5.1.

Teoría de vibraciones III

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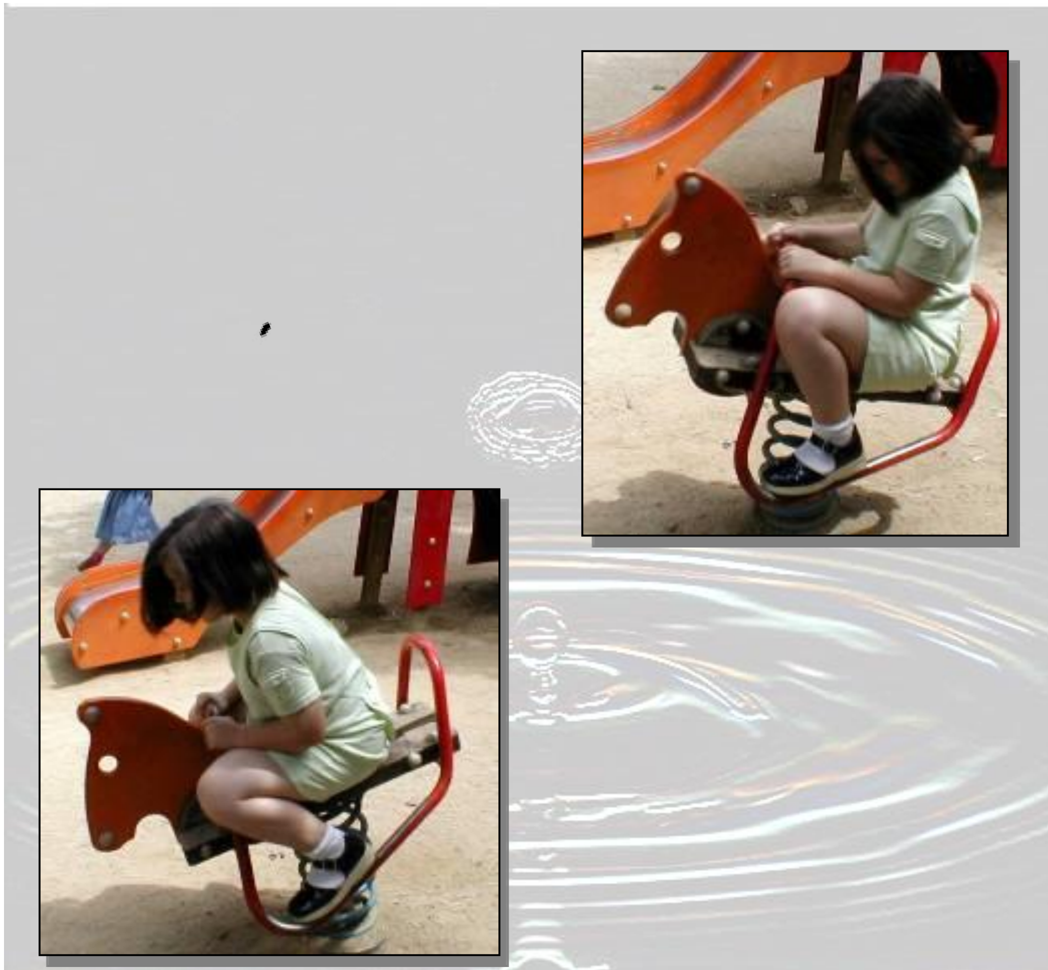

Actualizado 13/09/2011 RAFA
ED12 14/02/2013

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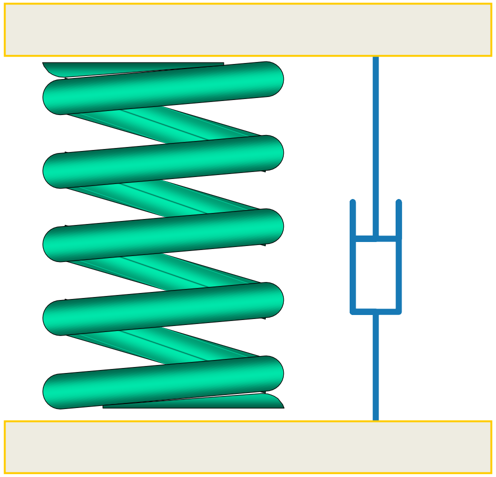
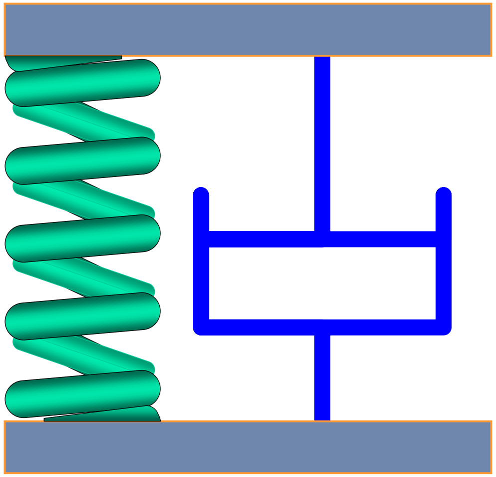
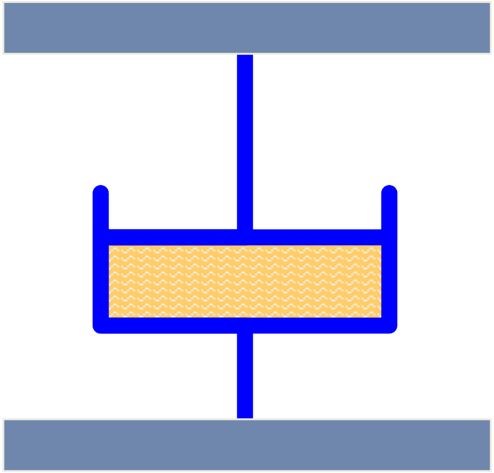
Aisladores



viscoelásticos



viscosos



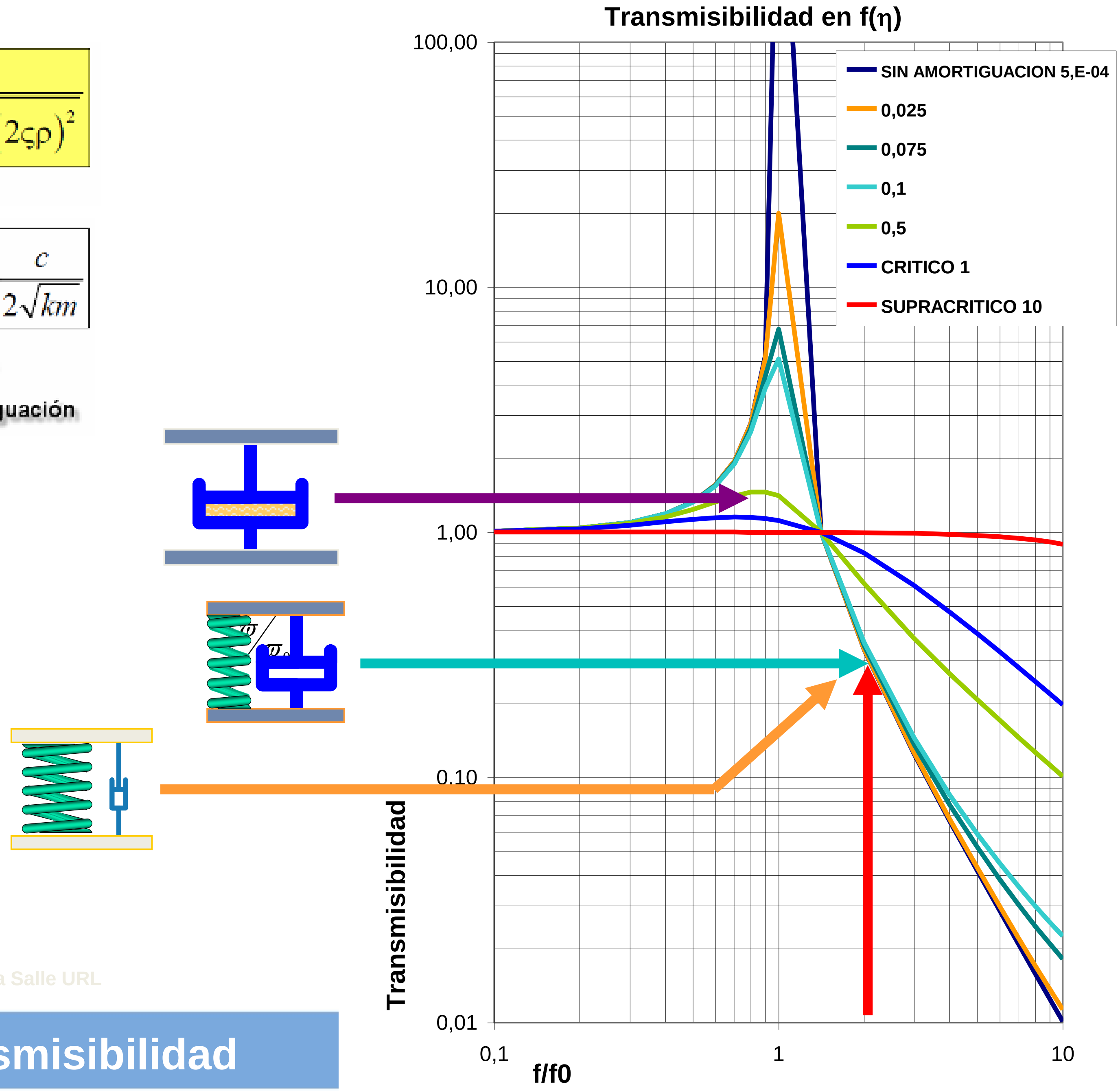
$$H(\rho) = \frac{x_0}{F_0} = \frac{1/k}{\sqrt{\left(1-\rho^2\right)^2 + \left(2\varsigma\rho\right)^2}}$$

Respuesta armónica

$$\rho = \frac{\omega}{\omega_0}, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \varsigma = \frac{c}{2\sqrt{km}}$$

ρ Razón de frecuencias

ς Coeficiente de amortiguación



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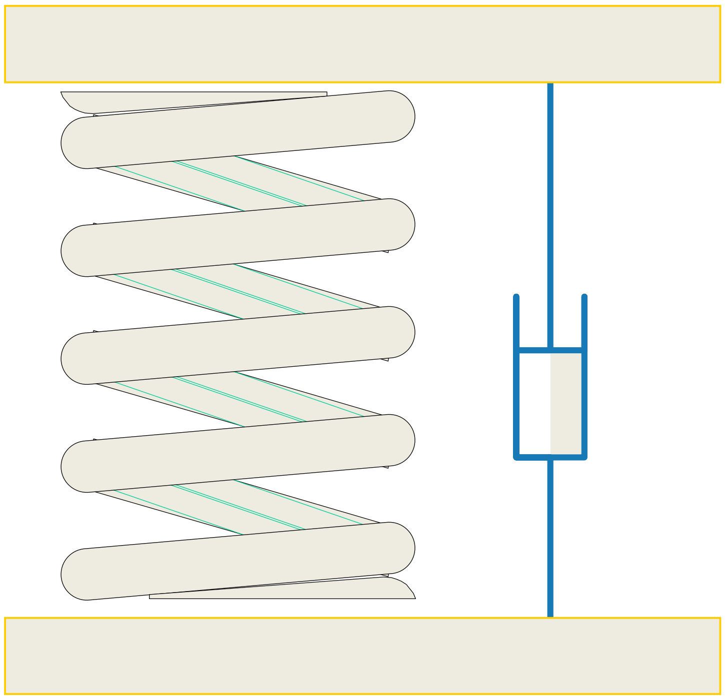
Transmisibilidad

Aisladores



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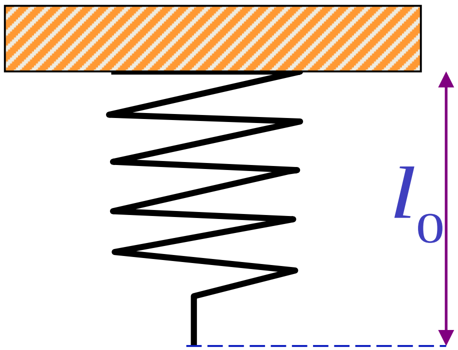
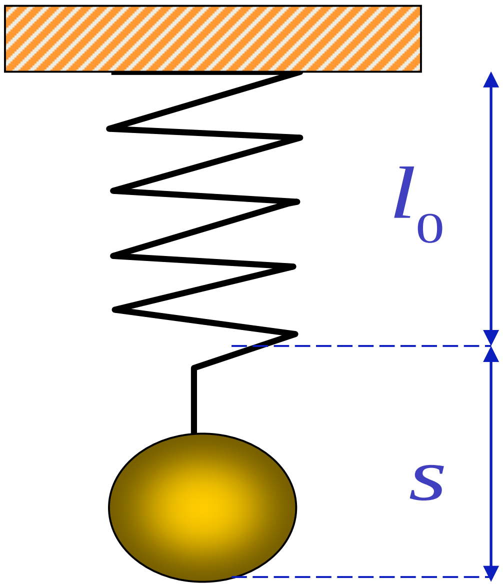
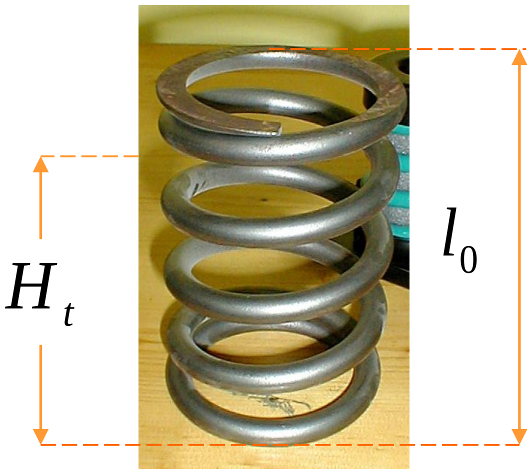


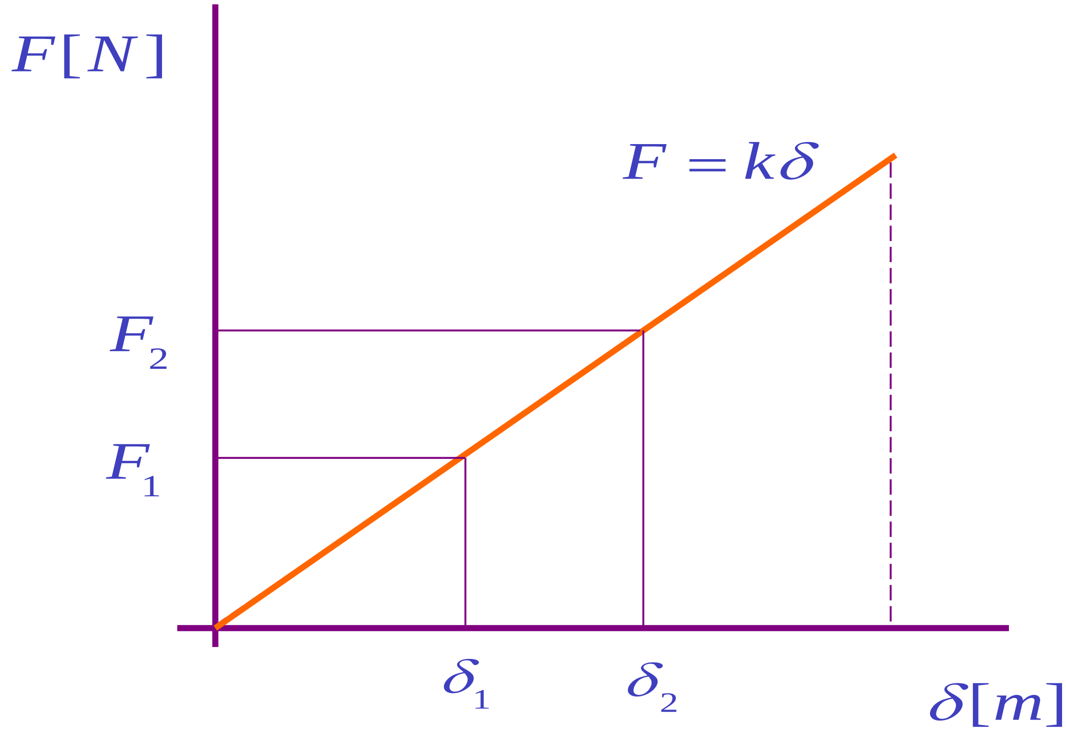
$$F = -k\delta$$



$$P = mg$$

$$mg - ks = 0 \qquad \sum F = 0$$





$$\delta = l_0 - H_t$$

$$K = \frac{dF}{d\delta}$$

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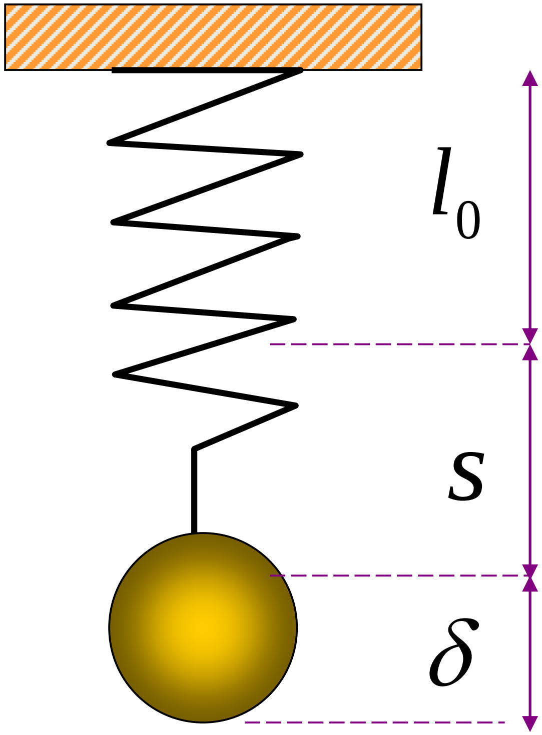
Tensión = 100N

δ tensión = 0,5 m

K?

[N/m]



$$m\frac{d^2x}{dt^2} = mg - ks - k\delta$$

$$m\frac{d^2x}{dt^2} = -k\delta$$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}\delta}$$

$$\boxed{m\frac{d^2x}{dt^2} + k\delta = 0}$$

$$\sum F \neq 0$$

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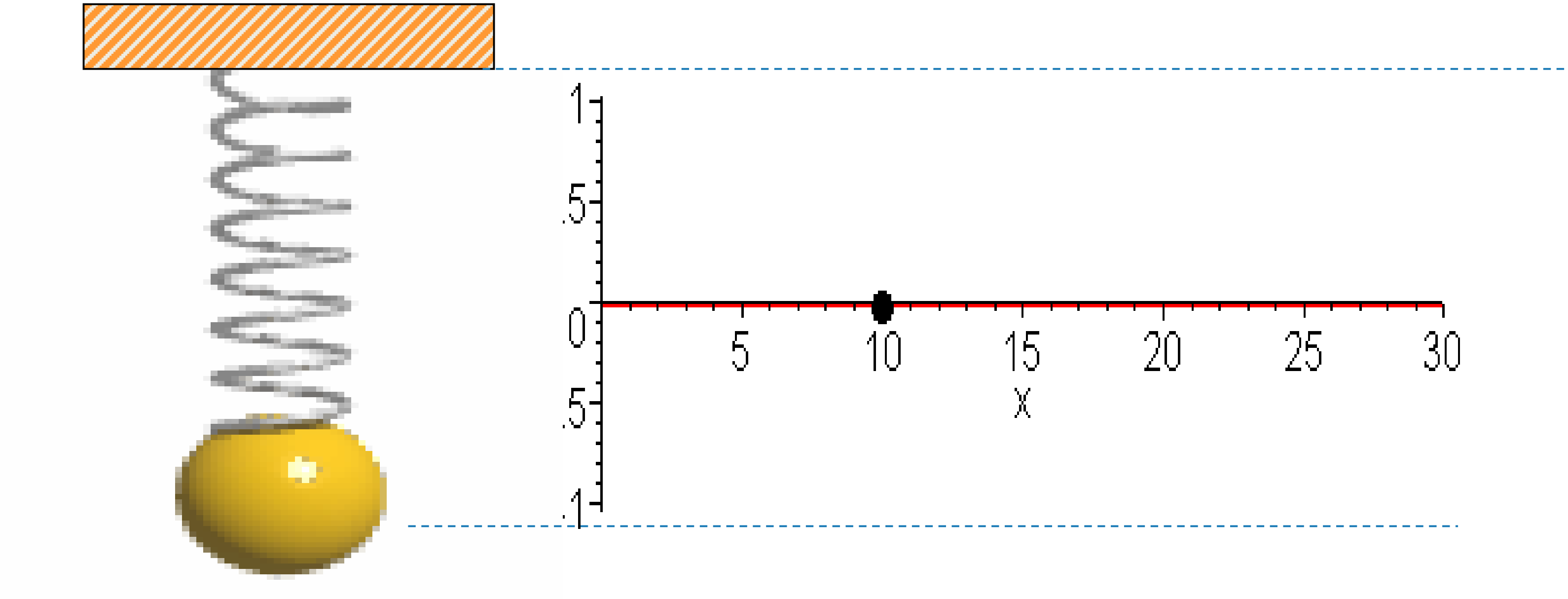

$$x(t) = A sen\, \varpi t$$

$$\varpi = \frac{2\pi}{T}$$ ***rad/s***

$$f = \frac{1}{T}$$ ***cps=Hz***

2π Hz= rad/s

rad/s /2π= Hz

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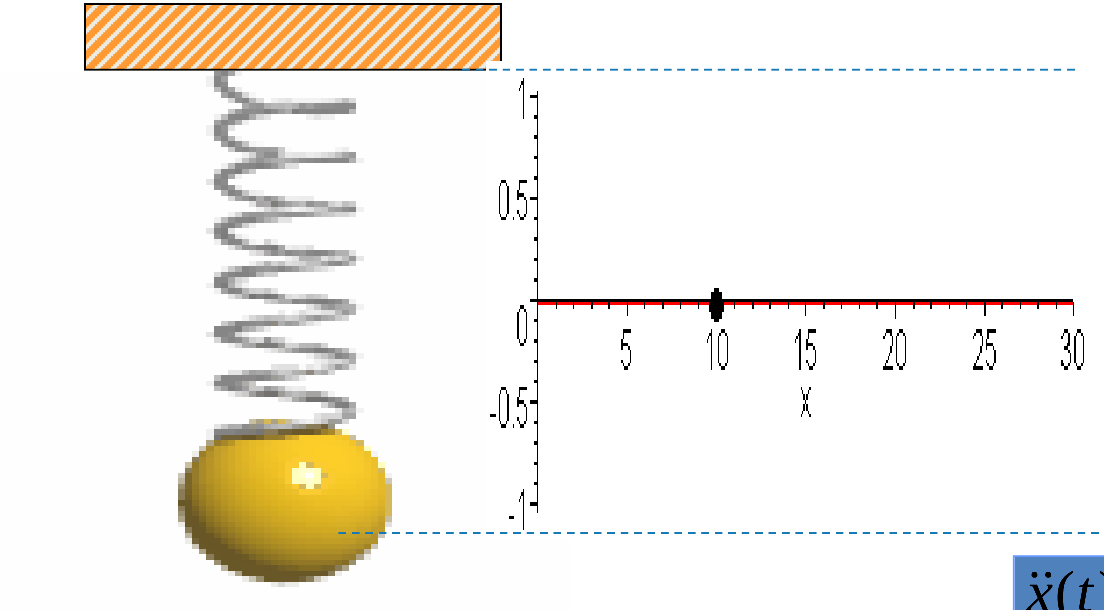


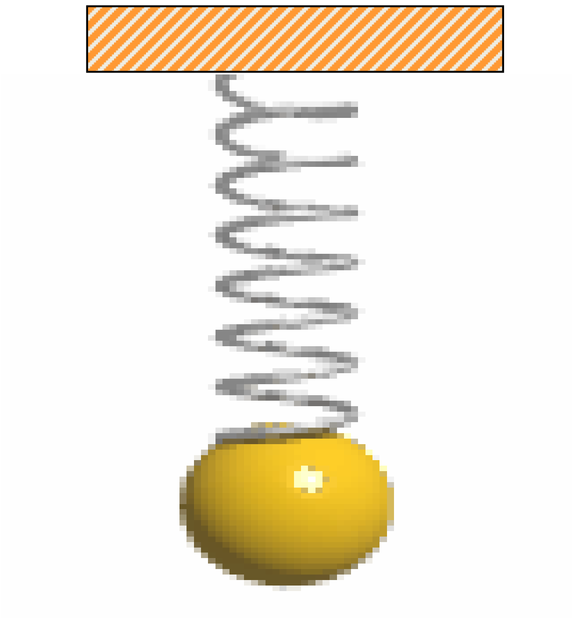
$$x(t) = A sen(\varpi_0 t + \varphi)$$
$$\dot{x}(t) = A\varpi_0 \cos(\varpi_0 t + \varphi)$$
$$\ddot{x}(t) = -A\varpi_0^2 sen(\varpi_0 t + \varphi)$$

$$\boxed{m\ddot{x}(t) = -kx(t)}$$

$$\overbrace{-mA\varpi^2 \sin(\varpi_0 t + \varphi)}^{\ddot{x}(t)} = -k\overbrace{A\sin(\varpi_0 t + \varphi)}^{x(t)}$$

$$-mA\varpi^2 = -kA$$

$$\boxed{\varpi^2 = \frac{k}{m}}$$



$$\frac{d^2x}{dt^2} = -\boxed{\frac{k}{m}}\,\delta$$

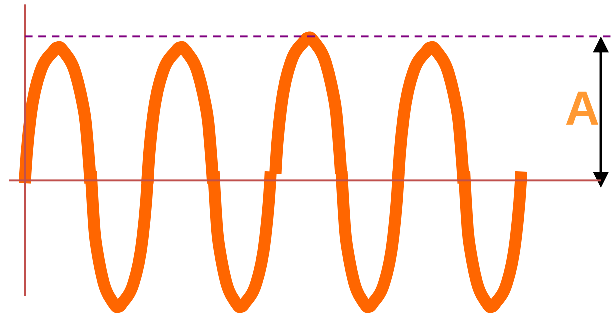
$$\varpi^2 = \frac{k}{m}$$

$$\varpi = {}^{2\pi}\!/\!{}_{T}$$

$$f = {}^{1}\!/\!{}_{T}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

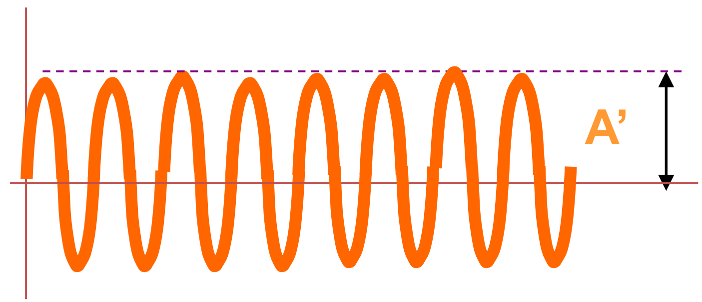
m K



Muelle blando

Fn

K m



Muelle duro

Fn’

A >A’ **Fn < Fn’**

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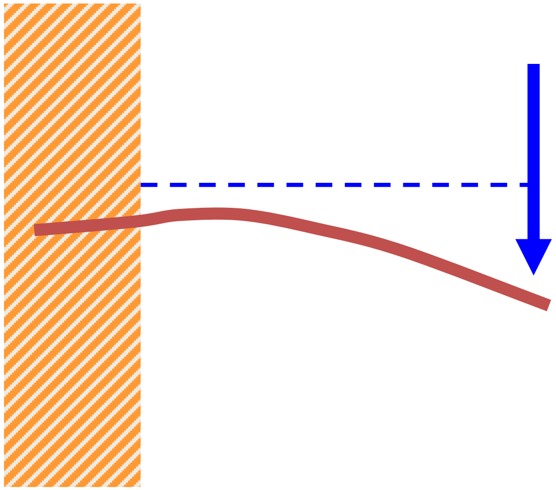
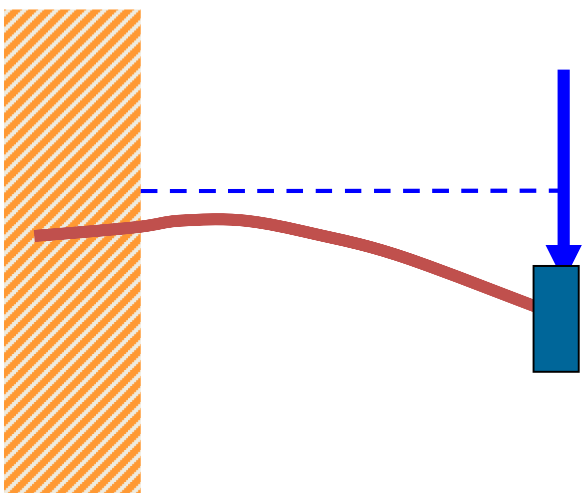
Vigas en voladizo



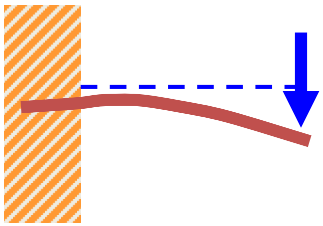
$$k = \frac{3EI}{L^3}$$

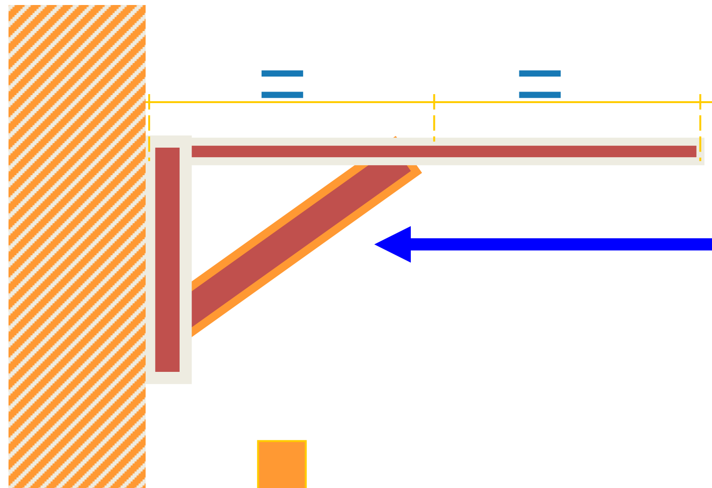
- *K=Rigidez elástica del sistema*
- *E=Módulo de elasticidad*
- *I=Momento de inercia*
- *L=longitud de la viga*
- *ω=Frecuencia angular propia*
- *μ=masa excéntrica*



$$\varpi = \sqrt{\frac{K}{m_v + 0{,}23m}}$$

Vigas en voladizo



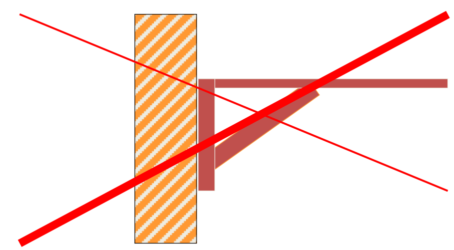


Δ Rigidez

$$\left.\begin{array}{l} k = \dfrac{3EI}{L^3} \\ L' < L \end{array}\right\} k' > k$$

∇ Amplitud

Δ Frecuencia



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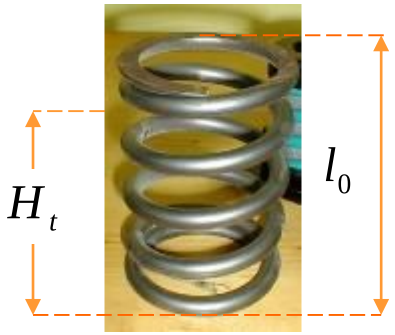
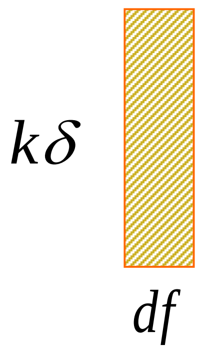
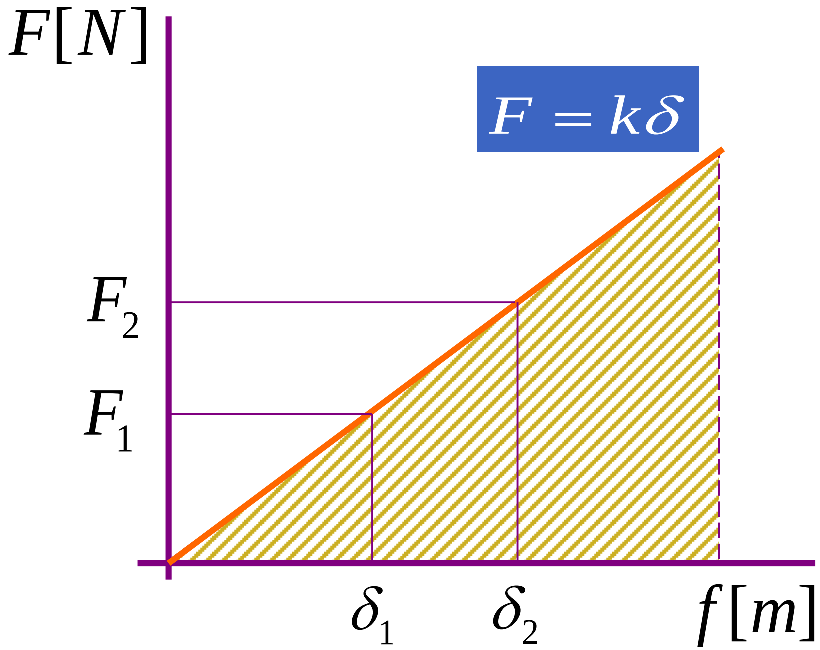


$$\int_n^0 F(\delta)d\delta = \int_n^0 k\delta d\delta = 1/2k\delta_0^2 - 1/2k\delta_n^2$$

$$\boxed{\Delta W_0^n = 1/2k\delta^2}$$

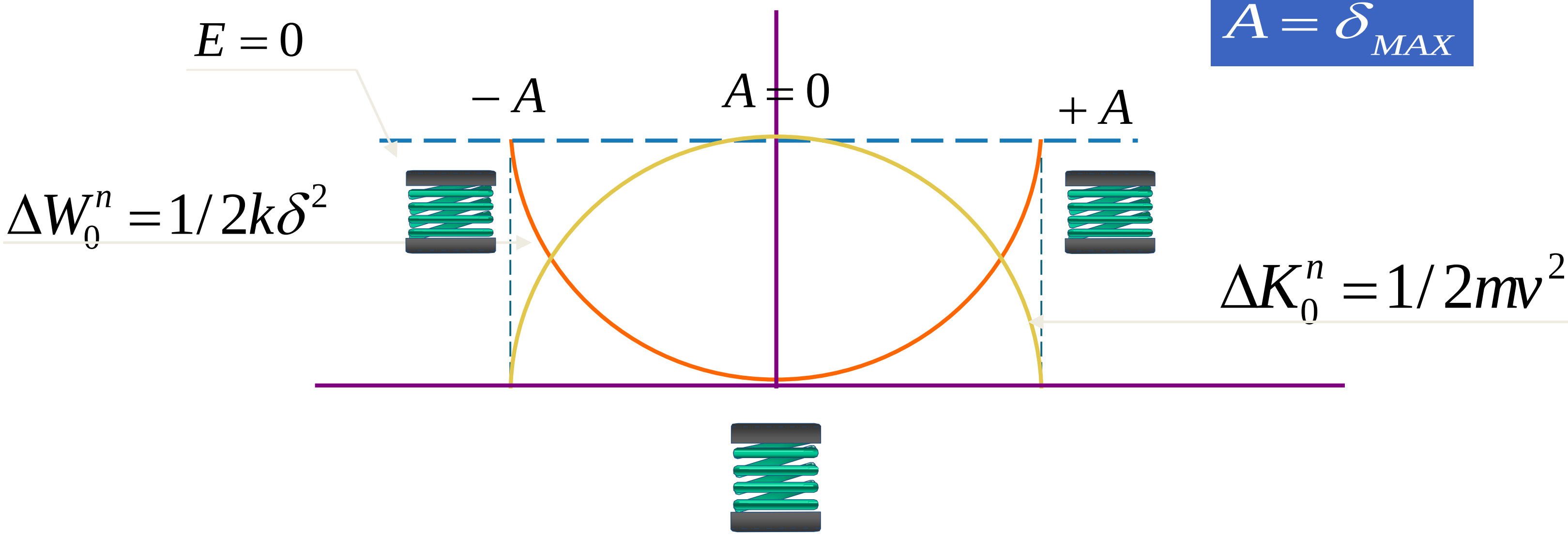




muelles lineales

$$E = K + W = 1/2mv^2 + 1/2k\delta^2 \quad (1)$$

$$A = \delta_{MAX}$$



$$E = 1/2mv^2 + 1/2k\delta^2 \Rightarrow 1/2\,k\boxed{A}^2 = 1/2mv^2 + 1/2k\delta^2 \quad (2)$$

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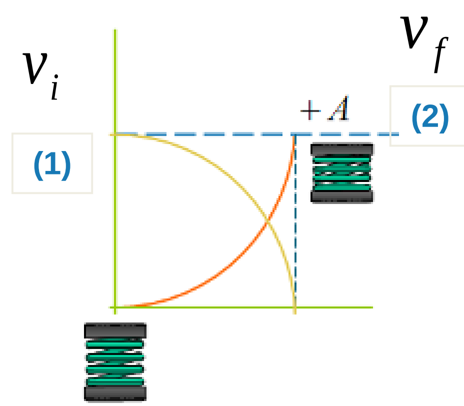
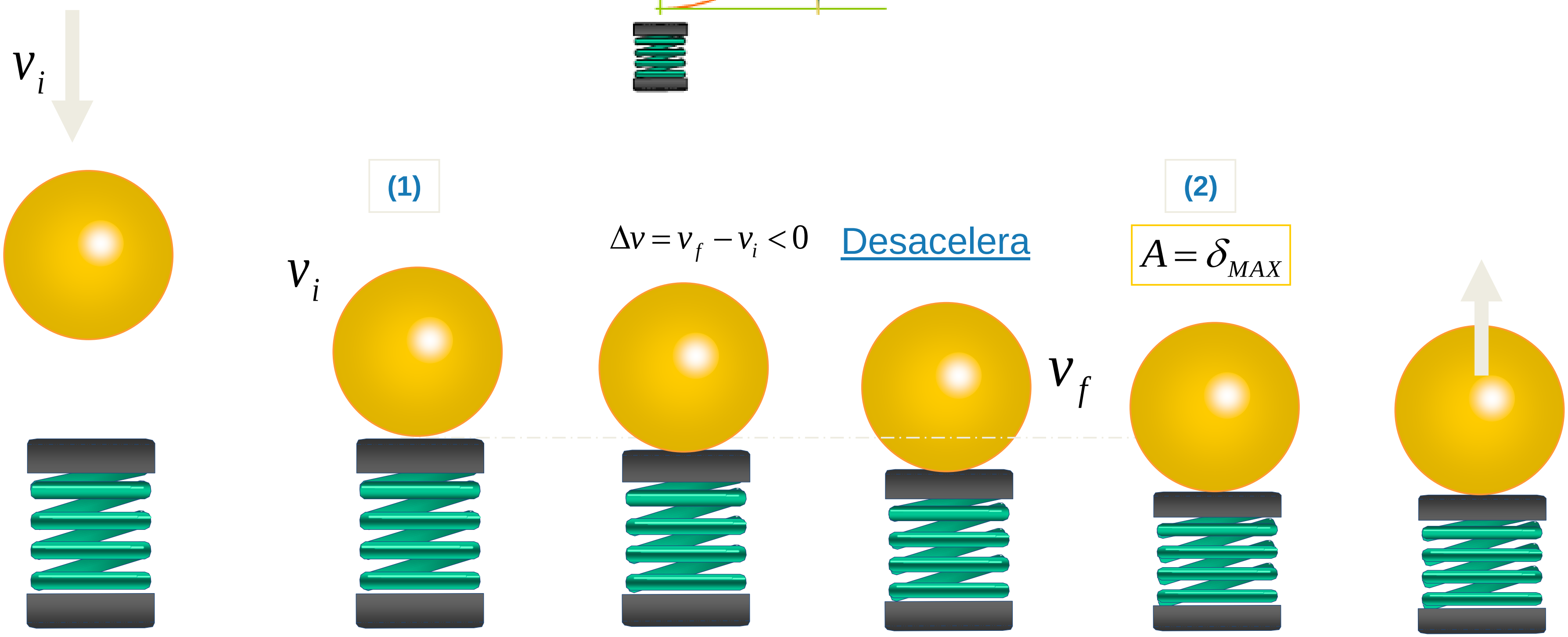
$$\Delta K_0^n = 1/2 m v^2$$

$$\Delta W_0^n = 1/2 k \delta^2$$





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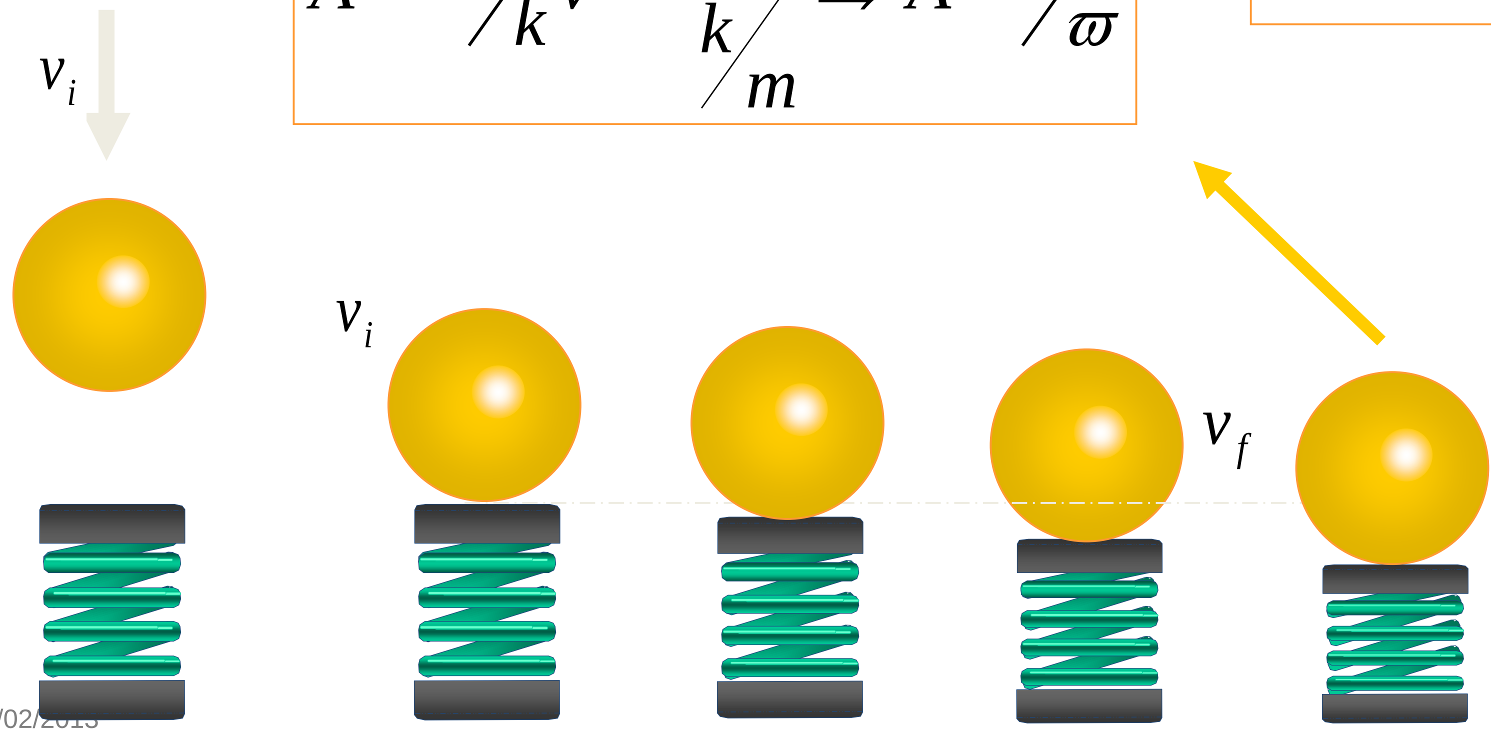
Choque contra un muelle

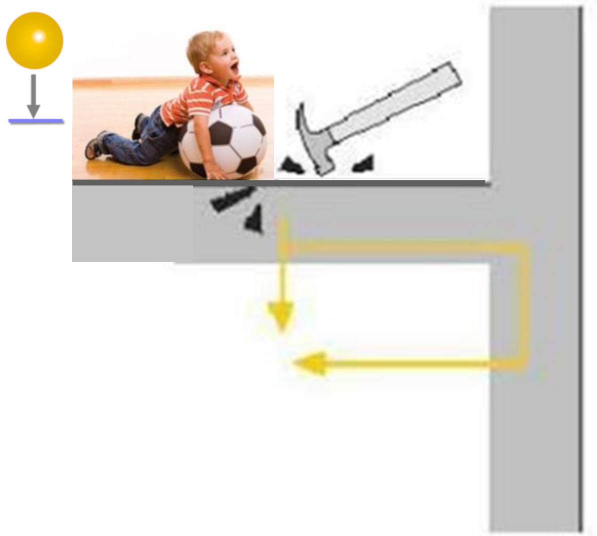
$$1/2mv^2 = 1/2kA^2$$

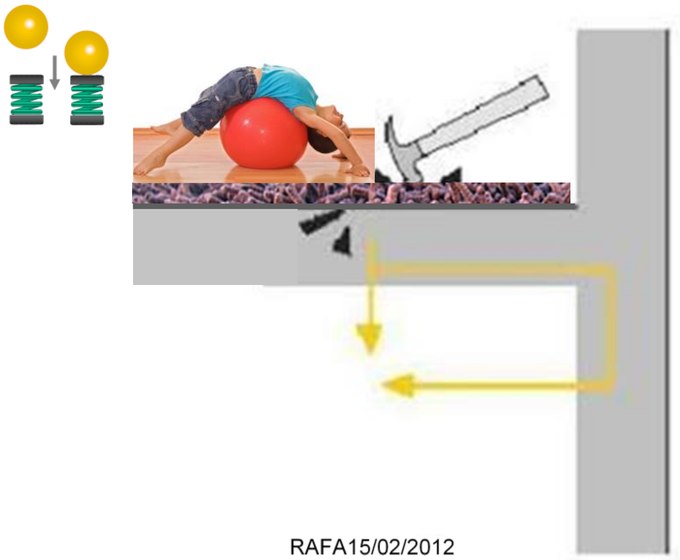
$$A^2 = {}^{m}\!/_{k}\, v^2 = \frac{v^2}{{}^{k}\!/_{m}} \Rightarrow A = {}^{v}\!/_{\varpi}$$

$$A = {}^{v}\!/_{\varpi}$$



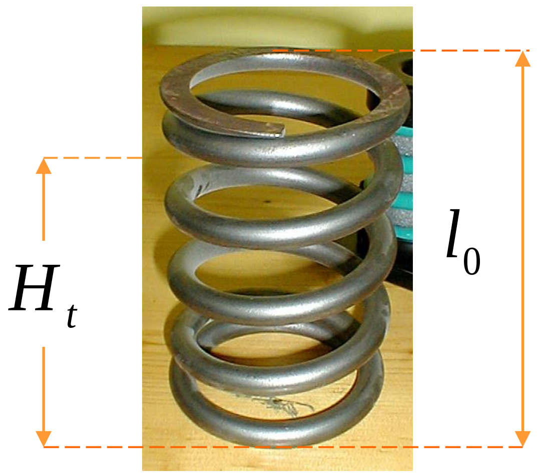
Choque contra un muelle

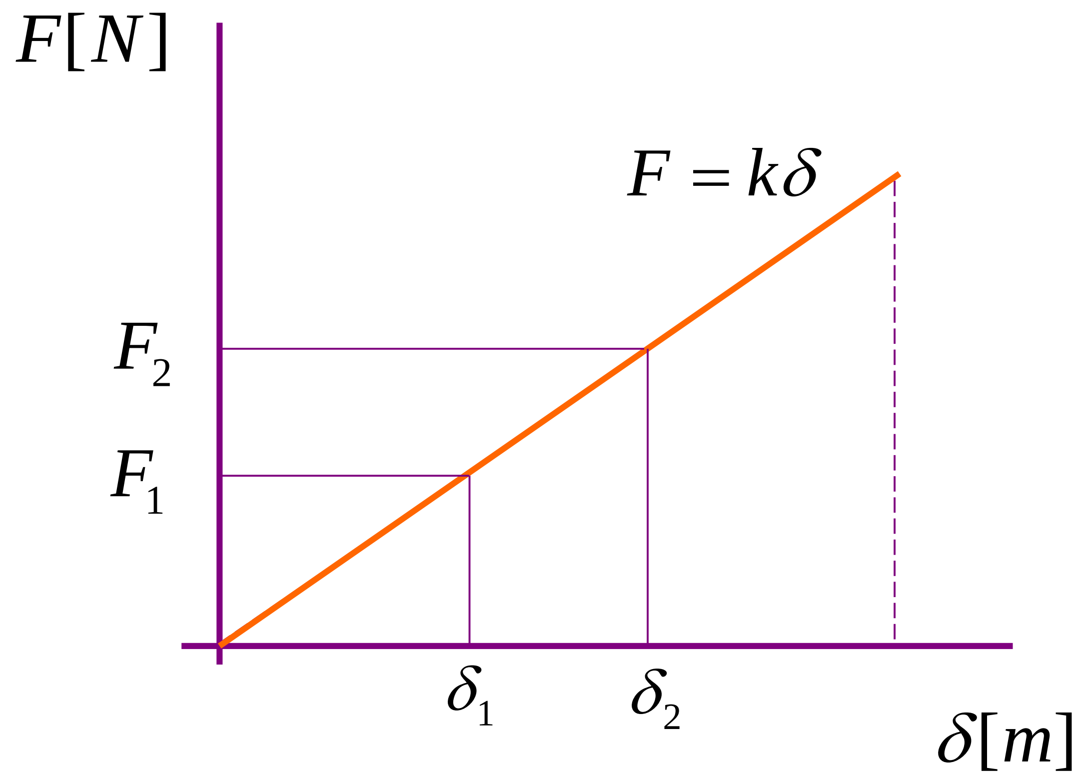




Muelles lineales

ojo!





$$\delta = l_0 - H_t$$

$$K = \frac{dF}{d\delta}$$

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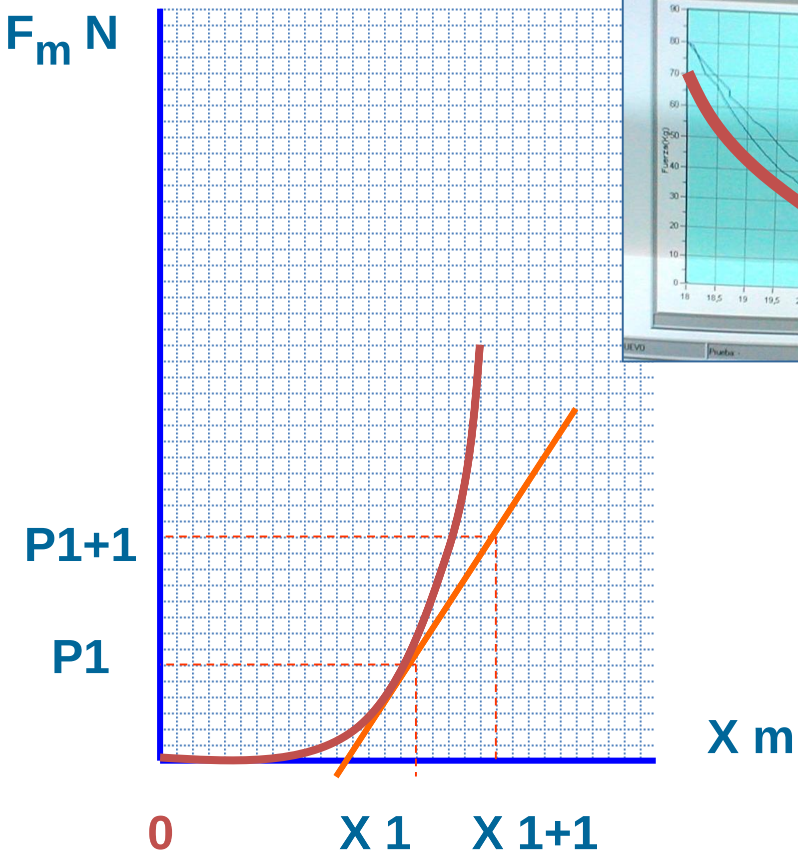
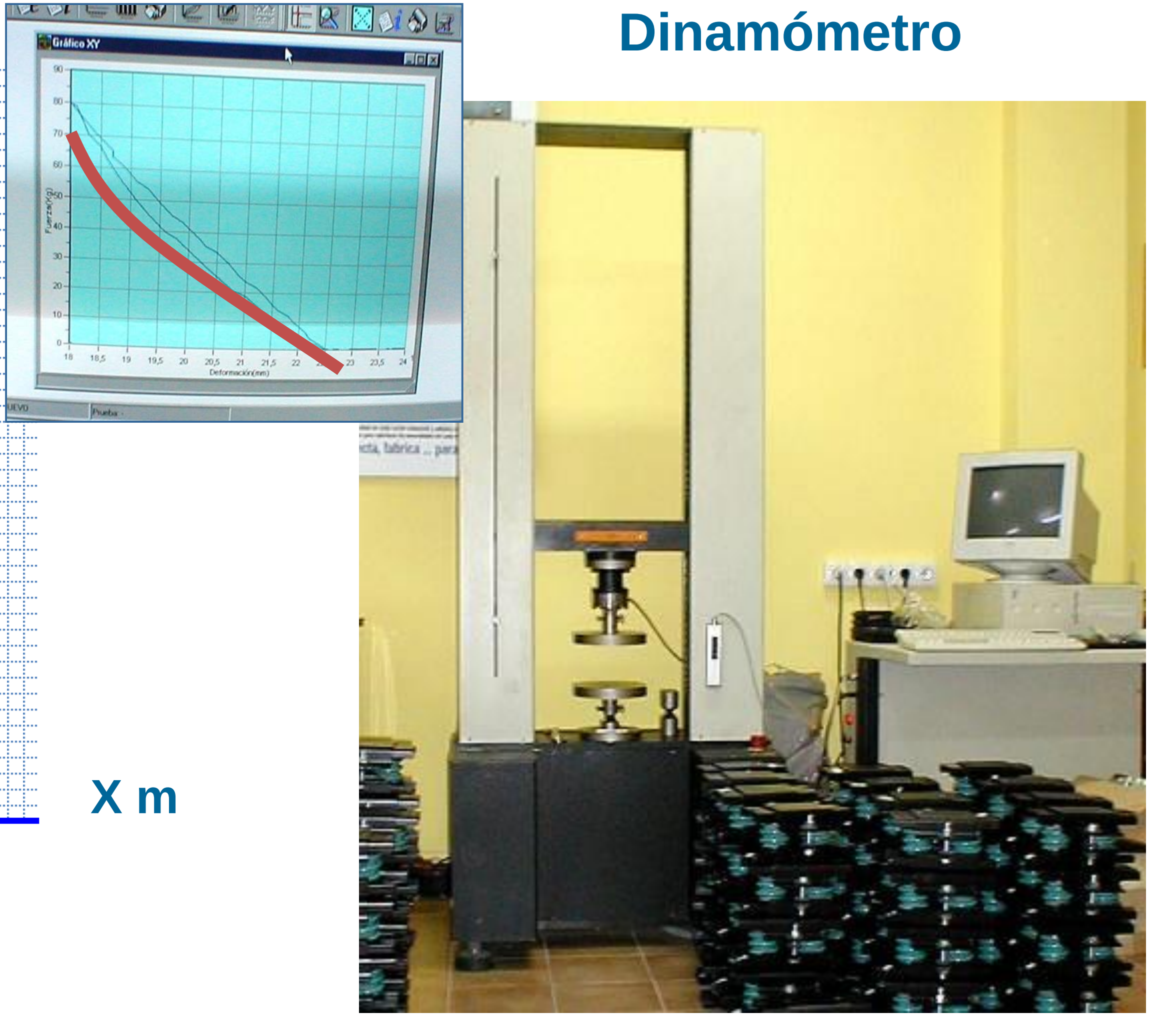
Muelles NO lineales



Dinamómetro



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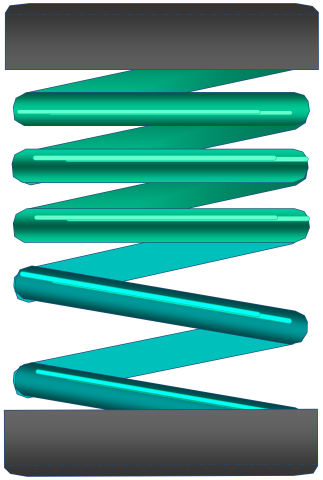
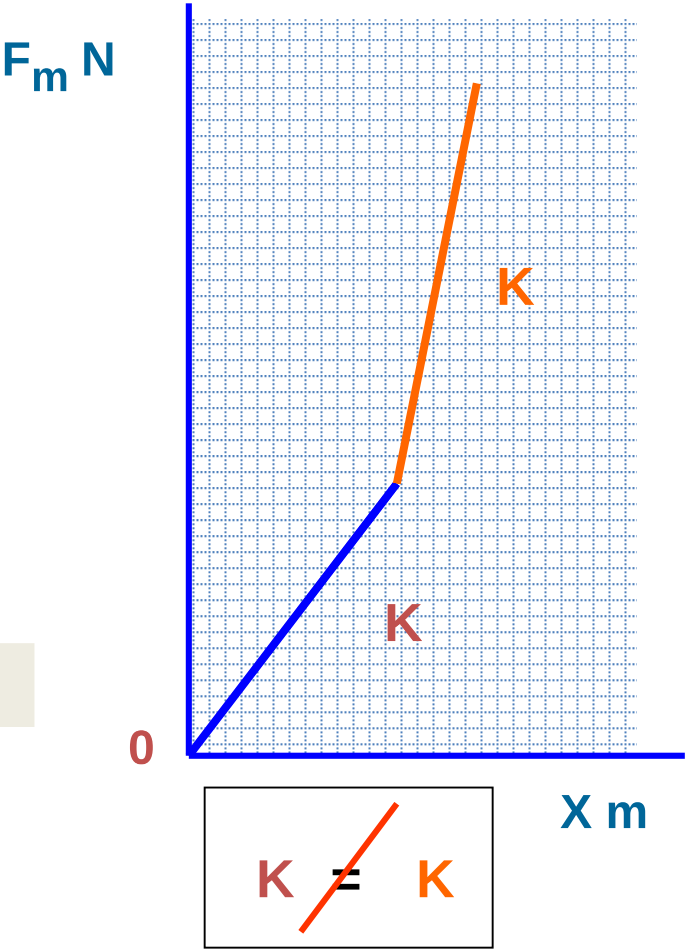
Ejemplos Muelles NO lineales

Muelles cónicos



Muelles bielásticos



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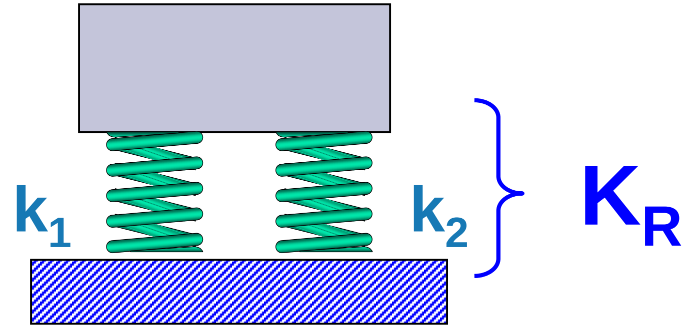
Centre Comercial Espais

Sistemas de montajes de muelles aisladores

Fuente: VIBCON Aisladores VIB



$$F_1 = K_1 \cdot \delta$$

$$F_2 = K_2 \cdot \delta$$

$$F_R = F_1 + F_2 = (K_1 + K_2) \cdot \delta$$

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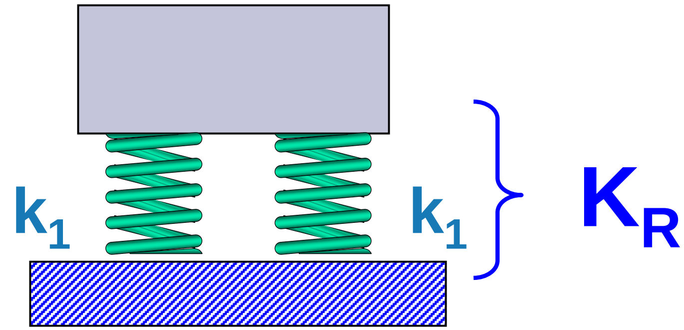
Montajes de aisladores en PARALELO

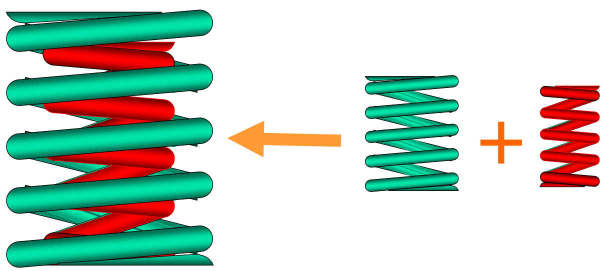
$$K_R = 4K$$

Sistemas en "paralelo"



$$K_R = 2K_1$$

Muelle Bogie



Suspensión coaxial

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Fuente: ALEGI (Labein) VITORIA Enero-Julio 1997

10/2003

Cámara horizontal

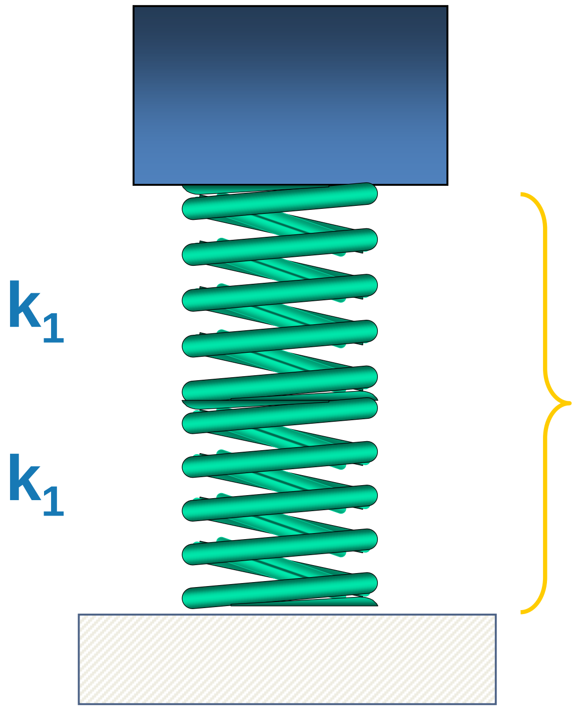
$$F = K_1 \cdot \delta_1$$

$$F = K_2 \cdot \delta_2$$

$$\delta_R = \delta_1 + \delta_2 = \frac{F}{K_1} + \frac{F}{K_2} = F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

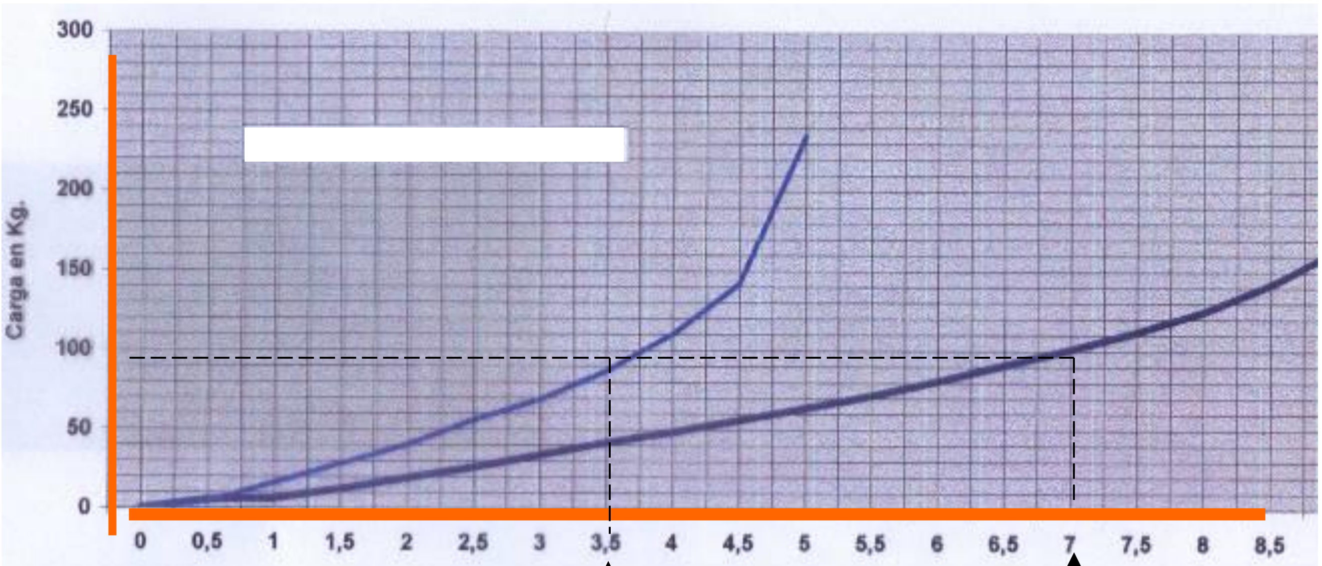
$$\frac{1}{K_R} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\boxed{K_R = \frac{K_1 \cdot K_2}{K_1 + K_2}}$$



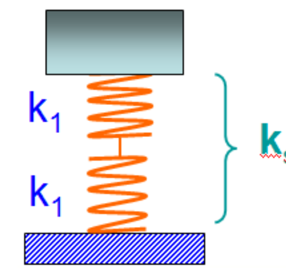
$$K_R = \frac{1}{2} K_1$$

Amortiguadores Viscoelásticos



Como un sistema en serie

$$K_R = \frac{1}{2} K_1$$



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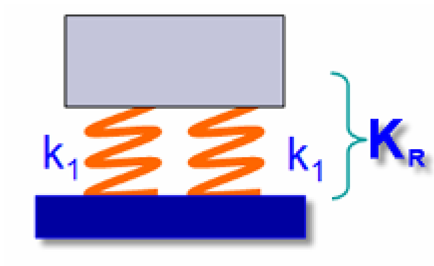
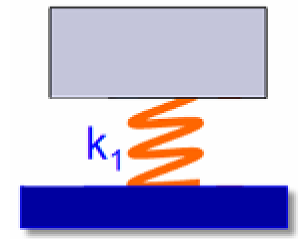
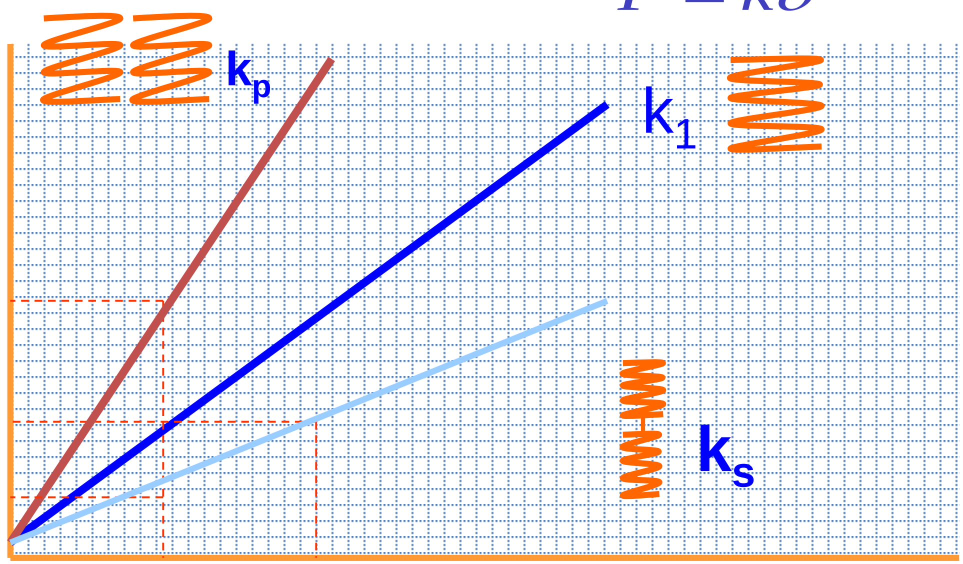
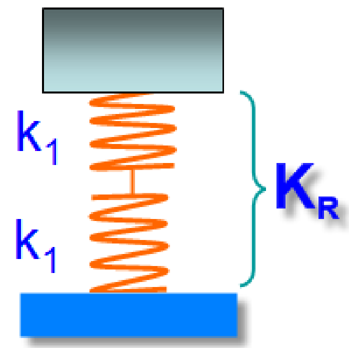




$$K_R = 2K_1$$

$$F = k\delta$$





$$K_R = \frac{1}{2} K_1$$

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$f_0 = f_0'$

"Resonancia"

Fuente: GYMSA 10/2003

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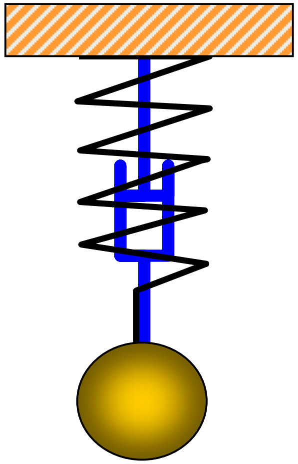
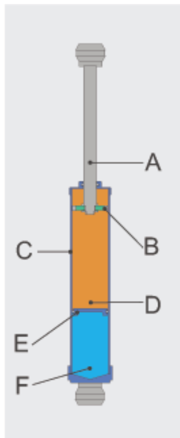
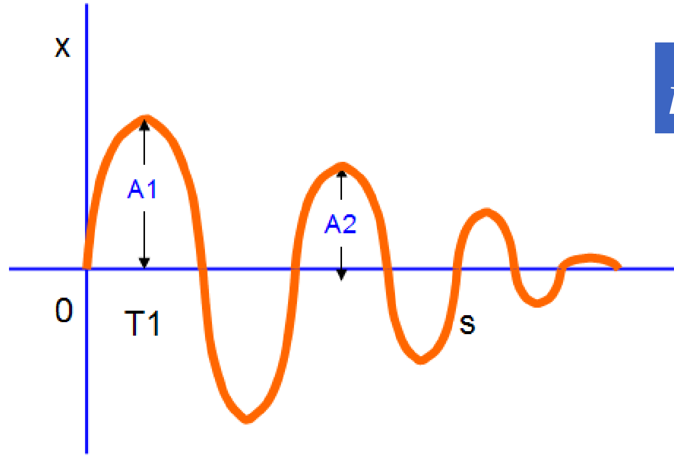
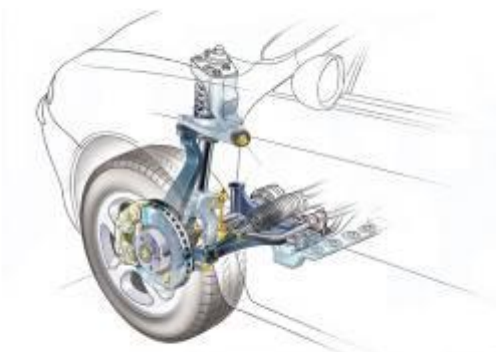






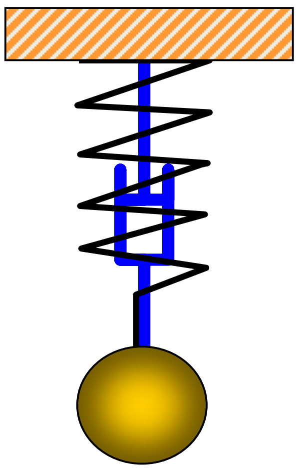
$$m\frac{d^2x}{dt^2}+k\delta+c\frac{dx}{dt}=0$$

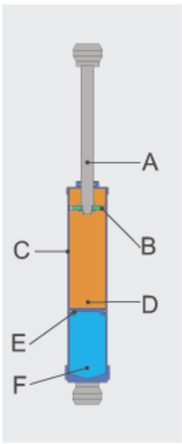
$$\boxed{m\ddot{x}+c\dot{x}+kx=0}$$

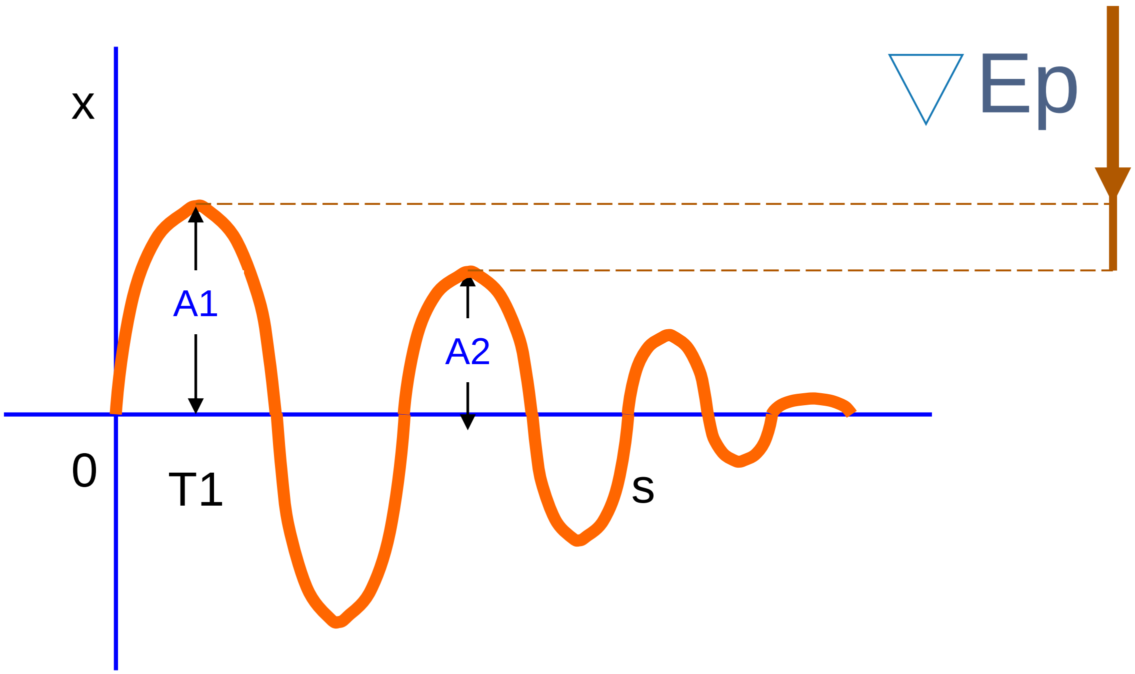


$$\boxed{\varpi_n{}'=\varpi_0\sqrt{1-\varsigma}}$$

$$\boxed{f_n{}'=f_n\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$







$$f_n' = f_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

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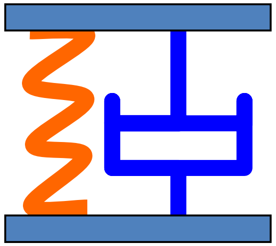
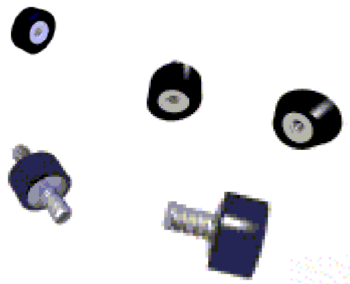
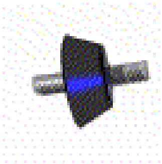
ERIA
LOPEZ
Juan Carl

Fuente: Hotel Sunway (Sitges-2002)

viscoelásticos







Fuente: THE RUBBER GROUP–S.A (03/2012)

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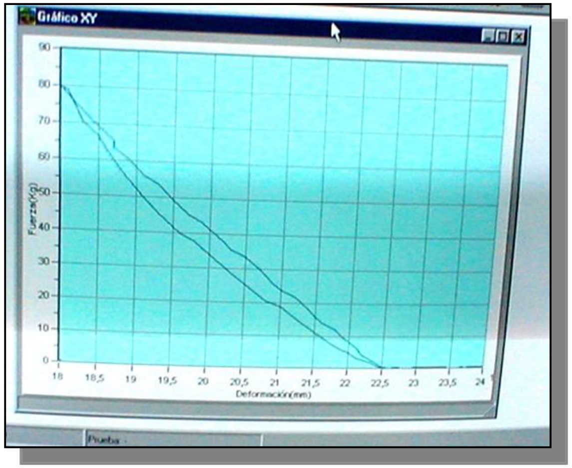
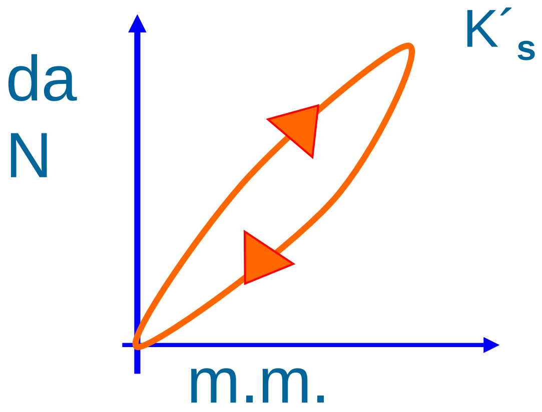
Solamente se puede conocer empíricamente





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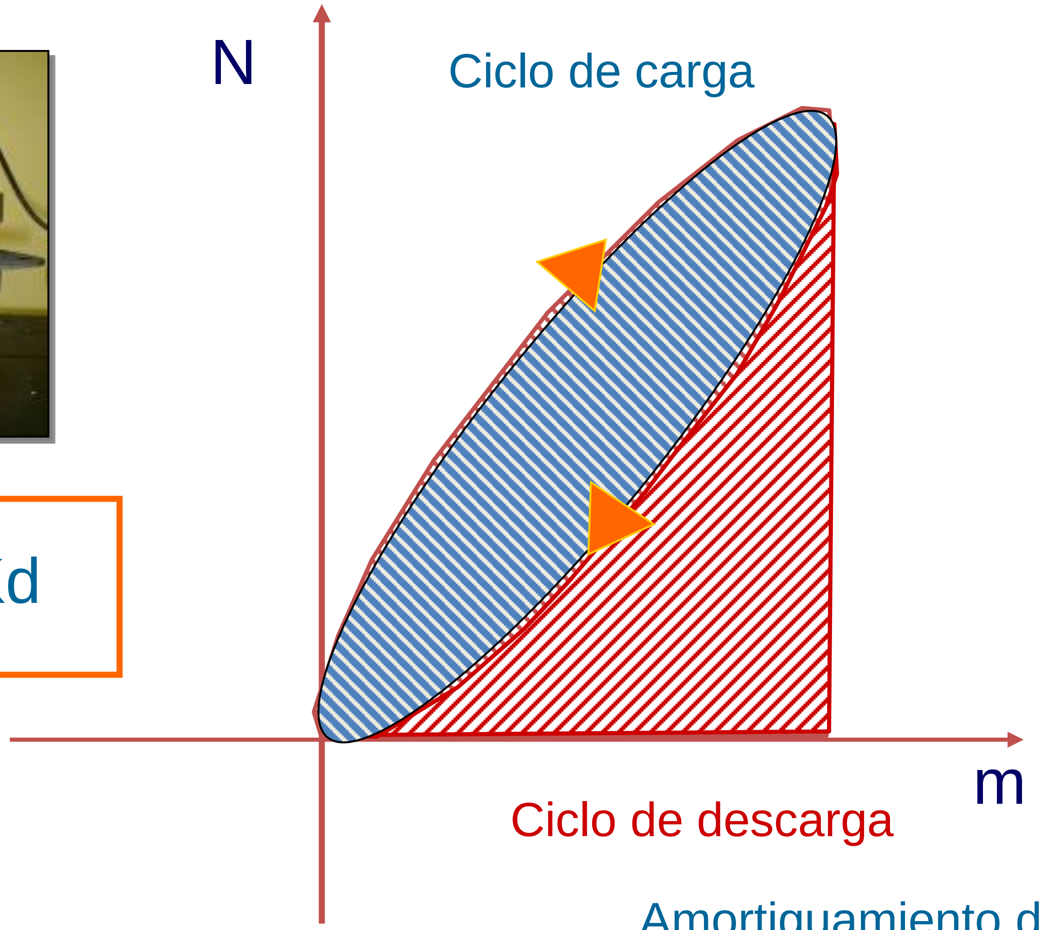

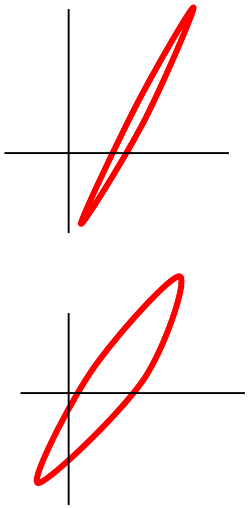

Ks $\cancel{\cong}$ Kd

Amortiguamiento de Histéresis

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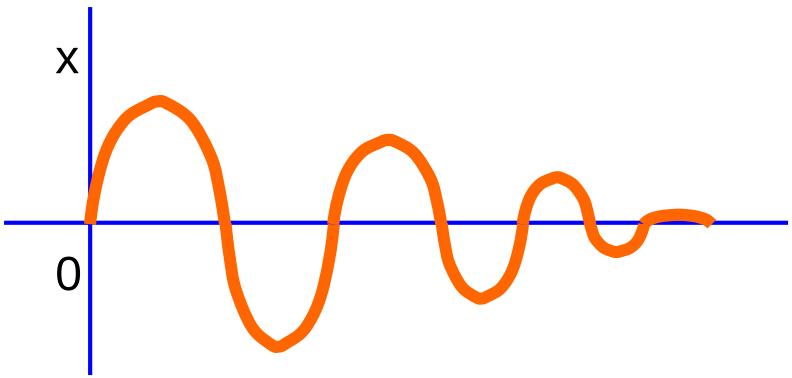
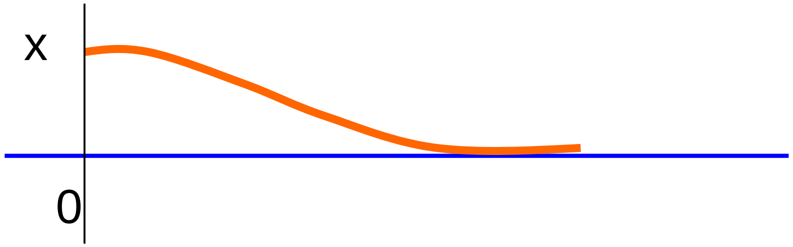
$$\varsigma = \frac{c}{c_c}$$

$$\boxed{\varsigma = \frac{c}{2\sqrt{km}}}$$

Factor de Amortiguación

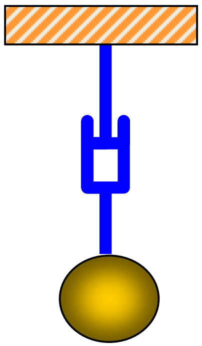


$\zeta < 1$ Infracrítico



$\zeta = 1$ Crítico

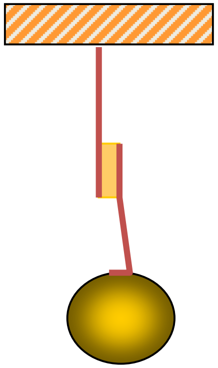
$\zeta > 1$ Supercrítico



Amortiguamiento viscoso

$$F_c = c\,\frac{dx}{dt}$$

C:cte. amortiguación **Nm/s**



Amortiguamiento seco o de Coulomb

$$F_c = \mu N$$

Amortiguamiento de Histéresis (η)

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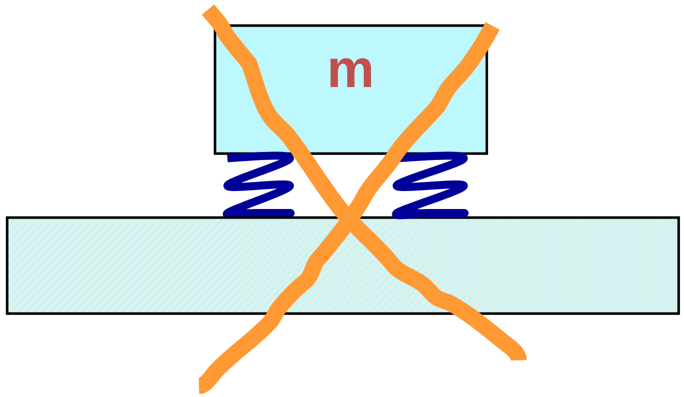
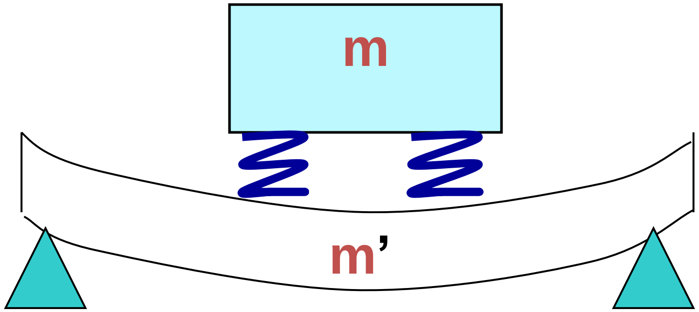
Según la teoría vibraciones

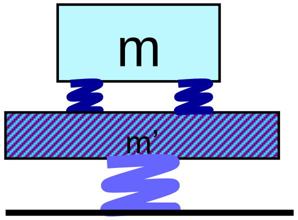


$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Los forjados son elásticos

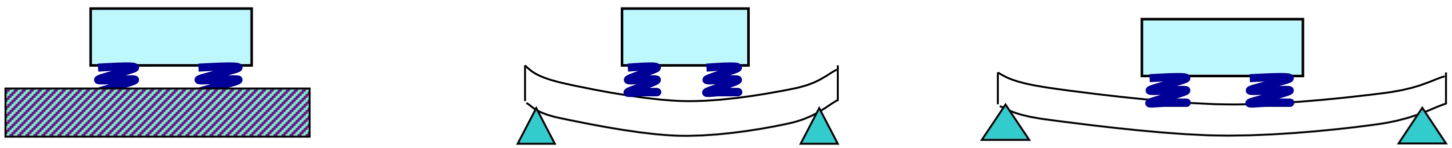


$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m+m'}}$$



Clase de solera	Luz entre apoyos de la solera	Frecuencia natural estimada Hz
S	**En sótano, sobre terreno**	**9**
F6	**Luces hasta 6 metros**	**7**
F9	**Luces hasta 9 metros**	**6**

Fuente: **Den Hartog**



S **F6** **F9**

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Los forjados son elásticos

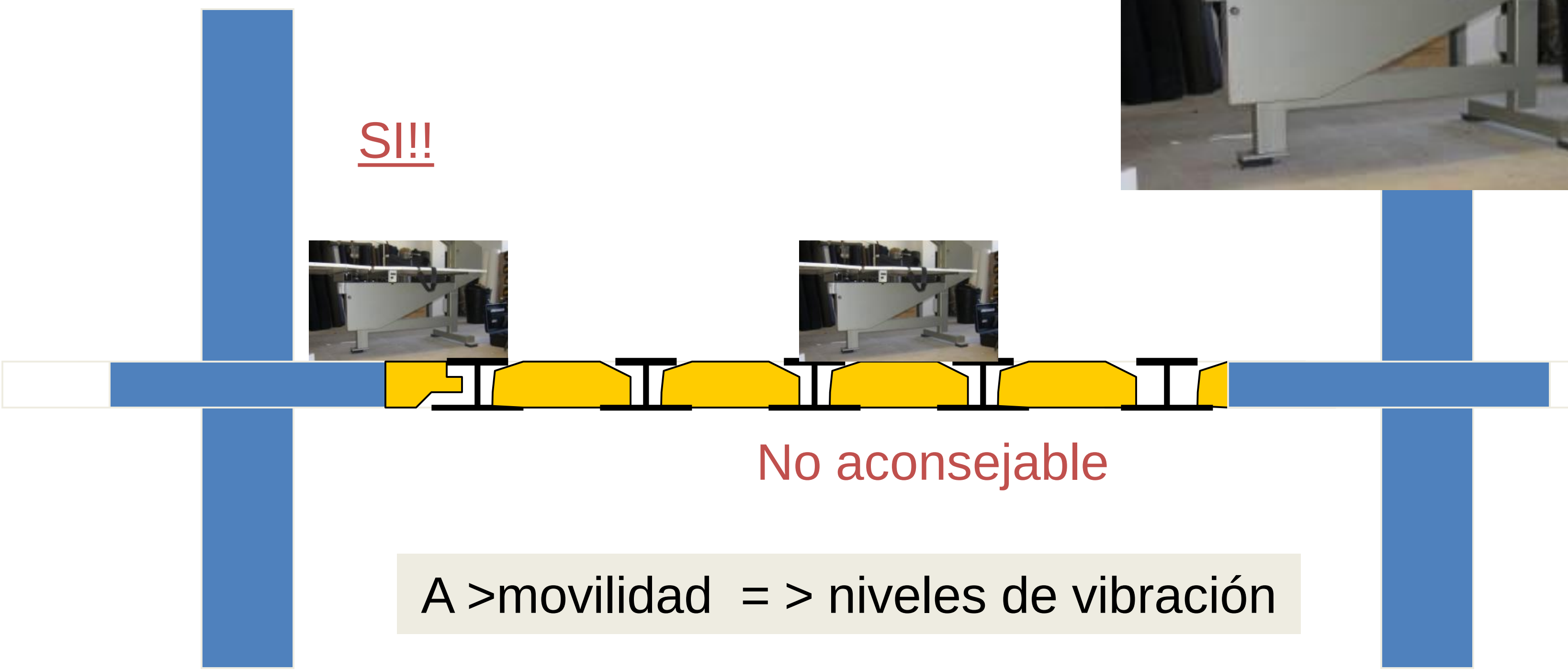
Ejemplos:Imprentas Máquinas de impresión cuatricomías

*Con colocar unas alfombrillas de caucho no es suficiente

Fuente: MATIMPRES Viladecans 2/2004

Ejemplo: cortadoras textiles de cinta

Colocar las máquinas cerca
de los capiteles de pilares



Fuente: CERVERA C/Hector 2/2004

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0. Estrategias de actuación

1. Grado de libertad

3. Montaje antivibratorio

4. Vibraciones forzadas

5. Tipos de MA

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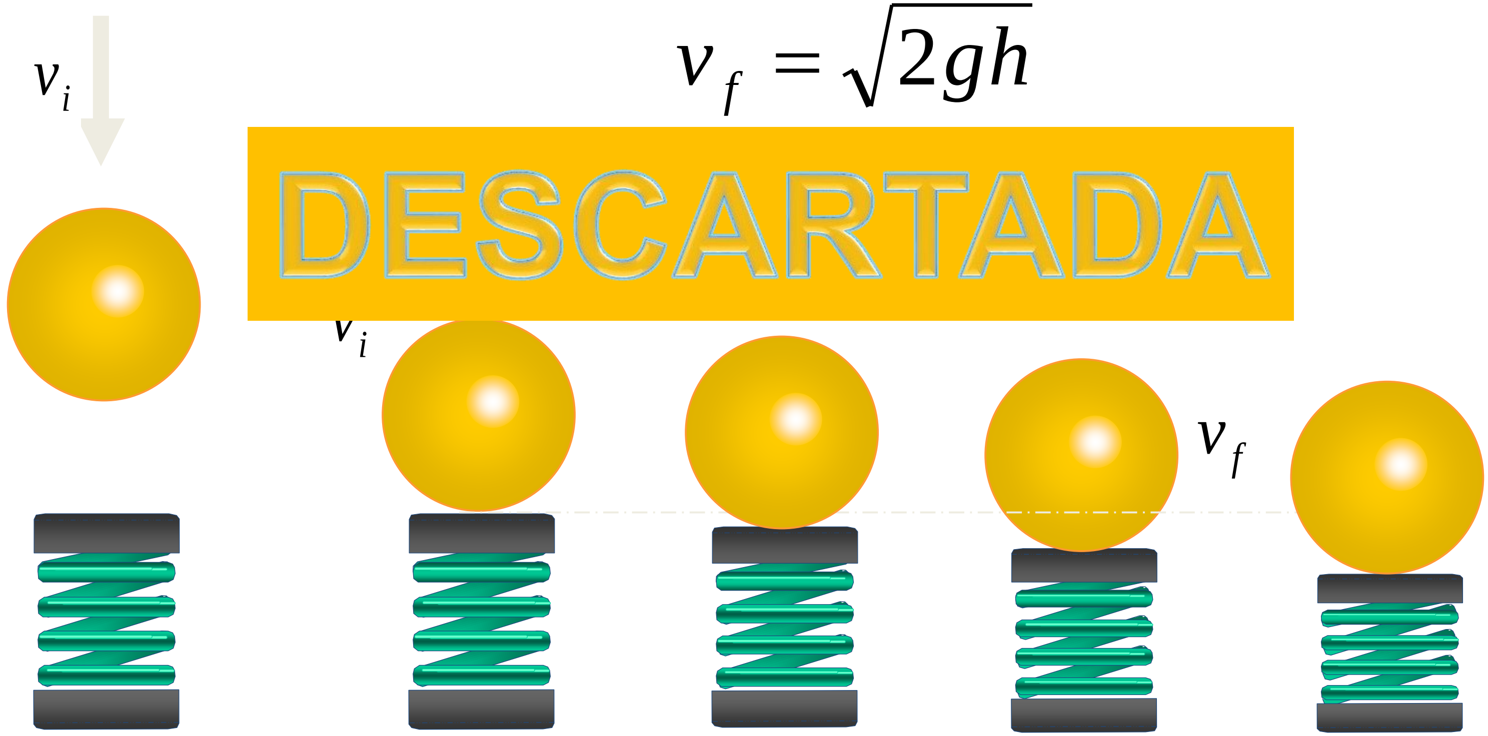
$$A = {}^{v}\!/_{\varpi}$$

$$v_f^{\,2} = v_i^{\,2} \cdot 2a\left(x_f - x_i\right)$$

$$v_f^{\,2} = 0 \cdot 2g\left(h\right)$$

$$v_f = \sqrt{2gh}$$

DESCARTADA



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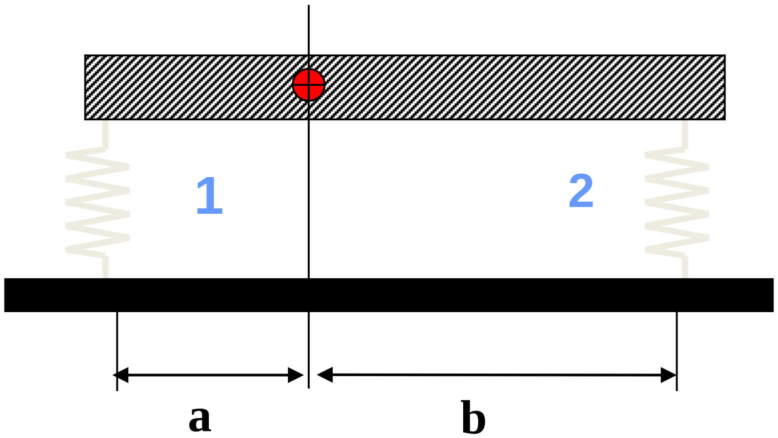
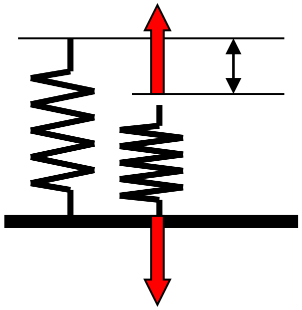
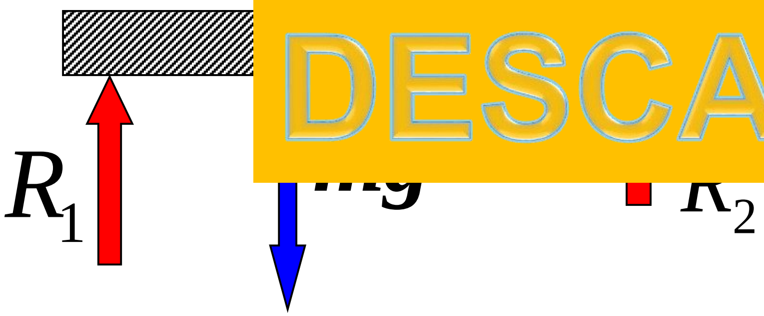


$$F = k\delta$$





DESCARTADA

...s: $R_1 + R_2 = mg$

Equilibrio de momentos: $R_1 a = R_2 b$

$$k_1 \delta a = k_2 \delta b$$

$$\frac{k_1}{k_2} = \frac{b}{a}$$

El muelle que está más cerca del centro de gravedad tiene que ser más duro

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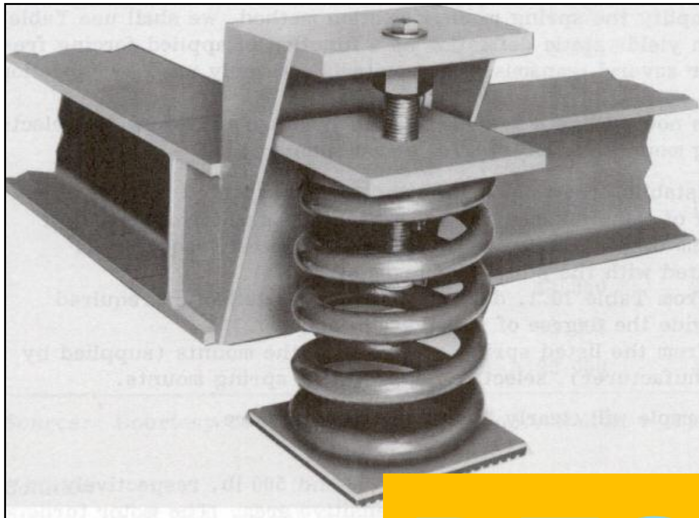
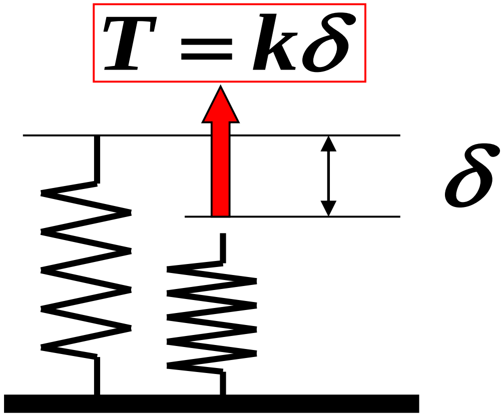
Ana





orizontalidad

DESCARTADA

Misma horizontalidad=misma fn

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Ana