An Investigation into the Use of Delay Coordinate Embedding Technique with MIMO ANFIS for Nonlinear Prediction of Chaotic Signals

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Abstract. This paper presents an investigation into the use of the delay coordinate embedding technique with multi-input multi-output (MIMO) adaptive-network-based-fuzzy-inference system (ANFIS) to learn and predict the continuation of chaotic signals ahead in time. Based on the average mutual information and global false nearest neighbors techniques, the optimal values of the embedding dimension and the time delay are selected to construct the trajectory on the phase space. The MANFIS technique is trained by gradient descent algorithm. First, the parameter set of the membership functions is generated with the embedded phase space vectors using the back-propagation algorithm. Second, fine-tuned membership functions that make the prediction error as small as possible are built. The model is tested with both periodic and the Mackey-Glass chaotic time series. Moving root-mean-square error is used to monitor the error along the prediction horizon.

1 Introduction

A time series is a sequence of regularly sampled quantities out of an observed system. It provides a useful basis for discovering some of its underlying characteristics, such as periodicity, stochastic distribution, etc. Many time- and frequency-domain prediction methods have been proposed 1. However, in practice, nonlinear chaotic time series are regularly observed in various natural phenomena 13. Since 1970's, chaotic time series prediction has been a popular subject 4 for understanding and controlling the chaotic behaviors to advantage, such as the stock market forecasting 5. Basically, prediction of chaotic time series requires a representative model. In recent years many new prediction approaches, such as the fuzzy predictor 6-13 and time-delay embedding technique 14, have emerged. They provide new insight into this type of systems not available with the traditional methods, such as linear regression technique and auto-regressive integrated moving average models, etc.

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As discussed in 9, the adaptive-network-based fuzzy inference system (ANFIS) with multi-input and single-output in 11 that uses Gaussian membership functions and employs hybrid back-propagation learning of a chaotic time series gives the smallest root-mean-square errors (RMSE) among various fuzzy predictors. However, similar to the other single scale chaotic time series prediction methods, the prediction horizon is usually limited by the fast-varying components, as the methods are based on the finite neighborhood relationships in the prediction. Nevertheless, it forms the best basis for further enhancement. On the other hand, time delay coordinate embedding methods 14-17 use the relationships between the delay coordinates of a point and the points that appear at some time later in the phase space. Its trajectories behave with quasi-periodicity. The nearest trajectories can contribute to the neighboring set with more than one point, resulting in an increased weighting of the contribution coming from the nearest trajectories.

This paper investigates the use of delay coordinate embedding technique with multi-input multi-output (MIMO) ANFIS to learn and predict the continuation of chaotic signals ahead in time. The methodology hybridizes the advantages of ANFIS and the time-delay coordinate embedding technique. The resulting model has better predictive performance with fewer membership functions. Based on the average mutual information and false nearest neighbors technique, the minimum time-delay embedding dimension is selected and phase space is constructed. The ANFIS is trained by gradient descent algorithm. First, the parameter set of the membership functions is generated with the embedded phase space vectors using the back-propagation algorithm. Second, fine-tuned membership functions that make the prediction error as small as possible are built. Section II describes the construction of the chaos-embedded phase space. Section III describes the operations of the MIMO-ANFIS technique. Section IV shows the procedures of the prediction algorithm. Section V gives the simulation results using the proposed method and the one in 11 having different number of membership functions to predict the Mackey-Glass chaotic time series. Moving root-mean-square error is used to monitor the error along the prediction horizon. Section VI contains some concluding remarks.

2 Construction of the Chaos Embedded Phase Space

Section V gives the simulation results using the proposed method and the one in 11 having different number of membership functions to predict the Mackey-Glass chaotic time series. Moving root-mean-square error is used to monitor the error along

A chaotic time series shows stochastic behavior in the time- and frequency-domain and deterministic behaviors in phase space structure 1. The trajectory of the attractors will repeat on the phase space with same initial conditions. Consider a time series $x(1), x(2), x(3), \ldots x(N)$, an embedded vector $y(i)$ is defined as

$$y(i) = [x[i+(D-1)\tau] \quad \ldots \quad x(i+\tau) \quad x(i)] \tag{1}$$

where $1 \le i \le N-(D-1)\tau$. $y(i)$ is a D-dimensional vector consisting of past signal samples, in which D is referred to the embedding dimension. τ is the time delay of the samples. $y(i)$ represents one point on a D-dimensional phase space R^D. A trajectory Y

$$Y = [\,y(1) \quad y(2) \quad \ldots \quad y(N-(D-1)\tau)]^T \tag{2}$$

will be formed on R^D. Theoretically, an embedding of the original trajectory can be obtained for sufficiently large value of D with any value of τ. In practice, if D is too large, noise in the data may reduce the density of points defining the attractor and increase the level of contamination of the data unnecessarily 18. However, if D is too small, the attractor will be folded. Moreover, the choice of τ is another crucial parameter to establish data correlation in the embedded vector. If τ is too small, the elements in each embedded vector will not be independent enough. On the contrary, if τ is too large, the relationships among elements in each embedded vector behave as a set of random data statistically 1. Thus, optimal values of D and τ have to be determined in order to efficiently extract the original behaviors of the chaotic system. In this paper, optimal values of τ and D are selected by using the average mutual information20 and the global false nearest neighbors 1, respectively.

A. Determination of τ

Average mutual information $I(\tau)$ is used to determine the nonlinear autocorrelation between $x(i)$ and $x(i+\tau)$ in order to form coordinates in a time delay vector. That is,

$$I(\tau) = \sum_{i=1}^{N-\tau} \sum_{j=\tau+1}^{N} P(x(i), x(j)) \log_2 \frac{P(x(i), x(j))}{P(x(i))\, P(x(j))} \,. \tag{3}$$

where $P(x(i))$ and $P(x(j))$ are the individual probability densities in values of $x(i)$ and $x(j)$, respectively and $P(x(i), x(j))$ is the joint probability density in values of $x(i)$ and $x(j)$.

The procedures are started with a value of one for τ and the corresponding value of $I(\tau)$ is calculated by (3). τ is then incremented until there is a sign change in the variation of $I(\tau)$ with respect to τ. That is, τ is chosen when

$$\mathrm{sgn}[I(\tau) - I(\tau-1)] \neq \mathrm{sgn}[I(\tau+1) - I(\tau)] \,. \tag{4}$$

B. Determination of D

For a given value of τ, a statistical technique is used to determine the minimum value of D that can unfold the reconstructed attractor. The procedures are as follows,

1) A dimension of 2 is firstly chosen as the initial guess.

2) All embedded vectors are formulated by (1) with the assumed phase space dimension.

3) The Euclidean distance between one vector and the others is calculated and a matrix containing all the distances (say $\vec{M}$) for the given dimension is formulated. For example, the distance between the vectors $y(i)$ and $y(j)$ is equal to

$$m_{i,j} = m_{j,i} = \sqrt{\sum_{k=1}^{D} [x(i-(k-1)\tau) - x(j-(k-1)\tau)]^2} \tag{5}$$

where $1 \le i \le N-(D-1)\tau$ and $1 \le j \le N-(D-1)\tau$. $m_{i,j}$ is a matrix element at *i*th row and *j*th column of $\vec{M}$.

4) By considering the smallest row element in each column of $\vec{M}$, a pair of embedded vectors is chosen for the corresponding column. For example, for a generic column *g*, an embedded vector pair of $[y(g), y(h)]$ is chosen if the element in the *h*th row is the smallest along the column. This procedure is performed for all columns in $\vec{M}$.

5) Two modified embedded vectors $y'(g)$ and $y'(h)$ are formulated with a higher dimension of (D + 1). That is,

$$y'(g) = [x(g-D\tau) \quad x[g-(D-1)\tau] \quad \ldots \quad x(g+\tau) \quad x(g)] \tag{6a}$$

$$y'(h) = [x(h-D\tau) \quad x[h-(D-1)\tau] \quad \ldots \quad x(h+\tau) \quad x(h)] \tag{6b}$$

6) A normalized value $\rho(D)$ showing the change in the distance of $[y(g), y(h)]$ and the distance of $[y'(g), y'(h)]$ is used to test the existence of any fold in the trajectory between $y(g)$ and $y(h)$ with D dimensions. That is,

$$\rho(D) = \frac{|x(g-D\tau) - x(h-D\tau)|}{m_{g,h}} \tag{7}$$

If $\rho(D)$ is larger than a threshold value Φ_{thres}, such as 10, it implies that a fold exists between $y(g)$ and $y(h)$. The embedded pair $[y(g), y(h)]$ is a false neighbor. An index λ is used to count the number of false neighbors in $\vec{M}$ for the dimension D. Thus,

$$\text{if } \rho(D) > \Phi_{thres}, \; \lambda(D) = \lambda(D) + 1. \tag{8}$$

7) The above procedures will be repeated from step 2) with a new dimension of (D + 1). Theoretically, the maximum value of D is up to an integer of ($\frac{N-2}{\tau}$). In this method, D is chosen when λ is in the first minimum. That is, it is in the condition of

$$\operatorname{sgn}[\lambda(D) - \lambda(D-1)] \neq \operatorname{sgn}[\lambda(D+1) - \lambda(D)]. \tag{9}$$

3 Operations of the MIMO-ANFIS

Prediction of chaotic time series using fuzzy neural system has been investigated in 21. In this paper, we showed that better performance can be achieved if prediction is done in embedding phase space instead of time domain. A Multi-Input Multi-Output Adaptive Neural-Fuzzy Inference System (MANFIS) is developed for predicting the chaotic time series in embedding phase space. Based on the ANFIS model 11, the

MANFIS is extended to generate multi-dimensional vector. A fuzzy rule set is applied to model the system, which maps precisely the input vectors (embedding phase space) to the output vectors. The fuzzy rule set consists of a series of IF-THEN rules operating on some fuzzy variables. These fuzzy variables are described by the corresponding membership functions, which is tuned by a gradient descent algorithm using collections of input-output vector pairs. Knowledge acquisition is achieved by multiplying the fuzzy quantities.

The topology of MANFIS is showed in Fig. 1. It can be partitioned, according to functionality, into the following sections.

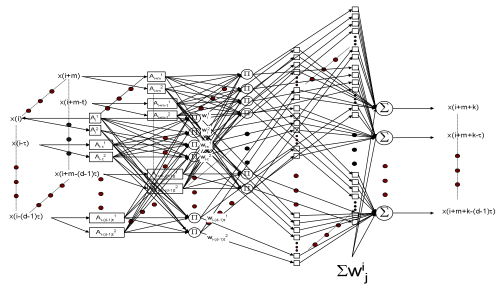

Fig. 1. Proposed Structure of the MANFIS

A. The Input Section

Input to the MANFIS is a matrix with the embedding phase vector as columns and the chaotic time series as rows. An embedding phase space vector, $\overline{y}(i)$, is reconstructed for every single point on the time series $\{x(1), x(2), \cdots x(N)\}$. These vectors are reconstructed by embedding time delay method described in Sec. *II*. If d is the dimension of the embedding phase space, input to the MANFIS is a $(m+1) \times d$ matrix. For instance, if the embedding time delay is T, the phase space vector is given by:

$$[x(i), x(i-T), \cdots, x(i-(d-1)\cdot T)], \qquad \forall i \in [1+(d-1)\cdot T, N] \tag{10}$$

and, the input matrix is:

$$\begin{bmatrix} x(i) & x(i+1) & \cdots & x(i+m) \\ x(i-T) & x(i+1-T) & \cdots & x(i+m-T) \\ \vdots & \vdots & \vdots & \vdots \\ x(i-(d-1)\cdot T) & x(i+1-(d-1)\cdot T) & \cdots & x(i+m-(d-1)\cdot T) \end{bmatrix} \tag{11}$$

where k is the number of prediction steps, and $i \in [1+(d-1)\cdot T, N-m-k]$.

B. The Fuzzifier Section

To be able to utilize fuzzy reasoning on the prepared input data, knowledge representation should be applied to each of the elements of the matrix. This is done by feeding the matrix elements to the fuzzifier that consists of two fuzzy membership functions. Each of these membership functions represents a linguistic label, which will be used in the fuzzy rule to generate the corresponding knowledge. Fuzzifying the matrix elements will quantify how important such a value is inside the particular linguistic label. The fuzzy values will then be used in the knowledge acquisition and reasoning sections. Gaussian distribution fuzzifier is used as the membership function MANFIS. The equation of the membership function is as follows,

$$A_o^p = \mu_p\left(x_o; c_p, \sigma_p\right) = \exp\left(-\frac{1}{2}\left(\frac{x_o - c_p}{\sigma_p}\right)^2\right) \tag{12}$$

where, o is the number of element of the input matrix,
p is the number of membership functions,
c and σ are parameters that determine the center and width of the membership function.

The knowledge, which is represented by linguistic labels, is vital to the fuzzy reasoning part , hence, the output of the whole system. Therefore, the choice of membership function and its distribution will have a direct impact on the overall system behavior and performance. Thus, c and σ must be tuned carefully with gradient decent algorithm.

C. The Knowledge Acquisition Section

Since it is very difficult to acquire enough knowledge from the chaotic time series to construct the rule-base, no IF-THEN rule set is used in MANFIS. Instead of relating the linguistic qualifiers with IF-THEN rules, weighted sum of products of the fuzzy values is used. The following equation is used to quantify the knowledge contributed by each element of the embedding phase space vector.

$$w_o^p = \prod_{i+g-(d-1)T \le o \le i+g} A_o^p , \tag{13}$$

where g is the number of columns in the input matrix, $(i+g-(d-1)T) \le o \le (i+g)$, $0 \le g \le m$ $1 \le p \le 2$. In fact, w_o^p represents the firing strength of a rule.

D. The Knowledge Reasoning Section

$$f_u^g = \sum_{v=i+g-(d-1)T}^{i+g} a_u^v x(v) \tag{14}$$

where $i+g-(d-1)T \le u \le i+g$, $0 \le g \le m$,
f_u^g is calculated using all elements in the g^{th} vector,
a_u^v is the consequent parameter set,
$x(v)$ is the v^{th} element in g^{th} vector of input matrix.

$$OK = F_{g,u}^{p,h} = w_h^p \cdot f_u^g \tag{15}$$

where $1 \le p \le 2$, $(i+g-(d-1)T) \le u \le (i+g)$, $(i+g-(d-1)T) \le h \le (i+g)$, $0 \le g \le m$.

E. The Defuzzifier Section

The function of this section is to map from the fuzzy set to the real-value points by calculating the centroid of each fuzzy variable. In the MANFIS, it is the output vector elements, which are calculated in (16):

$$OD_j = \frac{\sum\limits_{\substack{0\le g\le m \\ p=1\sim 2 \\ i+m-(d-1)T\le u\le i+m \\ j+g-(d-1)T\le h\le j+g}} F_{g,u}^{p,h}}{\sum\limits_{\substack{o=(i+g-(d-1)T)\sim(i+g) \\ p=1\sim 2 \\ g=0\sim m}} w_o^p} \tag{16}$$

where $j=(i+m+k)\sim(i+m-(d-1)T+k)$, F is calculated from (15) and the output vector matrix is

$$\begin{bmatrix} x(i+m+k) \\ x(i+m+k-T) \\ \vdots \\ x(i+m+k-(d-1)\cdot T) \end{bmatrix}, i\in\left[(d-1)\cdot T, N-m-k\right] \tag{17}$$

F. The learning algorithm

The MANFIS is trained with data obtained in both time domain and phase space. To achieve accurate prediction gradient descent or back-propagation algorithm is used to tune the membership functions and the consequent parameters. σ and c are the parameters of the membership functions, adjustment to these parameters are determined according to the gradient between the actual and expected output. That is,

$$\sigma_i^p(t+1) = \sigma_i^p(t) - \eta\frac{\partial E_i}{\partial \sigma_i(t)} \tag{18}$$

$$c_i^p(t+1) = c_i^p(t) - \eta\frac{\partial E_i}{\partial c_i(t)} \tag{19}$$

where p is the number of membership functions,
i is the number of node,
η is a constant determining the learning rate,
E is the error measure for the train data.

G. Simulation results and discussion

In order to show that the performance of MANFIS can be improved by applying embedding phase space transformation to the input data, two types of simulations have been carried out. The first one is a periodic time series and the second one is a chaotic time series (Mackey-Glass chaotic time series 20). The input data will be represented in both time domain and embedding phase space domain. Different numbers of training sets (100, 300, and 500) are used. The corresponding prediction errors are compared.

1. *Periodic time series*

The periodic time series equation is under investigation in

$$x(t) = \frac{1}{w}\sum_{i=a}^{b}\sin(k \cdot i \cdot t) \tag{20}$$

where w =5, k =0.01, a =1 and b =5. The time series is shown in Fig. 2.

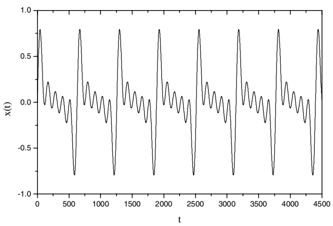

Fig. 2. Periodic Time Series of (20)

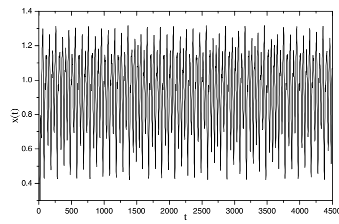

Fig. 3. Mackey-Glass chaotic time series

2. *Chaotic time series: Mackey-Glass equation*

The following Mackey-Glass equation (21) has been shown to be chaotic in 19 and 20 is investigated in this paper. The time series is shown in Fig. 3.

$$\frac{dx(t)}{dt} = \frac{0.2 \cdot x(t-\tau)}{1 + x^{10}(t-\tau)} - 0.1 \cdot x(t) \tag{21}$$

where $\tau = 17$, $x(0) = 1.2$.

It is a time-delay ordinary differential equation, which displays well-understood chaotic behavior with dimensionality dependent upon the chosen value of the delay parameter. The time series generated by the Mackey-Glass equation has been used as a test bed for a number of new adaptive computing techniques.

3. *Error Estimation:*

To compare the accuracy we compute the normalized mean squared error (NMSE):

$$\text{NMSE}(N) = \frac{\sum_{k\in\Lambda}\left(actual_k - prediction_k\right)^2}{\sum_{k\in\Lambda}\left(actual_k - mean_\Lambda\right)^2} \approx \frac{1}{\hat{\sigma}_\Lambda^2}\frac{1}{N}\sum_{k\in\Lambda}\left(x_k - \hat{x}_k\right)^2 \tag{22}$$

where x_k is the k^{th} point of the series of length N. $\hat{x}_k$ is the predicted value, and $mean_\Lambda$ and $\hat{\sigma}_\Lambda^2$ denote the sample average and sample variance of the actual values (targets) in Λ.

4. *Comparisons of the prediction errors*

Several simulations on predicting the periodic function in (20) and the chaotic signals in (21) have been performed. All MANFIS coefficients and the prediction results are shown in Fig. 4 – 15 and in Table I & Table II, in which *mf* represents the number of membership functions used, *ts* represents the number of set used, and *te* represents the number of training epoch spent periodic. The following observations can be noted.

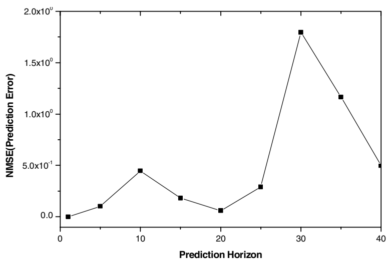

Fig. 4. Periodic time Series Prediction in Time Domain. (mf=2,ts=100,te=500).

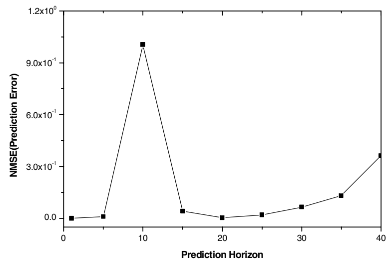

Fig. 5. Periodic time Series Prediction in Time Domain. (mf=2,ts=300,te=500).

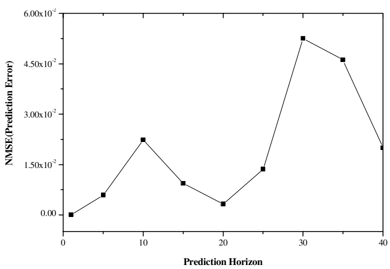

Fig. 6. Periodic time Series Prediction in Time Domain. (*mf*=2,*ts*=500,*te*=500).

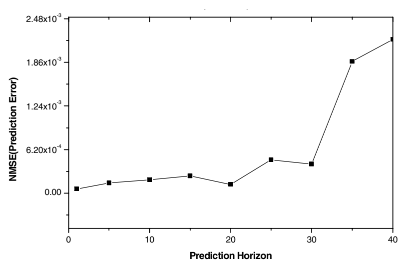

Fig. 7. Periodic time Series Prediction in Phase Space Domain. (*mf*=2,*ts*=100,*te*=500).

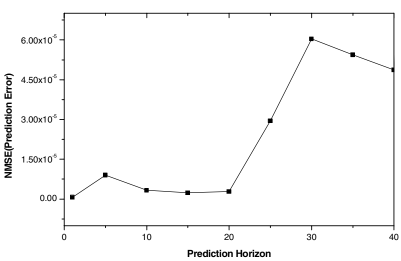

Fig. 8. Periodic time Series Prediction in Phase Space Domain.(mf=2,ts=300,te=500).

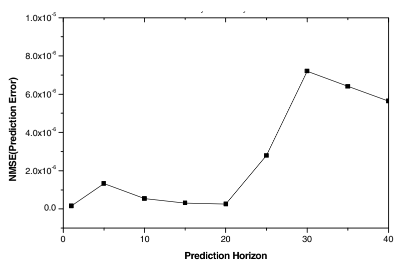

Fig. 9. Periodic time Series Prediction in Phase Space Domain. (mf=2,ts=500,te=500).

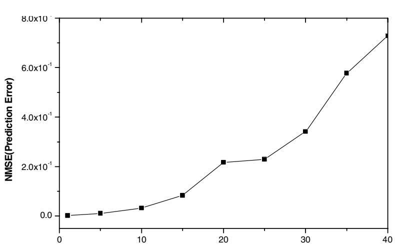

Fig. 10. Chaotic time Series Prediction in Time Domain. (mf=2,ts=100,te=500).

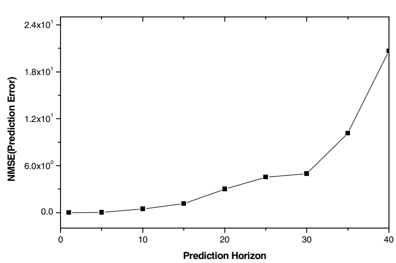

Fig. 11. Chaotic time Series Prediction in Time Domain. (mf=2,ts=300,te=500)/

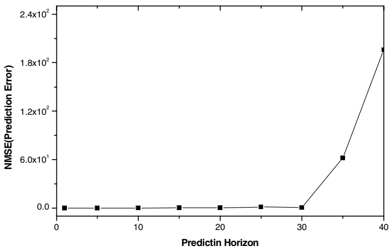

Fig. 12. Chaotic time Series Prediction in Time Domain. (mf=2,ts=500,te=500).

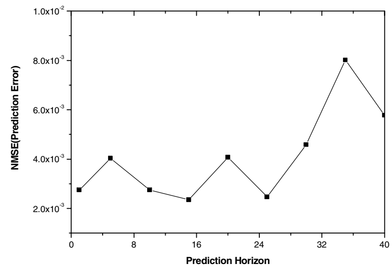

Fig. 13. Chaotic time Series Prediction in Phase Space Domain. (mf=2,ts=100,te=500).

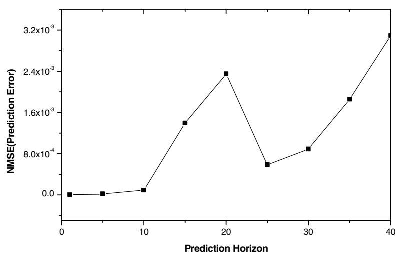

Fig. 14. Chaotic time Series Prediction in Phase Space Domain. (mf=2,ts=300,te=500).

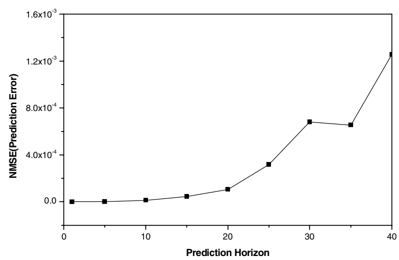

Fig. 15. Chaotic time Series Prediction in Phase Space Domain. (mf=2,ts=500,te=500).

a. Periodic time series prediction

Figs. 4 - 6 show the prediction errors of the periodic time series using the method in 10 with different values of *mf*, *ts*, and *te*. Figs. 7 - 9 show the ones using the proposed method with the same simulation parameters. Compared Figs. 4 – 6 with Figs. 7 – 9, respectively, the prediction errors with embedding phase space preprocessing are lower many times than the ones without embedding phase space preprocessing using the same simulation parameters.

In other words, less number of training sets is required in MANFIS with embedding phase space to achieve the same error in MANFIS without embedding phase space. For example, comparing Fig. 6 and Fig. 7, the former one has 500 training sets and the latter one has 100 training sets. However, the former one gives a maximum error of 0.0525 over the prediction horizon, whilst the latter one gives a maximum error of 0.00218 only.

b. Chaotic time series prediction

Figs.10 - 12 show the prediction errors of the chaotic time series using the method in 10. Figs.13 - 15 show the ones using the proposed method with the same simulation parameters. The prediction errors without embedding phase space preprocessing are high. Even if the training set is large, the chaotic properties of the Mackey-Glass series cannot be predicted. The maximum error in the prediction horizon is 61.973 in Fig. 12. However, with the use embedding phase space, the MANFIS is able to keep the maximum prediction errors below 0.0080 in Fig. 13, 0.0031 in Fig. 14, and 0.00013 in Fig. 15, respectively.

4 Conclusions

It is very difficult to perform accurate prediction on nonlinear or chaotic series such as Mackey-Glass time series. It has been proved that only adaptive fuzzy system cannot give satisfactory prediction results. The use of delay coordinate embedding technique with simple adaptive fuzzy system, as proposed in this paper, can enhance the prediction. The structure of a multi-input multi-output ANFIS (MANFIS) with two membership functions has been investigated. The system was trained with backpropagation learning algorithm. Simulation results show that prediction accuracy of a nonlinear system can be significantly improved by preprocessing the time series data with delay coordinate embedding technique. Moreover, the required training set can also be reduced.

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