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Time series prediction using Lyapunov exponents in embedding phase space

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Abstract

This paper describes a novel method of chaotic time series prediction, which is based on the fundamental characteristic of chaotic behavior that sensitive dependence upon initial conditions (SDUIC), and Lyapunov exponents (LEs) is a measure of the SDUIC in chaotic systems. Because LEs of chaotic time series data provide a quantitative analysis of system dynamics in different embedding dimension after embedding a chaotic time series in different embedding dimension phase spaces, a novel multi-dimension chaotic time series prediction method using LEs is proposed in this paper. This is done by first reconstructing a phase space using chaotic time series, then using LEs as a quantitative parameter to predict an unknown phase space point, after transferring the phase space point to time domain, the predicted chaotic time series data can be obtained. The computer simulation result of simulation showed that the proposed method is simple, practical and effective.

1. Introduction

Time series prediction is an important practical problem. Application of time series prediction can be found in the areas of economic and business planning, inventory and production control, weather forecasting, signal processing, control and many other fields: knowing the past, how to

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predict the future? Time series themselves exhibit reasonably well-understood behaviors. Often, a time series is composed of a long-term trend plus various periodic and random components. The prediction of the random component is often the focus of the time series predicting problem. The apparently random component of a time series usually falls into one of two categories. In the first case, the apparently random component is truly random; that is, the measurements are drawn from some underlying probability distribution. The second class of apparently random behavior in time series is not random at all, but rather, chaotic. A chaotic time series is characterized by values that appear to be randomly distributed and nonperiodic but are actually the result of a completely deterministic process. The deterministic behavior in a chaotic time series is usually due to underlying nonlinear dynamics. In fact, chaotic time series display some stochastic behavior in time domain and display some determined behavior in embedding phase space.

Nonlinear and chaotic time series analysis and prediction is becoming a more and more reliable tool for the study of complicated dynamics from measurements. Therefore, analysis and prediction of chaotic time series take a very important role in chaos research including chaos control and chaos application. The aim of chaotic time series prediction is to accurately predict the short-term evolution of the chaotic system.

Before the 1920s, predicting was done by simply extrapolating the series through a global fit in the time domain [1]. The beginning of "modern" time series prediction might be set at 1927 when Yule invented the auto-regressive technique in order to predict the annual number of sunspots. For the half-century following Yule, the reigning paradigm remained that of linear models driven by noise.

State-space reconstruction method is by the analysis of the geometry of the embedded data to discovering the relationship between past and future points in a time series. Farmer and Sidorowich [2] report on a direct implementation of this idea, and discuss the scaling of prediction error for several artificial and experimental time series. Their idea is to recognize that any manifold is locally linear. After embedding a time series in a state space using delay co-ordinates, they "learn" the induced nonlinear mapping using a local approximation. This method allows us to make a short-term prediction of the future behavior of a time series, using information based on past values. Casdagli [3] compares local linear approximations to the dynamics with global approximations in embedding phase space, in the form of radial basis function.

Different from the above methods, a new method of predicting the chaotic time series is proposed in this paper. The method follows the Taken's [4] method to reconstruct embedding phase space using chaotic time series, firstly. Then the characteristic of chaotic behavior is used to predict the unknown phase space point. After recovering the predicted phase space point to time domain, the required predicting time series data can be obtained. The main difference between this paper and [2,3] is the following aspects:

(1) In the proposed method, the fundamental characteristic of chaos is used to estimate the unknown points in embedded phase space.
(2) Multi-dimension embedding phase space method is used to improve the accuracy of prediction.

This paper is organized as follows: Section 2 is introduction to the fundamental characteristic of a chaotic system and Lyapunov exponents. Section 3 is embedding phase space reconstruction method. Section 4 is the proposed prediction method with computer simulation and error analysis of the proposed method. In Section 5, the conclusion is given.

2. Chaos and its fundamental characteristic

Chaos begins with the realization that quite simple mathematical equations can model quite complex and erratic behavior. In mathematical as well as physical systems, a tiny difference in input can result in overwhelming differences in output. This phenomenon is known as *sensitive dependence on initial conditions*. This means that any uncertainty in the initial state of the given system, no matter how small, will lead to rapidly growing errors in any effort to predict future behavior. This makes any long-term prediction of the trajectory of a real chaotic system impossible.

There are three fundamental characteristics of chaos: (1) an essentially continuous and possibly banded frequency spectrum that resembles random noise; (2) sensitivity to initial conditions, that is, nearby orbits diverge very rapidly; and (3) an ergodicity and mixing of the dynamical orbits which in essence implies the wholesale visit of the entire phase space by the chaotic behavior and a loss of information.

In order to quantitatively characterize a chaotic attractor, the *Lyapunov exponents* is a useful tool, which provides an important measure for the sensitivity of the chaotic orbit to its initial conditions. Lyapunov exponents describe, in a logarithmic scale, the growth or shrinkage rate of small perturbations in different directions in the phase space of the orbits. When at least one Lyapunov exponent is positive, the attractor is extremely sensitive to its initial conditions and hence becomes chaotic after a transient period of time.

For the formal definition of Lyapunov exponents, consider a specific one-dimensional map given by $x_{n+1} = f(x_n)$. The difference between two initially nearby states after nth steps is written as $f^n(x+\varepsilon) - f^n(x) \approx \varepsilon \cdot e^{n \cdot \lambda}$. For small ε, this expression becomes $\lambda \approx (1/n)\log_e |\mathrm{d}f^n/\mathrm{d}x|$. The λ is the Lyapunov exponent. Therefore the Lyapunov exponent gives the stretching rate per iterate. The signs of the Lyapunov exponents provide a qualitative picture of a system's dynamics. One-dimensional maps are characterized by a signal Lyapunov exponent, which is positive for chaos, zero for a marginally stable orbit, and negative for a periodic orbit.

The magnitudes of the Lyapunov exponents quantify an attractor's dynamics in information theoretic terms. The exponents measure the rate at which system processes create or destroy information; thus the exponents are expressed in bits of information/s or bits/orbit for a continuous system and bits/iteration for a discrete system [5]. Therefore, the ability of the prediction of proposed method depends on the value of the Lyapunov exponent, i.e. depends on the complex of the chaotic time series.

3. Embedding phase space reconstruction

Generally, the observed chaos phenomenon in real world is in chaotic time series data format. Thus, analysis of chaotic time series take a very important role in chaos research. Reconstruction phase space method using known chaotic time series data was found by Taken [4]. After embedding the like random chaotic time series data to phase space, chaos attractor can be observed. Chaotic time series display some stochastic behavior in time domain, at the same time the determined behavior of chaotic time series was shown in phase space structure. Therefore, chaotic time series in phase space can be analysed and predicted. Takens's embedding method [6] is described as follows:

From observed time series $x(t)$, data vector $Y(t) = F(x(t), x(t-T), \ldots, x(t-(D-1)T)$ is generated, where T is the time delay; this vector indicates one point of a D-dimensional reconstructed phase space R_D, where D is call embedding dimension. Therefore a trajectory can be drawn in the D-dimension reconstructed phase space by changing T. Assume that the target system is a deterministic dynamical system and that the observed time series is obtained through an observation system corresponding to C_1 continuous mapping from the state space of dynamical system to the one-dimension Euclidean space R. Then, the reconstructed trajectory is an embedding of the original trajectory when the D value is sufficiently large. If any attractor has appeared in the original dynamical system, another attractor, which retains the phase structure of the first attractor, will appear in the reconstructed state space. In order that such reconstruction achieves embedding, it has been proven that the dimension D should satisfy the condition: $D > 2m + 1$ where m is the state space dimension of the original dynamical system. However, this is a sufficient condition. Depending on the data, embedding can be established even when D is less than $2m + 1$.

In the embedding method, there are two parameters, embedding dimension and time delay. Abarbanel [7] gives us a good suggestion on how to select those two parameters. Time delay does not strongly affect reconstruct phase space and Lyapunov exponents estimation. One approach to estimating this value is to select the frequency (1/time scale) that corresponds to a dominant power spectral feature.

The reconstructed phase consists of points in D-dimension phase space. This number D should not be so small that the reconstruction is topologically incorrect. Memory constraints are the biggest problem with picking a value for D-dimension that is larger than the minimum acceptable value. An embedding could be obtained, if the embedding dimension is chosen to be greater than twice the dimension of the underlying attractor. However, because attractors reconstructed using smaller values of embedding offer yield reliable Lyapunov exponents, if embedding dimension is chosen too large, the noise in the data will tend to decrease the density of points defining the attractor, it making embedding trajectory harder to find replacement points. Noise is an infinite dimensional process that, unlike the deterministic component of the data, fills each available phase space dimension in a reconstructed attractor. Increasing embedding dimension past what is minimally required has the effect of unnecessarily increasing the level of contamination of the data.

Therefore, in this paper, a novel multi-dimension embedding phase space method to improve the accuracy of chaotic time series prediction is proposed for predicting chaotic time series. This method will be introduced in Section 4. As shown in Figs. 1 and 2, Rossler attractor is used as an example to show the result of embedding the Rossler time series data to different phase space.

4. Chaotic time series prediction

4.1. The proposed algorithm

The proposed chaotic time series prediction method has three steps. The first step is embedding time domain time series data to embedding phase space. The second step is calculate unknown

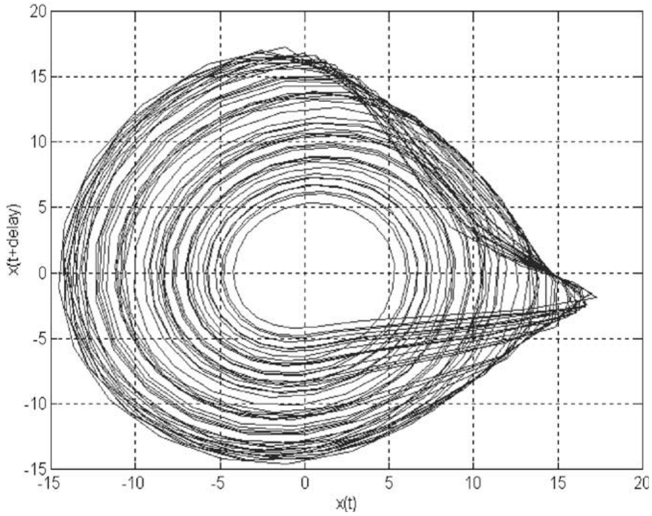

Fig. 1. Rossler attractor in 2D embedding phase space.

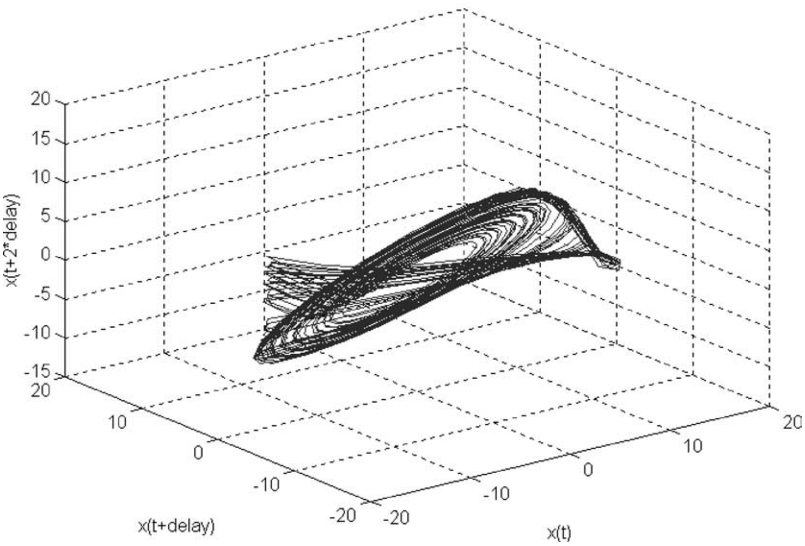

Fig. 2. Rossler attractor in 3D embedding phase space.

phase space points by chaos behavior. The third step is recover prediction phase space point to time domain and get the predicted time series value.

Given a chaotic time series data set $x(1), x(2), x(3), \ldots, x(N)$, N is the length of the data set. While embedding those known chaotic time data to D-dimension phase space, the time delay T can be used to get $[N-(D-1)T]$ phase space point $Y(I), I \in [1 \quad [N-(D-1)T]]$:

$$
\begin{aligned}
&Y(1) = [x(1) \quad x(1+T) \quad \ldots \quad x(1+(D-1)T)];\\
&Y(2) = [x(2) \quad x(2+T) \quad \ldots \quad x(2+(D-1)T)];\\
&Y(3) = [x(3) \quad x(3+T) \quad \ldots \quad x(3+(D-1)T)];\\
&\ldots\ldots\ldots\\
&Y(I) = [x(I) \quad x(I+T) \quad \ldots \quad x(I+(D-1)T)].
\end{aligned} \tag{1}
$$

Then, $N-(D-1)T$ embedding phase space points can be got. Based on those points, a point, which is the nearest to the $Y(N-(D-1)T)$ point, can be found. The point is marked as $Y(\min\_\mathrm{dist})$. The distance between $Y(N-(D-1)T)$ and $Y(\min\_\mathrm{dist})$ can be calculated and be named as the initial distance Dist_0. If Dist_1/Dist_0 has a little change in every evolution step, the distance Dist_1 between $Y(N-(D-1)T+1)$ and $Y(\min\_\mathrm{dist}+1)$ can be estimated, which follows the $\mathrm{Dist\_1} = \mathrm{Dist\_0} \cdot 2^{K\cdot\lambda}$, where λ is the Lyapunov exponent and K is the number of steps from Dist_0 to Dist_1. Then, Dist_1 can be obtained. Knowing the position of $Y(\min\_\mathrm{dist}+1)$ point, the position of $Y(N-(D-1)T+1)$ point can be calculated. Because $Y(N-(D-1)T+1)$ is composed as $[x(N-(D-1)T+1 \quad x(N-(D-1)T+T+1 \quad \ldots \quad x(N+1)]$, the value of $x(N+1)$ can be predicted. By the same methods, more long-term predictions can be obtained (the length of prediction depends on the chaos system behavior and value of Lyapunov exponent which was described in Section 2).

It can be shown in phase space map in Fig. 3. that $(N-(D-1)T)$ phase space points can be embedded when N chaotic time series data were given. Thus, No. $(N+1)$ time series data, i.e. prediction $(N-(D-1)T+1)$ phase space point, can be predicted.

Based on No. $(N-(D-1)T)$ phase space point ①, the nearest point of No. $(N-(D-1)T)$ phase space point ③ can be built from all known phase space point. After calculating the distance of point ① and point ③, the initial distance of this iteration Dist_0 can be obtained. Following the trajectory of embedding phase space, the distance Dist_1 between ② and ④ point can be calculated

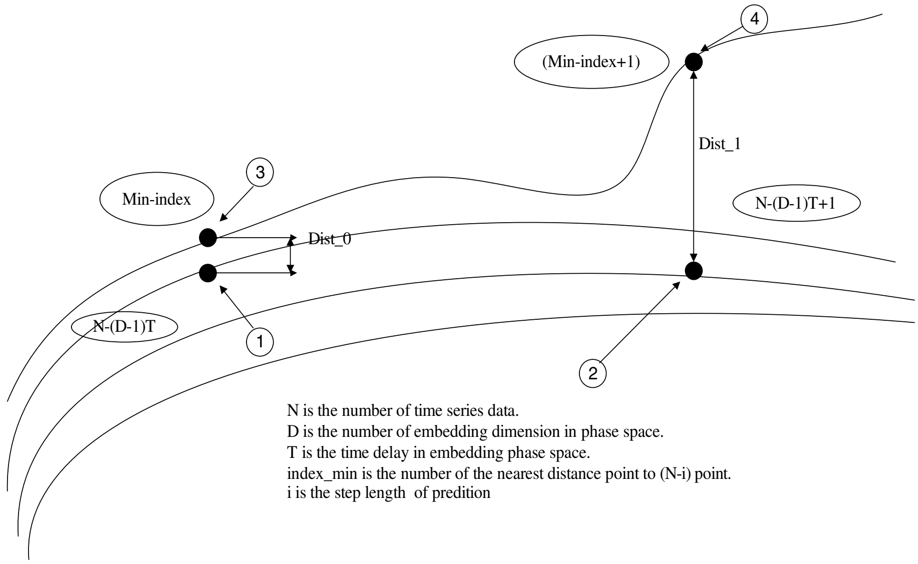

Fig. 3. Chaotic time series trajectory in embedding phase space.

by chaotic behavior. Then, calculated the ④ point position and obtained the time series predicting data is the chaotic time series point of predicted.

Another new idea of the proposed prediction method is multi-dimension phase space chaotic time series prediction method. The main basic points of the method are: (1) the same chaotic time series data can be embedded in different embedding dimensions with the different behavior of the chaotic attractor. (2) Embedding different dimension phase space can use the same chaotic time series data set. (3) The same data point in time series set takes different effect in different phase space. Because of the above three characters, the proposed multi-dimension method can be used to improve the accuracy of prediction.

The steps are:

(1) Predict the same chaotic time series data in different dimension embedding phase spaces.
(2) Calculate the total average prediction value from the prediction values in different dimension embedding phase spaces.

From error theory, it can be known that: the random errors will diminish as $1/\sqrt{M}$ with repeated measurements (M is the number of repeat). The prediction processing is equal to a measurement process. Prediction value is equal to the estimated value. Time series data is equal to the real value. The prediction error is equal to the random error. Multi-dimensional embedding phase space prediction is equal to the repeated measurements. By this method, the proposed method can diminish the prediction error and obtain more accuracy of the predicted result. The proposed algorithm is listed in Table 1.

4.2. Error analysis

From the above analysis, the error sources of the proposed prediction method are shown in Table 2.

Table 1
Proposed prediction algorithm

Step 1	Load a known time series data set $x(t)$ and the length of the data set is N;
Step 2	Choose the time delays, T;
Step 3	Choose the dimension of reconstructed phase space, D;
Step 4	Calculate the Lyapunov exponent denoted as "*lyexps*" based on know N length data set, i.e. finite-size Lyapunov exponents; If all *lyexps* < 0, then go to step ten; If there is at least a *lyexp* > 0 then next step;
Step 5	Reconstruct phase space based on the chaotic time delay and embedding dimension;
Step 6	Find the nearest point min_dist to the last known phase space point $Y(N-(D-1)T)$;
Step 7	Calculate the distance between min_dist point and the last point $Y(N-(D-1)T)$;
Step 8	Calculate the distance between (min_dist $+ 1$) point and $Y(N-(D-1)T+1)$ point;
Step 9	Get the coordinate of (min_dist $+ 1$) point in embedding phase space and predict the $x(N+1)$ value in time domain using Lyapunov exponent;
Step 10	Use different embedding dimension to calculate from step four to step nine and get more accurate prediction value using some calculate technique;
Step 11	Stop.

Table 2
Error analysis

No.	Error source	Solve method
1	The hypothesis of Dist_1/Dist_0 has a little changed in every evolution step	Use local and stepping Lyapunov exponent replace globe Lyapunov exponent in proposal prediction method
2	In the D embedding dimension, the chaotic time series data cannot display all behavior of the chaos attractor	Take the multi-dimension method to predict the chaotic time series
3	The accurate of value Lyapunov exponent The accurate refer to the ability accurate of Lyapunov exponent quantitative chaos behavior	There are many proposal methods of calculate Lyapunov exponents from chaotic time series data. We expect a more better method be proposal in near future. Or, use another nonlinear method, e.g. neural networks, fuzzy logic to predict chaotic time series in embedding phase space. This part will be reported in the successive papers

4.3. Computer simulation

In order to prove the validation of the proposed method, logistic chaotic system equation (2) is used as an example to test the proposed prediction method.

$$x(t+1) = 4.0 \cdot x(t) \cdot (1 - x(t)), \tag{2}$$

where $x(0) = 0.36$.

A chaotic time series data file was generated by Logistic equation (2) and had been introduced in Wolf's paper [5]. The chaotic time series data file includes 512 time series data points. The setting parameters are the following: the time delay is 1 and the D-dimension is 2. The logistic chaotic time series data in time domain is shown in Fig. 4.

Reconstructed phase space in two-dimension using the logistic chaotic time series is shown in Fig. 5.

The proposed chaotic time series prediction method is described in Table 1 *step five* to *step nine*. The predicted results are obtained in Figs. 6–8.

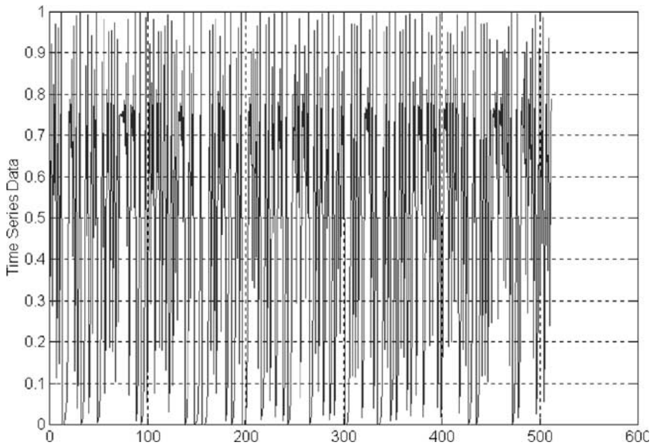

Fig. 4. Logistic chaotic time series in time domain.

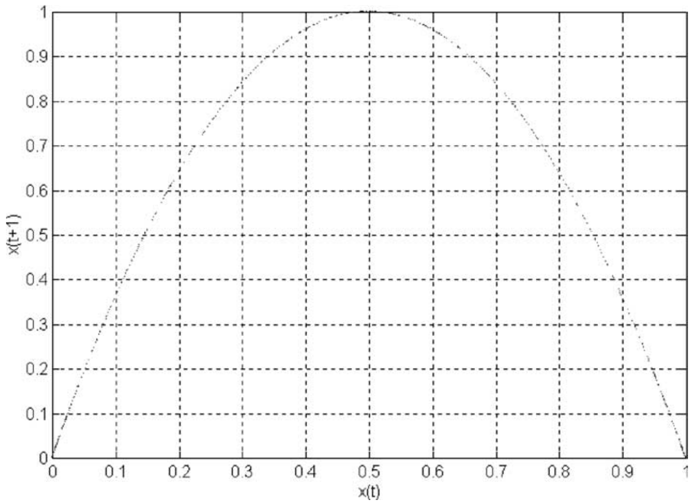

Fig. 5. Logistic chaotic time series in 2D embedding phase space.

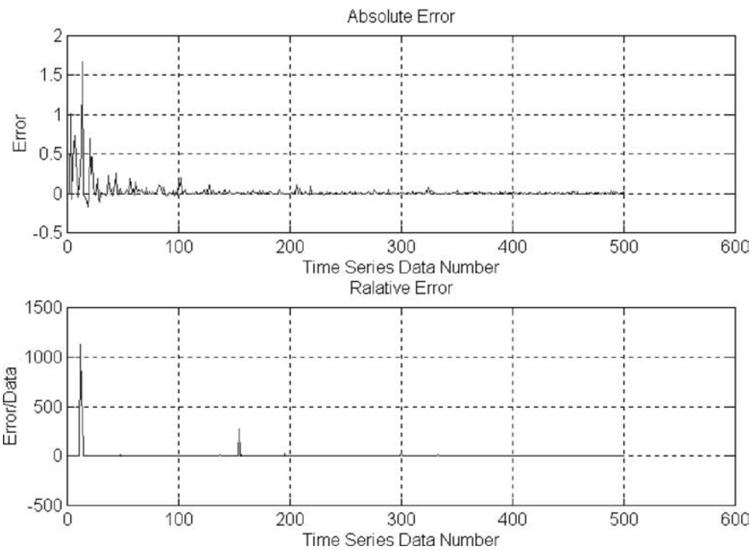

Fig. 6. Prediction absolute error and relative error for logistic chaotic time series.

The mean square error (MSE) (Eq. (3)) is used to evaluate the proposed prediction method.

$$\mathrm{MSE} = \sum_{i=1}^{N} (\mathrm{prediction\_value}(i) - \mathrm{real\_value}(i))^2 \cdot p_i, \tag{3}$$

where prediction_value(i) is ith chaotic time series point prediction value, real_value(i) is ith chaotic time series point real value, p_i is the ith chaotic time series point probability, N is the total number of predicted chaotic time series points.

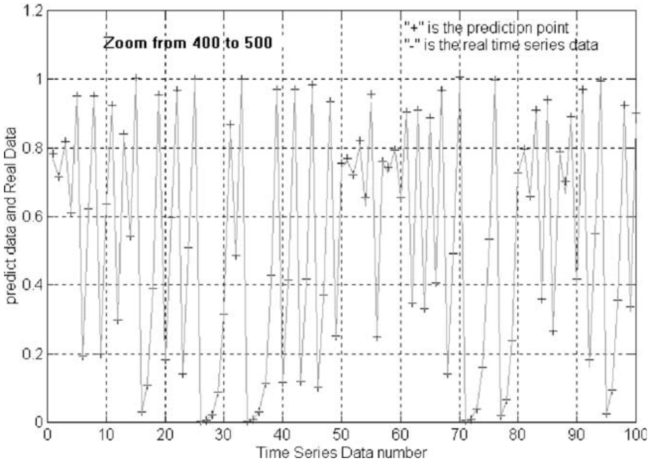

Fig. 7. Predicted result and real logistic chaotic time series data.

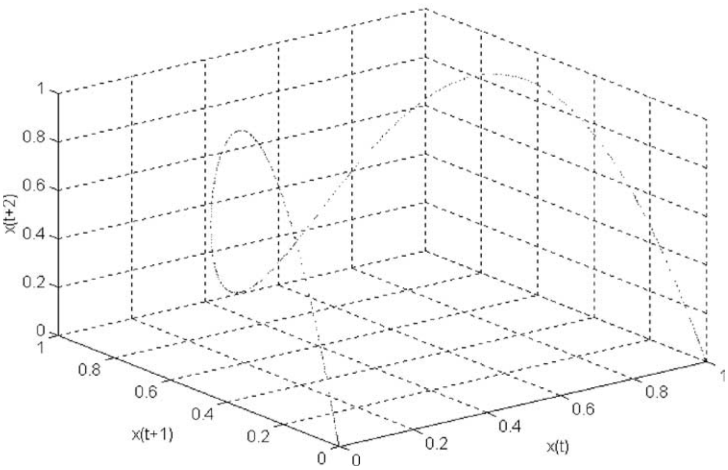

Fig. 8. Logistic chaotic time series in 3D embedding phase space.

$$\text{Absolute error sum is AES} = \sum_{i=1}^{K} \text{prediction\_value} - \text{real\_value} = 12.4515$$

$$\text{Relative error sum is RES} = \sum_{i=1}^{K} \frac{\text{prediction\_value} - \text{real\_value}}{\text{real\_value}} = 2586.6$$

$$\text{Average absolute error is AAE} = \frac{\text{AES}}{k} = 0.024903 \text{ here } k = 500 \text{ points}$$

Average relative error is $\mathrm{ARE} = \frac{\mathrm{RES}}{k} = 5.1732$ here $k = 500$ points

Mean square error is $\mathrm{MSE} = 0.0144$

Relative mean square error is $\mathrm{MSE} = \frac{\sum_{i=1}^{500}\left(\frac{\text{prediction\_value}(i)-\text{real\_value}(i)}{\text{real\_value}(i)}\right)^2}{500} = 3754.3$

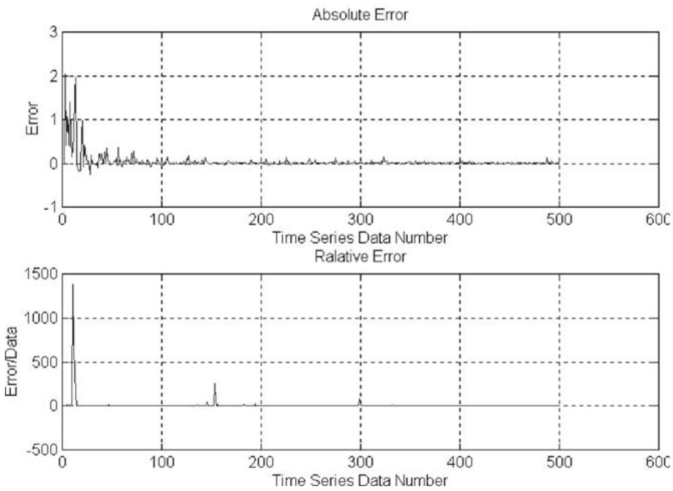

Fig. 9. Prediction absolute error and relate error for logistic chaotic time series.

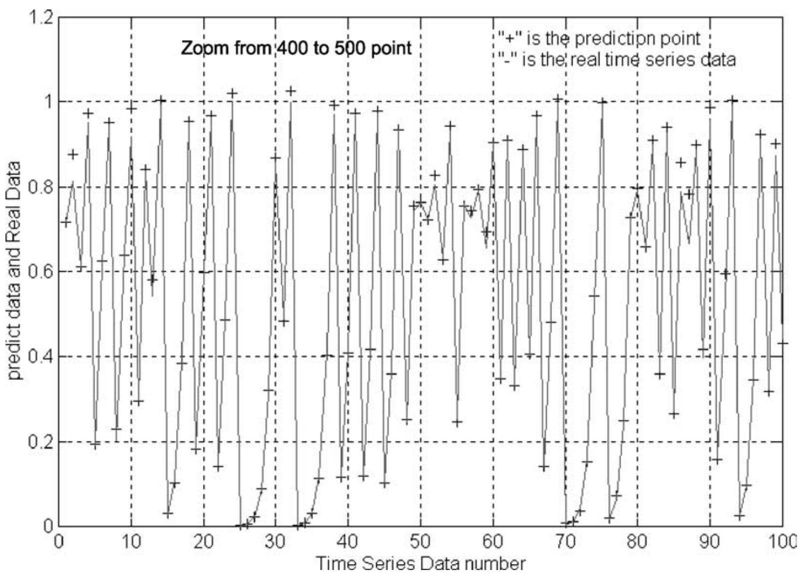

Fig. 10. Prediction result and real logistic chaotic time series.

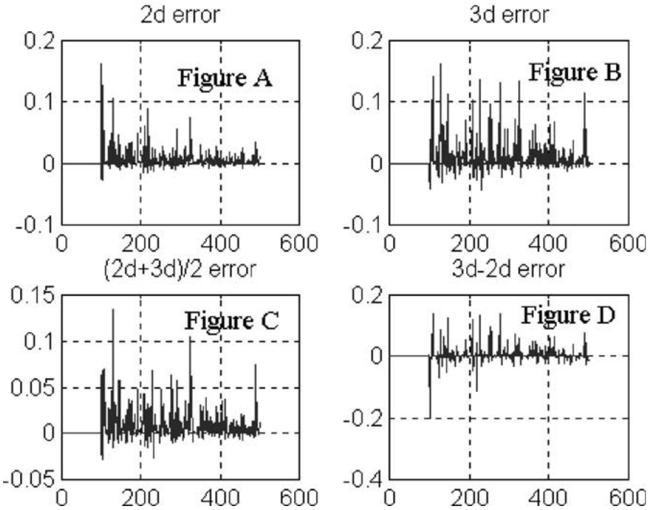

Fig. 11. Prediction error and phase dimension. (A) The prediction error in 2D phase space, (B) is the prediction error in 3D phase space, (C) is the prediction error using multi-dimensional method, (D) shows difference between A and B.

To *step ten*, the proposed prediction method is extended to multi-dimension embedding phase space. In three-dimension phase space, the parameters are as follows: the time delay is 1 and the D-dimension is 3. The time domain data is shown in Figs. 9–11:

$$\text{AES} = 21.1541 \quad \text{AAE} = 0.0423082 \text{ in 500 points} = 0.03875$$
$$\text{RES} = 3200.2 \quad \text{ARE} = 6.4004 \text{ in 500 points} = 5259.09$$

Fig. 11 is the prediction error with a different embedding dimension. In Fig. 11, A is the prediction error which is got from 2D embedding dimension phase space. B is the prediction error which is got from 3D embedding dimension phase space. C is the prediction error which uses multi-dimensional method to predict the time series. D shows the difference between A and B. From these figures, it can be obtained that the error in C is the minimum. These simulation result demonstrates that the proposed multi-dimension method is a good method for chaotic time series prediction. Table 3 shows the different phase dimension and prediction error.

From the simulation result, it can be obtained that multi-dimension prediction method can avoid the prediction error, which was bring by selected single embedding dimension, in spite of multi-dimension prediction value may not be the best prediction value. However, because there is no answer on "how to select an embedding dimension", the multi-dimension prediction method is a better prediction path now.

Table 3
Dimension of phase space and prediction error

Dimension phase space	AAE	ARE	MSE	RMSE
2	0.024903	5.1732	0.0144	2754.3
3	0.042308	6.4004	0.03875	5259.09
Multi-dimension	0.0090	1.2614	0.000318	161.17

5. Conclusion

In this paper, a novel chaotic time series prediction method in embedding phase space is presented. It is based on the fundamental characteristic of chaotic behavior that sensitive dependence upon initial conditions (SDUIC) and Lyapunov exponents (LEs) is a measure of the SDUIC in chaotic system, which is done by first reconstructing a phase space using chaotic time series, then using LEs as quantitative parameters to predict an unknown phase space point, and after transferring the phase space point to time domain, the predicted chaotic time series data can be obtained. The result of computer simulation shows:

(1) Chaotic time series display some stochastic behavior in time domain and display some determined behavior in embedding phase space. This behavior can be used for predicting the chaotic time series in the proposed prediction method.
(2) It is feasible that predicting chaotic time series based on the fundamental characteristic of a chaotic system which is its sensitivity to the initial state.
(3) Multi-dimension embedding phase space method predicts chaotic time series and can improve the accuracy of prediction.
(4) From the prediction result, it can be obtained that the more the number of time series data the better the result, because the nearest phase space point can be obtained more precisely from larger data set.
(5) The proposed prediction method is a good way for predicting chaotic time series.

In future research, some novel intelligent technology will be used to predict chaotic time series in embedding phase space (EPS), e.g. genetic algorithms (GAs) [8–11], recurrent neural networks (RNN) [12,13] and fuzzy logic (FL) [14]. They are not needed to calculate Lyapunov exponents for predicting chaotic time series data. The same results will be reported and be compared with the proposed method [15] in the successive papers.

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