Logistics

- TA Office Hours:
  - Mon & Wed 18:30-19:30pm (in person (Uris 465) or by Zoom).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 General Equilibrium and Welfare Theorems[1]

1.1 Important concepts:

1. Definition 1: Private-ownership economy. We move from pure exchange economy to a private-ownership economy by introducing a set of producers.
   (a) L commodities, aggreagte endowments $\bar{\omega} \in \mathbb{R}^L$.
   (b) Firms: $j = 1, ..., J$ with nonempty and closed production set $Y_j \in \mathbb{R}^L$
   (c) Consumers: $i = 1, ..., I$ with consumption set $X_i \subset \mathbb{R}^L$, preferences $\succeq_i$ and endowment $\omega_i \in X_i$. Consumer $i's$ ownership of firm j is $\theta_{ij} \in [0, 1]$ and $\sum_i \theta_{ij} = 1$.
2. Definition 2. Walrasian equilibrium for a private-ownership economy: denotes an allocation (x^*, y^*) and a price vector p^* such that:
   (a) (Profit maximization): For all j, $p^* \cdot y_j^* \geq p^* \cdot y_j$ for all $y_j \in Y_j$
   (b) (Utility maximization): For all i, x_i^* is $\succeq_i -maximal$ in the budget set:
   $$\left\{ x_i \in X_i : p^* \cdot x_i \leq p^* \cdot \omega_i + \sum_i \theta_{ij} p^* \cdot y_j^* \right\}$$
   (c) (Markets clear): $\sum_i x_i^* = \bar{\omega} + \sum_j y_j^*$
3. Definition 3: [Walrasian Quasi-equilibrium] A Walrasian quasi-equilibrium for a private-ownership economy denotes an allocation (x^*, y^*) and a price vector p^* such that:
   (a) (Profit Maximization): For all $j, p^* \cdot y_j^* \geq p^* \cdot y_j \; for \; all \; y_j \in Y_j$.
   (b) ("Expenditure Minimization")[2]: For all i, we have $p^* \cdot x_i^* \leq p^* \cdot \omega_i + P_i \theta_{ij} p^* \cdot y_j^*$ and for all $x_i' \in X_i$ such that $x_i' \succeq_i x_i^*$ we have p $* \cdot p^* \cdot x_i' \geq p^* \cdot \omega_i + P_i \theta_{ij} p^* \cdot y_j^*$.
   (c) (Markets Clear): $\sum_i x_i^* = \bar{\omega} + \sum_j y_j^*$

[1] These notes borrow from Jaden Chen's notes from 2020.
[2] when $\succeq_i$ is LNS, this condition is equivalent to x_i^* minimizes expenditure on $\{x_i : x_i \succeq_i x_i^*\}$

4. Theorem 1: [Existence of Walrasian equilibrium in private-owenership Economy] Suppose:
   (a) For all i, X_i is compact and convex, $\succeq_i$ is continuous and convex.
   (b) For all j, Y_j is compact and convex, $0 \in Y_j$;

   Then there is a Walrasian equilibrium.
5. Theorem 2: [First Welfare Theorem] Suppose $\succeq_i$'s are locally nonsatiated for all i and (p^*, x^*, y^*) is Walrasian equilibrium with $p^* \geq 0$, then (x^*, y^*) is Pareto optimal.
6. Theorem 3: [Second Welfare Theorem] Suppose every Y_j is convex and every preference relation $\succeq_i$ is convex and locally nonsatiated. Then for every Pareto optimal allocation (x^*, y^*), there is a way to reallocate endowments and shareholdings, such that (p^*, x^*, y^*) is Walrasian quasi-equilibrium where $p^* \neq 0$.

1.2 Questions:

1. Consider a private ownership economy with two individuals, Mr. 1 and Mr. 2 and two goods x and y. Consumers cannot consume in negative quantities. Mr. 1 and Mr. 2 have the following utility functions:

$$u_1 = x_1 - \gamma y_2$$
$$u_2 = \frac{1}{2} log x_2 + \frac{1}{2} log y_2$$

   where $\gamma \in [0, 1)$. Each consumer has endowment of 1 unit of each good. Let good x be the numeraire good and denote the price of good y by p.
   (a) Find the CE allocation and price for this economy.
   (b) For what values of γ is the CE Pareto optimal?
   (c) Does First Welfare Theorem hold here? Why?
2. (Welfare Economics Problem 2)
3. In a two-person, two good exchange economy, individuals A and B have the following preferences:

$$u^A(x, y) = min\{x, 2y\}$$
$$u^B(x, y) = min\{3x, y\}$$

   where x and y are two goods. The aggregate endowment is $(12, 12)$. Answer the following using an Edgeworth box:

(a) Find the set of Pareto optima.

(b) Assume A has all of good x and B all of good y. Describe the competitive equilibria.

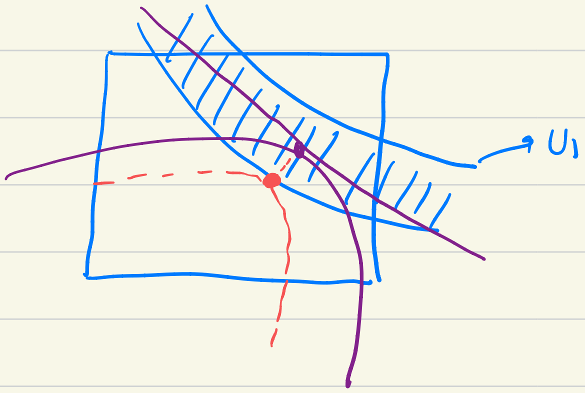
(c) Which endowment allocations have competitive equilibria with positive prices for both goods?

- It is better to assume preferences not utility functions. (General Rule).
- It is sometimes more convenient to assume on the utility functions

1st WT: most important thing is $\succsim$ is LNS.

If not then the CE may not be P.O.

Counterexample:



- Is a CE but not

corner 1 is the same ut.

corner 2 is better off

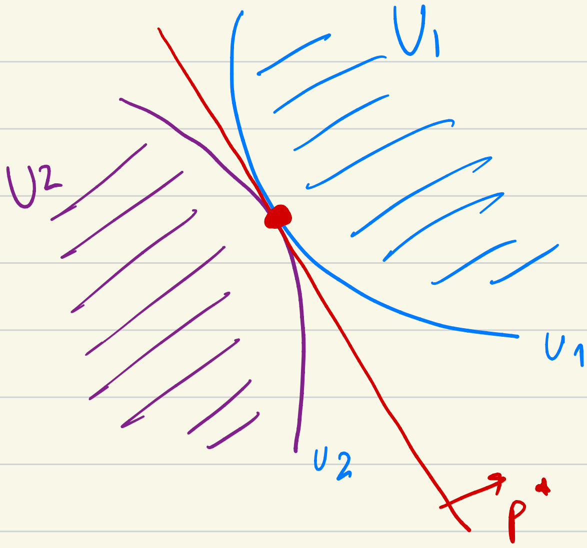
↓

Not PO!

CE is not PO.

2nd WT: $\succsim$ is LNS. Most important thing is $\succsim$ is CONVEX

Because we need to use the Separated Hyperplane theorem!!

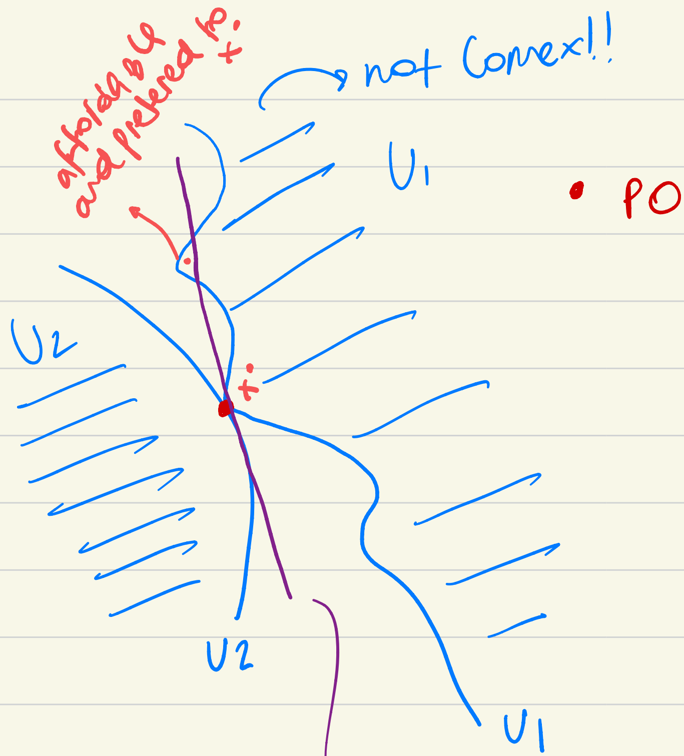


- PO,

Upper contour set of person 1 and 2 are convex → SHT

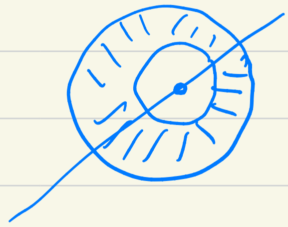
we can find a plane that separates the convex sets

If not convex, it is not able to find supporting planes!



You are gonna intersect either with U1 or U2, so PO is not a CE!!

(If they are closed it's not enough



Both closed but not separated HT)

Problems

sounds like exchange ...

1. Consider a private ownership economy with two individuals, Mr. 1 and Mr. 2 and two goods x and y. Consumers cannot consume in negative quantities. Mr. 1 and Mr. 2 has the following utility functions:

$$u_1 = x_1 - \gamma y_2$$
$$u_2 = \frac{1}{2}\log x_2 + \frac{1}{2}\log y_2$$

where $\gamma \in [0, 1)$. Each consumer has endowment of 1 unit of each good. Let good x be the numeraire good and denote the price of good y by p.

(a) Find the CE allocation and price for this economy.

(b) For what values of γ is the CE Pareto optimal?

(c) Does First Welfare Theorem hold here? Why?

$$\begin{cases} U_1 = x_1 - \gamma y_2 & e_1 = (1,1) \\ & e_2 = (1,1) \\ U_2 = \frac{1}{2}\log x_2 + \frac{1}{2}\log y_2 \end{cases}$$

$\rightarrow$ increasing in both x_2, y_2

if $p_2 = 0 \rightarrow$ good 1 ∞ $\rightarrow$ market can not clear! because we have a finite endowment.

as long as there is a CE $\rightarrow$ we can guarantee $p > 0$

Super Important!

- as long as there is 1 consumer who has strongly monotone preferences and $\exists$ CE $\rightarrow$ eq prices are > 0 !!
- as long as 1 consumer whose preferences are monotone $\rightarrow$ price vector ≥ 0 if $\exists$ CE but it can not be negative.

(a):

$$\begin{cases} \max & x_1 - \gamma y_2 \\ \text{st} & p x_1 + y_1 \leq p+1 \\ & x_1, y_1 \geq 0 \end{cases}$$

Linear utility function!

[normalize $p_x = p$; $p_y = 1$]

Walras law holds in eq after we set $y_1 = 0$.

$$\Longrightarrow \quad x_1 = \frac{p+1}{p} \quad ; \quad y_1 = 0$$

$$\begin{cases} \max & \frac{1}{2}\log x_2 + \frac{1}{2}\log y_2 \\ \text{st} & p x_2 + y_2 \leq p+1 \quad (\lambda) \end{cases}$$

since Inada cond satisfied ∴ drop $x_2, y_2 \geq 0$

$$[x_2] \quad \frac{1}{2x_2} = p\lambda \qquad\qquad x_2 = \frac{1}{2p\lambda}$$

$$[y_2] \quad \frac{1}{2y_2} = \lambda \qquad\qquad y_2 = \frac{1}{2\lambda}$$

∵ utility convex ∴ SOC ✓

∵ LNS ∴ plug in the budget constraint and get

$$\frac{1}{2\lambda} + \frac{1}{2\lambda} = p+1 \longrightarrow \lambda = \frac{1}{p+1}$$

$$\therefore \quad x_2 = \frac{p+1}{2p} \qquad y_2 = \frac{p+1}{2}$$

Mkt clearing

$$\begin{cases} x_1 + x_2 = 2 \\ y_1 + y_2 = 2 \end{cases} \Longrightarrow \quad p = 3$$

$\therefore$

$$\boxed{\begin{array}{ll} x_1^\circ = 4/3 & y_1^\circ = 0 \\ x_2^\circ = 2/3 & y_2^\circ = 2 \end{array}}$$

(b) Values of γ st CE is Pareto optimal.

Method 1: use definition of PO.

We want $(x_1^\circ, x_2^\circ, y_1^\circ, y_2^\circ)$ to be PO

At (x_2°, y_2°); $v_2 = \frac{1}{2} \log 2/3 + 1/2 \log 2 = \bar{U}$

$$\begin{cases} \max\limits_{x_1, x_2, y_2} & x_1 - \gamma y_2 \\ \text{st} & 1/2 \log x_2 + 1/2 \log y_2 \geq \bar{U} \\ & 0 \leq x_1 + x_2 \leq 2 \\ & 0 \leq y_2 \leq 2 \end{cases}$$

allocate combination of x, y st consumer 2 gets $\bar{U}$ already and consumer 1 gets the max possible given that

We want to find the range of γ such that

$(x_1^\circ, x_2^\circ, y_2^\circ)$ solves the problem

$\Rightarrow \quad \gamma \leq 1/3 \qquad \therefore \quad \gamma \in [0, 1/3]$

how to solve? By K-T

Method 2: graphically by Edgeworth Box in the eq.

$$u_1 = x_1 - \gamma \underbrace{y_2}_{2-y_1 \text{ by MC}} = x_1 - \gamma(2 - y_1) = x_1 + \gamma y_1 - 2\gamma$$

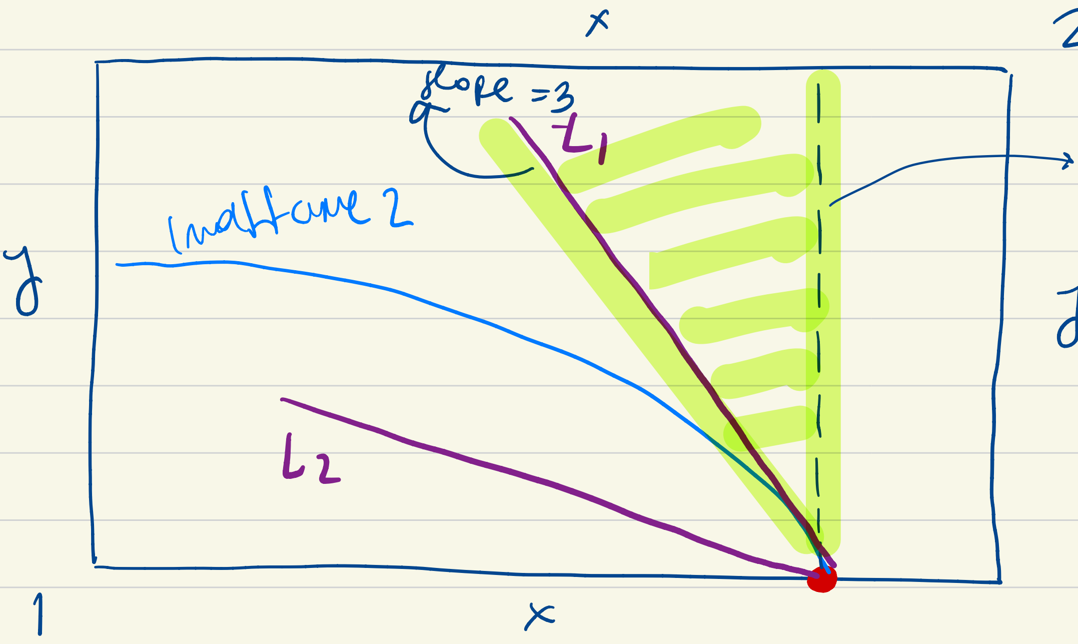
(covers 1 dealing money in eq.)

$$u_2 = \frac{1}{2}\log(x_2) + \frac{1}{2}\log(y_2)$$

Eq: $x_1^\circ = 4/3 \quad y_1^\circ = 0$

$x_2^\circ = 2/3$

$y_2^\circ = 2$



If $L1 \to$ PO

If $L2 \to$ NPO

$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

$\frac{1}{\gamma} = \frac{3}{1}$

$x_1 + \gamma y_1 - 2\gamma = \bar{U}$

$x_1 - 2\gamma - \bar{U} = -\gamma y_1$

slope: $-\frac{1}{\gamma}$

convex 1 indiff curve

$-\frac{1}{\gamma} \in [-3; -\infty) \to \gamma\ (\frac{1}{3}, +\infty)$

within this range consumer 1 we have PO.

$\therefore \frac{1}{\gamma} \in [3, \infty] \to \gamma \in [0, \frac{1}{3}]$

c) 1st welfare theorem holds?

No, because we have externality.

One assumption at the very beginning is violated
↓
assumption everybody utility depends on his or her own consumption

we need NO EXTERNALITIES!!

It should hold for any γ if we say it holds.

If person 1 a strategic hunter → he would consume more of J_1 to give consumer 2 less J_2
→ but here you take J_2 as given when taking decision.

ex ante he does not take the

ex post → Yes!

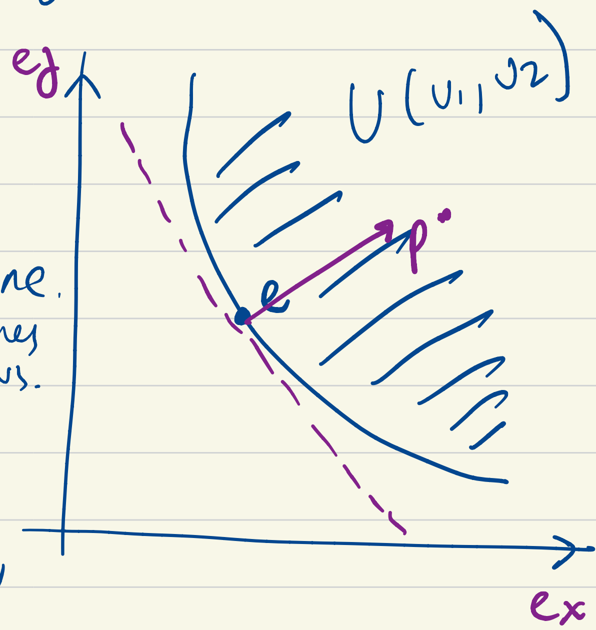
[Be explicit about your assumption]

Problem 2

(a) If $\succsim$ are LNS, at a CE price vector, expenditure on $U(u_1, u_2)$ is minimized at agg endowment.

2 commodities: x, y

2 consumers: $1, 2$



I will assume $\succsim$ are LNS and monotone.

monotone implies LNS.

If $\exists\, e'$ st $e' \in U(u_1, u_2)$ and $p^* e' < p^* e$. $\because e' \in U(u_1, u_2)$,

By definition there are c_1', c_2' such that:

$$\begin{cases} c_1' + c_2' \leq e' \\ u_1(c_1') \geq u_1(c_1^*) \\ u_2(c_2') \geq u_2(c_2^*) \end{cases}$$

[some write =] only need LNS

$\because \succsim$ is monotone, we have $p^* \geq 0$.

is not that we endow the economy with new endowment

$$\text{So } p^* c_1 + p^* c_2 \leq p^* e' < p^* e$$

prices are the same p^*

[monotone]

$$\therefore p^* c_1 + p^* c_2 < p^* e_1 + p^* e_2$$

So there is some $i \in \{1, 2\}$ such that $p^* c_i' < p^* e_i$

But notice that $u_i(c_i') \geq u_i(c_i^\circ)$, violating LNS.

(b) utility strictly quasi-concave ; $U(v_1, v_2)$ strictly convex set

Assume that $\succsim$ is lower-semi continuous. ($\succsim$ is cont)

If $e', e'' \in U(v_1, v_2)$. WTS $\alpha e' + (1-\alpha) e'' \in \text{int}\, U(v_1, v_2)$

for $\alpha \in (0,1)$ (ie $\exists$ open set $O \subset U(v_1, v_2)$ st $\alpha e' + (1-\alpha) e'' \in O$)

Suppose $c_i' \succsim c_i^\circ$ and $c_i'' \succsim c_i^\circ$ where c_i' and c_i'' are allocation of e' and e''.

$\because$ $\succsim$ is strictly convex ($u(\cdot)$ is strictly quasiconcave)

$\therefore$ $\alpha c_i' + (1-\alpha) c_i'' \succsim c_i^*$. Define $P(c_i^\circ)$ as the strict upper contour set of c_i°

$P(c_i^\circ) = \{c_i : c_i \succsim_i c_i^\circ\}$

$\therefore$ $\alpha c_i' + (1-\alpha) c_i'' \in P(c_i^\circ)$

Define $P \equiv P(c_1^\circ) + P(c_2^\circ) = \{c : c = c_1 + c_2 ; c_1 \in P(c_1^*), c_2 \in P(c_2^*)\}$

$\because$ $\succsim$ is lower semi continuous.

$\therefore$ $P(c_i^\circ)$ is open (since strict lower semi-continuity)

$\therefore$ P is open $\subset U(v_1, v_2)$ (sum of two open sets)

Since $\alpha c_i' + (1-\alpha) c_i'' \in P(c_i^*)$ $\forall i$, then

$\alpha (c_1' + c_2') + (1-\alpha)(c_1'' + c_2'') \in P$

$\therefore \quad \alpha e' + (1-\alpha) e'' \in P$

$\therefore \quad \alpha e' + (1-\alpha) e'' \in \text{int } U(u1, u2)$