Logistics

- TA Office Hours:
  - Mon & Wed 18:30-19:30pm
  - In person (Uris 465) or by Zoom (same link as before).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 Simple Leontief Model [1]

1.1 Important concepts:

- **Definition 1: Input requirements (or activity) matrix A:**

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

  where:

  - a_{ij} denotes the amount of good i needed to produce 1 unit of good j.
  - $a_{ij} \geq 0$
  - for each i there exists a j such that $a_{ij} > 0$.
  - Column vector $A^j = \begin{bmatrix} a_{1j} \\ \vdots \\ \vdots \\ a_{nj} \end{bmatrix}$ describes the amount of all goods required to produce 1 unit of good j.
  - Example:

$$A = \begin{bmatrix} 1/2 & 1/4 \\ 1/4 & 1/2 \end{bmatrix}$$

- **Definition 2: Productive matrix.** Input matrix A is productive if there exists a $x^* \geq 0$ such that $x^* >> Ax^*$.

[1] These notes are based on Jaden Chen's notes from 2020 and Abhi Ananth notes from 2021.

  - Example (non- productive matrix):

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- **Theorem 1:** If A is productive, then for any $y \geq 0$, the system $(I - A)x = y$ has a non-negative solution.
- **Theroem 2:** A is productive if and only if $(I - A)$ has full rank and $(I - A)^{-1}$ is non-negative.

Now, the above analysis imposes no constraint on production, so when A is productive you can produce any amount you want. Indeed, the missing ingredient we need for the model to be useful is labor, the primary factor of production. The labor requirement vector is given by $a_0 = (a_{01}, ..., a_{0n})$, where a_{0j} is the amount of labor needed to produce one unit of output j. Let L be the supply of labor in the economy. It now makes sense to describe what we can produce:

- **Definition 3:** The **set of feasible net output** is:

$$Y = \left\{ y \in \mathbb{R}^n : a_0 \cdot (I - A)^{-1} y \leq L, y \geq 0 \right\}$$

We have a constraint on how much we can produce. A natural question then would be how much should we produce? To do this we need to think about costs and revenue, so we need prices. Let $p = (p_1, ..., p_n)$ be the prices of goods and let w denote the wage rate.

- **Definition 4:** The **profit** from 1 unit of good i would be:

$$\pi_i = p_i - (wa_{0i} + p_1 a_{1i} + \cdots + p_n a_{ni})$$

  Rearranging and putting this in vector form gives the rate of profit $\pi = p \cdot (I - A) - wa_o$. With gross output vector x, the profit is $p \cdot x$.

- **Theroem 3:**
  - If A is productive and $a_0 \gg 0$;

  OR:

  - If A is productive and irreducible and $a_0 > 0$;

  Then there is a $(w^*, p^*) \gg 0$ such that $\pi^* = 0$ and $p^* = w^* a_0 (I - A)^{-1}$.

1.2 Questions:

1. Suppose the activity matrix for a Simple Leontief Model is $A = \begin{bmatrix} 1/3 & 1/4 \\ a & 1/2 \end{bmatrix}$ and labor input requirements are $a_0 = (1/3, 1/2)$.

   (a) For what values of a is A productive?

   (b) Let $L = 5$, describe the set of feasible net outputs in terms of a.

   (c) Let $w = 1$, describe the set of equilibrium prices in therms of a.

2. *(From LM problem set)* Are the following matrices productive? Prove your answers.

$$A_1 = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.2 & 0.4 \\ 0.2 & 0.4 & 0.3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.1 & 0.5 \end{bmatrix}$$