}_b^{-1}}{1 + 29 \text{Re}_b^{-1}} \quad (5.47)$$

On Figure 5.7, we plot the values of  $C_L$  obtained by the formulation of Shi *et al.* different values of the non-dimensional wall distance  $L_R$  (extending down to  $L_R = 1$ ) and non-dimensional shear rate Sr.

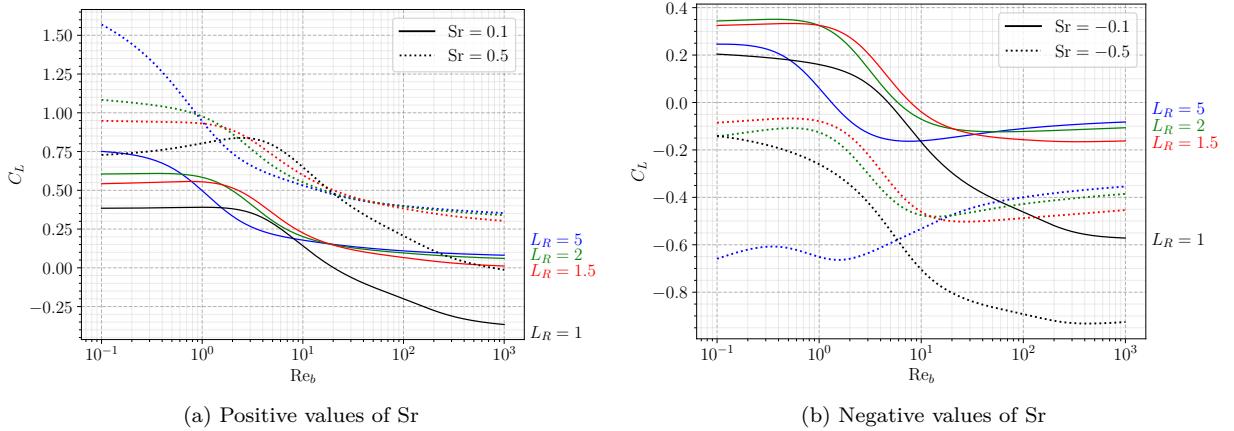


Figure 5.7:  $C_L$  computed using Shi *et al.* correlation.

We can see that the magnitude of the lift coefficient globally increases with the wall distance when  $\text{Sr} > 0$  and that negative lift values are easily reached when  $\text{Sr} < 0$ . This means that correlations for unbounded medium may overestimate the lift experienced by the bubble compared to the situation with a wall.

The extension to the case  $L_R = 1$  may be more questionable compared to the drag since the bubble touching the wall will stop any flow in between, leading to inertial and shear regimes that would be significantly different due to the redirection of the liquid at the bubble's foot towards the bulk. In particular, we can see that the values reached for  $L_R = 1$  on Figure 5.7 are not following the general trend of simulated  $L_R$ :

- Negative values of  $C_L$  are reached with positive  $Sr$  at high  $Re_b$  while getting close to the wall seemed to tend to a value of  $C_L \geq 0$  at high  $Re_b$ ;
- Magnitude of  $C_L$  with negative  $Sr$  are not coherent with the observed trend down to  $L_R = 1.5$ .

This observation suggest that we should include the effect of the wall using the lift of Shi *et al.* by limiting its use to  $L_R = 1.5$  contrary to the drag which extension to  $L_R = 1.0$  was coherent and validated.

**Remark :** In PWR conditions, taking  $Sr \approx 0.7$  with  $Re_b \approx 500$  for the static bubble on a wall leads to  $C_L \approx 0.45$  both with Mei & Klausner (Eq. 5.37) and Shi *et al.* (Eq. 5.39). For a bubble that would slide at 90% of the local liquid velocity, this gives  $Sr \approx 7$  and  $Re_b \approx 50$  yielding  $C_{L,Mei} \approx 4$  and  $C_{L,Shi} \approx 2.8$ .

### 5.2.7 Inertia Force

The Inertia force originates from various effects (bubble growth, freestream and bubble acceleration, etc.) and includes both added mass and Tchen forces and is expressed as presented in Magnaudet & Eames (2000) [20]:

$$\overline{F_I} = \rho_L V_b \underbrace{\left( \frac{\partial \overline{U_L}}{\partial t} + \overline{\nabla}(\overline{U_L}) \cdot \overline{U_L} \right)}_{\text{Liquid inertia or Tchen force}} + \underbrace{\frac{d}{dt} (\rho_L C_{AM} V_b (\overline{U_L} - \overline{U_b}))}_{\text{Added Mass force } \overline{F_{AM}}} \quad (5.48)$$

Since we consider a steady and quasi-parallel liquid flow, we respectively have:

$$\frac{\partial \overline{U_L}}{\partial t} = 0 \text{ and } \overline{\nabla}(\overline{U_L}) \cdot \overline{U_L} = 0 \quad (5.49)$$

Thus only remains the added mass force to express in the considered force balance. In the next subsections, we detail former approaches to tackle the added mass derivation and propose a more rigorous to re-evaluate the added mass coefficients.

#### 5.2.7.1 Former Approaches

In previous mechanistic models, the derivation of the added mass force was conducted with different approaches. In particular, some authors chose to rely on the Rayleigh-Plesset Equation (RPE) for a growing hemispherical bubble in a quiescent flow to obtain the reaction force from the liquid, oriented perpendicularly to the wall:

$$\overline{F_{AM,RPE}} = -\rho_L \pi R^2 \left[ R \ddot{R} + \frac{3}{2} \dot{R}^2 \right] \overline{e_y} \quad (5.50)$$

Then, assuming a bubble inclination angle  $\theta_i$ , this force was projected along the  $x$  axis to obtain an Added Mass force parallel to the wall that hinders departure. The inclination angle value is often empirical and used for data fitting [4, 22, 28, 44].

$$\overline{F_{AM,RPE}} = -\rho_L \pi R^2 \left[ R \ddot{R} + \frac{3}{2} \dot{R}^2 \right] (\sin(\theta_i) \overline{e_x} + \cos(\theta_i) \overline{e_y}) \quad (5.51)$$

This approach is questionable on different aspects. First, the RPE assumes a moving boundary in a quiescent unbounded liquid, which is physically far from the real situation of a bubble growing on a wall in a boiling flow. Moreover, the subsequent projection along the different directions regarding an unknown angle is hardly reasonable if  $\theta_i$  is chosen arbitrarily. Values of  $\theta_i$  selected by different authors are mentioned in Table 5.2.

On the other hand, some authors [11, 13, 38] considered two distinct contributions:

- Hemispherical bubble growth in a stagnant liquid, leading to Eq. 5.51 including the inclination angle  $\theta_i$  ;
- Spherical bubble growth in an uniform unbounded and inviscid liquid flow, which yields a detaching Added Mass term due to the interaction of bubble growth with the external flow:

$$\overline{F_{AM,U}} = \frac{3}{2} \rho_L V_b \frac{\dot{R}}{R} U_L \bar{e}_x \quad (5.52)$$

This last term is usually called a "bulk growth force". By including the effect of the liquid flow, this approach can be considered as closer to the reality. However, it relies on two separate derivations associated to different physical considerations.

### 5.2.7.2 Proposed Approach

To tackle the added mass derivation in a proper way, we propose to follow the approach of Lamb [16] (also presented by Milne Thomson [25] or Van Winjaarden [41]). By solving the potential flow around a bubble and its image, we can obtain the total liquid kinetic energy  $E_L$  that corresponds to a situation where a bubble is at a given distance from a wall (represented by the line normal to the line of centers of the bubbles).

Then we can use Lagrange equation to compute the resulting forces along a given coordinate  $q$ :

$$F_{AM,q} = -\frac{\partial}{\partial t} \left( \frac{\partial E_L}{\partial \dot{q}} \right) + \frac{\partial E_L}{\partial q} \quad (5.53)$$

$$(5.54)$$

This method was also used by Duhar [5] who developed an asymptotic expression of  $E_L$  to compute the addd mass coefficient when the bubble approaches the wall. Here, we express the liquid kinetic energy by relying on the work of Van Der Geld [10] who derived  $E_L$  in the case of a full or truncated spherical bubble laying on a wall and facing an uniform flow parallel to the wall of velocity  $U_L$  (Eq. 5.55). If the bubble slides at a velocity  $U_b = \dot{x}$ , it sees a liquid velocity  $U_{rel} = U_L - \dot{x}$ .

$$E_L = \frac{\rho_L V_b}{2} \left( \alpha \dot{y}^2 + \text{tr}(\beta) \dot{R}^2 + \psi \dot{R} \dot{y} + \alpha_2 (U_L - \dot{x})^2 \right) \quad (5.55)$$

where  $(x, y)$  are the coordinates of the bubble's center and  $\alpha, \text{tr}(\beta), \psi, \alpha_2$  are polynomials of  $R/y = 1/L_R$  derived by Van Der Geld for  $1 < R/y < 2$  i. e.  $0.5 < L_R < 1$ , corresponding to contact angles  $0^\circ < \theta < 60^\circ$ .

For each polynomial expression ( $\alpha$  is used as an example), we note  $n$  its degree and write:

$$\alpha = \sum_{k=0}^n \alpha_k \left( \frac{R}{y} \right)^k \text{ and } \tilde{\alpha} = \sum_{k=0}^n k \alpha_k \left( \frac{R}{y} \right)^k \quad (5.56)$$

This allows to express the following derivatives:

$$\frac{\partial \alpha}{\partial y} = -\frac{1}{y} \tilde{\alpha} \text{ and } \frac{\partial \alpha}{\partial t} = \left( \frac{\dot{R}}{R} - \frac{\dot{y}}{y} \right) \tilde{\alpha} \quad (5.57)$$

Noticing that the derivatives of the polynomials along  $x$  will be 0 and injecting  $E_L$  in Eq. 5.53 and ?? allows to express the added mass force in  $x$  and  $y$  directions. If we express it using geometrical ratios  $\frac{R}{y} = \frac{1}{F_1}$ ,  $\frac{\dot{y}}{R} = F_2$  and  $\frac{\dot{R}}{R} = F_3$ , we can obtain:

$$F_{AM,x} = \rho_L V_b \left[ \left( 3\alpha_2 + \left( 1 - \frac{F_2}{F_1} \right) \tilde{\alpha}_2 \right) \frac{\dot{R}}{R} U_{rel} - \alpha_2 \frac{\partial U_b}{\partial t} \right] \quad (5.58)$$

$$\begin{aligned} F_{AM,y} = -\rho_L V_b & \left[ \left( 3F_2\alpha + \frac{3}{2}\psi + \left( 1 - \frac{F_2}{F_1} \right) F_2 \tilde{\alpha} + \left( 1 - \frac{F_2}{F_1} \right) \frac{\tilde{\psi}}{2} + \frac{F_2}{F_1} \frac{\tilde{\alpha}}{2} + \frac{1}{F_1} \frac{\text{tr}(\tilde{\beta})}{2} + \frac{F_2}{F_1} \frac{\tilde{\psi}}{2} \right) \frac{\dot{R}^2}{R} \right. \\ & \left. + \left( F_3\alpha + \frac{\psi}{2} \right) \ddot{R} + \frac{1}{F_1} \frac{\tilde{\alpha}_2}{2} \frac{U_{rel}^2}{R} \right] \end{aligned} \quad (5.59)$$

In the case of a truncated sphere,  $F_1 = \frac{y}{R} = \cos(\theta) = L_R$ . If we suppose that the bubble keeps a nearly constant contact angle during its lifetime, we can further write  $F_1 = F_2 = F_3 = \cos(\theta) = L_R$ , which simplifies the forces in:

$$F_{AM,x} = \rho_L V_b \left[ 3\alpha_2 \frac{\dot{R}}{R} U_{rel} - \underbrace{\alpha_2}_{C_{AM,x}} \frac{\partial U_b}{\partial t} \right] \quad (5.60)$$

$$F_{AM,y} = \rho_L V_b \left[ - \left( 3 \underbrace{\left( L_R \alpha + \frac{\psi}{2} \right)}_{C_{AM,y1}} + \underbrace{\frac{\tilde{\alpha}}{2} + \frac{1}{L_R} \text{tr}(\tilde{\beta}) + \frac{\tilde{\psi}}{2}}_{C_{AM,y2}} \right) \frac{\dot{R}^2}{R} \right. \quad (5.61)$$

$$\left. - \underbrace{\left( L_R \alpha + \frac{\psi}{2} \right)}_{C_{AM,y1}} \ddot{R} + \underbrace{\frac{-1}{L_R} \frac{\alpha_2}{2} \frac{U_{rel}^2}{R}}_{C_{AM,y3}} \right] \quad (5.62)$$

On Figure 5.8, we plot the values of the added mass coefficients against the values of  $L_R$ .

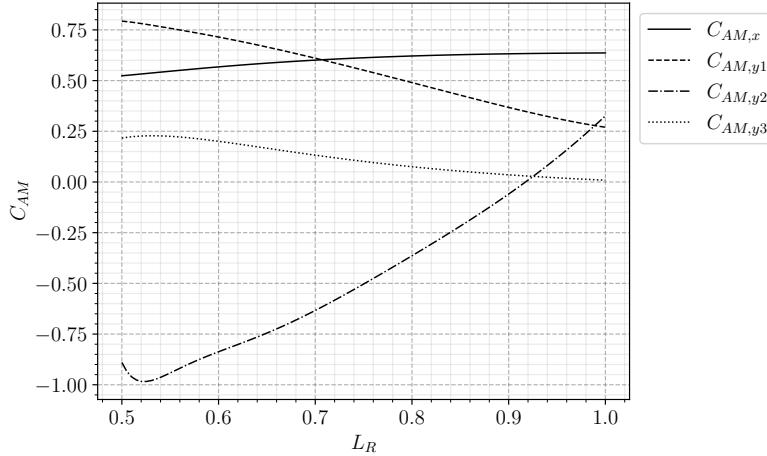


Figure 5.8: Values of the computed added mass coefficients in Eq. 5.60 and 5.62.

For the case of a spherical bubble laying on a wall ( $L_R = 1$ ), we finally have:

$$F_{AM,x} = \rho_L V_b \left[ 3C_{AM,x} \frac{\dot{R}}{R} U_{rel} - C_{AM,x} \frac{\partial U_b}{\partial t} \right] \quad (5.63)$$

with  $C_{AM,x} \approx 0.636$ .

$$F_{AM,y} = \rho_L V_b \left[ - (3C_{AM,y1} + C_{AM,y2}) \frac{\dot{R}^2}{R} - C_{AM,y1} \ddot{R} + C_{AM,y3} \frac{U_{rel}^2}{R} \right] \quad (5.64)$$

with  $C_{AM,y1} \approx 0.27$ ,  $C_{AM,y2} \approx 0.326$  and  $C_{AM,y3} \approx 8.77 \times 10^{-3}$ .

Parallel to the wall, the coupled term  $\frac{\dot{R}}{R} U_{rel}$  in Eq. 5.63 promotes detachment and sliding of the bubble if  $U_{rel} > 0$  e. g. if the bubble is attached to its nucleation site. This contradicts the aforementioned approach where solely projecting the RPE on both axes lead to an Added-Mass term related to bubble growth that only hinders the departure by sliding. Moreover, Eq. 5.64 exhibits a term induced by the relative velocity that acts as a lift force, which seems to rarely appear in other approaches.

**Remark :** The derived values of the added mass coefficients are only valid for  $0.5 < L_R < 1$  as previously mentioned. When the bubble leaves the wall, added mass calculations of Duhar [duhar\_phd] would be more appropriate.

Those theoretical results highlight the importance of conducting a rigorous approach when possible to deriving those transient aspects of the force balance. Otherwise, some terms may be missing and lead to contradictory physical conclusions.

In the spirit of avoiding to introduce extra empirical terms, we keep the Added Mass force as presented in Eq. 5.63 and 5.64 and consider no projection along the inclination angle.

### 5.2.8 Force Balance Summary

Writing Newton's second law, we have the total force balance over the bubble in both directions:

$$\rho_V \frac{\partial V_b U_{b,x}}{\partial t} = -\pi R \sigma f_{C,x}(\theta, d\theta) + V_b (\rho_L - \rho_V) g + \frac{1}{2} C_D \rho_L \pi R^2 |U_L - U_b| (U_L - U_b) \\ + \rho_L V_b \left[ 3C_{AM,x} \frac{\dot{R}}{R} (U_L - U_b) - C_{AM,x} \frac{\partial U_b}{\partial t} \right] \quad (5.65)$$

$$\rho_V \frac{\partial V_b U_{b,y}}{\partial t} = -\pi R \sigma f_{C,y}(\theta, d\theta) + 2\pi R \sigma \sin(\theta)^2 + \frac{1}{2} C_L \rho_L \pi R^2 (U_L - U_b)^2 \\ + \rho_L V_b \left[ -(3C_{AM,y1} + C_{AM,y2}) \frac{\dot{R}^2}{R} - C_{AM,y1} \dot{R} + C_{AM,y3} \frac{(U_L - U_b)^2}{R} \right] \quad (5.66)$$

Those force balances will respectively be used later to study the departure by sliding (along  $x$ ) and the lift-off from the wall (along  $y$ ).

On Table 5.2, we sum up some of the mentioned mechanistic approaches and their models along with the proposed force balance.

### 5.2.9 Liquid Velocity

To compute the liquid velocity and shear rate at bubble center height, we use the wall law of Reichardt [27], which describes the velocity profile from the viscous sublayer to the logarithmic region.

$$U_L^+ = \frac{1}{\kappa} \ln(1 + \kappa y^+) + c \left( 1 - e^{-y^+/\chi} + \frac{y^+}{\chi} e^{-y^+/3} \right) \quad (5.67)$$

$$U_L = U_L^+ U_\tau$$

with  $\kappa = 0.41$ ,  $\chi = 11$  and  $c = 7.8$ .

$$\frac{\partial U_L^+}{\partial y^+} = \frac{1}{1 + \kappa y^+} + \frac{c}{\chi} \left( e^{-y^+/\chi} + \left( 1 - \frac{y^+}{3} \right) e^{-y^+/3} \right) \quad (5.68)$$

$$\frac{\partial U_L}{\partial y} = \gamma = \frac{U_\tau^2}{\nu_L} \frac{\partial U_L^+}{\partial y^+}$$

The friction velocity is computed using Mac Adams correlation [23].

$$U_\tau = \sqrt{\frac{\tau_w}{\nu_L}} \quad (5.69)$$

$$\tau_w = 0.018 \text{ Re}_{D_h}^{-0.182} \frac{G_L^2}{\rho_L} \quad (5.70)$$

	Klausner (1993) [13]	Thorncroft (2001) [38]	Sugrue (2016) [34]
Forces	$\bar{F}_B = \frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$	$\frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$	$\frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$
	$\bar{F}_C = \text{Eq. ??, } r_w = 0.045 \text{ mm}$	$\text{Eq. ??, } r_w = R \sin(\theta_d)$	$\text{Eq. ??, } r_w = 0.025R$
	$\bar{F}_{CP} = \text{Eq. 5.16, } R_c = 5R$	Neglected	$\text{Eq. 5.16, } R_c = 5R$
	$C_D = \frac{16}{\text{Re}_b} \left[ 1 + \frac{3}{2} \left( \left( \frac{12}{\text{Re}_b} \right)^n + 0.796^n \right)^{1/n} \right], n = 0.65$	$C_D = \frac{16}{\text{Re}_b} \left[ 1 + \left( \frac{8}{\text{Re}_b} + \frac{1}{2} \left( 1 + \frac{3.315}{\sqrt{\text{Re}_b}} \right) \right)^{-1} \right]$	$C_D = \frac{16}{\text{Re}_b} \left[ 1 + \frac{3}{2} \left( \left( \frac{12}{\text{Re}_b} \right)^n + 0.796^n \right)^{1/n} \right], n = 0.65$
	$\bar{F}_L = C_L = 2.74\sqrt{\text{Sr}}$ $\times \left[ \text{Re}_b^{-2} + \left( 0.24\sqrt{\text{Sr}} \right)^4 \right]^{\frac{1}{4}}$	$C_L = 0.71\sqrt{\text{Sr}}$ $\times \left[ \left( \frac{1.15J(\varepsilon)}{\sqrt{\text{Re}_b}} \right)^2 + \left( \frac{3\sqrt{2\text{Sr}}}{8} \right)^2 \right]^{\frac{1}{2}}$	$C_L = 2.74\sqrt{\text{Sr}}$ $\times \left[ \text{Re}_b^{-2} + \left( 0.24\sqrt{\text{Sr}} \right)^4 \right]^{\frac{1}{4}}$
	$\bar{F}_{AM} = \frac{3}{2}\rho_L V_b \frac{\dot{R}}{R} U_L \bar{e}_x - \rho_L \pi R^2 \left( \frac{3}{2}\dot{R}^2 + R\ddot{R} \right)$ $\times (\cos(\theta_i) \bar{e}_y + \sin(\theta_i) \bar{e}_x), \theta_i = 10^\circ$	$2\pi\rho_L R^2 \dot{R} U_L \bar{e}_x - \rho_L \pi R^2 \left( \frac{3}{2}\dot{R}^2 + R\ddot{R} \right)$ $\times (\cos(\theta_i) \bar{e}_y + \sin(\theta_i) \bar{e}_x), \theta_i = 45^\circ$	$-\rho_L \pi R^2 \left( \frac{3}{2}\dot{R}^2 + R\ddot{R} \right)$ $\times (\cos(\theta_i) \bar{e}_y + \sin(\theta_i) \bar{e}_x), \theta_i = 10^\circ$
	Mazzocco (2018) [22]	Ren (2020) [28]	Present model
Forces	$\bar{F}_B = \frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$	$\frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$	$\frac{4}{3}\pi R^3 (\rho_L - \rho_V) \bar{g}$
	$\bar{F}_C = \text{Eq. ??, } r_w = R/15$	$\text{Eq. ??, } r_w = 0.2R$	$\text{Eq. ??, } r_w = R \sin(\theta)$
	$\bar{F}_{CP} = \text{Eq. 5.16, } R_c = 5R$	$\text{Eq. 5.16, } R_c = 5R$	$\text{Eq. 5.16, } R_c = R$
	$C_D = 1.13 \frac{24}{\text{Re}_b} \left( 1 + 0.104 \text{Re}_b^{0.753} \right)$	$C_D = \frac{16}{\text{Re}_b} \left[ 1 + \frac{3}{2} \left( \left( \frac{12}{\text{Re}_b} \right)^n + 0.796^n \right)^{1/n} \right], n = 0.65$	$C_D = C_{D,U} (1 + \Delta C_D)$ $C_{D,U} \text{ by Eq. 5.29, } \Delta C_D \text{ by Eq. 5.31}$
	$\bar{F}_L = C_L = 2.61$	$C_L = 2.74\sqrt{\text{Sr}}$ $\times \left[ \text{Re}_b^{-2} + \left( 0.24\sqrt{\text{Sr}} \right)^4 \right]^{\frac{1}{4}}$	$C_L \text{ by Shi et al. [31]}$
	$\bar{F}_{AM} = -\frac{1}{4}\pi\rho_L K^4 (\cos(\theta_i) \bar{e}_y + \sin(\theta_i) \bar{e}_x),$ $\sin(\theta_i) = 0.2, \cos(\theta_i) = 1$	$-\rho_L \pi R^2 \left( \frac{3}{2}\dot{R}^2 + R\ddot{R} \right)$ $\times (\cos(\theta_i) \bar{e}_y + \sin(\theta_i) \bar{e}_x), \theta_i = 15^\circ$	$\frac{F_{AM,x}}{\rho_L V_b} = C_{AM,x} \left[ 3 \frac{\dot{R}}{R} U_{rel} - \frac{\partial U_b}{\partial t} \right],$ $C_{AM,x} = 0.636, F_{AM,y} \text{ by Eq. 5.64.}$

Table 5.2: Summary of different force-balance mechanistic approaches.

### 5.3 BUBBLE GROWTH

#### 5.3.1 Introduction

In order to properly represent the bubble dynamics, it is mandatory to model the evolution of the bubble radius over time *i.e.* the bubble growth law. Since the bubble radius  $R$  and its derivatives  $\dot{R}$  and  $\ddot{R}$  appear in the force balance (notably in the expression of the added mass force 5.62), failing to predict the evolution of the bubble with time will definitely result in an erroneous force balance.

The problem of the bubble growth during its lifetime on the wall, including the sliding phase, is still an open question that aims to cover various types of heat transfer mechanisms. First, two growth regimes exist for a boiling bubble:

- Inertial growth, which occurs at the beginning of nucleation for low temperature difference between liquid and vapor. The evolution of the bubble size can be solved by the mass and momentum balances, through the solution of Rayleigh (1917) [rayleigh\_1917]. The form of the bubble growth is usually  $R(t) = A \times t$  (Mikic & Rohsenow, [mikic\_1969]).
- Heat diffusion growth happening post the inertial phase. This type of bubble growth has been widely studied by different authors [mikic\_1969, 26, 30, 48] and is usually of the form  $R(t) = B\sqrt{t}$ , derivable using the energy balance around the bubble.

Those two regimes can be compared using the non-dimensional time  $t^+$  defined as:

$$t^+ = \frac{A^2}{B^2} t \quad (5.71)$$

where

$$A = \sqrt{b \frac{h_{LV} \rho_V (T_L - T_V)}{\rho_L T_{sat}}} \text{ and } B = \sqrt{\frac{12}{\pi} \eta_L \text{Ja}} \quad (5.72)$$

with  $b = \frac{2}{3}$  for a bubble in an infinite liquid medium and  $b = \frac{\pi}{7}$  for a spherical bubble laying on a wall. So that when  $t^+ \ll 1$ ,  $R(t) = At$  (inertial growth) and when  $t^+ \gg 1$ ,  $R(t) = B\sqrt{t}$  (heat diffusion growth). A general solution asymptotically covering the two regimes has been derived by Mikic & Rohsenow:

$$R^+ = \frac{2}{3} \left[ (t^+ + 1)^{3/2} + t^{+3/2} - 1 \right], \quad R^+ = \frac{R}{B^2/A}, \quad t^+ = \frac{t}{B^2/A^2} \quad (5.73)$$

In most cases associated to wall nucleation and boiling flows, experimental observations showed that bubbles' lifetime is long enough to be mostly of heat conduction nature [15, 21, 47].

### 5.3.2 Heat Diffusion in Uniformly Superheated Liquid

The analytic derivation of a bubble growth law in a pure heat diffusion regime has been tackled by various authors, mostly for the case of a bubble in a uniformly superheated and quiescent liquid. An often referred solution is the work of Plesset & Zwick (1954) [26] who found an asymptotic solution for high values of Ja:

$$R(t) = \frac{2\sqrt{3}}{\sqrt{\pi}} \text{Ja} \sqrt{\eta_L t} \quad (5.74)$$

This result was generalized by Scriven (1959)[30] who derived:

$$R(t) = 2\mathcal{F}(\text{Ja}) \text{Ja} \sqrt{\eta_L t} \quad (5.75)$$

where  $\mathcal{F}$  is implicitly defined by assuming  $\frac{\rho_L}{\rho_V} \gg 1$ :

$$\mathcal{F}(\text{Ja}) = \frac{F}{2\text{Ja}^2}, \text{ and } \text{Ja} = F \exp\left(\frac{3}{2}F\right) \int_1^\infty \frac{1}{x^2} \exp\left(-\frac{F}{x} - \frac{F}{2}x^2\right) dx \quad (5.76)$$

which falls back to  $\mathcal{F}(\text{Ja}) \rightarrow \frac{\sqrt{3}}{\sqrt{\pi}}$  when  $\text{Ja} \gg 1$ .

The general formulation of  $\mathcal{F}$  has been verified by Legendre *et al.* [17] with Direct Numerical Simulation of spherical bubble growth in a quiescent superheated liquid.

Usually, most authors are accepting  $R(t) = K \text{Ja}_w \sqrt{\eta_L t}$  for the bubble growth. With  $K$  usually expressed as  $K = \frac{2b}{\sqrt{\pi}}$  with  $b$  being used as an adjustable constant depending on the flow conditions and the fluid, or derived analytically as presented before ( $b = \sqrt{3}$  [26],  $b = \frac{\pi}{2}$  [7],  $1 \leq b \leq \sqrt{3}$  [48],  $b = 1.56$  [43],  $b = 0.24$  [42], etc.).

When the bubble presents a relative velocity with the ambient liquid, the disturbance of the thermal boundary layer around the liquid-vapor interface will impact its growth. This phenomenon has been numerically studied by Legendre *et al.* [17] who found that the ratio between the growth rate  $\dot{R}$  and the relative velocity  $U_{rel}$  was controlling the growth regime as follows:

- If  $\frac{\dot{R}}{U_{rel}} \gg 1$ , the regime is close to the static heat diffusion and correspond to the Scriven formulation (Eq. 5.75) ;
- If  $\frac{\dot{R}}{U_{rel}} \ll 1$ , the relative velocity impacts the thermal boundary layer formation and leads to a growth matching the solution of Ruckenstein (1958) [ruckenstein\_1958] where the Nusselt number is:

$$\text{Nu} = 2\sqrt{\frac{\text{Pe}(t)}{\pi}}, \quad \text{Pe}(t) = \text{Pr}_L \times \text{Re}_b(t) \quad (5.77)$$

In this case, the bubble growth is accelerated and  $R \propto t^{2/3}$ .

### 5.3.3 Microlayer Evaporation

In addition to the traditional heat diffusion from superheated liquid to the bubble through the liquid-vapor interface, bubble growth can also be enhanced by the evaporation of a so-called "microlayer". This term denotes a very thin layer of liquid (typically  $\sim \mu\text{m}$  [15]) which is trapped between the heated wall and the bubble base, as shown on Figure ???. The existence of this microlayer has now been supported by both experimental visualizations [koffman\_plessset\_1983, chen\_haginiwa\_2017, chen\_hu\_2020, 15] and numerical investigations [guion, 2, 39].

Parallel to the heat diffusion approach, some authors computed the bubble growth by considering a pure microlayer evaporation regime. A well-known model of this type has been derived by Cooper & Lloyd in 1969 [cooper\_lloyd\_1969] and considers the wall thermal properties so that:

- $R(t) = 2.5 \frac{\text{Ja}}{\sqrt{\text{Pr}_L}} \sqrt{\eta_L t}$  if  $\lambda_w \gg \lambda_L$  ;
- $R(t) = \frac{2}{\sqrt{\pi}} \sqrt{\frac{\lambda_w \rho_w c_{p,w}}{\lambda_L \rho_L c_{p,L}}} t$  if  $\lambda_w \ll \lambda_L$

**Remark :** The parameter  $\sqrt{\frac{\lambda_w \rho_w c_{p,w}}{\lambda_L \rho_L c_{p,L}}}$  that accounts for the wall properties is the same used in the correlation of Ünal for the maximum bubble diameter (Eq. 5.3).

The microlayer is also often taken into account for HFP modeling by enhancing the boiling heat flux through a computation of the microlayer volume [kommajosyula, demarly].

However, the presence of a liquid microlayer beneath the nucleated bubble is not assured for every boiling conditions. Indeed, experimental observations recently realized by Kossolapov [15] showed that the microlayer only existed for pressures below 3 bars when using water as working fluid. Moreover, Direct Numerical Simulations of Urbano *et al.* [39] where a full coupling between mass, momentum and energy balance was achieved managed to detect whether if the bubble grows in a contact-line regime or if a microlayer appears. They proposed a criterion based on the capillary and Jakob numbers determining the formation of a microlayer :

$$\frac{\text{JaCa}}{(\theta - \theta_0)^3} > \frac{1}{A^3}, \quad \theta_0 = 5^\circ, \quad A = 313 \quad (5.78)$$

**Remark :** Computing the capillary number using  $\dot{R} = \frac{K \text{Ja}}{2} \sqrt{\frac{\eta_L}{t}}$  as the interface velocity, we use Urbano *et al.* criterion to compute the time  $t_{max}$  after which the microlayer would cease to exist. Applying this to PWR conditions ( $K = 1$ ,  $\Delta T_w = 5K$ ) yields  $t_{max} < 10^{-11}\text{s}$  for  $6^\circ \leq \theta \leq 90^\circ$ , meaning that there is no time during the bubble growth during which a microlayer could grow. This agrees with the observation that increasing pressure would lead to microlayer disappearance.

We will not further detail the study of the microlayer regime since its existence is very unlikely if not impossible in the pressurized conditions typical of a PWR.

### 5.3.4 Bubble Growth in Subcooled Flow Boiling

If we consider the full problem of bubble growth in subcooled flow boiling, the analytic expressions presented above may fall out of their validation range since extra physical phenomena will be at stake. This more generic type of growth lack of proper theoretical derivations due to the complexity of the considered system (turbulence, condensation, convection, etc.). That is why authors trying to represent such complex bubble growth often combine different heat transfer mechanisms such as:

- Evaporation due to conduction from the superheated liquid near the bubble base ;

- Evaporation of the liquid microlayer ;
- Condensation on top of the bubble when it reaches subcooled liquid ;
- Convective heat transfer due to relative velocity between the bubble and the liquid.

To our knowledge, such models always consider empirical or fitted parameters. For instance Yoo *et al.* [42] wrote for a sliding bubble:

$$\frac{\partial R}{\partial t} = \underbrace{\gamma \text{Pr}_L^{-0.5} \text{Ja}_w \sqrt{\frac{\eta_L}{t}} \frac{A_{ML}}{A_b}}_{\text{Microlayer}} + \underbrace{(1-f) \frac{b}{\sqrt{\pi}} \text{Ja}_w \sqrt{\frac{\eta_L}{t}}}_{\text{Superheated liquid}} - \underbrace{\frac{f \Delta T_L C}{1 - \rho_V / \rho_L} R}_{\text{Subcooled convection}} \quad (5.79)$$

where  $\gamma = \sqrt{\frac{\lambda_w \rho_w c_{p,w}}{\lambda_L \rho_L c_{p,L}}}$ ,  $\frac{A_{ML}}{A_b} = 1.22 \gamma^{-0.79} \exp(-0.204 \text{Ja}_w)$ ,  $f = 0.5$ ,  $b = 0.24$  and  $C = 0.1$ .

Their model was validated against low pressure sliding of boiling bubbles for different fluids (Water [21], FC87 [37], R113 [yoo\_exp\_2016]). They account for wall properties through the parameter  $\gamma$  in the microlayer term while assuming that 50% of the bubble faces subcooled liquid ( $f = 0.5$ ) and condenses following the formulation of Levenspiel [19].

Zhou *et al.* [47] also proposed a similar modeling of the bubble growth, validated on their own measurements for boiling water at low pressure:

$$\frac{\partial R}{\partial t} = \underbrace{\frac{1}{C} \text{Pr}_L^{-0.5} \text{Ja}_w \sqrt{\frac{\eta_L}{t}}}_{\text{Microlayer}} + \underbrace{\sqrt{\frac{3}{\pi}} \text{Ja}_T \sqrt{\frac{\eta_L}{t}} \min\left(\frac{y_{sat}}{2R}, 1\right)}_{\text{Superheated liquid}} - \underbrace{\frac{\eta_L}{2R} \text{Ja}_L (2 + 0.6 \text{Re}_b^{0.5} \text{Pr}_L^{0.3}) \max\left(\frac{H - y_{sat}}{2R}, 0\right)}_{\text{Subcooled convection}} \quad (5.80)$$

where  $C = 1.45$ ,  $\text{Ja}_T$  is the Jakob number taken at  $\min(\bar{T} - T_{sat}, 0)$ ,  $\bar{T}$  the average liquid temperature around the bubble,  $H = R(1 + \cos(\theta))$ .

While they consider a constant coefficient for the microlayer evaporation, they propose a finer modeling of the condensation term by evaluating the height  $y_{sat}$  at which  $T_L = T_{sat}$  using the turbulent wall law of Kader [kader\_wall\_law\_1969]. The condensation is modeled by the Ranz & Marshall correlation [ranz\_marshall] that accounts for the relative velocity through the bubble Reynolds number.

**Remark :** As mentioned before, those model rely on numerous empirical parameters due to the variety of considered phenomena. In particular, microlayer evaporation is systematically considered which could be questioned regarding the observations made in Subsection 5.3.3.

### 5.3.5 Analytic Approach of Bubble Growth in a Linear Thermal Boundary Layer

In this Subsection, we propose an analytic derivation of bubble growth for a truncated sphere laying on a wall in a boundary layer with a linear temperature profile. The considered geometrical and thermal definitions are depicted on Figure 5.9.

We consider an established single-phase thermal boundary layer of thickness  $\delta$ . When the bubble start to grow, an other boundary layer will of thickness  $\delta_b$  grow between the liquid-vapor interface and the surrounding liquid which temperature depends on the wall distance  $y$ .

The liquid temperature is assumed to follow a linear profile:

$$T_L(y) = T_w + \frac{T_{L,bulk} - T_w}{\delta} y \quad (5.81)$$

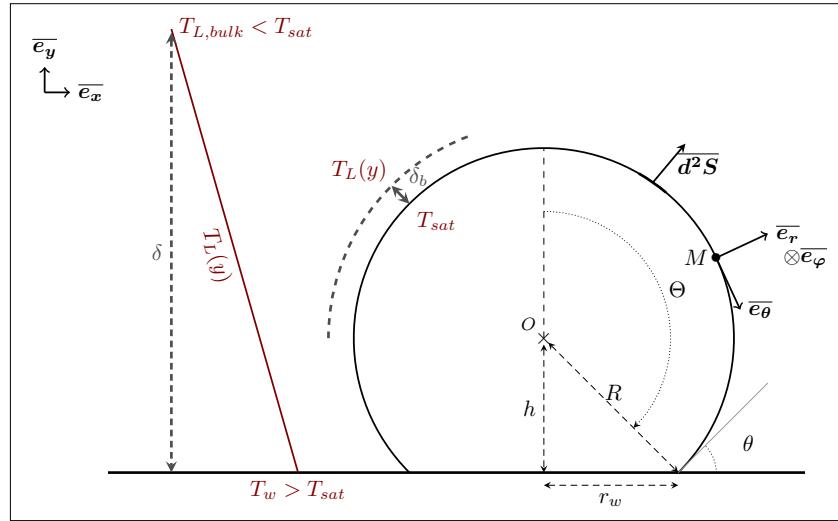


Figure 5.9: Studied geometry

Assuming that the vapor stays at a temperature close to  $T_{sat}$ , the radial component of the temperature gradient at the bubble's interface can be expressed as:

$$\nabla(T) \cdot \bar{e}_r = \frac{\partial T}{\partial r}(R, \theta, \phi) \approx \frac{T_L(y) - T_{sat}}{\delta_b} \quad (5.82)$$

Applying Fourier's law to the liquid close to the bubble to estimate the heat flux density vector  $\bar{j}_Q = -\lambda_L \nabla(T)$ . Between  $t$  and  $t + dt$  the heat exchanged through  $d^2S$  is:

$$d^2Q_b \approx -\frac{\lambda_L}{\delta_b} \left[ \Delta T_w R^2 \sin(\theta) - \frac{\Delta T_w + \Delta T_L}{\delta} R^3 [\cos(\theta) - \cos(\Theta)] \sin(\theta) \right] d\theta d\varphi \quad (5.83)$$

Then assuming that  $\delta_b$  is constant between  $t$  and  $t + dt$ , the total heat flux can be expressed:

$$Q_b = \frac{2\pi\lambda_L R^2}{\delta_b} (1 + \cos(\theta)) \left[ \Delta T_w - \frac{R}{2\delta} (\Delta T_w + \Delta T_L) (1 + \cos(\theta)) \right] \quad (5.84)$$

Writing the mass balance by considering that the heat flux contributes solely to phase change:

$$\frac{\partial V_b}{\partial t} = \frac{Q_b}{\rho_V h_{LV}} \quad (5.85)$$

$$V_b = \frac{4}{3}\pi R^3 f_V, \quad f_V = \frac{1}{4} (2 - \cos(\theta)) (1 + \cos(\theta))^2 \quad (5.86)$$

Writing this in terms of bubble radius:

$$\frac{\partial R}{\partial t} = \frac{\text{Ja}_w \eta_L}{2\delta_b f_V} (1 + \cos(\theta)) \left[ 1 - \frac{R}{2\delta} \left( 1 + \frac{\text{Ja}_L}{\text{Ja}_w} \right) (1 + \cos(\theta)) \right] \quad (5.87)$$

Which reduces to the following differential equation:

$$\frac{\partial R}{\partial t} + aR = b \quad (5.88)$$

$$a = \frac{\text{Ja}_w \eta_L}{4\delta_b f_V} \left( 1 + \frac{\text{Ja}_L}{\text{Ja}_w} \right) (1 + \cos(\theta))^2 \quad \text{and} \quad b = \frac{\text{Ja}_w \eta_L}{2\delta_b f_V} (1 + \cos(\theta)) \quad (5.89)$$

Solutions of this differential equation depend on the hypothesis over  $\delta$  and  $\delta_b$ . If we assume that the bubble grows in a fully established liquid flow then  $\delta$  can be considered as constant.

When the bubble will start to nucleate, the liquid-vapor interface will delimit a frontier through which a transient heat transfer between the vapor at constant temperature  $T_{sat}$  and liquid at  $T_L(y)$  will occur. To estimate the associated local boundary layer thickness  $\delta_b$ , we can rely on the solution of semi-infinite transient conduction as treated in Del Valle & Kenning [delvalle\_quenching] or Mikic & Rohsenow [mikic\_1970]:

$$\delta_b = \sqrt{\eta_L t} \quad (5.90)$$

The differential equation Eq. 5.88 becomes:

$$\frac{\partial R}{\partial t} + a(t) R = b(t) \quad (5.91)$$

$$a(t) = \frac{Ja_w \sqrt{\eta_L}}{4\delta f_V \sqrt{t}} \left(1 + \frac{Ja_L}{Ja_w}\right) (1 + \cos(\theta))^2 = K_a t^{-1/2} \quad (5.92)$$

$$b(t) = \frac{Ja_w \sqrt{\eta_L}}{2f_V \sqrt{t}} (1 + \cos(\theta)) = K_b t^{-1/2} \quad (5.93)$$

With the initial condition  $R(t=0) = 0$ , the solution to this differential equation is:

$$R(t) = R_\infty \left(1 - e^{-2K_a \sqrt{t}}\right) \quad (5.94)$$

$$R_\infty = \frac{K_b}{K_a} = \frac{2\delta}{\left(1 + \frac{Ja_L}{Ja_w}\right)(1 + \cos(\theta))} \quad (5.95)$$

This type of bubble growth presents interesting properties. First, it degenerates to the uniformly superheated liquid solution when  $t \rightarrow 0$ :

$$R(t) \underset{t \rightarrow 0}{\sim} \frac{1 + \cos(\theta)}{f_V} Ja_w \sqrt{\eta_L t} \quad (5.96)$$

with a purely geometrical growth constant depending on the contact angle, equal to 2 for the spherical case.

Moreover, this growth law accounts for the liquid subcooling and thus presents an equilibrium radius  $R_\infty$  when  $t \rightarrow \infty$ , corresponding to the bubble size at which the vaporization from the superheated liquid is exactly compensated by the condensation at the bubble top.

To the best of our knowledge, this simple bubble growth law has never been proposed in the literature. However, this equation has some limitations :

- It requires the knowledge of the liquid thermal boundary layer thickness  $\delta$  which estimation can be tricky ;
- This law can't be applied if  $T_{L,bulk} > T_{sat}$ .

**Remark :** It is worthy to note that this solution is derived solely using the energy balance at the liquid-vapor interface. No momentum balance was used when solving this physical problem, which can be considered as a limit of the approach.

### 5.3.6 Comparison with DNS Results

To assess the validity of Eq. 5.94, we will compare the radius time profile with DNS results by Urbano *et al.* [urbano\_2019] who simulated the same physical situation as depicted in Figure 5.9 for pool boiling. They also solved the heat conduction in the wall and studied the growth dynamics depending on the values of  $\Delta T_L$  and  $\Delta T_w$  as well as the equilibrium diameter reached by the bubble.

In their analysis, Urbano *et al.* derived the same equilibrium radius as in Eq. 5.95 by equating the condensation and vaporization heat fluxes. By comparing with the equilibrium radius reached in their simulations, they found that a corrective factor  $C = 1.15829$  was needed to correct Eq. 5.95. This difference could be explained by the heat conduction in the wall that is not accounted for in the theoretical approach.

DNS results obtained for three couples of subcooling  $\Delta T_L$  and superheat  $\Delta T_w$  are used for comparison. Results are displayed on Figure 5.10 with and without the corrective factor on  $R_\infty$  suggested by Urbano *et al.*

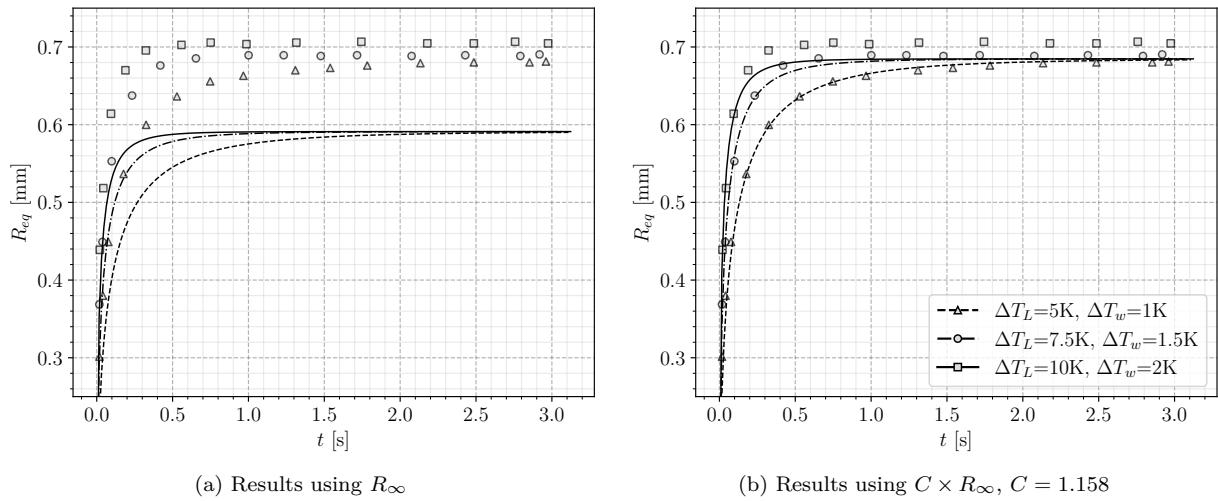


Figure 5.10: Comparison with DNS results of Urbano *et al.* [urbano\_2019] ( $\delta = 3\text{mm}$  and  $\theta = 50^\circ$ ).  
Lines : Model predictions - Markers : DNS

The analytical formulation of the bubble growth matches very well with the DNS results when the equilibrium radius is corrected. The different growth regime induced by the pairs  $(\Delta T_L, \Delta T_w)$  are correctly captured by the model. DNS results present different equilibrium radius values when the subcooling and superheat changes, which can not be accounted for by the model.

**Remark :** Those results are encouraging and validate the modeling of  $\delta_b$  with the semi-infinite conduction model (Eq. 5.90).

### 5.3.7 Comparison with Experimental Measurements

To further evaluate the proposed model, we compare the result with experimental measurements of bubble radius in vertical boiling of water at atmospheric pressure by Maity [21]. The choice of  $\delta$  is adapted to each case in order to observe the behavior of the model.

In addition, we also plot the predictions by the heat diffusion solution  $R = K \text{Ja}_w \sqrt{\eta_L t}$  with  $K = \frac{2b}{\sqrt{\pi}}$  and  $1 \leq b \leq \sqrt{\pi}$ . A solution with an optimized value of  $K$  is also represented. The models of Mazzocco and Yoo *et al.* are also compared.

The results are presented on Figure ??.

The model globally reproduces the experimental results better than the other models. In particular, the progressively damped growth rate when the bubble start to face colder liquid seems to correctly captures

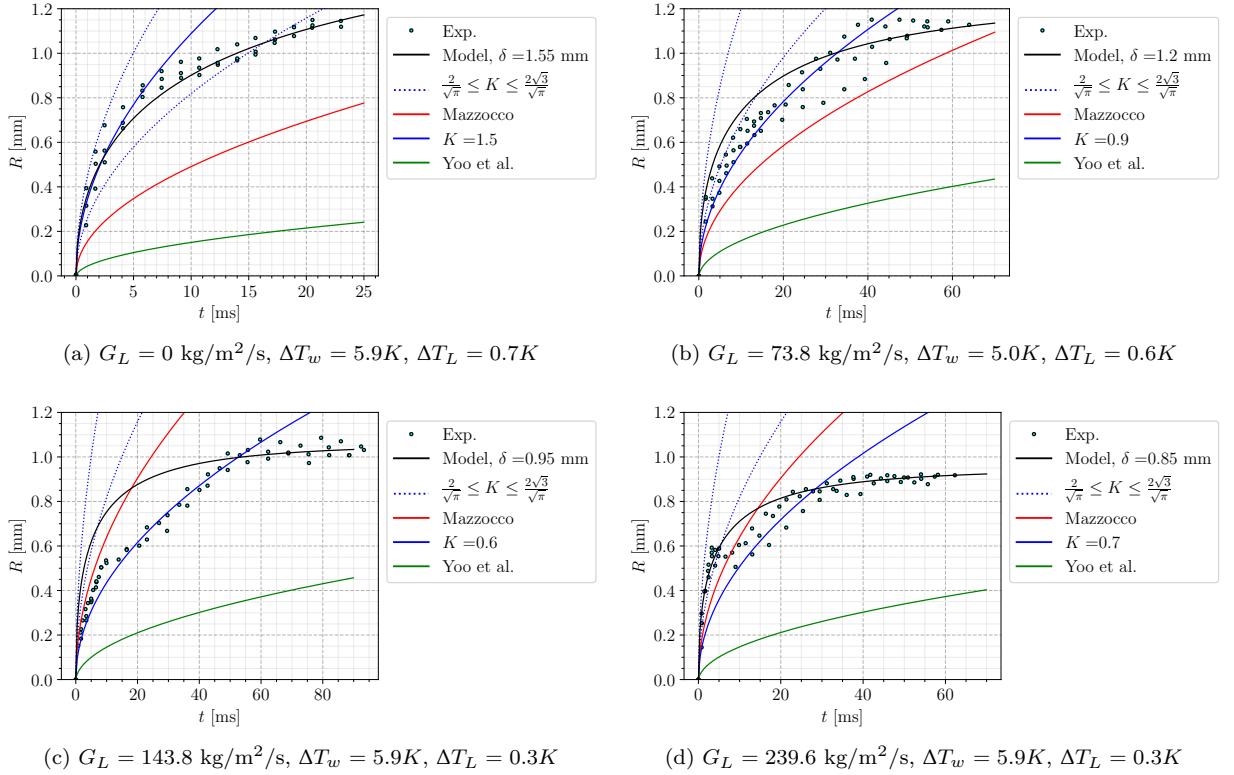


Figure 5.11: Comparison with experimental measurements of Maity [21].

the nonlinear experimental growth. Values of  $\delta$  needed to produce those results were between 0.85 mm and 1.55mm, which reasonably agrees with measurements of Maity in his experiment for horizontal flow giving  $\delta$  roughly between 1 mm and 1.5 mm.

On the other hand, the fitted value of  $K$  is often lower than the lower bound  $\frac{2}{\sqrt{\pi}}$  suggested by Zuber [48]. This is a consequence of the subcooled flow which deviates from the uniformly superheated liquid from which those values were derived. This fitted profile manages to capture some stages of bubble growth but can't predict the asymptotic behavior where bubble reaches a quasi-constant radius.

Finally, we see that the model of Yoo *et al.* systematically underestimates the bubble radius. We suspect this could come from the assumption that half of the bubble faces subcooled liquid, which is hardly reasonable especially at early growth stages.

## 5.4 DEPARTURE BY SLIDING

### 5.4.1 Non-Dimensional Analysis

To tackle the departure by sliding, we rely on force balance parallel to the wall in Eq. 5.65. Before departure, the bubble grows on its nucleation site while staying immobile, thus with a sliding velocity  $U_b = \frac{\partial U_b}{t} = 0$ . The force balance parallel to the wall becomes:

$$-\pi R \sigma f_{C,x} + \frac{4}{3} \pi R^3 (\rho_L - \rho_V) g + \frac{1}{2} C_D \rho_L \pi R^2 U_L^2 + \frac{4}{3} \pi R^3 \rho_L 3C_{AM,x} \frac{\dot{R}}{R} U_L = 0 \quad (5.97)$$

We can note that the departure by sliding is promoted by the buoyancy, the drag and the added mass forces. Only the capillary force keeps the bubble attached to its nucleation site, which will be discussed later.

Re-writing Eq. 5.97 in non-dimensional form by dividing the LHS by the Added Mass force yields:

$$-\frac{1}{2} \frac{f_{C,x}}{K^2 C_{AM,x}} \frac{1}{\text{Ca}} \frac{\text{Pr}_L}{\text{Ja}_w^2} + \frac{1}{3} \frac{1}{K^2 C_{AM,x}} \frac{\text{Re}_b}{\text{Fr}} \frac{\text{Pr}_L}{\text{Ja}_w^2} + \frac{1}{8} \frac{C_D}{K^2 C_{AM,x}} \text{Re}_b \frac{\text{Pr}_L}{\text{Ja}_w^2} + 1 = 0 \quad (5.98)$$

where we have the following non-dimensional numbers:

$$\begin{aligned} \text{Re}_b &= \frac{2 R U_L}{\nu_L} ; \text{Fr} = \frac{\rho_L U_L^2}{(\rho_L - \rho_V) g R} ; \text{We} = \frac{\rho_L U_L^2 R}{\sigma} ; \text{Eo} = \frac{(\rho_L - \rho_V) g R^2}{\sigma} ; \\ \text{Ja}_w &= \frac{(T_w - T_{sat}) \rho_L c_{P,L}}{\rho_V h_{LV}} ; \text{Pr}_L = \frac{\nu_L}{\eta_L} ; \frac{\dot{R}}{U_L} = \frac{K^2 \text{Ja}_w^2}{\text{Pr}_L \text{Re}_b} ; \text{Ca} = \frac{\mu_L U_L}{\sigma} \end{aligned}$$

Eq. 5.98 exhibits terms that can be used to compare the magnitude of each detaching forces. We can obtain the following conditions:

$$\text{Added Mass greater than Drag if: } \frac{\text{Ja}_w^2}{\text{Pr}_L} > \frac{1}{8} \frac{C_D}{C_{AM,x}} \frac{1}{K^2} \text{Re}_b \quad (5.99)$$

$$\text{Added Mass greater than Buoyancy if: } \frac{\text{Ja}_w^2}{\text{Pr}_L} > \frac{1}{3} \frac{1}{C_{AM,x} K^2} \frac{\text{Re}_b}{\text{Fr}} \quad (5.100)$$

$$\text{Drag greater than Buoyancy if: } \text{Re}_b > \frac{16}{3} \frac{1}{C_D} \frac{\text{Eo}}{\text{Ca}} = \text{Re}_c \quad (5.101)$$

Those boundaries can be plotted on a  $(\text{Ja}_w^2/\text{Pr}_L ; \text{Re}_b)$  map for a given fluid and bubble diameter  $D$ . An example of such a map is presented on Figure 5.12. This allows to visualize the operating conditions under which each of the detaching forces will be dominant. Logically, Buoyancy dominates for low  $\text{Re}_b$  regimes contrary to Drag. Added Mass dominates when values of  $\text{Ja}_w^2/\text{Pr}_L$  are high *i.e.* when bubble grows rapidly.

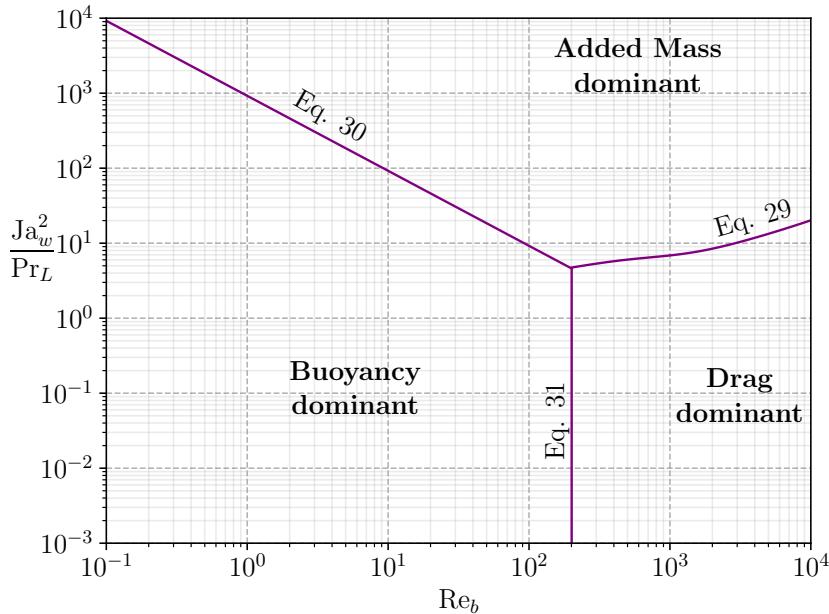


Figure 5.12: Dominance map regarding departure by sliding. Boundaries plotted for water at 1 bar and  $D_d = 0.5\text{mm}$ . ( $K = 2$ )

#### 5.4.1.1 Influence of Pressure

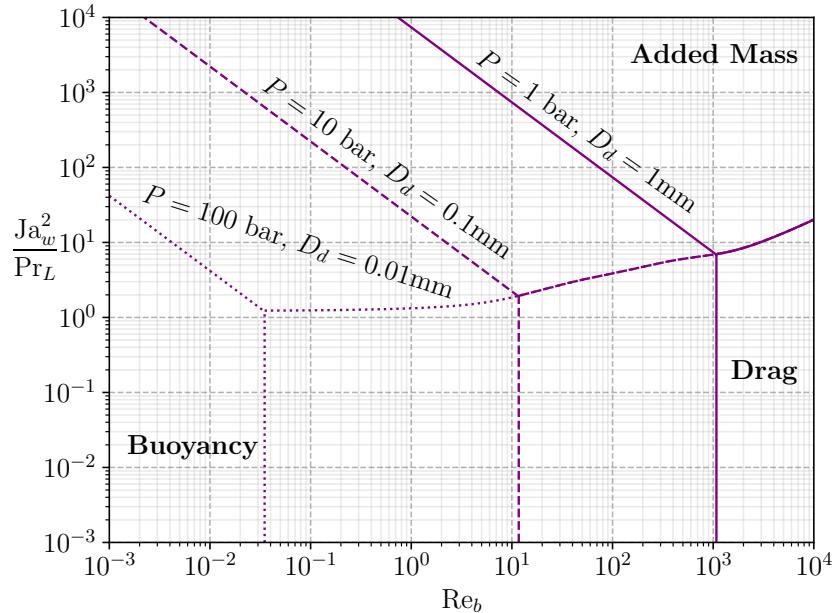
On Figure 5.13a, we draw the dominance map for 3 different pressures and associated orders of magnitude of bubble departure diameter [14].

The impact of pressure is mostly seen through the decrease of bubble departure diameter. As pressure increases, Buoyancy force decreases while Drag and Added Mass forces display much larger dominance zones. The competition between those two terms mainly relies on the competition between liquid flow velocity and wall superheat or heat flux.

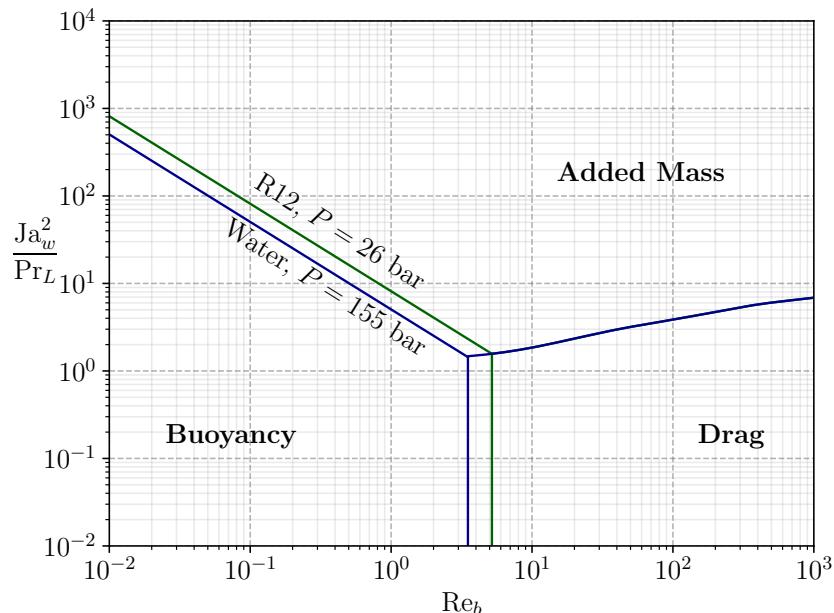
### 5.4.1.2 Comparison between Fluids

On Figure 5.13b, we compare the dominance zones for R12 at 26 bar and water at 155 bar. Moderately pressurized R12 (10 to 30 bar) has often been used as a simulating fluid to mimic water in PWR since it has the same density ratio and Weber number for instance.

Assuming that the conservation of Weber and Boiling numbers may lead to similar bubble departure diameters, we can observe that the boundaries between the two fluids are very close. This qualitatively indicates that R12 shall present bubble departure by sliding mechanisms similar to what happens in PWR, which could comfort the confidence one may have in extrapolating the observations done using the simulating fluid to industrial applications.



(a) Dominance map plotted for water at different pressures and bubble departure diameters. ( $K = 2$ )



(b) Dominance map for R12 as simulating fluid for PWR.  $D_d = 0.05 \text{ mm}$  is chosen according to R12 measurements from Garnier *et al.* [9] who observed bubbles of  $\sim 0.1 \text{ mm}$  diameter after lift-off. ( $K = 2$ )

Figure 5.13: Examples of qualitative analysis using the non-dimensional regime map

### 5.4.2 Application to Experimental Data

Now we want to apply this non-dimensional approach to experimental measurement in order to determine the actual bubble departure by sliding regimes. We rely on 7 experiments in which bubble departure diameters in vertical flow boiling were measured. The operating conditions are gathered in Table 5.3.

Author	Fluid	$D_h$ [mm]	$P$ [bar]	$G_L$ [kg/m <sup>2</sup> /s]	$\Delta T_L$ [K]	$\phi_w$ [kW/m <sup>2</sup> ]	$\Delta T_w$ [K]	$D_d$ [mm] ( $N_{mes}$ )
Thorncroft [37] (1998)	FC-87	12.7	N.A.	0 - 319	0.99 - 3.27	2.83 - 11.8	0.54 - 6.89	0.094 - 0.237 (10)
Maity [21] (2000)	Water	20	1.01	0 - 239.6	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)
Chen [3] (2012)	Water	3.8	1.2 - 3.35	214 - 702	14.5 - 30.3	83.6 - 334	N.A.	0.549 - 2.255 (22)
Sugrue [34] (2014)	Water	16.6	1.01	250 - 400	10 - 20	50 - 100	2 - 6	0.229 - 0.391 (16)
Guan [11] (2014)	Water	9	1.01	87.3 - 319.2	8.5 - 10.5	68.2 - 104	4.5 - 8.5	0.62 - 1.85 (12)
Ren [28] (2020)	Water	3.8	2 - 5.5	488.4 - 1654	28.7 - 51	160.7 - 643.2	N.A.	0.045 - 0.111 (42)
Kossolapov [15] (2021)	Water	11.8	19.9 - 39.8	500 - 1500	10	178 - 613	N.A.	0.01 - 0.047 (11)

Table 5.3: Bubble departure diameters data sets in vertical flow boiling

If the value of  $\Delta T_w$  is not available in the considered data-set, we estimate it  $\Delta T_w$  using Frost & Dzakowic correlation [8].

$$\Delta T_w = \text{Pr}_{L,sat} \sqrt{\frac{8\sigma\phi_w T_{sat}}{\lambda_L \rho_V h_{LV}}} \quad (5.102)$$

Comparisons with bubble growth profile from Kossolapov [15] showed that values of  $K$  respectively around 7 and 15 were needed to match experimental measurements at 20 bar and 40 bar. This means Eq. 5.102 probably underestimates the wall superheat on those cases. We will thus further adopt a corrective factor of 7 at 20 bar and 15 at 40 bar to use more realistic values of  $K$ .

To place experimental measurements on the non-dimensional map, we need a bubble detachment diameter value  $D_d$  to plot the dominance zones. Since measured  $D_d$  vary significantly in each experiment, we draw the boundaries for the maximum and minimum values of  $D_d$  as shown on Figure 5.14a. If the considered data covers different pressures, boundaries for each pressure are plotted to exhibit its impact (Figures 5.14d, 5.14e and 5.14f). We chose a value of  $K = 1$  to draw the boundaries.

The Figure 5.14 shows that for most of the low pressure experiments, the detaching forces are the Added Mass and the Buoyancy. Smaller bubbles are mainly detached under the effect of the Added Mass force (Figures 5.14c, 5.14d and 5.14e). When the bubbles detach at higher diameters, the impact of the Buoyancy force naturally increases and is comparable to the Added Mass force (Figures 5.14a and 5.14b).

When the pressure increases, we observe that the experimental measurements gradually move towards the Drag dominant zone as seen on Figures 5.14e and 5.14f. This main difference in the dynamic regime when bubble departs by sliding arises from multiple effects:

- The decrease of  $\rho_L/\rho_V$  with pressure, thus reducing  $J_{aw}$  and the impact of the detaching Added Mass term ;
- The higher liquid mass fluxes in Kossolapov experiments, increasing the impact of the Drag ;
- The decrease of  $D_d$  with pressure, reducing the magnitude of Buoyancy.

However, we see that some measurements lie close to the Added Mass / Drag boundary (Figure 5.14f), indicating that the Added Mass force still plays a significant role for bubble detachment. This means

that regardless of the operating pressure, the detaching term associated to the coupling between bubble growth and outer liquid flow should not be neglected in the force balance (Eq. 5.63).

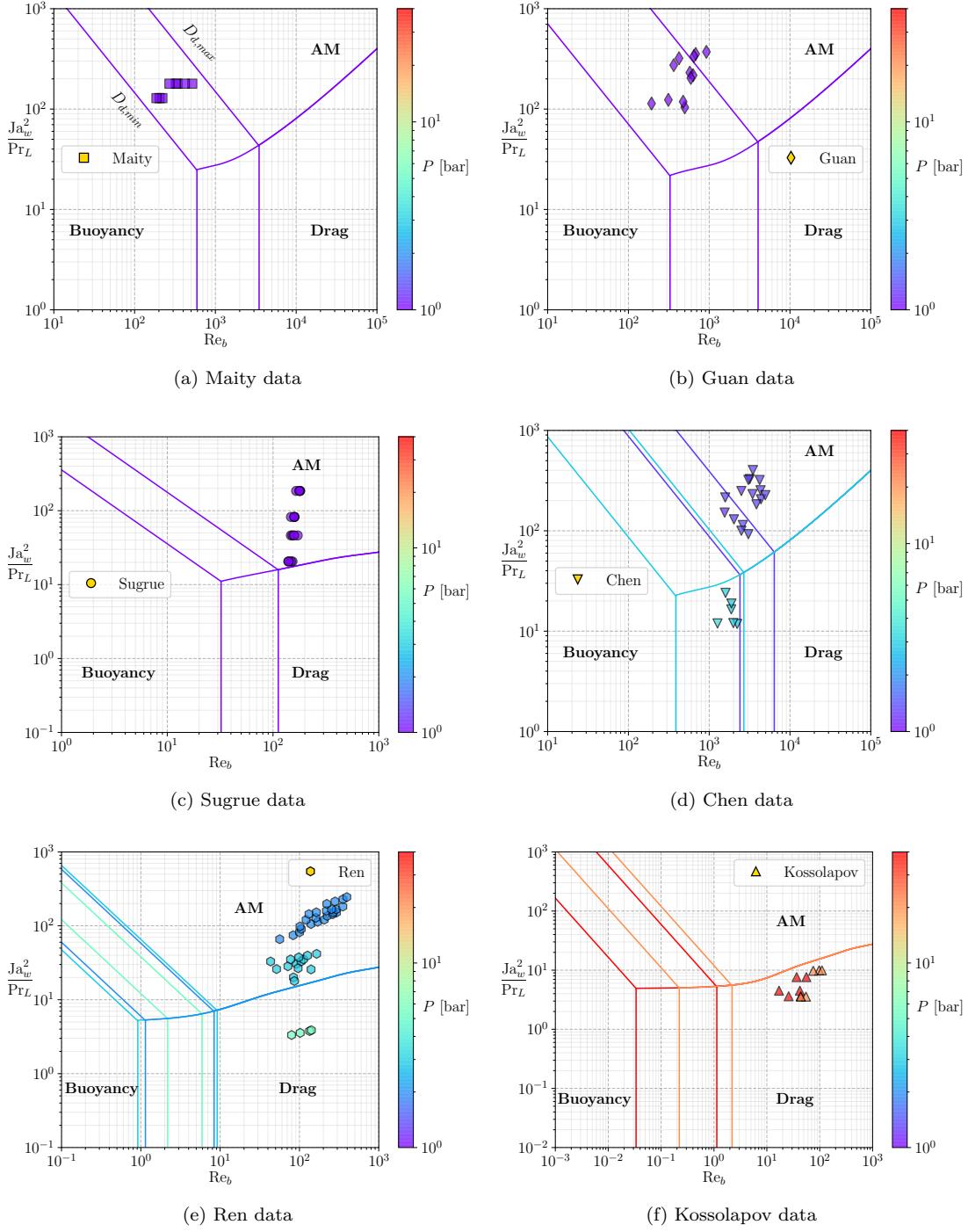


Figure 5.14: Dominance maps for each water data sets from Table 5.3.

### 5.4.3 Departure Diameter Prediction

#### 5.4.3.1 About the Use of Empiricism

As previously mentioned, the case of bubble detachment in vertical flow boiling is particular since only one force maintains the bubble attached to its nucleation site: the Capillary force (Eq. ??). Its expression depends on the contact angle  $\theta$ , the angle half-hysteresis  $d\theta$  and the bubble foot radius  $r_w$  (or ratio to bubble diameter  $r_w/R$ ) and is thus very sensitive to those values. Paradoxically, those terms are among

the least precisely known due to the difficulty of measurement and associated uncertainties. For instance, conducting precise evaluations of the contact angle near the bubble base through optical techniques can be challenging because of the strong temperature gradients close to the heated surface.

Consequently, empirical choices have to be made in order to set a value to those parameters, often by relying on data-fitting or approximate measurements in other conditions. For instance, contact angles are often taken as arbitrary average values [28] or measurements in room conditions [34] and applied over a whole set of experiments. This is questionable since contact angle is unlikely to remain unchanged over different operating conditions and surfaces with varying roughness and properties [33].

However, no better information except those given by the authors can be used to evaluate the Capillary force since no generic model exist to compute the contact angle and hysteresis. In this work, admitting a significant uncertainty (typically 5°, as in Guan [11]), we will use the following values for the contact angles :

- $\theta_u = 25.3^\circ$  and  $\theta_d = 6.6^\circ$  for Thorncroft data (measured values for FC-87 on nichrome [38]) ;
- $\theta_u = 50^\circ$  and  $\theta_d = 40^\circ$  for Maity data (measured average contact angles for each bubble during its lifetime [21]) ;
- $\theta_u = 130^\circ$  and  $\theta_d = 65^\circ$  for Chen data (chosen values in their study following measurements for water on stainless steel at high temperature by Kandlikar *et al.* [12]) ;
- $\theta_u = 91^\circ$  and  $\theta_d = 8^\circ$  for Sugrue data (measured values at room temperature [36]) ;
- $\theta_u = 75^\circ$  and  $\theta_d = 30^\circ$  for Guan data (measured average value through experimental visualizations [11]) ;
- $\theta_u = 45^\circ$  and  $\theta_d = 36^\circ$  for Ren data (chosen values in their study [28]) ;
- $\theta = 80^\circ$  for Kossolapov data (typical contact angle for water on ITO [15]) and  $d\theta = 1^\circ$  assuming that the very small bubbles at high pressure are nearly not tilted.

Similarly, the bubble foot radius  $r_w$  is often empirically assumed to be either constant [13] proportional to the bubble radius [22, 34] or to follow a linear or logarithmic law of  $R$  [11, 47]. That is why we chose to use the truncated sphere hypothesis (Eq. 5.14) to compute  $r_w$  using  $R$  and  $\theta$ .

Finally, we would like to acknowledge that the empiricism to evaluate those parameters represents one of the biggest flaws of the force-balance approach. Indeed, such a model aims to detect small sign changes in a sum of a few  $\mu\text{N}$  of forces that are decades larger as pointed out by Bucci *et al.* [1]. Mechanistic models are thus strongly sensitive to any extra parameter included in the modeling of the forces.

#### 5.4.3.2 Growth Constant Values

Since the value  $K \approx 2$  represents an upper bound for the growth constant in a quiescent uniformly superheated liquid [26] and that values below 1 can be a better fit for bubble growth in subcooled flow boiling (Figure 5.18), we propose a simple law that includes an influence of the liquid subcooling and velocity:

$$K = \frac{2}{(1 + \text{Re}_\tau)^{0.3} (1 + \text{Ja}_L)} \quad (5.103)$$

where  $\text{Re}_\tau = U_\tau L_c / \nu_L$ .

This expression naturally degenerates to  $K = 2$  for quiescent saturated liquid and yields values between 0.15 and 1.99 for the cases of Table 5.3.

#### 5.4.3.3 Predictions

We consider the non-dimensional force balance before departure.

$$C_{AM,x} K^2 \frac{\text{Ja}_w^2}{\text{Pr}_L} + \frac{1}{3} \frac{\text{Re}_b}{\text{Fr}} + \frac{1}{8} C_D \text{Re}_b = \frac{1}{2} \frac{f_{C,x}}{\text{Ca}} \quad (5.104)$$

Since we only have the capillary term hindering departure as a first approach, we can suppose that departure is reached when:

$$C_{AM,x}K^2 \frac{\text{Ja}_w^2}{\text{Pr}_L} + \frac{1}{3} \frac{\text{Re}_b}{\text{Fr}} + \frac{1}{8} C_D \text{Re}_b > \frac{1}{2} \frac{f_{C,x}}{\text{Ca}} \quad (5.105)$$

which is similar to considering that the other forces overcome the Capillary force.

On Figure 5.15, we show the predictions obtained with the proposed modeling and those obtained with Mazzocco's recent model [22].

The model have an acceptable trend on some experimental sets, but strong overestimation occur on the cases of Sugrue. Moreover, we observe significant underestimations on the data of Ren at 2 bar and Thorncroft.

Mazzocco's model provides a good accuracy on the data of Sugrue, Guan, Maity and Ren (2 bar). However, we observe very large overestimation over Thorncroft's measurements and significant underestimation on Chen, Ren (3 and 5 bar) and Kossolapov measurements.

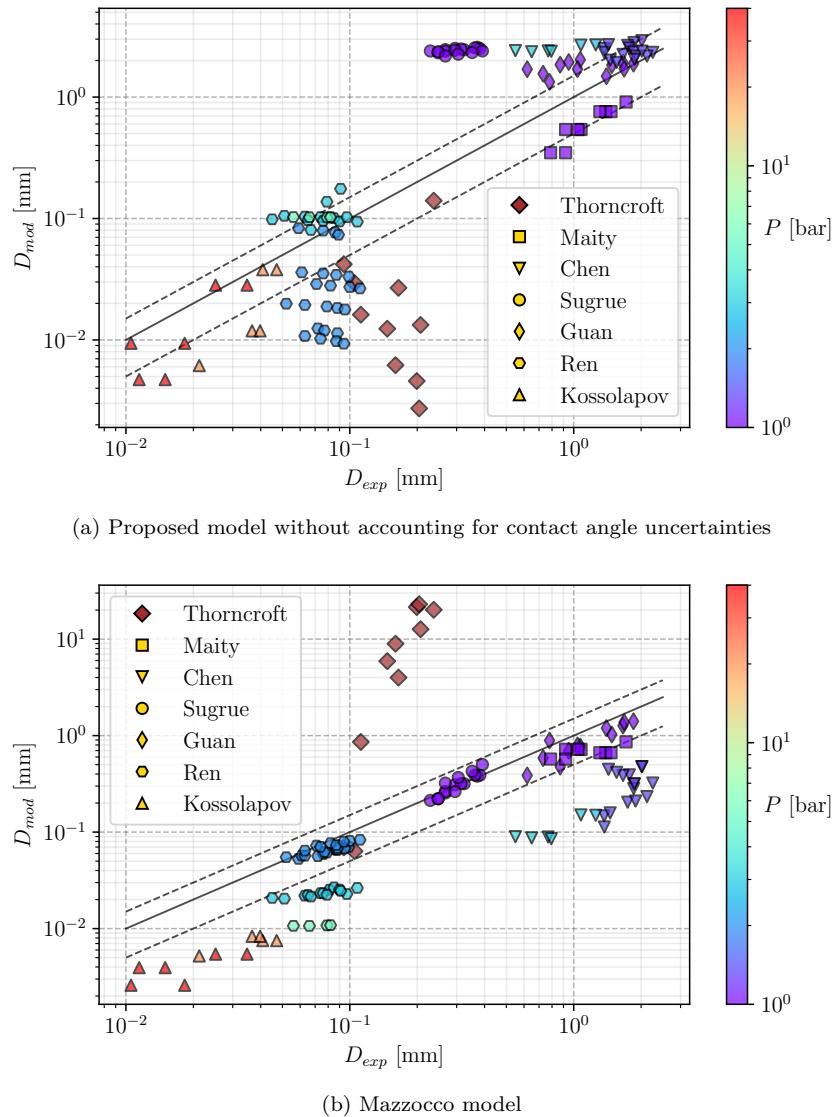


Figure 5.15: Predicted bubble departure diameters

#### 5.4.4 Discussion and accounting for parameters uncertainties

The aforementioned errors observed for the proposed model may originate from various reasons:

- The contact angle proposed for Sugrue cases is high with a large hysteresis, suggesting strongly deformed and flattened bubbles under the truncated sphere hypothesis. Based on images from Sugrue's work [35], a comparison between a real bubble with the assumed shape is presented on Figure 5.16. This shows a huge difference which indicates that the contact angle and hysteresis

values may be overestimated. Using the available images, the ratio of the bubble diameter to the apparent bubble foot would lead to an average contact angle  $\theta \approx 20^\circ$  for a truncated sphere. Noting that a larger inclination is observed for the bubbles under higher mass fluxes leads us to suppose a value  $d\theta \approx 15^\circ$ . This represent a similar inclination to contact angle ratio ( $d\theta/\theta$ ) compared to the initially proposed values. The resulting new shape is also presented on Figure 5.16 and seem to better represent the actual bubble.

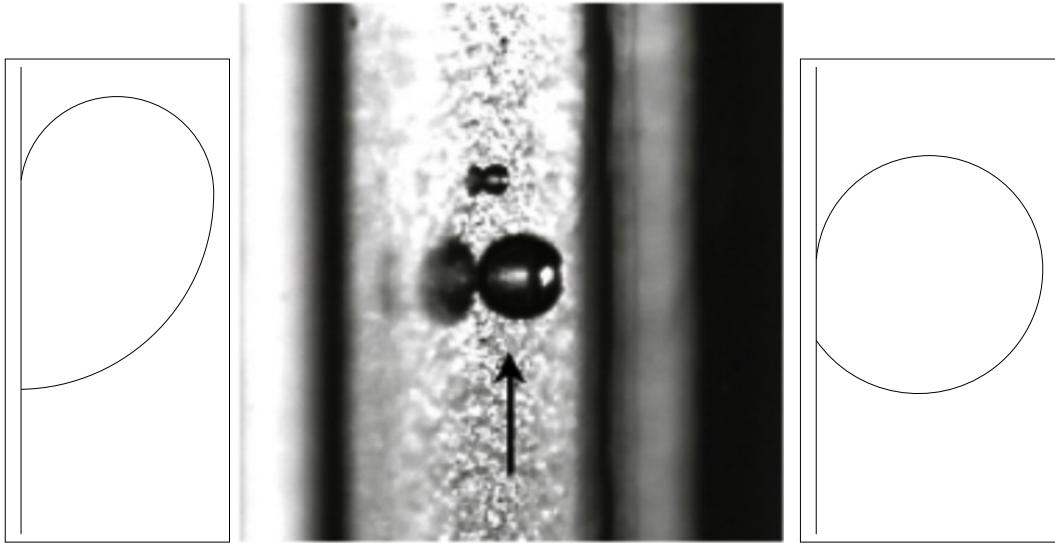


Figure 5.16: Initially assumed, real and reassessed bubble shape for Sugrue cases (picture adapted from [35]).

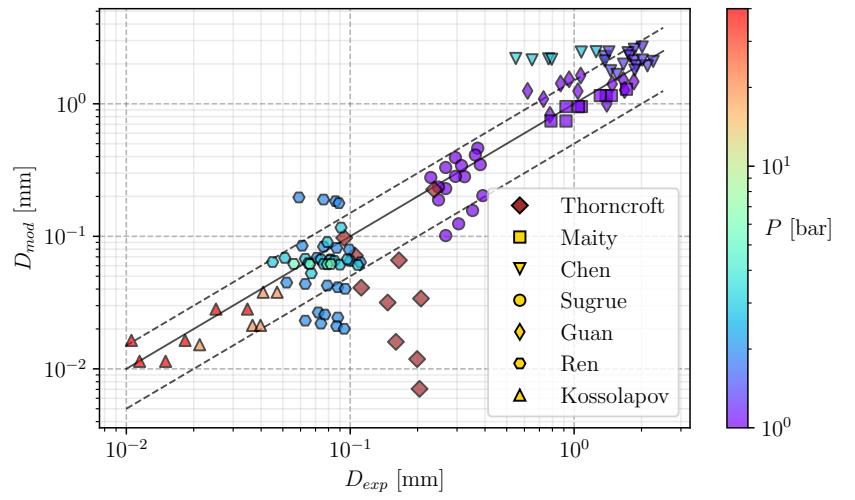
- For cases where limited under and overestimation is observed, we may allow to account for an uncertainty as high as  $5^\circ$  for the average contact angle  $\theta$  and half-hysteresis  $d\theta$ .
- As mentioned earlier, applying the same contact angle and hysteresis over a wide range of measurements is a strong assumption, especially for cases where different pressures and bubble diameter variations are observed. Thus, we may slightly distinguish the applied values of  $\theta$  and  $d\theta$  for different pressures within a given experiment, keeping a change no larger than  $5^\circ$ .
- Kossolapov cases at  $G_L = 500 \text{ kg/m}^2/\text{s}$  are better predicted. Cases under higher mass fluxes (1000 and  $1500 \text{ kg/m}^2/\text{s}$ ) present underestimation that could come from the value of  $d\theta$ . At such mass fluxes, the Weber number can be up to a decade higher and bubbles may thus accept a larger inclination before detachment.
- Cases of Ren and Chen rely on chosen values for  $\theta$  and  $d\theta$  and not on measured ones. They are therefore subject to strong uncertainties. We can note that the values for Chen cases are significantly high.
- The proposed growth law is still rather simple and may miss significant information, especially regarding bubble size and fluid properties such as the Prandtl number.
- Errors on Thorncroft cases may be linked to uncertainties regarding FC-87 properties. Indeed, we use the values given at  $T_{sat} = 29^\circ$  at 1 bar in his work [37]. However, the saturation temperature indicated in his test matrix is close to  $40^\circ$  which means that measurements were conducted at a higher pressure, for which we do not have FC-87 properties.

Therefore, using modified values of  $\theta$  and  $d\theta$  among experimental data sets with no more than a  $5^\circ$  change (except for Sugrue cases reassessed values) leads to predictions on Figure 5.17.

The predictive capacity of the model is significantly enhanced, especially on Sugrue's cases which tends to indicate that the contact angle reassessment was justified under the truncated sphere hypothesis. Table 5.4 summarizes the average errors obtained with the present model and Mazzocco's one.

The proposed model achieves an overall better predictive capability even when excluding measurements from Thorncroft on which Mazzocco's model strongly overestimates the departure diameter. Mazzocco's model is still better on Sugrue and Guan cases since it was built and validated using those measurements. It better predicts results from Ren but only for the 2 bar cases while it underestimates the departure

Author	$\theta$ [°]	$d\theta$ [°]
Thorncroft	20	14
Maity	45	10
Chen	92.5	27.5
Sugrue	20	15
Guan	47.5	17.5
Ren (2 bar)	45.5	8.5
Ren (3 bar)	37.5	3.5
Ren (5 bar)	35.5	3.5
Koss. (500 kg/m²/s)	80	0.5
Koss. (1000 kg/m²/s)	80	1
Koss. (1500 kg/m²/s)	80	1.5



(a) Modified contact angle and hysteresis values.

(b) Predicted bubble departure diameters.

Figure 5.17: Proposed model performance while accounting for contact angle uncertainties

Author	Mazzocco	Present model
Thorncroft	4874%	60.6%
Maity	39.7%	13.8%
Chen	83.8%	69.9%
Sugrue	9.73%	26.22%
Guan	25.5%	36.8%
Ren	40.32%	44.1%
Kossolapov	78.3%	24.1%
Total	442%	43%
Total (without Thorncroft)	46.58%	41.4%

Table 5.4: Average relative error reached by the models.

diameter for higher pressures. Those results are a coupled effect of his optimized growth law along with the imposed value of  $r_w/R$  and the use of the inclination angle to hinder departure as mentioned in 5.2.7. The approach demonstrated the importance and the strong influence of the contact angle and hysteresis. A small change of their value (staying in the uncertainty range of 5°) allowed to reach reasonable predictions over a large range of bubble departure diameters with the model of this paper, using a reduced number of empirical parameters.

## 5.5 SLIDING PHASE

### 5.5.1 Modeling

After departure, bubbles slide over a distance  $l_{sl}$  which scales the impact of the sliding phenomenon over the wall heat transfer. Achieving good prediction of bubble sliding velocity is then important if one wishes to correctly quantify its impact.

Following the force balance framework presented in Section ??, we can write Newton's second law parallel to the wall for the sliding bubble.

$$\rho_V \frac{d(V_b U_b)}{dt} = -\pi R \sigma f_{C,x} + \frac{4}{3} \pi R^3 (\rho_L - \rho_V) g + \frac{1}{2} C_D \rho_L \pi R^2 U_L^2 + \frac{4}{3} \pi R^3 \rho_L \left[ 3C_{AM,x} \frac{\dot{R}}{R} U_{rel} - C_{AM,x} \frac{dU_b}{dt} \right] \quad (5.106)$$

This equation can be re-written to express the bubble acceleration.

$$\left( 1 + \frac{\rho_L}{\rho_V} C_{AM,x} \right) \frac{dU_b}{dt} = \left( \frac{\rho_L}{\rho_V} - 1 \right) g + \frac{3}{8} \frac{C_D}{R} \frac{\rho_L}{\rho_V} (U_L - U_b) |U_L - U_b| + 3 \frac{\dot{R}}{R} \left[ C_{AM,x} \frac{\rho_L}{\rho_V} (U_L - U_b) - U_b \right] - \frac{3}{4} \frac{\sigma}{\rho_V} \frac{f_{C,x}}{R^2} \quad (5.107)$$

Then, we numerically solve this equation from the moment when  $R \geq R_d$  using a first order Euler scheme for a duration close to the experimental sliding time. Next sections compare obtained results against low and high pressure data.

### 5.5.2 Low Pressure Sliding

Maity [21] provided simultaneous measurements of bubble radius and velocity over time for three liquid mass fluxes in vertical boiling. To assess the validity of Eq. 5.107, we modify the growth constant  $K$  in order to roughly match experimental radius measurements. The goal is to verify if the force balance allows a good prediction of bubble velocity provided a correct bubble growth. The contact angles were kept the same as in 5.4.3 since Maity provided average values over the bubble lifetime.

Results are displayed on Figure 5.18. The model seems to fairly good predict bubble sliding velocity for the 3 cases. The moment of departure is a bit underestimated as previously observed (Figure 5.15).

The biggest discrepancy is observed for the case at  $G_L = 143.8 \text{ kg/m}^2/\text{s}$ . The slope of the velocity profile is close to the experiments, but the bubble reaches a nearly constant acceleration too rapidly which yields an approximately constant overestimation of 0.1 m/s.

The  $G_L = 239.6 \text{ kg/m}^2/\text{s}$  is well predicted regarding the velocity. However, the growth profile was difficult to match since measurements exhibit significant changes in growth regime after departure, which is probably due to the bubble being large enough to be impacted by the bulk flow. A finer model for bubble growth could be of interest here.

We can note that values of  $K$  between 0.5 and 1 were used to better fit the bubble radius time profile.

### 5.5.3 High Pressure Sliding

In his work, Kossolapov [15] conducted measurements of radius and sliding length over thousands of individual bubbles and then provided the associated statistical distributions. To compare our model with his measurements, we took the upper and lower bounds of  $R$  and  $l_{sl}$  over time and plotted the associated bands of measured values as shown on Figure 5.19 and 5.20.

Comparisons were done for cases at 20 bar and 40 bar and 3 different values of  $G_L$ . The value of  $d\theta$  for the simulations was kept really small ( $2^\circ$  at 20 bar and  $0.5^\circ$  at 40 bar) since bubble tilt is supposed to reduce during sliding because the relative velocity regarding the liquid flow is diminishing. Moreover, higher pressure means smaller bubbles that are even more unlikely to present a significant contact angle hysteresis. We also want to mention that neglecting the capillary term in Eq. 5.107 had a minor impact over the results except that the bubble accelerates a little bit faster.

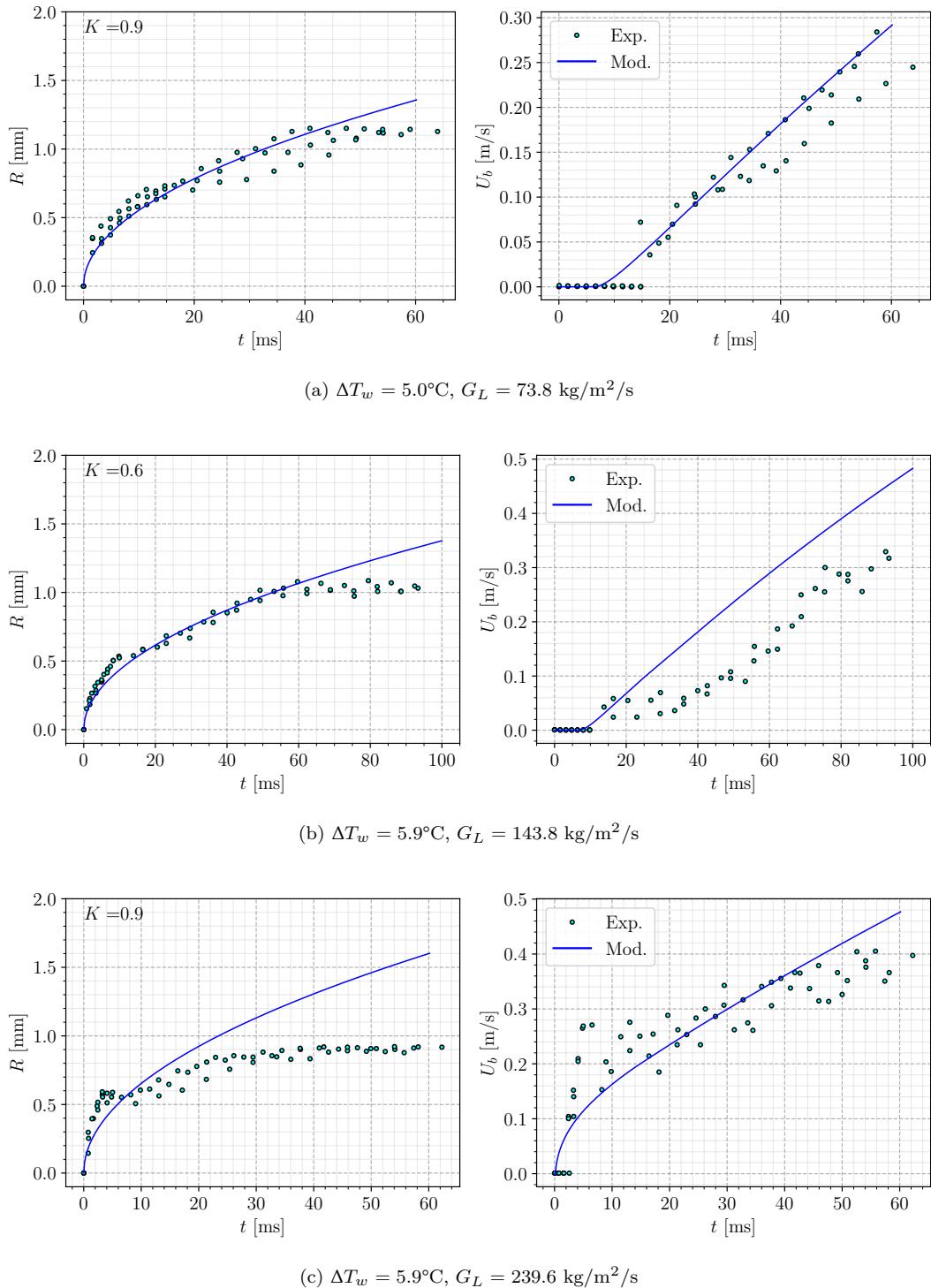
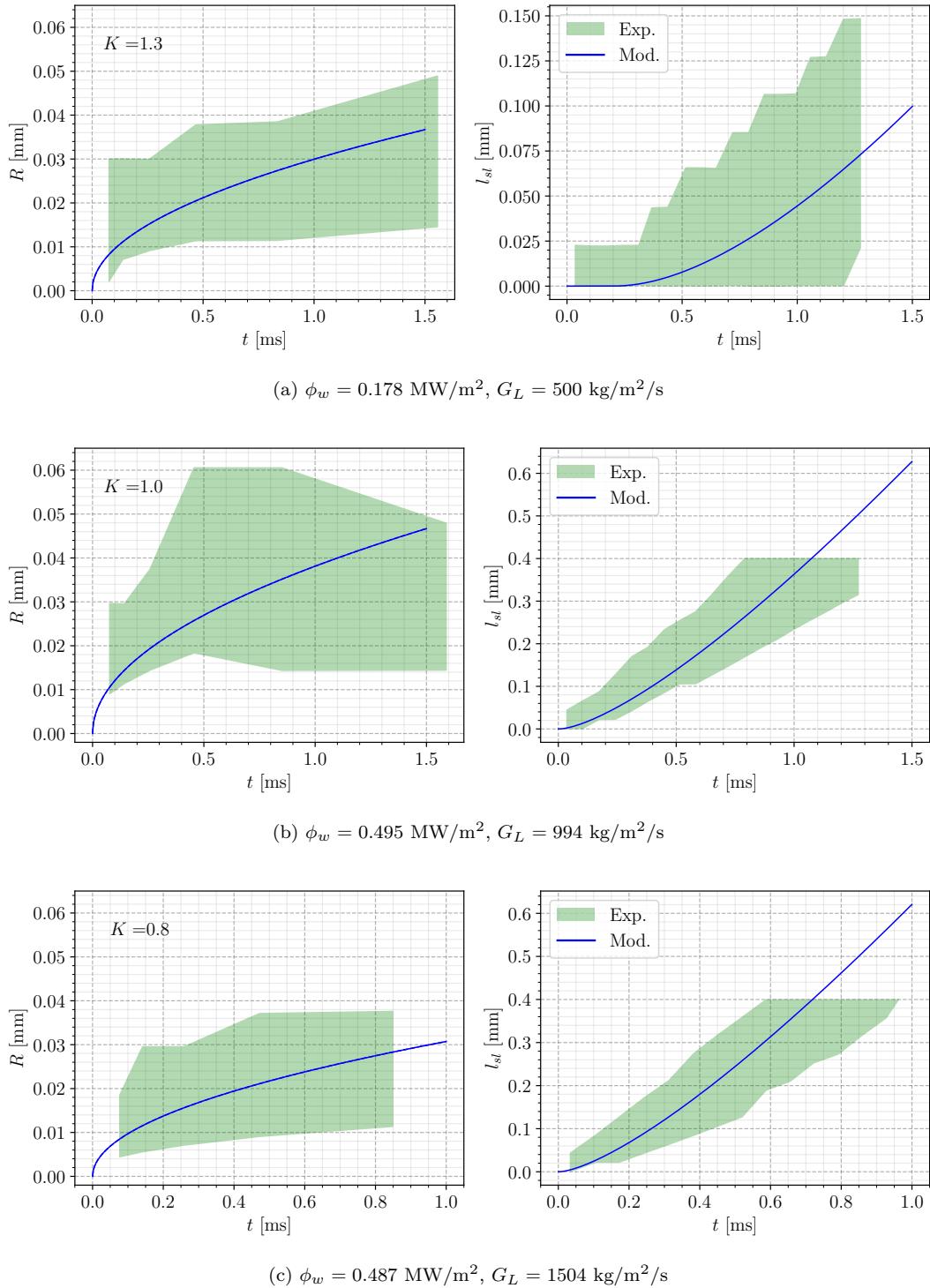


Figure 5.18: Bubble sliding velocity predictions on Maity cases

The obtained results are in good agreement with the sliding length profile vs. time, which means bubble sliding velocity is well predicted for those cases.

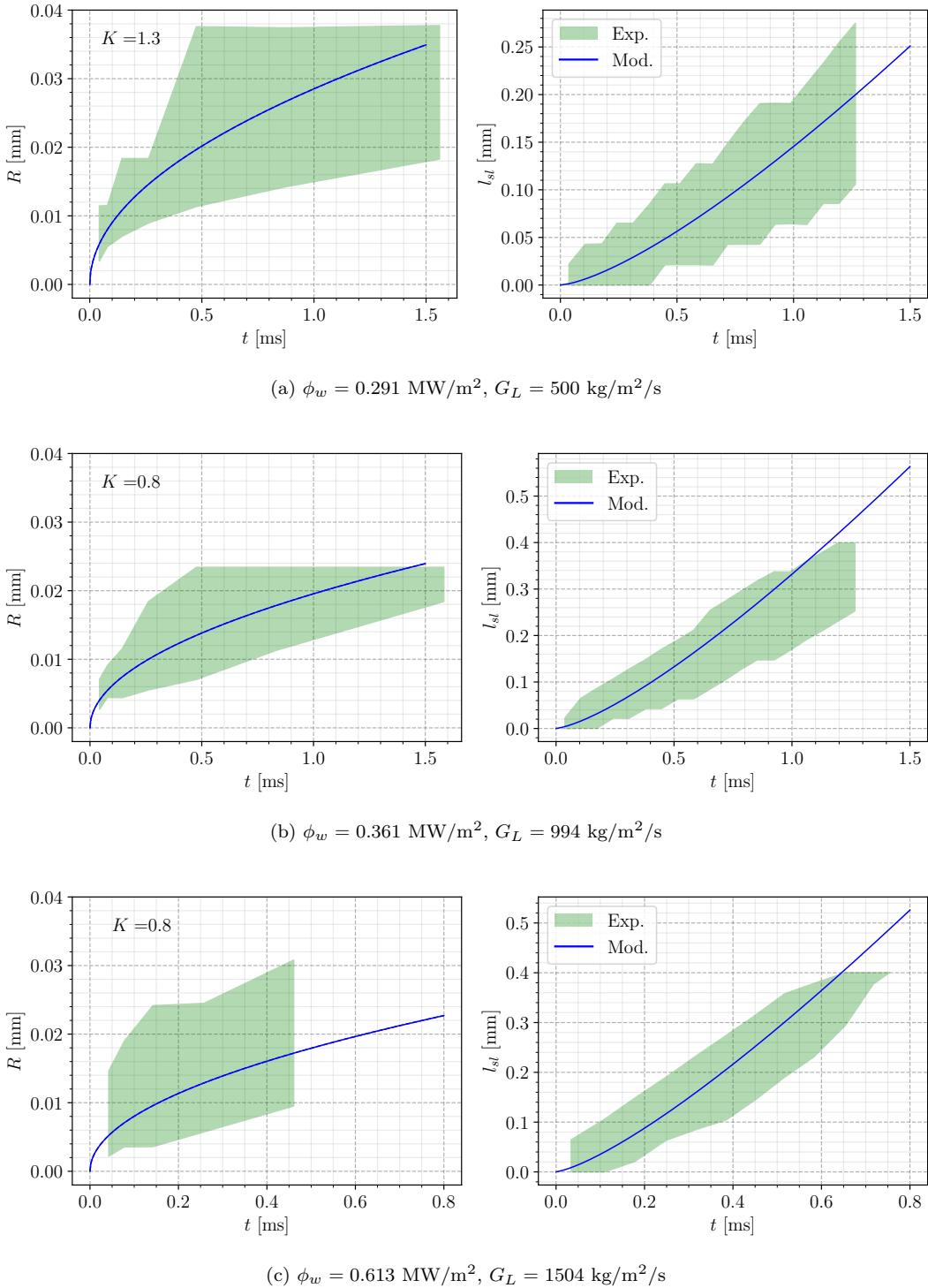
Once the estimation of  $\Delta T_w$  using Eq. 5.102 is corrected as mentioned in 5.4.2, values of  $K$  between 0.8 and 1.3 reasonably fits the bubble radius measurements.

Figure 5.19: Bubble sliding length predictions on Kossolapov cases -  $P = 20 \text{ bar}$ 

## 5.6 CONCLUSION

In this work, we proposed a revisited force balance for a single bubble in a vertical upward boiling flow. This force balance was then used to study bubble departure by sliding and compared with bubble departure diameter and sliding velocity measurements. The main highlights of this study are:

- The use of a recent correlation to compute the Drag coefficient thanks to DNS results of Shi *et al.* [31].
- Reassessed computation of the Added Mass force and associated coefficients from the expression of the liquid kinetic energy proposed by Van Der Geld [10].

Figure 5.20: Bubble sliding length predictions on Kossolapov cases -  $P = 40 \text{ bar}$ 

- A global force balance that avoids including extra empirical parameters. We notably get rid of empirical choices regarding bubble foot radius and do not include an arbitrary bubble inclination angle to create an Added Mass term hindering departure.
- A non dimensional approach leading to force regime maps to qualitatively determine the dynamic regime in which bubbles are departing from the nucleation site. It shows that the detaching Added Mass term due to external liquid flow is rarely negligible and often dominates at low pressure. Increasing pressure mostly leads to Drag dominant regimes.
- Bubble departure diameter predictions are achieved with a reasonable accuracy over a large range of measured values from 7 data sets. This could be reached by accepting an uncertainty of  $5^\circ$  for the

values of the contact angle  $\theta$  and half-hysteresis  $d\theta$  to which the model was sensitive. This indicates that using fixed values for large data sets seems incorrect and that extra empiricism may not be needed if a proper modeling of those parameters was achieved.

- Bubble sliding velocity simulations showed a good agreement with experimental observations both at low and high pressure provided a correct bubble growth profile.

The limitations of the developed approach lie mainly in the simple and empirical law to estimate the growth constant  $K$  (Eq. 5.103). This could be enriched by a clean modeling of the bubble growth including effects such as condensation, microlayer evaporation and impact of the external liquid flow. Existing models rely on empirical values which thus reduces their general applicability outside of their validation range. For instance, studies conducted by Zhou [47] and Yoo [42] could be used by enriching their modeling with finer results to get rid of data fitting. DNS results such as those regarding bubble growth in a flowing superheated or subcooled liquid by Legendre *et al.* [17] could be of interest in that prospect.

Finally, the precise estimation of the contact angle and hysteresis remains a critical parameter to predict departure by sliding as demonstrated throughout this study. Local measurements of those values and their evolution with operating conditions would be a very valuable information in that regard. The article of Song & Fan [33] that sums up existing modeling and experimental measurements provides a good overview of the problem and identifies the associated challenges that are still to be tackled.

## 5.7 BUBBLE LIFT-OFF

# 6

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## CLOSURE LAWS FOR THE HEAT FLUX PARTITIONING MODEL

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### 6.1 SINGLE-PHASE HEAT TRANSFER COEFFICIENT

The choice of a proper correlation to compute the single-phase heat transfer coefficient is a first but inevitable step to build a HFP model. Indeed, if the single-phase convection term is badly computed, the resulting boiling model will fail to predict the wall temperature.

For instance, if the liquid convective HTC is overestimated, it would result in a delayed increase of the boiling and quenching heat fluxes which would in turn lead to an overprediction of the wall temperature. To assess existing correlations for the single-phase HTC, we will use wall temperature measurements extracted from experimental boiling curves for water where  $T_w \leq T_{sat}$ . They correspond to the single-phase part of the experimental data later used to assess the HFP model.

The chosen data are presented on Table 6.1.

Author	$D_h$ [mm]	$P$ [bar]	$G_L$ [kg/m <sup>2</sup> /s]	$\Delta T_L$ [K]	$\phi_w$ [MW/m <sup>2</sup> ]	$T_{sat} - T_w$ [K]	$N_{mes}$ [-]
Kossolapov [15] (2021)	12	10.5	500 - 2000	10	0.1 - 0.6	0.22 - 9.5	12
Richenderfer [richenderfer_phd] (2018)	15	1 - 5	1000 - 2000	10-20	0.1 - 0.63	1 - 18.7	13
Jens-Lottes [jens_lottes_data] (1951)	5.74	137.9	2617.5	53.3 - 92.2	0.91 - 2.37	0.33 - 44.1	15
Kennel [kennel_phd] (1948)	4.3 - 13.2	2 - 6.2	284 - 10 577	11.1 - 83.3	0.035 - 1.89	0.35 - 69	52

Table 6.1: Experimental data range of wall temperature measurements from the single-phase part of boiling curves.

On Figure 6.1, we compare the results of wall temperature prediction in the single-phase region obtained with the correlation of Dittus-Boelter (Eq. ??) and Gnielinski (Eq. ??).

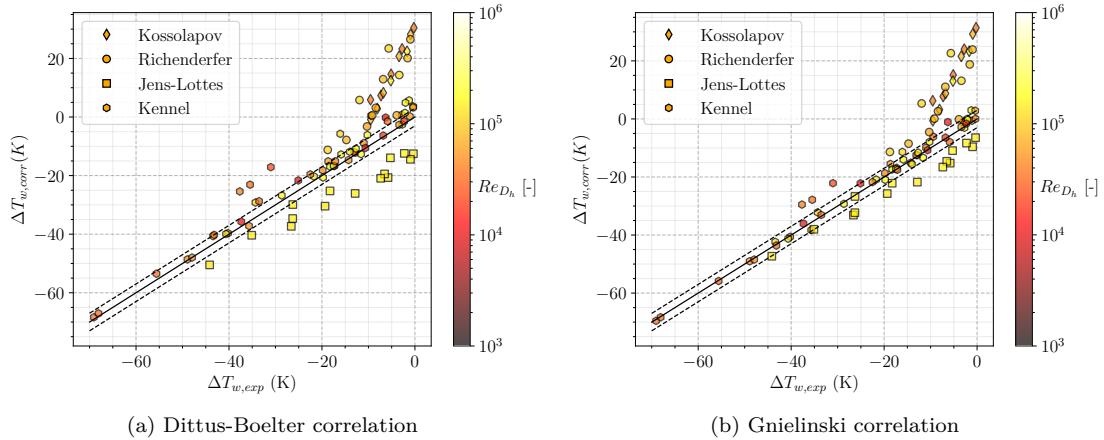


Figure 6.1: Predictive capability of wall temperature by single-phase heat transfer correlations.  $\pm 3K$  error bars indicated.

The two correlations are of similar efficiency regarding wall temperature predictions over the considered data sets. They both have very good agreement with Kennel data and clear overestimations of  $\Delta T_w$  on

Richenderfer and Kossolapov measurements. The slope difference compared to the parity implies that the correlations are predicting too small Nusselt numbers for those cases.

Regarding Jens-Lottes data, both model underestimate the wall temperatures, with better results achieved by Gnielinski correlation.

**Remark :** We tested different friction factor along with different values of wall roughness in the Gnielinski correlation and observed a negligible impact on the overall results. This allows to stay with a simple formulation for the friction coefficient.

The error obtained on Richenderfer and Kossolapov data can be explained by the definition of the HTC computed by Gnielinski correlation. Indeed, Gnielinski correlated a Nusselt number associated to a forced convection coefficient  $h_{fc,Gniel}$  in the case of a internal flow with a completely heated wall.

However, only one side of the channel is heated in Richenderfer ans Kossolapov experiments. If  $S_{heat}$  denotes this actual heated surface, then Gnielinski correlation estimates the HTC for a surface  $4S_{heat}$ . With the same imposed total heat power  $\Phi_w$  and bulk liquid temperature  $T_L$ , we have:

$$h_{fc,Gniel} = \frac{\Phi_w}{(T_{w,Gniel} - T_L) 4S_{heat}} \quad (6.1)$$

$$h_{fc,exp} = \frac{\Phi_w}{(T_{w,exp} - T_L) S_{heat}} \quad (6.2)$$

Writing  $T_{w,Gniel} = T_{w,real}$  then yields:

$$h_{fc,exp} = 4h_{fc,Gniel} \quad (6.3)$$

On Figure 6.2 we display the predictions of Gnielisnki correlation including this correction by a factor 4 on the HTC for Richenderfer and Kossolapov cases. We also test a 10% reduction on the HTC for Jens-Lottes cases to assess the error made by Gnielinski correlation.

On the same Figure, we also present predictions achieved with the local HTC estimation implemented in NCFD (Eq. ??), using a value of  $y^+ = 100$ .

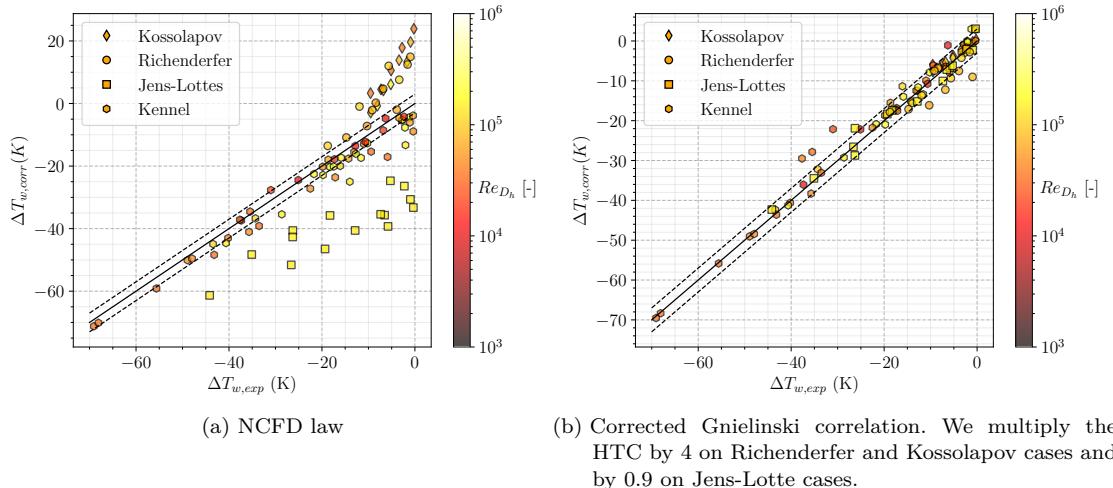


Figure 6.2: Predictive capability of wall temperature by NCFD law and Gnielinski correlation including corrections.  $\pm 3K$  error bars indicated.

The NCFD approach yields predictions similar to the 1D correlations (Figure 6.1) with larger underestimations on Jens-Lottes measurements. On the other hand, we see that applying a constant correction to the Gnielinski correlation (4 for Kossolapov and Richenderfer cases, 0.9 for Jens-Lottes cases) suffices to yield accurate predictions on the whole range of wall temperature measurements.

**Remark :** The NCFD law was tested without running CFD simulations. The equations were re-written in python to allow its testing outside of the whole code. The use of  $y^+ = 100$  as well as the Mac Adams correlation (Eq. ??) for the friction velocity  $U_\tau$  may induce a difference with the predictions that could be achieved by running a complete CFD computation of the considered cases.

The average errors obtained with each model are summed up on Table 6.2

Model	Kossolapov err. [K]	Richenderfer err. [K]	Jens-Lottes err. [K]	Kennel err. [K]
Dittus-Boelter	19.67	15.07	10.09	3.13
Gnielinski	20.31	14.06	6.09	1.74
NCFD law	15.52	9.25	23.69	3.36
Corrected Gnielinski	1.34	3.08	1.57	1.74

Table 6.2: Average errors achieved by the considered models on each data sets.

**Recalling that Gnielinski correlation was also providing good results on the DEBORA cases with R12 (Chapter ??), this further indicates it as a proper choice regarding single-phase HTC estimation in the HFP model.** However, we will later allow the use of the correction factors when needed to ensure a proper representation of the single-phase part when trying to assess the models associated to boiling.

## 6.2 SINGLE BUBBLE QUENCHING AREA

When computing the quenching heat flux, we need to provide the total wall area visited by a single bubble  $A_{q,1b}$  that will undergo quenching.

In wall boiling model that do not consider bubble sliding [**kurul\_podowski**, **ncfd\_hfp**, **shaver\_podowski**] the impacted area at bubble lift-off is often considered as :

$$A_{q,1b} = F_A \pi R_{lo}^2 \quad (6.4)$$

with  $F_A$  being an enhancement factor that accounts for the possibility that the bubble will induce quenching over a surface larger than its projected area.

$$A_{q,1b} = \begin{cases} \pi R_{lo}^2 & \text{if } l_{sl} \leq R_{lo} - R_d \\ \frac{1}{2} \pi R_d^2 + l_{sl} (R_d + R_{lo}) + \frac{1}{2} \pi R_{lo}^2 & \text{if } l_{sl} \geq R_{lo} + R_d \end{cases} \quad (6.5)$$

Which can be re-expressed by defining  $l_{sl}^* = \frac{l_{sl}}{R_{lo}}$  and  $A_{q,1b}^* = \frac{A_{q,1b}}{\pi R_{lo}^2}$

$$A_{q,1b}^* = \begin{cases} 1 & \text{if } l_{sl}^* \leq 1 - \frac{R_d}{R_{lo}} \\ \frac{1}{2} \left( 1 + \left( \frac{R_d}{R_{lo}} \right)^2 \right) + \frac{l_{sl}^*}{\pi} \left( 1 + \frac{R_d}{R_{lo}} \right) & \text{if } l_{sl}^* \geq 1 + \frac{R_d}{R_{lo}} \end{cases} \quad (6.6)$$

and we linearly interpolate those two expressions for the region where  $1 - \frac{R_d}{R_{lo}} \geq l_{sl}^* \geq 1 + \frac{R_d}{R_{lo}}$ .

## 6.3 GROWTH TIME

In this case, the average radius of the bubble over a growth time  $t_g$  (as needed in Subsection ??) is :

$$\overline{R(t)} = \frac{1}{t_g} \int_0^{t_g} R(t) dt = \frac{R_\infty}{t_g} \left[ \frac{e^{-2K_a \sqrt{t}} (2K_a \sqrt{t} + 1)}{2K_a^2} + t \right]_0^{t_g} \quad (6.7)$$

$$= \frac{R_\infty}{2K_a^2 t_g} \left( e^{-2K_a \sqrt{t_g}} (2K_a \sqrt{t_g} + 1) + 2K_a^2 t_g - 1 \right) \quad (6.8)$$

Moreover, if we consider a lift-off radius  $R_d$ , we can express the associated growth time  $t_g$  :

$$t_g = \left[ \frac{1}{K_a} \ln \left( \frac{1}{\sqrt{1 - \frac{R_d}{R_\infty}}} \right) \right]^2 = \left[ \frac{1}{K_a} \ln \left( \sqrt{1 - \frac{R_d}{R_\infty}} \right) \right]^2 \quad (6.9)$$

This expression could then be used as a closure relationship for  $t_g$  in the HFP model, meaning that this growth time will depend on the departure diameter closure for  $R_d$ .

### 6.3.1 Estimation of the thermal boundary layer thickness $\delta$

In order to fully close the modeling of the bubble growth, we have to compute the thermal boundary layer thickness  $\delta$ .

To do so, we test a first approach based on the hydrodynamic boundary layer profile. In the turbulent layer, the non-dimensional liquids velocity  $u_l^+$  is :

$$u_l^+ = \frac{1}{\kappa} \ln(y^+) + B, \text{ with } \kappa = 0.41 \text{ and } B = 5.25 \quad (6.10)$$

Since  $u_l^+(y^+) = u_l(y)/u_\tau$  and  $y^+ = yu_\tau/\nu_l$ , we can compute the height of the hydrodynamic boundary layer  $\delta_h$  where  $u_l(\delta_h) = 0.99u_{l,bulk}$  as :

$$\delta_h = e^{\kappa(0.99u_{l,bulk}\sqrt{\rho_l/\tau_w} - B)} \times \nu_l \sqrt{\frac{\rho_l}{\tau_w}} \quad (6.11)$$

where  $\tau_w$  is computed using the Mac Adams correlation (Eq. ??).

Finally, to get an approximation of the thermal boundary layer thickness, we consider a simple first approach as :

$$\delta = \frac{1}{Pr_l} \delta_h \quad (6.12)$$

Since the Prandtl number is the ratio of momentum diffusivity to thermal diffusivity, its inverse shall approximately scale the ratio between the thermal boundary layer and hydrodynamic boundary layer thickness.

### 6.3.2 Estimation of $t_g$ against experimental data

In order to assess the proposed approach to compute  $t_g$ , we compare the yielded results with data taken from Unal of maximum bubble growth.

Only 6 measurements of maximum growth time are available (**Je vais en ajouter d'autres par la suite**) but they are associated with the bubble lift-off diameter. Since those measurements were conducted with boiling water on stainless steel, we set the contact angle at  $\alpha = 80^\circ$ .

We also compare the predictions with the model proposed by Kommajosyula :

$$t_g = \left( \frac{D_d}{4K} \right)^2 \quad (6.13)$$

$$K = \frac{\sqrt{\eta_l} Ja_w}{0.804\sqrt{Pr_l}} + \chi 1.95 Ja_w \eta_l, \text{ with } \chi = A - B\zeta \quad (6.14)$$

where  $\zeta = Ja_l/Ja_w$  and both  $A = 1.55$  and  $B = 0.05$  have been fitted over the data of Klausner by Mazzocco *et al.*.

The results are displayed on Figure 6.3.

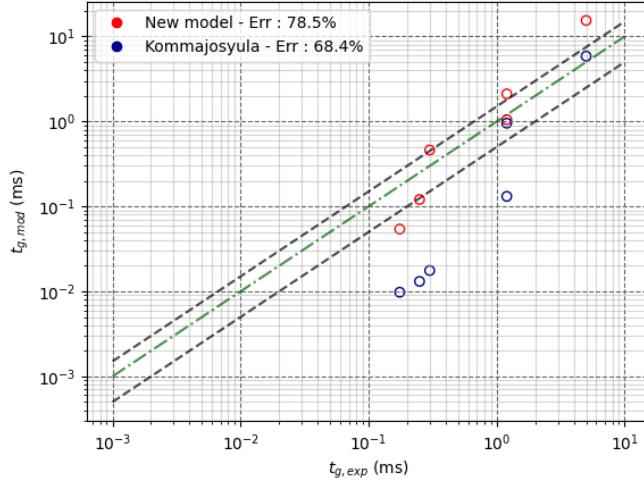


Figure 6.3: Comparison of the proposed model and Kommajosyula model against experimental data from Unal. Black lines represent the  $\pm 50\%$  error bars.

As we can see, the average error from our model is greater than Kommajosyula's one. However, this is mainly due to the highest measured growth time (upper right point). Without this measurement, the average error reaches 52.5%, indicating that a much greater number of experimental measurements should be needed properly evaluate each model.

On the other hand, we may consider that the proposed approach could be of interest since it is not based on data-fitted parameters while providing reasonable results.

Author	Fluid	$D_h$ [mm]	$P$ [bar]	$G_L$ [kg/m <sup>2</sup> /s]	$\Delta T_L$ [K]	$\phi_w$ [kW/m <sup>2</sup> ]	$\Delta T_w$ [K]	$D_d$ [mm] ( $N_{mes}$ )
Maity [21] (2000)	Water	20	1.01	0 - 239.6	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)
Kossolapov [15] (2021)	Water	20	20 - 40	500 - 1500	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)

Table 6.3: Bubble growth time data in vertical flow boiling

#### 6.4 BUBBLE WAIT TIME

The wait time between two nucleation events on an active site corresponds to the time needed for the thermal boundary layer to reconstruct after its disruption due do bubble departure from the nucleation site.

Author	Fluid	$D_h$ [mm]	$P$ [bar]	$G_L$ [kg/m <sup>2</sup> /s]	$\Delta T_L$ [K]	$\phi_w$ [kW/m <sup>2</sup> ]	$\Delta T_w$ [K]	$D_d$ [mm] ( $N_{mes}$ )
Basu <i>et al.</i> [basu_heat] (2005)	Water	20	1.01	0 - 239.6	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)
Richenderfer <i>et al.</i> [richenderfer_investigations_2018] (2018)	Water	20	1.01	0 - 239.6	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)
Kossolapov <i>et al.</i> [15] (2021)	Water	20	1.01	0 - 239.6	0.3 - 0.7	N.A.	5 - 5.9	0.788 - 1.71 (9)

Table 6.4: Bubble wait time data in vertical flow boiling

Formulation of Basu *et al.* [**basu\_heat\_2005**]:

$$t_w = 139.1 \Delta T_w^{-4.1} \quad (6.15)$$

Formulation of Kommajosyula [**Komma**]:

$$t_w = 0.061 \frac{\text{Ja}_L^{0.63}}{\Delta T_w} \quad (6.16)$$

**Remark :** Kommajosyula's correlation yields a zero wait time when bulk liquid approaches saturation. This behavior is questionable

Mikic & Rosenhow analytical expression [**mikic**]:

$$t_w = \frac{1}{\pi \eta_L} \left[ \frac{(\Delta T_w + \Delta T_L) R_c}{\Delta T_w - T_{sat} \left( \frac{1}{\rho_V} - \frac{1}{\rho_L} \right) \frac{2\sigma}{h_{LV} R_c}} \right]^2 \quad (6.17)$$

A relevant

## 6.5 NUCLEATION SITE DENSITY

The Nucleation Site Density is among the most influencing parameters over the HFP models predictions, particularly regarding wall temperature. Indeed, its value directly controls the density of bubbles generated at the heater and therefore impacts both the boiling ( $\phi_e$ ) and quenching ( $\phi_q$ ) heat fluxes to the first order. Being able to come up with correct predictions of the NSD is thus critical if one wishes to properly capture the thermal behavior of the boiling surface.

However, the value of  $N_{sit}$  is actually influenced by many parameters being either linked to thermal-hydraulics (wall temperature, pressure, operating fluid) or the heater material (roughness, wettability). That is why its value is often estimated through empirical correlations, for which many different expression have been proposed over the years since the end of the XX<sup>th</sup> century.

One of the firstly identified behavior of the NSD was its power dependency with the wall superheat ( $N_{sit} \propto \Delta T_w^m$ ), which is form adopted in the correlation of Lemmert & Chawla [**Lemmert**] :

$$N_{sit} = [210 (\Delta T_w - T_{sat})]^{1.8} \quad (6.18)$$

**Remark :** This law is used in the HFP model of Kurul & Podowski and NEPTUNE\_CFD to compute  $N_{sit}$ .

However, such an expression misses the influence of other parameters such as pressure, which has been proven to be strongly impacting the range of active cavities that can generate bubbles as shown on Figure 6.4 and induces a larger bubble density over the heater.

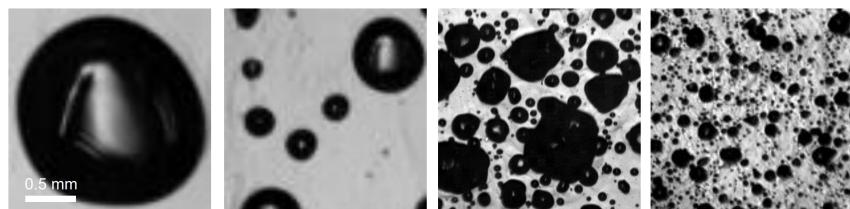


Figure 6.4: HSV Visualization of bubble density at various pressures adapted from Kossolapov [15] (left to right: 1.01 bar, 3 bar, 19.8 bar, 75.8 bar).

Moreover, experimental measurements such as in Borishanskii [**borishanskii**] showed that the power dependency on the wall superheat changes by increasing both with pressure and the superheat value itself.

This was accounted for by Hibiki & Ishii in 2003 [**hibiki\_ishii**] who came up with a new correlation that requires an estimation of the minimum activated cavity radius  $R_c$  :

$$N_{sit} = N_0 \left( 1 - \exp \left( -\frac{\theta^2}{8\mu^2} \right) \right) \left[ \exp \left( f(\rho^+) \frac{\lambda'}{R_c} \right) - 1 \right] \quad (6.19)$$

$$R_c = \frac{2\sigma \left( 1 + \frac{\rho_V}{\rho_L} \right) / P}{\exp \left( \frac{h_{LV}\Delta T_w}{R_g T_w T_{sat}} \right) - 1} \quad (6.20)$$

$$f(\rho^+) = -0.01064 + 0.48246\rho^+ - 0.22712\rho^{+2} + 0.05468\rho^{+3} \quad (6.21)$$

with  $R_g$  the perfect gas constant times the molar mass of the fluid,  $N_0 = 4.72 \times 10^5 \text{ m}^{-2}$ ,  $\mu = 0.722 \text{ rad}$ ,  $\lambda' = 2.5 \times 10^{-3} \text{ m}$  and  $\rho^+ = \log_{10} \left( \frac{\rho_L - \rho_V}{\rho_V} \right)$ .

**Remark :** This law is used in the HFP model of Gilman & Baglietto [**gilman\_baglietto**].

We can note that it also includes the value of the static contact angle  $\theta$  which can be used as a parameter to accounts for wall properties, since it is dependent on the wall roughness, wettability and the operating fluid.

Indeed, a high-wetting material (low values of  $\theta$ ) will allow smaller cavities to be flooded by the surrounding liquid, thus hindering non-condensable gases to be captured inside and become a potentially active nucleation site (Figure 6.5).

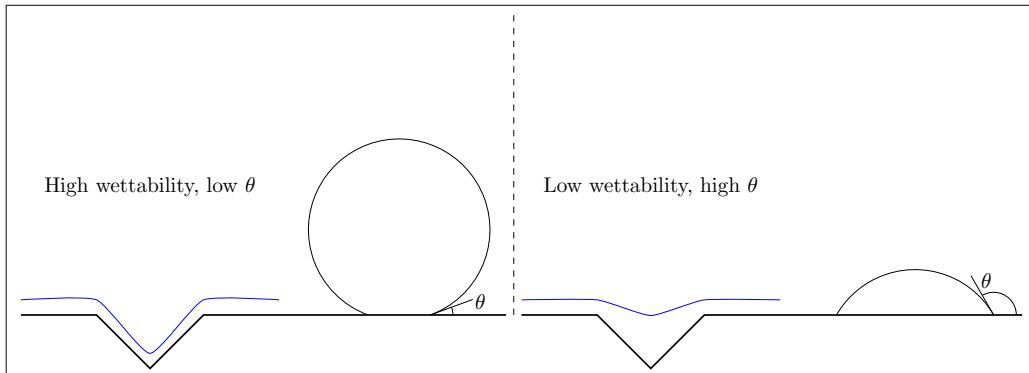


Figure 6.5: Sketch of the link between bubble contact angle and wettability / cavity flooding

This influence of the contact angle on the NSD was confirmed by experimental observations of Basu *et al.* [**basu\_nsit**] and was also included in a law correlated on their own measurements :

$$N_{sit} = \begin{cases} 0.34 [1 - \cos(\theta)] \Delta T_w^2 & \text{if } \Delta T_{w,ONB} < \Delta T_w < 15 \text{ K} \\ 3.4 \times 10^{-5} [1 - \cos(\theta)] \Delta T_w^{5.3} & \text{if } \Delta T_w > 15 \text{ K} \end{cases} \quad (6.22)$$

Similarly, Zhou *et al.* [**zhou\_nsd**] correlated their measurements, including an influence of the pressure:

$$N_{sit} = N_0 (1 - \cos(\theta)) [\exp(f(P) \Delta T_w) - 1] \quad (6.23)$$

$$f(P) = 0.218 \ln \left( \frac{P}{P_0} \right) + 0.1907 \quad (6.24)$$

with  $N_0 = 55\,395.26 \text{ m}^{-2}$  and  $P_0 = 1.01 \text{ bar}$ .

Finally, one of the most recent NSD correlation has been proposed by Li *et al.* in 2018 [**li\_new\_2018**] and validated over a large range of measurements by including a more realistic power law for  $\Delta T_w$ . It avoids the divergence of  $N_{sit}$  observed in Hibiki & Ishii law (Eq. 6.19) when reaching high pressure and

superheats. It also includes the impact of pressure and contact angle and its evolution with temperature *e.g.* its decrease close to 0 ° when approaching the critical temperature [song\_fan\_contact\_angle]:

$$N_{sit} = N_0 e^{f(P)} \Delta T_w^{A\Delta T_w + B} (1 - \cos(\theta)) \quad (6.25)$$

$$f(P) = 26.006 - 3.678e^{-2P} - 21.907e^{-P/24.0.65} \quad (6.26)$$

$$A = -2 \times 10^{-4} P^2 + 0.0108P + 0.0119 \quad (6.27)$$

$$B = 0.122P + 1.988 \quad (6.28)$$

$$1 - \cos(\theta) = (1 - \cos(\theta_0)) \left( \frac{T_c - T_{sat}}{T_c - T_0} \right)^\gamma \quad (6.29)$$

with  $P$  in MPa,  $\theta_0$  the contact angle at room temperature  $T_0$ , and default value being for water  $\theta_0 = 41.37^\circ$ ,  $T_c = 374^\circ\text{C}$   $T_0 = 25^\circ\text{C}$ ,  $\gamma = 0.719$ .

**Remark :** We can question the absence of bulk liquid velocity and temperature in the presented law since they should logically influence the nucleation process. However, this impact is rather limited as observed in experimental measurements of Zhou *et al.* and Kossolapov.

In order to assess existing NSD correlations and choose the most pertinent to include in a HFP model, we gather NSD measurements from 4 different authors. The different operating conditions of the chosen data sets are gathered on Table 6.5.

Author	Fluid	$P$ [bar]	$G_L$ [kg/m <sup>2</sup> /s]	$\Delta T_L$ [K]	$\Delta T_w$ [K]	$\theta_0$ [°]	$N_{mes}$ [-]
Zhou [zhou_nsit] (2020)	Water	1.21 - 3.12	482.7 - 1930.6	8 - 15	6.7 - 20.2	51	60
Richenderfer [richenderfer] (2018)	Water	1.01	500 - 1000	10	21.7 - 42.8	80	49
Kossolapov [15] (2021)	Water	1.01 - 75.8	500 - 2000	80	10	80°	63
Borishanskii [borishanskii] (1966)	Water	1.01 - 198	N.A.	N.A.	1.75 - 17.3	45	132

Table 6.5: Nucleation Site Density data in flow boiling

We then compare the predictions achieved by the model of Lemmert & Chawla (Eq. 6.18), Hibiki & Ishii (Eq. 6.19), Zhou *et al.* (Eq. 6.23) and Li *et al.* (Eq. 6.25). The comparison with measurements are presented on Figure 6.6.

The Lemmert & Chawla model appears to fail in predicting the NSD at high pressures. This is a logical drawback of its sole dependence on the wall superheat. More importantly it increasingly underestimates the NSD as pressure increases, which makes it a clearly unsuitable correlation to compute  $N_{sit}$  particularly for pressurized flows such as in PWR.

Altough the model of Zhou *et al.* includes a pressure term, its partial calibration on data covering a low range of pressure may explain the large error observed when compared to higher pressure measurements. On the contrary, models from Hibiki & Ishii and Li *et al.* seem to better reproduce the different trends with flow conditions, especially with pressure. The model from Li *et al.* achieves better predictions by avoiding to reach unphysically high values of  $N_{sit}$  at higher wall superheat compared to Hibiki & Ishii. This behavior is clear over Kossolapov data at high pressure, where both model lead to overestimations, the strongest discrepancy being associated to Hibiki & Ishii model.

Overall, the model of Li *et al.* is the most efficient with an acceptable agreement on most of the data of Borishanskii and Zhou *et al.*. The measurements of Richenderfer and Kossolapov fail to be precisely reproduced, but it shows a coherent trend and the most limited error when compared to other correlations.

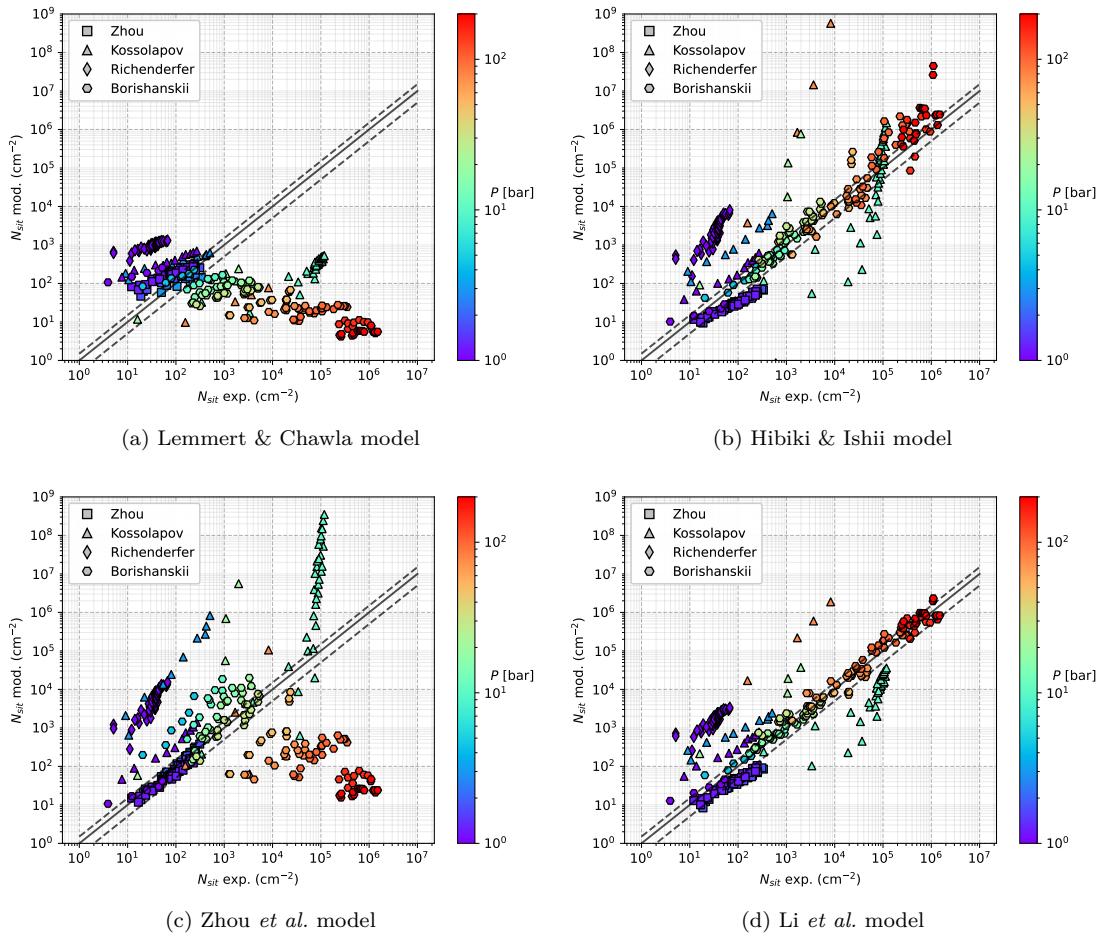


Figure 6.6: Predictions of the chosen models against the experimental data of Table 6.5 with  $\pm 50\%$  error bars.  
The contact angles

**Remark :** The coherency of NSD predictions is hard to ensure since we do not know the exact contact angle and boiling surface morphology in the experiments. This was pointed out by Richenderfer [[Richen\\_phd](#)] who observed significant variation in the NSD value depending on the heater, though keeping the same material (ITO). For instance, this may explain the fact that the NSD measured by Kossolapov at 10.5 bar is higher than any other pressure on his experiment, leading to both underpredictions and overpredictions of the model of Li *et al.* depending on the pressure.

All things considered, those comparisons show that the Nucleation Site Density remains among the most difficult quantity to evaluate because of its very large variations over experiments, boiling surfaces and flow conditions. Dedicated correlations are hardly precise outside of their establishment databases. However, it remains the best yet only way to compute  $N_{sit}$ . In that regard, the NSD correlation of Li *et al.* appears to be the most coherent choice.

## 6.6 CONSIDERATIONS ON BUBBLE INTERACTION AND NUCLEATION SITES DEACTIVATION

NSD correlation actually compute the total number of sites where bubbles can nucleate on a surface. However, they do not represent how important a nucleation site will be in term of bubbles generation compared to another.

In fact, Kossolapov has observed that each nucleation site has its own bubble nucleation frequency. Thus implying that some sites play a much greater role in wall nucleation compared to others. One can even consider that some nucleation sites can be neglected regarding their small impact on the whole phase change process.

In order to physically take into account this effect, Gilman considered a statistical approach, by assuming that nucleation site are randomly distributed over the heater surface (Complete Spatial Randomness). Then, considering that a nucleation site located under a bubble will be deactivated, one can express the number of sites actually contributing to bubble nucleation  $N_{sit,a}$  as :

$$N_{sit,a} = (1 - \mathcal{P}) N_{sit}, \text{ with } \mathcal{P} = 1 - e^{-N_b \pi (D_b/2)^2} \quad (6.30)$$

where  $N_b = t_g f N_{sit}$  is the actual average density of bubbles on the heater surface.

The resulting number of sites is a solution of the implicit equation on  $N_{sit}$ , which can be solved numerically or using the Lambert function (reciprocal of  $w \rightarrow we^{-w}$ ).

*Je vais détailler un peu plus cela avec des schémas clairs, ainsi qu'un exemple de comment cela pondère la densité de site totale, puisque cela permet d'éviter la divergence dans une loi telle que celle de Hibiki & Ishii à haute surchauffe pariétale*

### 6.6.1 Static suppression

If we suppose that the nucleation sites follow a homogeneous spatial Poisson process with an event density  $\lambda$ , we can express the probability of finding  $n$  sites within an area  $A$  as :

$$\mathcal{P}(N(A) = n) = \frac{(\lambda A)^n}{n!} e^{-\lambda A} \quad (6.31)$$

If we consider the actual number of bubble-generating sites  $N_b$ , those sites are holding a bubble over a fraction  $t_g f$  of the nucleation cycles in average. Thus, the actual density of bubbles held by the sites is :  $t_g f N_b$ . To derive  $N_b$  from the value  $N_{sit}$  provided by NSD correlations, we have to evaluate the probability of nucleation site overlapping, which corresponds to a distance  $r$  lower than  $R_d$  between two bubbles.

$$\mathcal{P}(r \leq R_d) = 1 - \mathcal{P}(N(\pi R_d^2) = 0) \quad (6.32)$$

$$= 1 - e^{-N_b t_g f \pi R_d^2} = \mathcal{P} \quad (6.33)$$

This probability thus represents the proportion of bubble that can't be geometrically accommodated on the surface. We can then evaluate  $N_b$  from  $N_{sit}$  as :

$$N_b = (1 - \mathcal{P}) N_{sit} \quad (6.34)$$

$$\Leftrightarrow N_b t_g f \pi R_d^2 e^{N_b t_g f \pi R_d^2} = N_{sit} t_g f \pi R_d^2 \quad (6.35)$$

$$\Leftrightarrow N_b = \frac{\mathcal{W}(N_{sit} \mathbf{A})}{\mathbf{A}} \quad \text{where } \mathbf{A} = t_g f \pi R_d^2 \quad (6.36)$$

### 6.6.2 Static coalescence

Now that the actual number of bubble-generating sites have been identified, we can consider other interaction phenomena that can occur on the boiling surface. For instance, if two sites are simultaneously generating a bubble at a distance  $d$  between  $R_d$  and  $2R_d$ , the bubbles will coalesce while growing up to the detachment diameter. To estimate the probability of having a bubble on a site in this distance range, we consider the probability density function of the nearest-neighbour in the case of a homogeneous spatial Poisson process  $f$  with an event density  $\lambda$ .

$$f(r) = 2\lambda\pi r e^{-\lambda\pi r^2} \quad (6.37)$$

The considered probability of interaction is then :

$$\mathcal{P}(R_d \leq r \leq 2R_d) = \int_{R_d}^{2R_d} f(r) dr \quad (6.38)$$

$$= e^{-\lambda\pi R_d^2} - e^{-4\lambda\pi R_d^2} \quad (6.39)$$

$$= e^{-\lambda\pi R_d^2} \left[ 1 - \left( e^{-\lambda\pi R_d^2} \right)^3 \right] \quad (6.40)$$

$$= \mathcal{P}_{coal,st} \quad \text{with} \quad \lambda = t_g f N_b \quad (6.41)$$

The density of bubble-generating sites that will lead to a static coalescence can then be estimated as :

$$N_{coal,st} = \mathcal{P}_{coal,st} N_b \quad (6.42)$$

If we suppose that coalescing bubbles will instantly lift-off due to the perturbation associated with the coalescence process, this yields an associated boiling flux :

$$\phi_{e,coal,st} = N_{coal,st} f \rho_V h_{LV} \frac{4}{3} \pi R_{coal,st}^3 \quad \text{where} \quad R_{coal,st} = \sqrt[3]{2} R_d \quad (6.43)$$

considering that the bubbles will merge approximately at  $R = R_d$ .

### 6.6.3 Sliding coalescence

Now that suppressed sites and sites that will lead to static coalescence have been identified, the remaining sites  $N_{sl} = (1 - \mathcal{P}_{coal,st}) N_b$  will generate sliding bubbles. While sliding, a single bubble swipes an area :

$$A_{sl,1b} \approx l_{sl,1b} \frac{D_d + D_{lo}}{2} \quad (6.44)$$

In this area, there are an average number of bubble-generating sites  $N_b A_{sl,1b}$  and an average number of growing bubbles on their sites  $t_g f N_b A_{sl,1b}$

Two situations can happen from the sliding process :

- The bubble slides without coalescing
- The bubble coalesces while sliding with a bubble growing on its site and lifts-off

Following the same approach from the static suppression, we can estimate the probability of finding no growing bubble over the sliding surface :

$$\mathcal{P}(N(A_{sl,1b}) = 0) = e^{-N_b t_g f A_{sl,1b}} \quad (6.45)$$

Thus, if a sliding bubble among the  $N_{sl}$  does not encounter any growing bubble, the sites on its sliding area will be wiped and thus be quenched by cold liquid. This means that those sites will be suppressed due to the sliding of other bubbles over them.

Among the  $N_b$  bubble generating sites we can identify 4 categories of sites :

- Sites generating bubbles which will slide without encountering any growing bubble on their path :

$$N_{sl,NC} = N_{sl} e^{-f t_g N_b A_{sl,1b}}$$

- Sites generating bubbles that will coalesce with a growing bubble during sliding :

$$N_{sl,C} = N_{sl} \left( 1 - e^{-f t_g N_b A_{sl,1b}} \right)$$

- Sites which will be suppressed by bubbles sliding without coalescing :

$$N_{sup,sl} = N_{sl,NC} N_b A_{sl,1b}$$

- Sites generating bubbles that will coalesce with a sliding bubble coming from upstream. Those bubbles are still in the growing phase up to detachment when they are coalescing with sliding bubbles. Therefore, there are equal to the number of sliding bubbles that will coalesce :

$$N_{g,C} = N_{sl,C}$$

This allows to finally write :

$$N_b = N_{sl,NC} + N_{sl,C} + N_{sup,sl} + N_{g,C} \quad (6.46)$$

$$= N_{sl} \left[ 2 - e^{-ft_g N_b A_{sl,1b}} (A_{sl,1b} N_b - 1) \right] \quad (6.47)$$

Which finally yields the total number of sliding bubbles :

$$N_{sl} = \frac{N_b}{2 - e^{-ft_g N_b A_{sl,1b}} (A_{sl,1b} N_b - 1)} \quad (6.48)$$

# 7

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## ASSEMBLING A NEW HEAT FLUX PARTITIONING MODEL

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### 7.1 GENERAL DESCRIPTION OF THE MODEL

The main goal of such a model is to provide a way to compute the wall temperature  $T_w$  resulting from the applied wall heat flux  $\phi_w$ , or the other way around.

In order to try to be as extensive as possible regarding the different heat transfer mechanisms at stake, the wall heat flux is supposed to be split between 4 different contributions (Figure 7.1) :

- A convective heat flux towards the liquid phase, unaffected by the presence of bubbles on the heater surface :  $\phi_{c,l}$
- A boiling heat flux, representing the energy removed from the wall to grow a bubble up to its lift-off diameter :  $\phi_b$
- A quenching heat flux, accounting for transient heat transfer to the liquid phase when bubbles slide or lift-off from the wall :  $\phi_q$
- A convective heat flux towards the vapor phase, representing the heat transfer occurring through the dry areas of the surface beneath the bubbles :  $\phi_{c,v}$

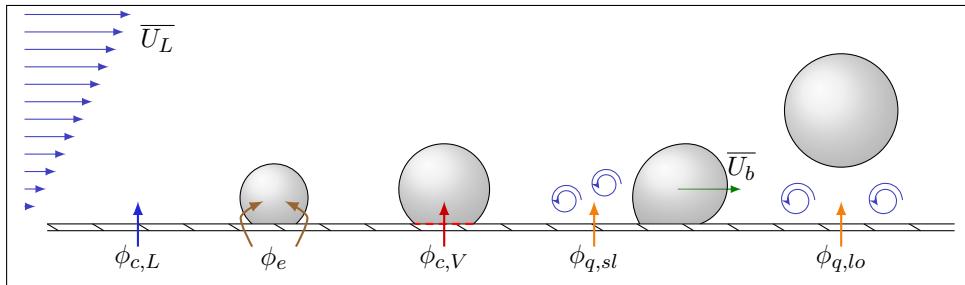


Figure 7.1: Sketch of the considered HFP

The supposed mechanisms yields the total wall heat flux partitioning (7.1) :

$$\phi_w = \phi_{c,l} + \phi_e + \phi_q + \phi_{c,v} \quad (7.1)$$

In the following subsections, we focus our analysis on each term to detail its modeling.

#### 7.1.1 Convective heat fluxes

The convective heat fluxes towards the liquid phase  $\phi_{c,l}$  and the vapor phase  $\phi_{c,v}$  can be written using an associated heat transfer coefficient (7.2) :

$$\phi_{c,l} = a_{c,l} h_{c,l} (T_w - T_l) \quad \text{and} \quad \phi_{c,v} = a_{c,v} h_{c,v} (T_w - T_v) \quad (7.2)$$

### 7.1.2 Boiling heat flux

The total energy associated with the nucleation of a bubble with a volume  $V_b$  can be expressed as  $V_b \rho_v h_{lv}$ . If one knows the nucleation frequency  $f$  at which bubbles are generated along with the nucleation site density on the heater surface  $N_{sit}$ , the resulting heat flux associated with the nucleation phenomenon can thus be written as (7.3) :

$$\phi_b = N_{sit} f V_b \rho_v h_{lv} \quad (7.3)$$

### 7.1.3 Quenching heat flux

The quenching heat flux accounts for the transient heat transfer which occurs when cold liquid is brought close to the wall when a bubble slides or lifts-off, thus disrupting the previously established thermal boundary layer.

DEL VALLE & KENNING have supposed that this kind phenomenon can be represented as a semi-infinite transient heat transfer between the liquid at  $T_l$  and the wall at  $T_w$ . Solving the conductive heat transfer problem yields an instantaneous heat flux expressed as Eq. 7.4.

$$\phi_q(t) = \frac{\lambda_l (\textcolor{red}{T}_w - \textcolor{blue}{T}_l)}{\sqrt{\pi \eta_l t}} \quad (7.4)$$

Therefore, we can average this heat flux over a time  $t_w$ , during which the quenching operates, and ponderating it both by the portion of the affected heater area  $a_q$  and the fraction of quenching time over a total bubble nucleation cycle  $t_w f$ , yielding :

$$\phi_q = \textcolor{orange}{a_q} \textcolor{blue}{t_w} f \frac{1}{\textcolor{blue}{t_w}} \int_0^{\textcolor{blue}{t_w}} \frac{\lambda_l (\textcolor{red}{T}_w - \textcolor{blue}{T}_l)}{\sqrt{\pi \eta_l t}} dt = \textcolor{orange}{a_q} \textcolor{blue}{t_w} f \frac{2 \lambda_l (\textcolor{red}{T}_w - \textcolor{blue}{T}_l)}{\sqrt{\pi \eta_l \textcolor{blue}{t_w}}} \quad (7.5)$$

### 7.1.4 Needed closure relationships

After expressing each heat flux components of the global partitioning, the resulting formulations yields a first list of parameters for which closure relationships (or at least precise definition) are needed. Terms previously highlighted in orange will be given a specific definition, terms in blue require a closure law, wall temperature is indicated in red.

The different terms needing further development are listed below :

- The fractions of the heater area ponderating convective and quenching heat transfers :  $a_{c,l}$ ,  $a_{c,v}$  and  $a_q(??)$
- The convective heat transfer coefficients :  $h_{c,l}$  and  $h_{c,v}$  (Section ??)
- The nucleation site density over the heater surface :  $N_{sit}$  (Section : ??)
- The nucleation frequency, which includes both the growth time  $t_g$  (Section ??) of a bubble and the waiting time  $t_w$  (Je dois encore proposer une modélisation pour  $t_w$ , à discuter) :  $f = 1 / (t_g + t_w)$
- The total bubble volume  $V_b$  (??) generated until its lift-off, thus including the modeling of the bubble lift-off diameter :  $D_{lo}$
- The phases temperature :  $T_l$  (??) and  $T_v$  (??)

Part IV  
TOWARDS THE INDUSTRIAL GEOMETRY

# 8

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## PROMOTEUR

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### 8.1 BOILING FREON IN A TUBE WITH MIXING VANES : DEBORA-PROMOTEUR EXPERIMENTS

In this section, we simulate upward boiling flows of R12 in a vertical tube equipped with mixing vanes and compare the outlet void fraction profile predicted by NEPTUNE\_CFD with measurements coming from the DEBORA-Promoteur experiment.

#### 8.1.1 *Description of the experiment*

In 2003, the wish to investigate boiling flows in complex geometries similar to those in PWR fuel assembly lead CEA and EDF to modify the DEBORA facility to introduce mixing vanes (MV) within the tube. This mixing device has been designed to have the same geometric properties as the mixing vanes attached to rod bundle grids (Figure 8.1).

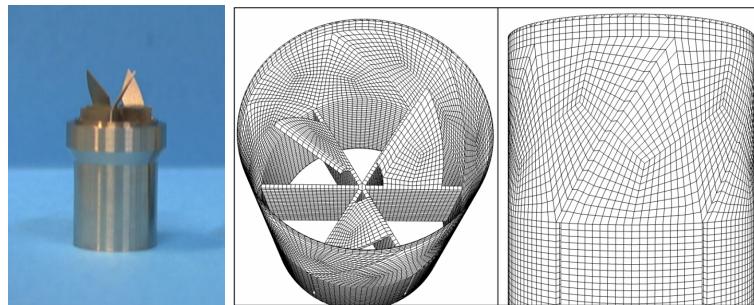


Figure 8.1: Picture of the mixing device (left) and its fine meshing (right).

Two series of measurements were conducted on this geometry :

- Campaign 4800 : measurements of  $\alpha$  using two optical probes, mixing device placed  $0.455m \approx 23.5D_h$  upstream the end of the heating length
- Campaign 5200 : measurements of  $\alpha$  and  $U_{G,z}$  using two optical probes, mixing device placed  $0.192m \approx 10D_h$  upstream the end of the heating length

The goal of those tests was to observe the impact of the mixing device on the void fraction profile. The induced rotation is expected to gather the bubbles at the center of the tube and enhance condensation for highly subcooled cases. Those expectations are confirmed when looking at experimental  $\alpha$  profiles on Figure 8.2. The strong differences compared to simple tube profiles could explain the gain on the CHF value in PWR thanks to the mixing grids. Cases are named following the same nomenclature as presented in Section 2.1.

#### 8.1.2 *Analysis of the Experimental Measurements*

ESTIMATION OF BUBBLE DIAMETER USING AGATE PROM MEASUREMENTS OF URAD

### 8.1.3 NEPTUNE\_CFD simulations of DEBORA-Promoteur cases

We simulated 3 cases for each position of the mixing device, covering different local thermodynamic quality near the vanes ( $x_{eq,MV}$ ) :

- 48G3P26W23Te65 & 52G3P26W23Te65 with  $x_{eq,MV} \approx -1\%$
- 48G3P26W23Te69 & 52G3P26W23Te69 with  $x_{eq,MV} \approx 4\%$
- 48G3P26W23Te75 & 52G3P26W23Te75 with  $x_{eq,MV} \approx 12\%$

Computations are conducted using two meshes for Te69 cases : a large one (M1) with 444 703 cells and a fine one (M2) with 3 487 627 cells. Results for void fraction profiles are shown on Figure 8.2.

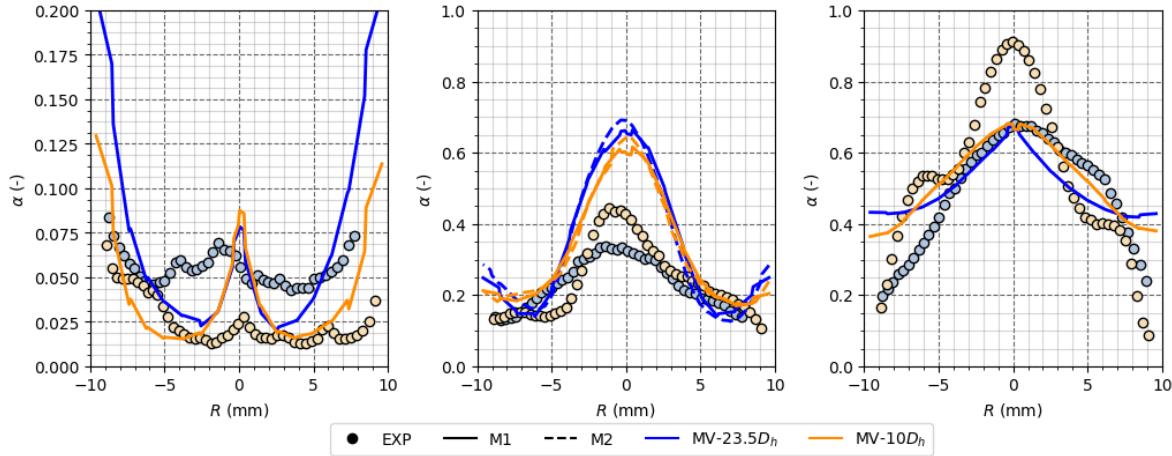


Figure 8.2: NCFD (lines) vs. Exp. (circles) -  $\alpha$  profiles for two MV positions ( $23.5D_h$  in blue,  $10D_h$  in orange)  
-  $T_{in} = 65^\circ\text{C}$  (left),  $T_{in} = 69^\circ\text{C}$  (middle),  $T_{in} = 75^\circ\text{C}$  (right) - Simulations using two meshes M1 (coarse) and M2 (fine) for  $T_{in} = 69^\circ\text{C}$ .

Quantitatively speaking, it seems that NEPTUNE\_CFD reproduces the effect of vapor accumulation at the center thanks to the pressure gradient generated by the swirl induced by the mixing vanes. The radial position of the core void fraction peak correctly matches the experimental one.

However, measured void fraction profiles are not predicted correctly. A particularly strong overestimation of the core void fraction is observed as well as close to the wall. The CMFD results tend to rapidly reach a core void fraction around 60% ( $T_{in} = 69^\circ\text{C}$  cases) and then flattens with increasing temperature ( $T_{in} = 75^\circ$  cases). This contradicts experimental observation where the void fraction profile globally rises when inlet temperature increases, except at the wall where no peak is observed due to bubble removing effect by the liquid's rotation. Moreover, the  $T_{in} = 75^\circ$  case with MV at  $10D_h$  experimentally shows local  $\alpha$  peaks at  $R \approx \pm 6\text{mm}$  which remain currently unexplained and not reproduced by the simulations.

To investigate what could be a potential origin for the core void fraction peak overestimation, we present in Section 8.2 single-phase flow simulations in the MV geometry.

## 8.2 LIQUID WATER FLOW IN A TUBE WITH MIXING VANES : AGATE-PROMOTEUR EXPERIMENT

In this penultimate section, we briefly investigate single-phase flow within the same geometry as Section 8.1.

### 8.2.1 Description of the experiment

In 2003, using the same experimental geometry as DEBORA-Promoteur cases (Section 8.1), Laser Doppler Velocimetry (LDV) measurements of velocity and turbulent fluctuations for an adiabatic single-phase flow of water were conducted. The outlet pressure was around  $P = 2$  bar with an inlet mass flux  $G \approx 3000 \text{ kg/m}^2/\text{s}$ . Measurements were conducted on 6 different diameters and repeated at various axial positions upstream and downstream the mixing vanes.

A first look at experimental measurements (Figure 8.3) shows that the vanes geometry induces significantly non-symmetric velocity profile. Moreover, we observe high turbulent fluctuations which maximum is located at the same radial position as the maximum radial velocity gradient.

### 8.2.2 NEPTUNE\_CFD simulations of AGATE-Promoteur case

On Figure 8.3, we present some of the results obtained with NEPTUNE\_CFD using the  $R_{ij} - \varepsilon$  SSG turbulence model on the M2 mesh, along with a smooth wall law and a rough wall law (roughness  $\epsilon = 0.01\text{mm}$ ). The turbulent fluctuations Root Mean Square (RMS) correspond, for instance, to  $\sqrt{\langle u_x'^2 \rangle}$  for the  $x$  direction where  $u_i'$  represents the fluctuating part of the velocity along component  $i$  and  $\langle \cdot \rangle$  the time-averaging operator. Subscripts  $R$  and  $A$  stand for radial and axial values ;  $U_0$  is the average inlet velocity.

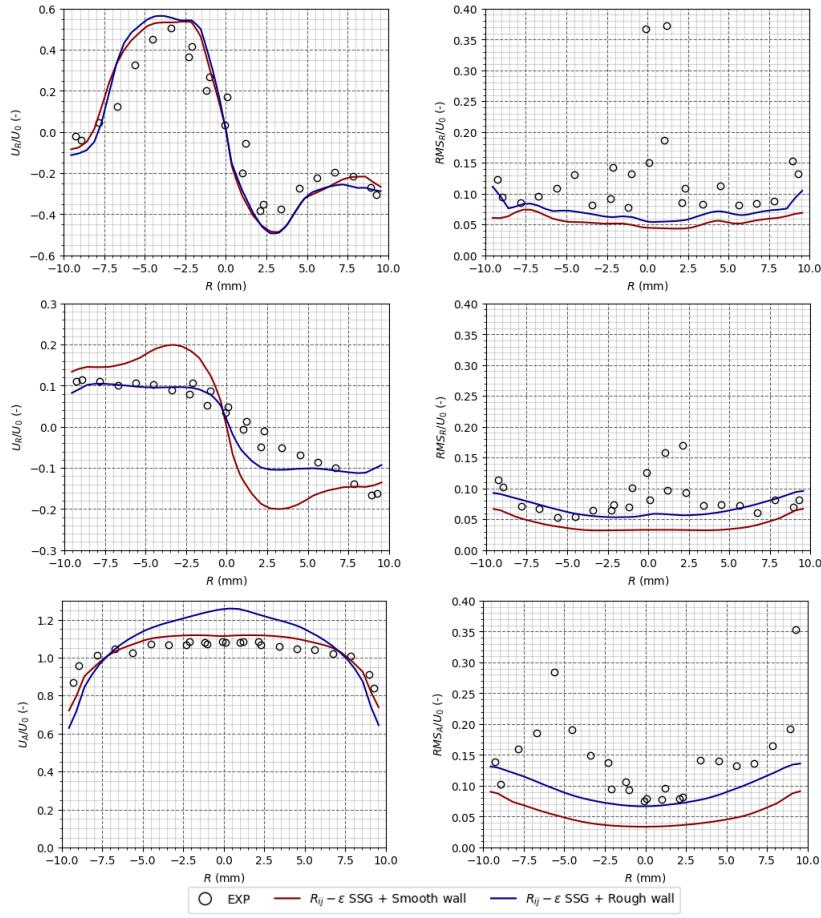


Figure 8.3: NCFD vs. Exp. - Top & Middle : Radial velocity and turbulent RMS ( $z = 30\text{mm}$  &  $z = 440\text{mm}$ ) - Bottom : Axial velocity and turbulent RMS ( $z = 440\text{mm}$ ).

Non-symmetric radial velocity profiles close to the MV are quite well reproduced by the simulations. However, far downstream the MV, it appears that the fluid's rotation is overestimated by the model with a smooth wall approach, while applying a roughness helps to reduce the magnitude of the swirl. Moreover, the radial turbulent fluctuations are better estimated by the rough wall approach at  $z = 440\text{ mm}$ . On the other hand, it seems that the rough wall approach deteriorates the axial velocity profile compared to the experiment. As shown on the bottom part of Figure 8.3, the smooth wall simulation returns a flat velocity profile closer to the experiment than the rough wall one which overestimates the core velocity peak.

Both simulations globally underestimate the turbulent fluctuations, which can have a significant influence over the observed discrepancies on velocity profiles since turbulence plays a key role to homogenize the fluid flow.

Those results finally highlight the fact that simulation of such rotating flows may need a particular wall approach to better capture the induced swirl and its dissipation. Correct prediction of turbulent

fluctuations would be of significant interest to ensure liquid velocity validation. Further investigations on boiling cases could possibly be improved by a roughness approach, which is the current correction used for two-phase wall laws (Subsection 1.1.6).

Part V  
CONCLUSION

Part VI  
APPENDIX

# A

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## DETAILS ON THE BUBBLE GROWTH CALCULATION

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To further study the bubble growth process, we will try to derive an analytic In this section, computations are conducted using the spherical coordinates  $(\bar{e}_r, \bar{e}_\theta, \bar{e}_\varphi)$  with coordinates  $(r, \theta, \phi)$ .

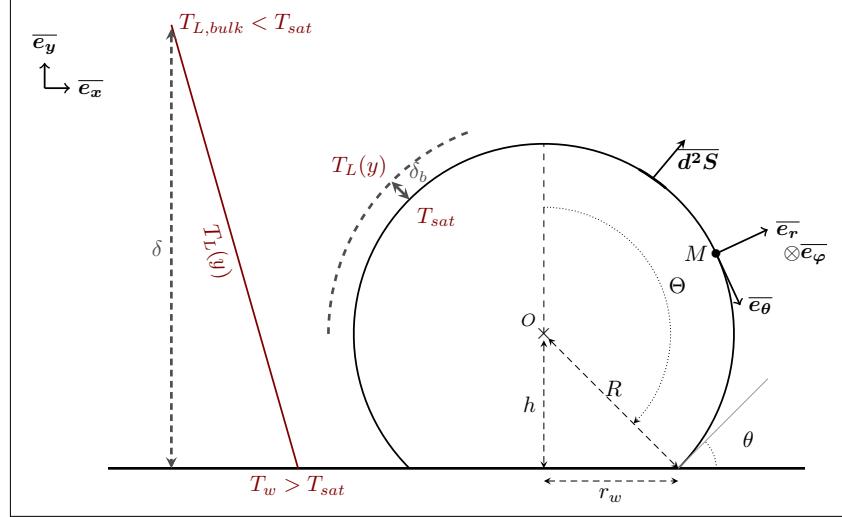


Figure A.1: Studied geometry

Geometrical definitions :

- $\Theta = \pi - \theta$  the angular portion of the truncated sphere in spherical coordinates ;
- $r_w = R\sin(\theta) = -R\sin(\Theta)$  the bubble foot radius ;
- $h = R\cos(\theta) = -R\cos(\Theta)$  the distance between the wall and the center of the bubble ( $> 0$  if  $\theta < \pi/2$ ,  $< 0$  otherwise) ;
- $R$  and  $V = \frac{4}{3}\pi R^3 f_V(\theta)$  the radius and the volume of the bubble ;
- $\bar{d}^2S = R^2\sin(\theta) d\theta d\varphi \bar{e}_r$  the surface vector in spherical coordinates ;
- $y = R\cos(\theta) + h = R(\cos(\theta) - \cos(\Theta))$  the distance to the wall in cartesian coordinates.

Thermal-hydraulics definitions :

- $\Delta T_L = T_{sat} - T_\infty$  et  $\Delta T_w = T_w - T_{sat}$  the subcooling of the liquid and the wall superheat respectively ;
- $\delta$  et  $\delta_b$  the flow boundary layer thickness and the bubble boundary layer thickness respectively ;
- $d^2Q_b$  the heat received by the bubble between  $t$  and  $t + dt$  through the surface  $d^2S$  ;
- $\lambda, \rho, c_p, \eta, h$  the thermal conductivity, density, heat capacity, thermal diffusivity and mass enthalpy respectively ( $l$  standing for liquid and  $g$  for gas) ;

- $\text{Ja}_w = \Delta T_w \rho_L c_{p,L} / (h_{LV} \rho_V)$  the wall superheat (or boiling) Jakob number and  $\text{Ja}_L = \Delta T_L \rho_L c_{p,L} / (h_{LV} \rho_V)$  the subcooled liquid (or condensation) one.

The thermal boundary layer of the flow is assumed to follow a linear profile, giving the expression :

$$T_l(y) = T_w + \frac{T_\infty - T_w}{\delta} y \quad (\text{A.1})$$

If we consider that the bubble stays at a temperature close to  $T_{sat}$ , the radial component of the temperature gradient at the bubble's interface yields :

$$\bar{\nabla}(T) \cdot \bar{er} = \frac{\partial T}{\partial r}(R, \theta, \phi) \approx \frac{T_l(y) - T_{sat}}{\delta_b} \quad (\text{A.2})$$

Applying Fourier's law to the liquid close to the bubble :  $\bar{j_Q} = -\lambda_l \bar{\nabla}(T)$ , then the bubble receives between  $t$  and  $t + dt$  through  $d^2S$  :

$$d^2Q_b = \bar{j_Q} \cdot (-\bar{d^2S}) \quad (\text{A.3})$$

$$= \lambda_l \frac{\partial T}{\partial r}(R, \theta, \varphi) R^2 \sin(\theta) d\theta d\varphi \quad (\text{A.4})$$

$$\approx \lambda_l \frac{T_l(y) - T_{sat}}{\delta_b} R^2 \sin(\theta) d\theta d\varphi \quad (\text{A.5})$$

$$= \lambda_l \frac{1}{\delta_b} \left[ T_w + \frac{T_\infty - T_w}{\delta} y - T_{sat} \right] R^2 \sin(\theta) d\theta d\varphi \quad (\text{A.6})$$

$$= \frac{\lambda_l}{\delta_b} \left[ \Delta T_w - \frac{\Delta T_w + \Delta T_l}{\delta} R [\cos(\theta) - \cos(\Theta)] \right] R^2 \sin(\theta) d\theta d\varphi \quad (\text{A.7})$$

$$= \frac{\lambda_l}{\delta_b} \left[ \Delta T_w R^2 \sin(\theta) - \frac{\Delta T_w + \Delta T_l}{\delta} R^3 [\cos(\theta) - \cos(\Theta)] \sin(\theta) \right] d\theta d\varphi \quad (\text{A.8})$$

The total heat flux received by the bubble can then be derived, supposing that  $\delta_b$  is constant all around the bubble between  $t$  and  $t + dt$  :

$$Q_b = \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\Theta} d^2Q_b \quad (\text{A.9})$$

$$= \frac{2\pi\lambda_l}{\delta_b} \left[ \int_0^\Theta \Delta T_w R^2 \sin(\theta) d\theta + \int_0^\Theta \frac{\Delta T_w + \Delta T_l}{\delta} R^3 \cos(\Theta) \sin(\theta) d\theta - \int_0^\Theta \frac{\Delta T_w + \Delta T_l}{\delta} R^3 \cos(\theta) \sin(\theta) d\theta \right] \quad (\text{A.10})$$

$$= \frac{2\pi\lambda_l}{\delta_b} \left[ \Delta T_w R^2 (1 - \cos(\Theta)) + \frac{\Delta T_w + \Delta T_l}{\delta} R^3 \left[ \cos(\Theta) (1 - \cos(\Theta)) - \frac{1}{4} (1 - \cos(2\Theta)) \right] \right] \quad (\text{A.11})$$

$$= \frac{2\pi\lambda_l R^2}{\delta_b} \left[ \frac{-R}{2\delta} (\Delta T_w + \Delta T_l) (1 + 2\cos(\alpha) + \cos^2(\alpha)) + \Delta T_w (1 + \cos(\alpha)) \right] \quad (\text{A.12})$$

$$= \frac{2\pi\lambda_l R^2}{\delta_b} (1 + \cos(\alpha)) \left[ \Delta T_w - \frac{R}{2\delta} (\Delta T_w + \Delta T_l) (1 + \cos(\alpha)) \right] \quad (\text{A.13})$$

Between  $t$  and  $t + dt$ , the bubble receives a  $Q_b dt$  energy amount through thermal diffusion. Assuming this energy solely contributes to evaporation of the surrounding liquid, the resulting mass of generated vapor is :

$$dm_g = \rho_g dV = \frac{Q_b dt}{h_{lg}} \quad (\text{A.14})$$

$$\text{then } \frac{dV}{dt} = \frac{Q_b}{\rho_g h_{lg}} \quad (\text{A.15})$$

Since  $V = \frac{4}{3}\pi R^3 f_V(\alpha)$ , we can write :

$$\frac{dV}{dt} = \frac{4}{3}\pi f_V(\alpha) 3R^2 \frac{dR}{dt} \quad (\text{A.16})$$

Then :

$$\frac{dR}{dt} = \frac{1}{4\pi R^2 f_V(\alpha)} \frac{1}{\rho_g h_{lg}} \frac{2\pi \lambda_l R^2}{\delta_b} (1 + \cos(\alpha)) \left[ \Delta T_w - \frac{R}{2\delta} (\Delta T_w + \Delta T_l) (1 + \cos(\alpha)) \right] \quad (\text{A.17})$$

$$= \frac{1}{2f_V(\alpha)} \frac{\Delta T_w}{h_{lg} \rho_g} \frac{\lambda_l}{\delta_b} (1 + \cos(\alpha)) \left[ 1 - \frac{R}{2\delta} \left( 1 + \frac{\Delta T_l}{\Delta T_w} \right) (1 + \cos(\alpha)) \right] \quad (\text{A.18})$$

$$= \frac{\text{Ja}_w \eta_l}{2\delta_b f_V(\alpha)} (1 + \cos(\alpha)) \left[ 1 - \frac{R}{2\delta} \left( 1 + \frac{\text{Ja}_l}{\text{Ja}_w} \right) (1 + \cos(\alpha)) \right] \quad (\text{A.19})$$

If we define :

$$a = \frac{\text{Ja}_w \eta_l}{4\delta_b \delta f_V(\alpha)} \left( 1 + \frac{\text{Ja}_l}{\text{Ja}_w} \right) (1 + \cos(\alpha))^2 \quad \text{et} \quad b = \frac{\text{Ja}_w \eta_l}{2\delta_b f_V(\alpha)} (1 + \cos(\alpha)) \quad (\text{A.20})$$

Finally :

$$\frac{dR}{dt} + aR = b \quad (\text{A.21})$$

Solutions of this differential equation depend on the hypothesis over  $\delta$  and  $\delta_b$ . In a first approach, we will consider that  $\delta$  does not vary during the whole bubble growth. This thickness could be expressed using boundary layer thickness correlations for example (laminar or turbulent depending on the Reynolds number).

On the other hand, we will consider 3 different choices to model  $\delta_b$  :

#### A.0.1 Case 1 : $\delta_b$ is constant

This is a **strong assumption** since it means that  $\delta_b$  does not depend on the temperature and the flow shear rate facing the bubble. In this case,  $a$  and  $b$  are constant, giving :

$$\frac{dR}{dt} + aR = b \quad (\text{A.22})$$

With the initial condition  $R(t = 0) = 0$  the solution of this differential equation is :

$$R(t) = \frac{b}{a} (1 - e^{-at}) \quad \text{where} \quad \frac{b}{a} = \frac{2\delta}{\left( 1 + \frac{\text{Ja}_l}{\text{Ja}_w} \right) (1 + \cos(\alpha))} = R_\infty \quad \text{is the final bubble equilibrium radius.} \quad (\text{A.23})$$

#### A.0.2 Case 2 : $\delta_b = \sqrt{\eta_l t}$

This expression derives from the growth of a boundary layer through pure diffusion, as studied by LEGENDRE *et al.*

Such a boundary layer thickness is also used when one writes the quenching heat flux (see Section 7.1 and DEL VALLE & KENNING work).

In this case, this means that the boundary layer around the bubble interface grows through diffusion only.

Thus, it yields :

$$\frac{dR}{dt}(t) + a(t)R(t) = b(t) \quad (\text{A.24})$$

$$a(t) = \frac{\text{Ja}_w \sqrt{\eta_l}}{4\delta f_V(\alpha) \sqrt{t}} \left(1 + \frac{\text{Ja}_l}{\text{Ja}_w}\right) (1 + \cos(\alpha))^2 = K_a t^{-1/2} \text{ and } b(t) = \frac{\text{Ja}_w \sqrt{\eta_l}}{2f_V(\alpha) \sqrt{t}} (1 + \cos(\alpha)) = K_b t^{-1/2} \quad (\text{A.25})$$

With the initial condition  $R(t=0) = 0$ , the solution is given by :

$$R(t) = R_\infty \left(1 - e^{-2K_a \sqrt{t}}\right) \text{ avec } R_\infty = \frac{K_b}{K_a} \quad (\text{A.26})$$

### A.0.3 Case 3 : $\delta_b = CR$

In this last case, the bubble boundary layer is assumed to be proportional to the bubble radius  $R$ , through a constant coefficient  $C$ . This yields :

$$a(t) = \frac{\text{Ja}_w \eta_l}{4\delta f_V(\alpha) CR(t)} \left(1 + \frac{\text{Ja}_l}{\text{Ja}_w}\right) (1 + \cos(\alpha))^2 = K'_a \frac{1}{R(t)} \text{ et } b(t) = \frac{\text{Ja}_w \eta_l}{2f_V(\alpha) CR(t)} (1 + \cos(\alpha)) = K'_b \frac{1}{R(t)} \quad (\text{A.27})$$

Giving :

$$\frac{dR}{dt}(t) + K'_a R(t) = K'_b \frac{1}{R(t)} \quad (\text{A.28})$$

Which the solution with the initial condition  $R(t=0) = 0$  is :

$$R(t) = R_\infty \left(W\left(-e^{-tK'_a^2/K'_b - 1}\right) + 1\right) \quad (\text{A.29})$$

With  $W$  being the LAMBERT function defined as the reciprocal function of  $w \rightarrow we^w$  on  $\mathbb{C}$ .

# B

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## DETAILS ON THE BUBBLE FORCE BALANCE

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Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl. Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

$$r_w = \frac{1}{2} (\sin(\theta_d) R + \sin(\theta_u) R) \quad (B.1)$$

$$= \frac{R}{2} (\sin(\theta_s + d\theta) + \sin(\theta - d\theta)) \quad (B.2)$$

$$= \frac{R}{2} \left[ 2\sin\left(\frac{\theta_s + d\theta + \theta_s - d\theta}{2}\right) \cos\left(\frac{\theta_s + d\theta - (\theta_s - d\theta)}{2}\right) \right] \quad (B.3)$$

$$= R\sin(\theta_s) \cos(d\theta) \quad (B.4)$$

$$\overline{F_{CP}} = \frac{2\sigma}{R_c} \frac{\pi d_w^2}{4} \overline{e_y} \quad (B.5)$$

$$= \frac{2\sigma}{R} \frac{\pi 4R^2}{4} \sin^2(\theta) \cos^2(d\theta) \overline{e_\perp} \quad (B.6)$$

$$= 2\sigma\pi R \underbrace{\sin^2(\theta) \cos^2(d\theta)}_{f_{cp}} \overline{e_\perp} \quad (B.7)$$

$$= 2\pi R\sigma f_{cp}(\theta, d\theta) \overline{e_\perp} \quad (B.8)$$

$$\overline{F_{CP}} \cdot \overline{e_\perp} = -d_w\sigma \frac{\pi}{\alpha_r - \alpha_u} (\cos(\alpha_a) - \cos(\alpha_r)) \quad (B.9)$$

$$= \pi 2R\sigma \frac{\sin(\alpha + d\alpha) + \sin(\alpha - d\alpha)}{2} \frac{\cos(\alpha + d\alpha) - \cos(\alpha - d\alpha)}{2d\alpha} \quad (B.10)$$

$$= 2\pi R\sigma \frac{2\sin(\alpha) \cos(d\alpha)}{2} \frac{(-2\sin(\alpha) \sin(d\alpha))}{2d\alpha} \quad (B.11)$$

$$= -2\pi R\sigma \underbrace{\sin^2(\alpha) \frac{\sin(2d\alpha)}{2d\alpha}}_{f_{s,\perp}} \quad (B.12)$$

$$= -2\pi R\sigma f_{s,\perp}(\alpha, d\alpha) \quad (B.13)$$

$$(B.14)$$

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Table B.1: Autem usu id.

$$\overline{F_s} \cdot \overline{e_{\parallel}} = -1.215d_w\sigma \frac{\pi(\alpha_r - \alpha_u)}{\pi^2 - (\alpha_r - \alpha_u)^2} (\sin(\alpha_r) + \sin(\alpha_u)) \quad (B.15)$$

$$= -1.215d_w\sigma \frac{\pi 2d\alpha}{\pi^2 - 4d\alpha^2} (\sin(\alpha + d\alpha) + \sin(\alpha - d\alpha)) \quad (B.16)$$

$$= -1.215 2R\sigma \frac{\pi d\alpha}{\pi^2 - 4d\alpha^2} (\sin(\alpha + d\alpha) + \sin(\alpha - d\alpha))^2 \quad (B.17)$$

$$= -2\pi R\sigma \frac{1.215d\alpha}{\pi^2 - 4d\alpha^2} 4\sin^2(\alpha) \cos^2(d\alpha) \quad (B.18)$$

$$= -2\pi R\sigma \underbrace{1.215 \frac{d\alpha}{(\frac{\pi}{2})^2 - d\alpha^2} \sin^2(\alpha) \cos^2(d\alpha)}_{f_{s,\parallel}} \quad (B.19)$$

$$= -2\pi R\sigma f_{s,\parallel}(\alpha, d\alpha) \quad (B.20)$$

## B.1 APPENDIX SECTION TEST

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue. Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

## B.2 ANOTHER APPENDIX SECTION TEST

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There is also a useless Pascal listing below: [Listing B.1](#).

Listing B.1: A floating example (`listings` manual)

---

```
1 for i:=maxint downto 0 do
begin
{ do nothing }
end;
```

---

---

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---

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*Paris, December 2022*

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Luc Favre

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