Chaotic Time Series Prediction Using a Neuro-Fuzzy System with Time-Delay Coordinates

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Abstract—This paper presents an investigation into the use of the delay coordinate embedding technique in the multi-input-multioutput-adaptive-network-based fuzzy inference system (MANFIS) for chaotic time series prediction. The inputs to the MANFIS are embedded-phase-space (EPS) vectors preprocessed from the time series under test, while the output time series is extracted from the output EPS vectors from the MANFIS. A moving root-mean-square error is used to monitor the error over the prediction horizon and to tune the membership functions in the MANFIS. With the inclusion of the EPS preprocessing step, the prediction performance of the MANFIS is improved significantly. The proposed method has been tested with one periodic function and two chaotic functions including Mackey-Glass chaotic time series and Duffing forced-oscillation system. The prediction performances with and without EPS preprocessing are statistically compared by using the t-test method. The results show that EPS preprocessing can help improve the prediction performance of a MANFIS significantly.

Index Terms—Chaotic time series prediction, neuro-fuzzy systems, time-delay coordinate embedding.

1 Introduction

A time series is a sequence of regularly sampled quantities out of an observed system. It is a useful source of extracts for discovering and studying the behaviors of the system such as periodicity and stochastic distribution. In addition, a reliable time series prediction method can help researchers model the system and forecast its behaviors. Since 1970s, many prediction methods in time or frequency domain have been proposed [1].

Among the different types of time series, chaotic time series can be commonly found in natural phenomena [2], [3], [4]. Thus, starting from 1980s, chaotic time series prediction has been a popular subject [5] for understanding and controlling the chaotic behaviors to advantage, such as the stock market forecasting [6]. The prediction of a chaotic time series requires a representative model. In recent years, many new prediction approaches, such as the wavelet networks [7], neural networks [8], [9], [10], hierarchical Bayesian approach [11], fuzzy [12], [13], [14], [15], [16], [17] and neuro-fuzzy predictor [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], and time-delay embedding technique [31], [32], [33], [34], have emerged. Also, past research has also been done based on a master-slave synchronization scheme for prediction of chaotic behavior [35], multiple model predictor and genetic algorithm to reconstruct piecewise chaotic dynamics [36], and an effective digital tracker for continuous-time chaotic orbit tracking [37]. They provide new insight into this type of systems not available in the traditional methods such as linear regression technique and autoregressive integrated moving average models.

As discussed in [18], the adaptive-network-based fuzzy inference system (ANFIS) with multi-input and single-output in [19], which uses Gaussian membership functions and employs hybrid backpropagation learning of a chaotic time series, gives the smallest root-mean-square errors among various fuzzy predictors. However, similar to the other single-scale chaotic time series prediction methods, the prediction horizon is usually limited by the fast-varying components, because the methods are based on the finite neighborhood relationships in the prediction. Nevertheless, it forms the best basis for further enhancement.

Time-delay coordinate embedding methods [31], [32], [33], [34] use the relationships between the delay coordinates of a point and the points that appear at some time later in the phase space. Its trajectories behave with quasiperiodicity. The nearest trajectories can contribute to the neighboring set with more than one point, resulting in an increased weighting of the contribution coming from the nearest trajectories.

This paper investigates the use of delay coordinate embedding technique with multi-input-multioutput-ANFIS (denoted by MANFIS) to learn and predict the continuation of chaotic signals ahead of time. The methodology hybridizes the advantages of ANFIS and the time-delay

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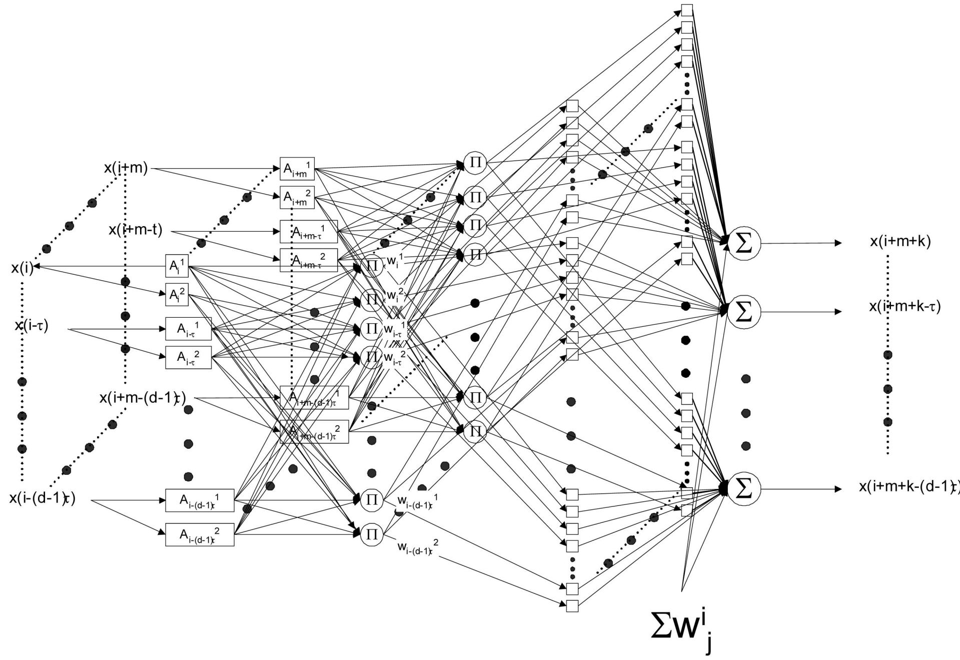

Fig. 1. Proposed structure of the MANFIS.

coordinate embedding technique. The resulting model has better predictive performance with fewer membership functions. Based on the average mutual information and false nearest neighbor method, the minimum time-delay embedding dimension is selected and phase space is constructed. The MANFIS is trained by gradient descent algorithm. First, the parameter set of the membership functions is generated with the embedded-phase-space (EPS) vectors using the backpropagation algorithm. Second, fine-tuned membership functions that make the prediction error as small as possible are built. The proposed method has been tested with one periodic function and two chaotic functions including Mackey-Glass chaotic time series and Duffing forced-oscillation system. The prediction performances without and with EPS preprocessing are compared statistically by using the t-test method [39]. The results show that EPS preprocessing can improve the prediction performance of the MANFIS.

2 Construction of the Embedded Phase Space

A chaotic time series generally exhibits stochastic characteristics in time or frequency domain. However, by using the coordinates with appropriate embedding dimension and time delay, a quasiperiodic attractor in the phase space can be derived [2]. Fig. 1 shows a chaotic time series and its trajectories in the EPS. Consider a time series $\{x(1), x(2), x(3), \ldots, x(N)\}$. An embedded phase vector $y(i)$ is

$$y(i) = [\, x(i) \quad x(i-\tau) \quad \ldots \quad x(i-(D-1)\tau)\,], \qquad (1)$$

where $\forall i \in [1+(D-1)\tau, N]$, D is the embedding dimension, and τ is the time delay, $y(i)$ is a vector in the D-dimensional phase space R^D. A trajectory Y in R^D is defined as

$$Y = \left[\, y^T(i) \quad y^T(i+1) \quad \ldots \quad y^T(i+m) \,\right]. \qquad (2)$$

In order to extract the behaviors of the time series in an efficient way, optimal values of D and τ have to be determined. In this paper, τ and D are determined by using the average mutual information in [31], [32], [33], and [34] and the global false nearest neighbors in [2], respectively.

3 Operations of the MANFIS

Traditional neuro-fuzzy-based time series prediction methods are typically based on sequential time series as the input vector. Such concept has also been used to determine the control parameters in a chaotic system [13]. This paper proposes the use of EPS to preprocess the input data to form the input vectors for the ANFIS model [18]. The method is further extended to multidimension matrix (MANFIS). Fuzzy rule sets are applied to map the EPS input matrix to the output vectors. The fuzzy rule set is composed of a series of IF-THEN fuzzy rules, the fuzzy parameters of the membership functions are tuned by using the gradient descent algorithm with a collection of input-output vector pairs. The structure of the proposed MANFIS is shown in Fig. 1 and its operations are described as follows:

3.1 The Input Section

The input of the MANFIS is a matrix with the embedding phase vector as the columns and the chaotic time series as the rows. Its dimension is $D \times (m+1)$. For a given time series of $\{x(1), x(2), \ldots, x(N)\}$, the EPS vector $y(i)$ is given by

$$y(i) = [\, x(i) \quad x(i-T) \quad \ldots \quad x(i-(D-1)\tau], \quad \forall i \in [1+(D-1)\tau, N]. \qquad (3)$$

Thus, the input matrix Y is

$$Y = \begin{bmatrix} y^T(i) & y^T(i+1) & \dots & y^T(i+m) \end{bmatrix}$$

$$Y = \begin{bmatrix} x(i) & x(i+1) & \dots & x(i+m) \\ x(i-\tau) & x(i+1-\tau) & \dots & x(i+m-\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x[i-(D-1)\tau] & x[i+1-(D-1)\tau] & \dots & x[i+m-(D-1)\tau] \end{bmatrix}, \tag{4}$$

where i is an integer, $i \in [1+(D-1)\tau, N-m-k]$, and k is the number of prediction steps.

3.2 The Fuzzifier Section

Each element in the input matrix is fed to a fuzzifier with two fuzzy membership functions, where $p = \{1, 2\}$, to perform the fuzzy reasoning. Each membership function corresponds to a linguistic label in the fuzzy rule system. In this paper, the Gaussian Fuzzifier is used, and the membership value $A_r^p[x(r)]$ is given by

$$A_r^p[x(r)] = \exp\left[-\frac{1}{2}\left(\frac{x(r) - c_r^p}{\sigma_r^p}\right)^2\right], \tag{5}$$

where r is the index of the element in the input matrix, p is the index for the membership functions $p = \{1, 2\}$, and c_r^p and σ_r^p are the center and the width of the membership function, respectively, which are tuned by the gradient decent algorithm in Section 3.5.

3.3 The Knowledge Acquisition and Reasoning Section

As it is difficult to acquire knowledge from the chaotic time series and to construct the fuzzy rule base, the weighted sum of the products of the fuzzy values or firing strength is used for knowledge acquisition. First, the firing strength of the elements in the EPS vector $w_p(g)$ is quantified by the product of the membership values A_r^p for the gth column of Y:

$$w_p(g,u) = \prod_{u=0}^{D-1} A_r^p[x(r)], \quad r = i + g - u\,\tau, \tag{6}$$

where $g \in \{0, m\}$ is the column number of Y, and $u \in \{0, D-1\}$ is the row number of Y.

Second, a first-order Sugeno model $f_p(g,u)$ is used to represent the consequence of the antecedent in (6). It is defined as

$$f_p(g,u) = \sum_{i=0}^{D-1} h_p(g,u,i)x(r), \quad r = i + g - u\,\tau, \tag{7}$$

where $h_p(g,u,i)$ is the consequent parameter of the element in gth column and uth row of Y.

Then, the knowledge contribution of the Sugeno model $F_p(q,g,u)$ of the element in gth column and uth row of Y is calculated by

$$F_p(q,g,u) = w_p(g,q) f_p(g,u), \quad q \in \{0, D-1\}. \tag{8}$$

3.4 The Defuzzifier Section

The function defuzzifier is used to map the output fuzzy set to a real-value corresponding to the centroid of the output fuzzy set. It generates the output z_q as

$$z_q = \frac{\sum_{u=0}^{D-1}\sum_{g=0}^{m}\sum_{p=1}^{2} F_p(q,g,u)}{\sum_{u=0}^{D-1}\sum_{g=0}^{m}\sum_{p=1}^{2} w_p(g,u)}. \tag{9}$$

The output vector $\hat{y}^T(i+m+k)$ is defined as

$$\hat{y}^T(i+m+k) = \begin{bmatrix} x(i+m+k) \\ x(i+m-\tau+k) \\ \vdots \\ x(i+m-(D-1)\tau+k) \end{bmatrix} = \begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_{D-1} \end{bmatrix}. \tag{10}$$

3.5 The Learning Algorithm

The MANFIS is trained with data obtained in both time domain and phase space. To achieve accurate prediction, gradient descent or the backpropagation algorithm [18] is used to tune the parameters σ_r^p, c_r^p, and $h_p(g,u,i)$. Adjustment of these parameters is determined by the gradient between the actual and the expected output. That is,

$$\hat{\sigma}_r^p = \sigma_r^p - \eta\frac{\partial E}{\partial \sigma_r^p}, \tag{11}$$

$$\hat{c}_r^p = c_r^p - \eta\frac{\partial E}{\partial c_r^p}, \tag{12}$$

$$\hat{h}_p(g,u,i) = h_p(g,u,i) - \eta\frac{\partial E}{\partial h_p(g,u,i)}, \tag{13}$$

where $\hat{\sigma}_r^p$, $\hat{c}_r^p$, and $\hat{h}_p(g,u,i)$ are updated values of σ_r^p, c_r^p, and $h_p(g,u,i)$, respectively, in the next iteration cycle, η is a constant determining the learning rate, and E is the error function as described in Section 4.4 for the training data.

4 A Comparative Study

Predictions of three time series, including one periodic and two chaotic time series, have been investigated. A comparative study into the prediction performances with and without EPS preprocessing as the input to the ANFIS has been performed. Different numbers of training sets have been taken. The results have been statistically compared by using the t-test method in [39] with the prediction horizon of 10, 20, 30, and 40.

4.1 Periodic Time Series

The periodic time series under investigation is

$$x(t) = \frac{1}{\omega}\sum_{i=a}^{b} \sin\kappa\, it, \tag{14}$$

where $\omega = 5$, $\kappa = 0.01$, $a = 1$, and $b = 5$.

4.2 Chaotic Time Series—Mackey-Glass Function

The first chaotic time series is *Mackey-Glass function* [40], [41]. It is expressed as

$$\frac{dx}{dt} = \frac{0.2x(t-\lambda)}{1 + x^{10}(t-\lambda)} - 0.1x(t), \tag{15}$$

where $\lambda = 17$ and $x(0) = 1.2$.

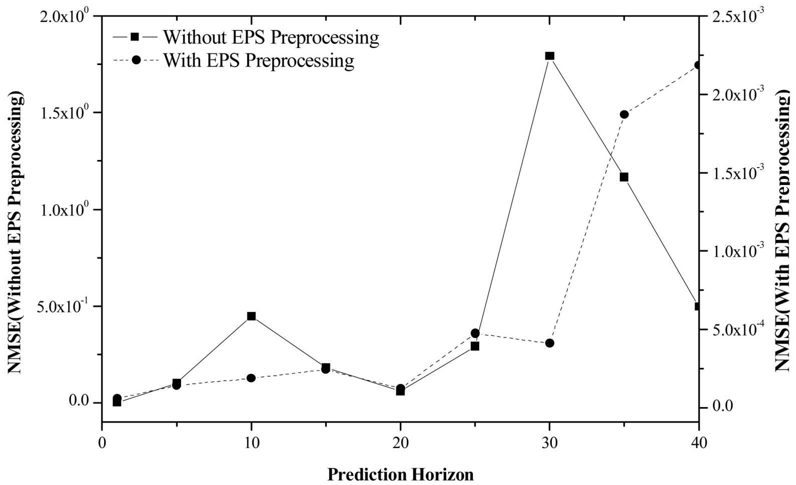

Fig. 2. Periodic time series prediction with $m_f = 2$, $t_s = 100$, and $t_e = 500$.

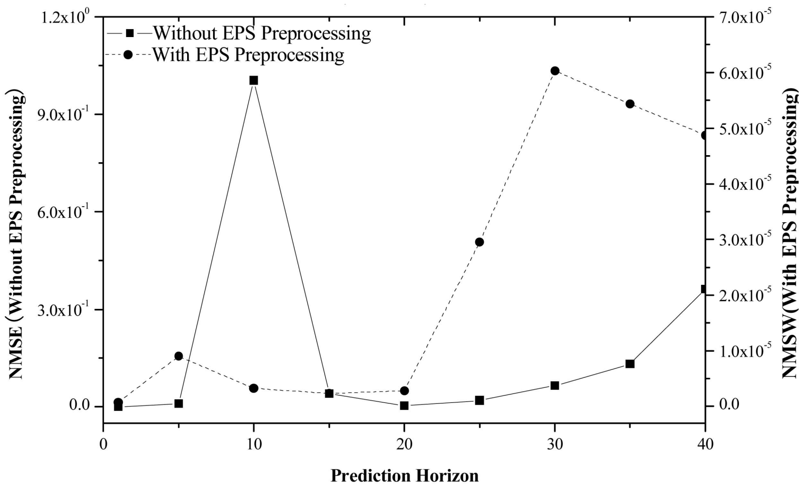

Fig. 3. Periodic time series prediction with $m_f = 2$, $t_s = 300$, and $t_e = 500$.

4.3 Chaotic Time Series—Duffing Forced-Oscillation System

The second chaotic time series is Duffing forced-oscillation system [41]. It is expressed as

$$\frac{dx_1}{dt} = x_2, \tag{16}$$

$$\frac{dx_2}{dt} = -0.1x_2 - x_1^3 + 12\cos t + u(t). \tag{17}$$

The system is chaotic when $u(t)$ is zero.

4.4 Estimation of the Prediction Error

The prediction error is studied by calculating the normalized mean squared error (NMSE), which is defined as

$$\mathrm{NMSE}(N) = \frac{\sum_{k\in\Lambda}[\mathrm{x}_k - \hat{x}_k]^2}{\sum_{k\in\Lambda}[\mathrm{x}_k - \bar{x}_k]^2} \approx \frac{1}{\hat{\sigma}_\Lambda^2}\frac{1}{N}\sum_{k\in\Lambda}(x_k - \hat{x}_k)^2, \tag{18}$$

where x_k and $\hat{x}_k$ are the actual and predicted kth point of the series of length N. $\hat{x}_k$ is the predicted value, and $\bar{x}_k$ and $\hat{\sigma}_\Lambda^2$ denote the sample average and sample variance of the actual values (targets) in Λ, respectively.

4.5 Comparisons of the Prediction Errors

Different predictions with the method using sequential time series in [18] and the proposed MANFIS for the periodic function in (14) and the chaotic signals in (15)-(17) have been performed. The coefficients used in the MANFIS and the prediction results are given in Figs. 2, 3, 4, 5, 6, 7, 8, 9, and 10, and Tables 1, 2, and 3, in which m_f is the number of membership functions used, t_s is the number of set used, and t_e is the number of training epoch spent. All testing results given in this section are the average performance after 10 runs. The training data used in each prediction is taken from different sections. For example, the first prediction is based on the first 100 data, the second prediction is based on the 101st to 200th data, the third prediction is based on the 201st to 300th data, and so on.

4.5.1 Periodic Time Series

As shown in Figs. 2, 3, and 4, the prediction errors with EPS preprocessing are lower than the ones without EPS preprocessing. In other words, less number of training sets is required in MANFIS with EPS to achieve the same error in MANFIS without EPS. For example, comparing the one with EPS in Fig. 2 and the one without EPS in Fig. 4, the former

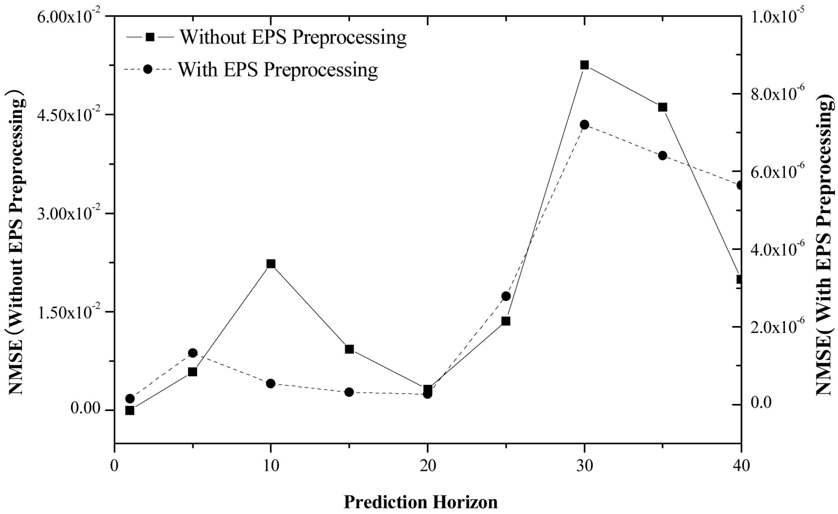

Fig. 4. Periodic time series prediction with $m_f = 2$, $t_s = 500$, and $t_e = 500$.

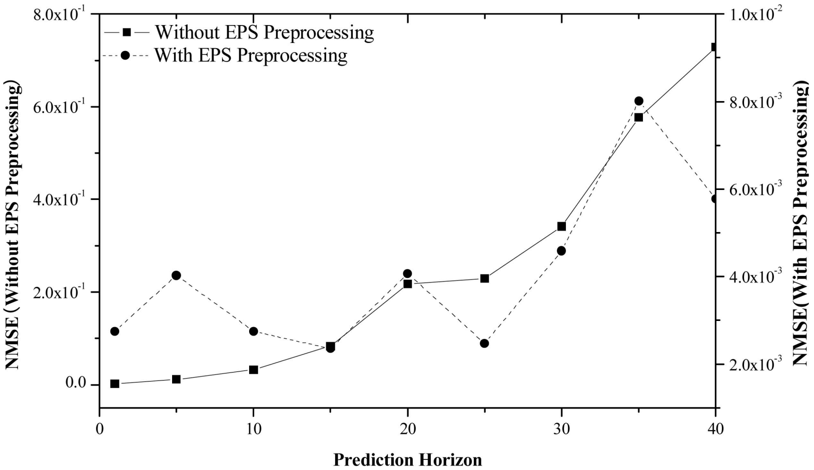

Fig. 5. Mackey-Glass time series prediction with $m_f = 2$, $t_s = 100$, and $t_e = 500$.

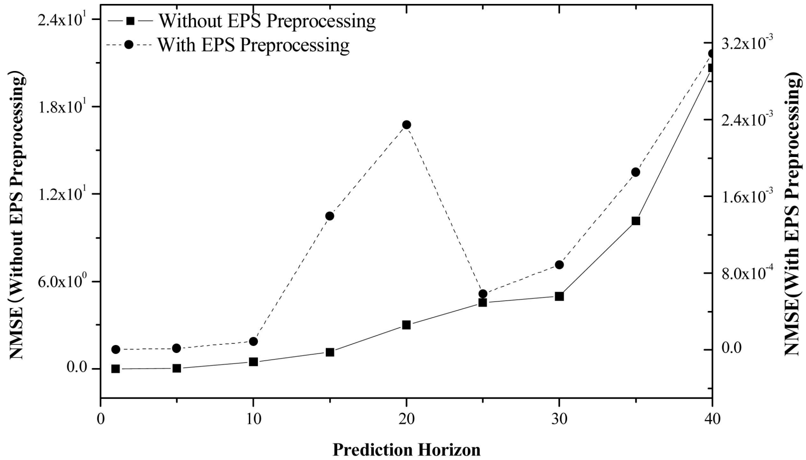

Fig. 6. Mackey-Glass time series prediction with $m_f = 2$, $t_s = 300$, and $t_e = 500$.

one has 100 training sets and the latter one has 500 training sets. However, the former one gives a maximum error of 0.00218 only over the prediction horizon, while the latter one gives a maximum error of 0.0525. As shown in Table 1, all t-test values are high, showing that EPS preprocessing can help improve the prediction performance.

4.5.2 Chaotic Time Series—Mackey-Glass Function

As shown in Figs. 5, 6, and 7, the prediction errors without EPS preprocessing are higher than the ones with the EPS preprocessing. Even if the training set is large, the chaotic properties of the Mackey-Glass series cannot

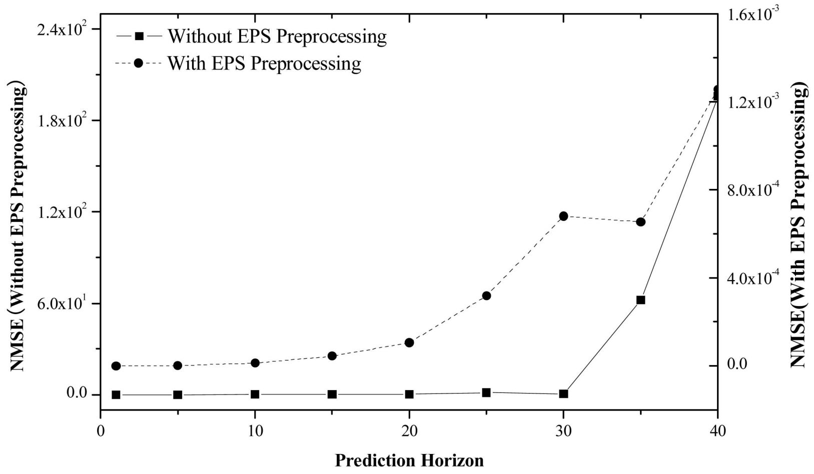

Fig. 7. Mackey-Glass time series prediction with $m_f = 2$, $t_s = 500$, and $t_e = 500$.

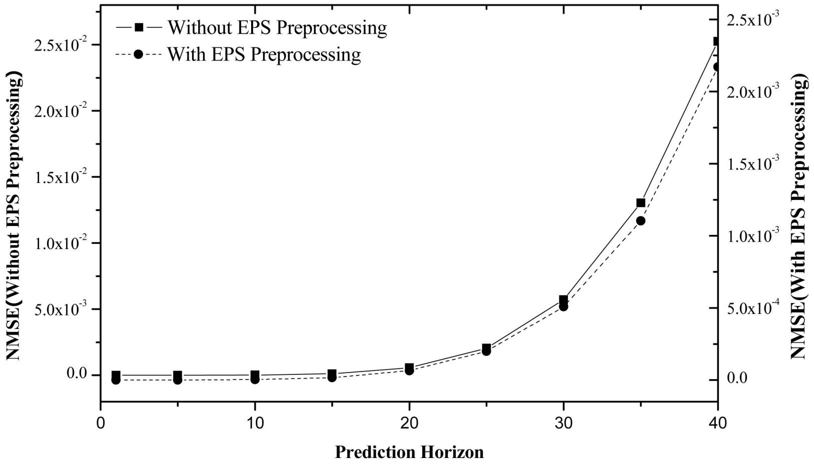

Fig. 8. Duffing forced-oscillation system time series prediction with $m_f = 2$, $t_s = 300$, and $t_e = 500$.

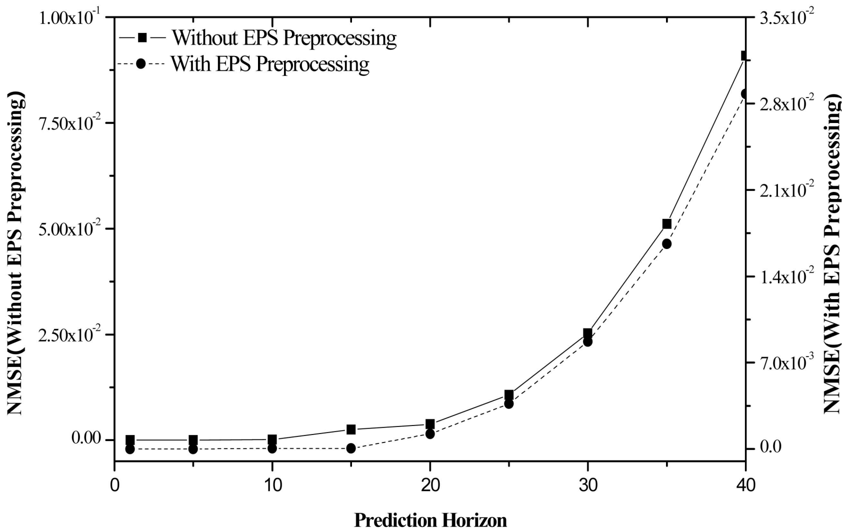

Fig. 9. Duffing forced-oscillation system time series prediction with $m_f = 2$, $t_s = 500$, and $t_e = 500$.

be predicted without EPS. The maximum error without EPS in the prediction horizon is 180 in Fig. 7. However, with EPS, the MANFIS is able to keep the maximum prediction errors below 0.00802 in Fig. 5, 0.00309 in Fig. 6, and 0.00126 in Fig. 7, respectively. Again, as shown in Table 2, all t-test values are high, showing that EPS preprocessing can help improve the prediction performance.

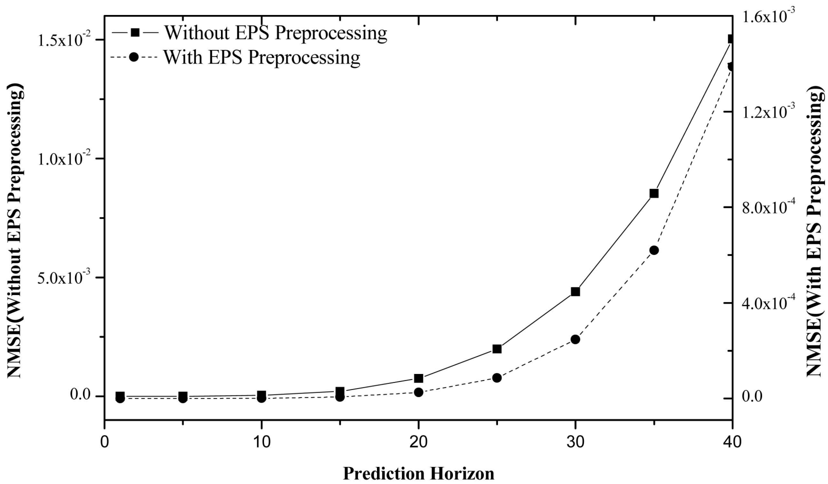

Fig. 10. Duffing forced-oscillation system time series prediction with $m_f = 2$, $t_s = 700$, and $t_e = 500$.

TABLE 1
T-Test between the Prediction Errors with and without EPS of the Time Series Given in (14)

t_s	Prediction horizon	Without EPS		With EPS		Without EPS-With EPS T-Test Value
		Mean	Std Dev	Mean	Std Dev	
100	10	4.47008×10^{-1}	9.53401×10^{-4}	1.89798×10^{-4}	5.32416×10^{-7}	14.8200199
	20	5.96940×10^{-2}	3.37876×10^{-4}	1.23389×10^{-4}	4.08304×10^{-7}	55.844846
	30	1.80549	1.31975×10^{-2}	4.17147×10^{-4}	3.09217×10^{-6}	43.256143
	40	5.06966×10^{-1}	7.08532×10^{-3}	2.24500×10^{-3}	3.53553×10^{-5}	22.524115
300	10	1.00623	9.32893×10^{-4}	3.31239×10^{-6}	2.11430×10^{-8}	34.1487494
	20	3.85700×10^{-3}	3.16403×10^{-5}	2.82016×10^{-6}	1.87428×10^{-8}	39.376101
	30	6.63450×10^{-2}	4.82062×10^{-4}	6.10008×10^{-5}	6.63836×10^{-7}	43.455821
	40	3.69552×10^{-1}	7.79141×10^{-3}	4.95678×10^{-5}	9.07756×10^{-7}	14.994707
500	10	2.24360×10^{-2}	1.33932×10^{-4}	5.45025×10^{-7}	2.34200×10^{-9}	53.261682
	20	3.29300×10^{-3}	4.49815×10^{-5}	2.63643×10^{-7}	1.30243×10^{-9}	22.872306
	30	5.34930×10^{-2}	7.18085×10^{-4}	7.28373×10^{-6}	7.61871×10^{-8}	23.560655
	40	2.03270×10^{-2}	4.71052×10^{-4}	5.70137×10^{-6}	4.13349×10^{-8}	13.664227

TABLE 2
T-Test between the Prediction Errors with and without EPS of the Time Series Given in (15)

t_s	Prediction horizon	Without EPS		With EPS		Without EPS-With EPS T-Test Value
		Mean	Std Dev	Mean	Std Dev	
100	10	3.29080×10^{-2}	1.66853×10^{-4}	2.76100×10^{-3}	2.46982×10^{-4}	57.305401
	20	2.19295×10^{-1}	2.46136×10^{-3}	4.08400×10^{-3}	1.07497×10^{-3}	27.645367
	30	3.46980×10^{-1}	5.13617×10^{-3}	4.63800×10^{-3}	5.22388×10^{-3}	21.073711
	40	7.45189×10^{-1}	1.57896×10^{-2}	5.89100×10^{-3}	1.18926×10^{-3}	14.807255
300	10	4.67812×10^{-1}	2.45290×10^{-3}	8.83348×10^{-5}	2.46882×10^{-3}	60.344756
	20	3.01679	1.91919×10^{-2}	2.35600×10^{-3}	6.99206×10^{-3}	49.674522
	30	5.00116	2.68399×10^{-2}	8.99574×10^{-4}	1.17239×10^{-3}	58.916386
	40	2.09829×10^{1}	3.67859×10^{-1}	3.13700×10^{-3}	3.12872×10^{-3}	18.035202
500	10	7.43610×10^{-2}	4.66344×10^{-4}	1.27982×10^{-5}	4.95538×10^{-4}	50.425709
	20	1.80021×10^{-1}	1.67905×10^{-3}	1.06061×10^{-4}	5.98423×10^{-3}	33.908296
	30	5.69207×10^{-1}	3.90436×10^{-3}	6.89168×10^{-4}	5.31792×10^{-4}	46.042429
	40	2.00467×10^{2}	3.45446	1.28200×10^{-3}	1.87380×10^{-2}	18.350977

TABLE 3
T-Test between the Prediction Errors with and without EPS of the Time Series Given in (16) and (17)

t_s	Prediction horizon	Without EPS		With EPS		Without EPS-With EPS T-Test Value
		Mean	Std Dev	Mean	Std Dev	
300	10	1.29901×10^{-5}	3.64395×10^{-6}	2.86635×10^{-6}	2.20001×10^{-6}	7.52107
	20	5.60263×10^{-4}	1.40878×10^{-4}	6.50711×10^{-5}	3.62333×10^{-5}	10.76518
	30	5.71000×10^{-3}	1.13475×10^{-3}	5.08991×10^{-4}	2.13568×10^{-4}	14.24384
	40	2.52500×10^{-2}	3.40987×10^{-3}	2.16992×10^{-3}	7.41134×10^{-4}	20.91588
500	10	1.71625×10^{-4}	1.54416×10^{-4}	4.02306×10^{-5}	1.63548×10^{-5}	2.67585
	20	3.79895×10^{-3}	3.11834×10^{-3}	1.22223×10^{-3}	3.04044×10^{-4}	2.60069
	30	2.52727×10^{-2}	1.71085×10^{-2}	8.72000×10^{-3}	2.29289×10^{-3}	3.03244
	40	9.08900×10^{-2}	6.05052×10^{-2}	2.87800×10^{-2}	9.28952×10^{-3}	3.20856
700	10	3.56087×10^{-5}	1.02674×10^{-6}	1.25373×10^{-6}	1.02146×10^{-6}	75.01221
	20	7.46392×10^{-4}	3.33840×10^{-5}	2.62211×10^{-5}	1.71206×10^{-5}	60.70089
	30	4.40300×10^{-3}	2.43450×10^{-4}	2.47351×10^{-4}	1.34161×10^{-4}	47.21257
	40	1.50280×10^{-2}	9.15494×10^{-4}	1.38752×10^{-3}	6.27955×10^{-4}	38.88513

4.5.3 Chaotic Time Series—Duffing Forced-Oscillation System

In this simulation example, the prediction errors without EPS preprocessing are higher than the ones with the EPS preprocessing as shown in Figs. 8, 9, and 10. The behavior of the Duffing forced-oscillation system cannot be predicted without EPS. The maximum error without EPS in the prediction horizon is about 0.02525 in Fig. 8, 0.09089 in Fig. 9, and 0.01503 in Fig. 10. However, with EPS, the MANFIS is able to keep the maximum prediction errors about 0.00217 in Fig. 8, 0.02878 in Fig. 9, and 0.00139 in Fig. 10, respectively. For the Duffing forced-oscillation system, the t-test results are shown in Table 3 and have successfully demonstrated the advantages of EPS preprocessing again.

5 Conclusions

A MANFIS with EPS preprocessing for chaotic time series prediction has been presented. By applying the delay coordinate embedding technique to form the input vector for the MANFIS, the system can efficiently learn and predict a chaotic time series. The proposed system has been tested with periodic and chaotic time series. The prediction accuracy can be significantly improved with the EPS preprocessing.

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