Logistics

- TA Office Hours:
  - Mon & Wed 18:30-19:30pm (in person (Uris 465) or by Zoom).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 Non-transferable Utility Matching[1]

1.1 Concepts

- Consider a two-sided market, that (for example) includes a set of men M and a set of women W.
- Each men and women has strict rank order preferences over all matches as well as the single status. WLOG, assume $\mid M \mid = \mid W \mid$.
- Represent preferences by numerical payoffs (utility), but that can not be traded, therefore the non-transferable utility.
- Denote $\mu(w, m) = 1$ if woman w is matched with man m, and $\mu(w, m) = 0$ otherwise.
- Every man can match with at most one woman, and every woman can match with at most one man, i.e., $\mu(w, m) = 1$ for at most one m.

The goal is to find a match, i.e. $m \longleftrightarrow w$ so that everyone is "happy" in the sense that no man and woman would want to switch partners. More formally, a match is stable if there is no m and w' such that: $m \longleftrightarrow w$, $m' \longleftrightarrow w'$ and $w' \succ_m w$ and $m \succ_{w'} m'$.

- Definition 1: ["Equilibrium"] A matching μ is stable if it is not blocked by any individual or any pair (m, w) of agents. (think of stability as "equilibrium").
- Theorem 1: [Existence of equilibrium-Gale Shapley] A stable matching always exists for every marriage market $(M, W, \succ)$ (way to find one: Deferred Acceptance Algorithm)

The Gale-Shapley Algorithm, also known as the Deferred Acceptance Algorithm (DAA) is as follows:

1. Each man proposes to his top-ranked choice.

[1]Based on Jaden Chen's notes from 2020, Abhi Ananth notes from 2021 and Lone Smith class notes.

2. Each woman keeps her top-ranked man among those that proposed to her and rejects the rest.

3. Each man who has been rejected proposes to his top-ranked choice among those who have not rejected him.

4. Each woman keeps her best proposal among the new ones and the one from previous round and rejects the rest.

5. The process ends when no man has a woman to propose to, and then proposals become matches.

- **Example: rappers and classic rock artists** *(Borrowed from Lones Smith notes)***.** *"Assume a contest in which a rapper that has to match with a classic rock artist to collaborate on a song. We represent preferences by the utilities below, and assume a zero utility for not matching.*

	Pink Floyd	*Bob Dylan*	*J.Hendrix*
Jay-Z	***6,9***	*12,12*	*18,15*
50 C	*4,16*	***8,18***	*12,20*
Snoop	*2,23*	*4,24*	***6,25***

The following algorithm arrives at this allocation: Rappers first propose, everyone asking their top rocker, J. Hendrix. Jimmy Hendrix accepts his most preferred option, Snoop. Then 50 C and Jay-Z ask their top remaining partner, Dylan, who in turn accepts his most preferred suitor 50 C. Finally, Jay-Z and Pink Floyd match.

Comparison with Transferable Utility Matching: *assume these payoffs are actually money, and thus transferable across agents (which utility normally is not). In this world of transferable utility (TU) matching, agents can be compensated his partner for an inferior match quality, or might pay for a better match. NTU presumes that match differences cannot be equalized by monetary transfers. The TU model formally introduces side payments (wages and profits).*

Applied to the rappers and classic rockers example, start with the unique stable NTU matching we found, but then allow transfers (it might help to think of these as "bribes"). In this case, the matching unravels. For Jay-Z is willing to offer Jimi Hendrix up to $18-6=12$ to match with him and sever his match with Pink Floyd. This strictly exceeds Jimi's loss of $25-15=10$ from doing this rematch. Thus, any payment to Jay-Z between 10 and 12 leaves both musicians willing abandon their partners for this new match. It is natural to ask then which matchings are immune to side payments, *namely, stable even in the presence of transfers. We claim that it is: (Jay-Z, Jimmy Hendrix); (50 C, Bob Dylan); (Snoop, Pink Floyd)."*[2]

- Theorem 2: [Optimality of the Equilibrium-Gale Shapley] Denote S as the set of stable matching,
  - $\mu_1 \succeq_M \mu_2$ if $\mu_1(m) \succeq_m \mu_2(m)$ for every $m \in M$;
  - A stable matching μ is M-optimal if $\mu \succeq_M \mu'$ for all $\mu' \in S$;
  - A stable matching μ is W-optimal if $\mu \succeq_W \mu'$ for all $\mu' \in S$;

  when all men and women have strict preferences, there always exists M-optimal stable matching and W-optimal stable matching (induced by M-proposing DAA and W-proposing DAA respectively). That means there is no other stable matching such that any man gets matched with a higher-ranked woman. Furthermore, M-optimal stable matching is W-worst stable matching, W-optimal stable matching is M-worst stable matching.

In previous discussion, we assume that individuals report their preferences frankly and we see that by running the algorithm, we can obtain a stable matching. However, in reality, individuals may have incentives to lie about their preferences in order to get a better match. Therefore, the next question to investigate is: are the matching algorithms "strategy-proof"? (i.e, all individuals will report their true preferences).

- Theorem 3: (Impossibility Theorem) No stable matching mechanism exists for which stating the true preferences is a dominant strategy for every agent.
- Theorem 4: In a marriage problem $(M, W, \succ)$, when M-optimal stable mechanism is employed
  - It is a dominant strategy for each man to state his true preferences;
  - when there is more than on stable matching, there will be an incentive for some woman to misrepresent her preferences.

[2] Borrowed frome Lone Smith notes: https://www.lonessmith.com/wp-content/uploads/2016/10/1-Matching-and-Double-Auctions.pdf.

1.2 Questions

1. In this matching problem we assume that getting married is strictly preferred by all men and women to staying single. Consider the following preferences for 5 men and 5 women:

m_1	2	3	4	5	1	w_1	1	2	3	5	4
m_2	3	4	5	1	2	w_2	2	1	4	5	3
m_3	5	1	4	2	3	w_3	3	2	5	1	4
m_4	3	1	2	4	5	w_4	4	5	1	2	3
m_5	1	5	2	3	4	w_5	5	1	2	3	4

   (a) Using the men-proposing DAA, find a stable match.

   (b) Using the women-proposing DAA, find a stable match.

   (c) When the men-proposing version is used, can women 1 be better off by not revealing her true preferences?

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m_5	1	5	2	3	4	w_5	5	1	2	3	4

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a) R1 (round 1) each man proposed to the most prefered women

R2 (round 2) " " 2nd most prefered women.

Round 1: m1 → 2   m2, m4 → 3   m3 → 5   m5 → 1

Round 2: w3 rejects m4.

(m1 w2) (m2, w3) (m3, w5) (m5, w1)

Round 3: m4 → w1

w1 rejects m4 (2nd time)

(m1, w2) (m2, w3) (m3, w5) (m5, w1)

Round 4: m4 → w2

w2 rejects m4 (3rd time) AGAIN

(m1, w2) (m2, w3) (m3, w5) (m5, w1)

Round 5: m4 → w4

w4 accepts m4

(m1 w2) (m2 w3) (m3 w5) (m5 w1) (m4 w4)

b) TFY.

c) Suppose that w1 misreports her preferences as:

(1 2 3 4 5)

Let's see what happens:

Run the algorithm again: (TFY)

We will get the following match:

(m1 w2) (m2 w3) (m5 w5) (m4 w4)
(m3 w1)

New matching w1 gets m3 $\longrightarrow$ she is better off!

Example that women have incentive to lie their preferences in man best matching.