Logistics

- TA Office Hours:
  - Mon & Wed 18:30-19:30pm
  - In person (Uris 465) or by Zoom (same link as before).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 General Leontief Model (continued)

1.1 Questions

1. *(Linear Economic Models PS, Q14)* **Ricardian model with transportation costs.**

   Transportation is a productive activity that turns apples in country i into apples in country j. In the competitive model, this technology will be owned by a firm just as the technology for making apples and the technology for making beer are owned by firms. The technology is known as the iceberg technology. For every unit shipped from country i, $0 < k < 1$ units arrive at country j. Otherwise, everything else is usual. Output of good A in country i is $q_i^A = \alpha_i l_i^A$, $q_i^B = \beta_i l_i^B$. The output of each good in country i is divided between home consumption and exports. The consumption of each good in country i is the sum of home production and imports. Both countries have identical utility, which is Cobb Douglas, in log form, $u(c^A, c^B) = \delta log c^A + (1-\delta) log c^B$. Parts 1 through 7 are questions having only to do with technology and the fact that each country must consume some of both goods in equilibrium. Parts 8 and 9 require facts from the demand side of the model.

   To take into account:

   - All profits must be non-positive.
   - A produced good must earn 0 profits.
   - A production activity that earns negative profits won't be used.
   - Supply equals demand.

   Since comparative advantages clearly matter, assume $\boldsymbol{\alpha_1/\beta_1 > \alpha_2/\beta_2}$.

   (a) How many goods does this model have?

   (b) Will any country both import and export the same good?

   (c) Will a country export the good in which it has a comparative disadvantage?

(d) Can any country totally specialize in production (of the good in which it has a comparative advantage) but not trade?

(e) Show that if no trade equilibrium exists, then $\alpha_1/\beta_1 < (1/k^2)\alpha_2/\beta_2$.

(f) Show that a necessary condition for an equilibrium with trade but no specialization is that $\alpha_1/\beta_1 = (1/k^2)\alpha_2/\beta_2$.

(g) Show that a necessary condition for one country to specialize is that $\alpha_1/\beta_1 \geq (1/k^2)\alpha_2/\beta_2$, and that strict inequality is a sufficient condition.

(h) In a trade equilibrium, do both countries have the same relative consumption c^A/c^B? How does this compare to the standard Ricardian model ($k = 1$)?

2 Hecksher-Ohlin-Vanek Model [1]

2.1 Questions

1. Suppose in a small open economy, world output prices for good 1 and 2 are p and 1, respectively. The production functions are:

$$
\begin{aligned}
q_1 &= k^{1/2} l^{1/2} \\
q_2 &= k^{3/4} l^{1/4}
\end{aligned}
$$

(a) Suppose both goods are produced with positive quantity, compute the equilibrium factor prices.

(b) Compute $\frac{\partial w}{\partial p}$ when both goods are produced.

(c) Suppose $p = 1$. If the endowment of capital and labor are both 100, do both firms operate? Compute factor prices.

(d) Suppose $p = 1$. If the endowment of capital and labor are 100 and 400 respectively, do both firms operate? Compute factor prices.

[1] These notes borrow form Jaden Chen's notes from 2020.

Q 14 Linear Models PS, parts 1 to 8

1) 3 activities ($\uparrow$ A, $\uparrow$ B, $\uparrow$ T) } 6 goods
2 countries

2) country both import and export the same good? (1 unit)

country 1 exports good A
country 1 imports good A

0 profit condition

$k P_{2A} - P_{1A} = 0$ [country 1]

$k P_{1A} - P_{2A} = 0$ [country 2]

combine: $P_{1A} = k P_{2A}$

$\longrightarrow k \cdot k P_{2A} - P_{2A} = 0$

$(k^2 - 1) P_{2A} = 0$

$k = 1$ X by assumption

$P_{2A} = 0$ impossible (both goods are consumed)

$\longrightarrow$ not possible that A export and import good 1.

3) will country export the good in which it has CD?

Suppose 1 exports good B (disadvantage one).
then country 2 imports good B $\longrightarrow$ from part 2, we know it can't export good 2
$\downarrow$
country 2 exports A

country 1 exports B
country 2 exports A

0 profit condition

$k P_{2B} - P_{1B} = 0 \longrightarrow P_{1B} = k P_{2B}$

$k P_{1A} - P_{2A} = 0 \longrightarrow P_{2A} = k P_{1A}$

From the profit max problem: $\max \; p_1^B \beta_1 l_i^B - w_1 l_i^B$

$$\left.\begin{aligned} kp_{2B} \quad & p_1^B \beta_1 = w_1 \\ & p_2^A \alpha_2 = w_2 \\ kp_{1A} \quad & \text{"} \end{aligned}\right|$$

And we don't know if the remaining good are produced:

$$p_1^A \alpha_1 \leq w_1$$
$$p_2^B \beta_2 \leq w_2$$

Comparing

$$\begin{cases} w_1 = k p_{2B} \beta_1 \geq p_1^A \alpha_1 \rightarrow \frac{1}{k} \frac{\alpha_1}{\beta_1} \leq \frac{p_{2B}}{p_{1A}} \\ w_2 = k p_{1A} \alpha_2 \geq p_2^B \beta_2 \rightarrow \frac{k \alpha_2}{\beta_2} \geq \frac{p_{2B}}{p_{1A}} \end{cases}$$

$$\rightarrow \frac{1}{k} \frac{\alpha_1}{\beta_1} \leq \frac{k \alpha_2}{\beta_2}$$

$$\rightarrow \frac{\alpha_1}{\beta_1} \leq k^2 \frac{\alpha_2}{\beta_2}$$

since $k \in (0,1)$

contradicts CA pattern:

$$\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}.$$

4) total specialization but not trade? no (eq ut of not consuming any given good is ∞!)

5) If no trade eq exists, then:

$$\frac{\alpha_1}{\beta_1} < \frac{1}{k^2} \frac{\alpha_2}{\beta_2}$$

If no trade ⟶ Both countries produce both goods domestically:

0 profit cond

↓

$$\left.\begin{aligned} p_{1A}\,\alpha_1 &= w_1 \\ p_{1B}\,\beta_1 &= w_1 \end{aligned}\right\} \quad \frac{p_{1B}}{p_{1A}} = \frac{\alpha_1}{\beta_1} \qquad \text{I}$$

$$\left.\begin{aligned} p_{2A}\,\alpha_2 &= w_2 \\ p_{2B}\,\beta_2 &= w_2 \end{aligned}\right\} \quad \frac{p_{2B}}{p_{2A}} = \frac{\alpha_2}{\beta_2} \qquad \text{II}$$

transportation costs (only care about 1 exporting A
2 exporting B)

negative profits:

$$k\, p_2^A - p_1^A < 0 \longrightarrow p_1^A > k\, p_2^A$$

$$k\, p_1^B - p_2^B < 0 \longrightarrow p_1^B < \frac{1}{k}\, p_2^B$$

↓

$$\text{III} \qquad \frac{p_{1B}}{p_{1A}} < \frac{1}{k^2}\,\frac{p_{2B}}{p_{2A}}$$

⟶ Combine I II III:

$$\boxed{\frac{\alpha_1}{\beta_1} = \frac{p_{1B}}{p_{1A}} < \frac{1}{k^2}\,\frac{p_2^B}{p_2^A} = \frac{1}{k^2}\,\frac{\alpha_2}{\beta_2}}$$

6) Trade but no specialization. (necessary condition)

$$\frac{\alpha_1}{\beta_1} = \frac{1}{k^2}\,\frac{\alpha_2}{\beta_2}$$

all technologies make 0 profits. (Part 5 but with =)

7) necessary condition for specialization

$$\frac{\alpha_1}{\beta_1} \geq \frac{1}{k^2} \frac{\alpha_2}{\beta_2}$$

Combine

A

$p_1^A \alpha_1 = w_1$

$p_1^A = k p_2^A$

$p_2^B \beta_2 = w_2$

B

$p_1^B \beta_1 \leq w_1$

$p_2^B = k p_1^B$

$p_2^A \alpha_2 \leq w_2$

8) In trade eq, do both countries have the same relative consumption? $\frac{c^A}{c^B}$? Compare to standard Ricardian model $k=1$.

consumer max problem:

$$\begin{cases} \max & \delta \log c_{iA} + (1-\delta) \log c_{iB} \\ \text{st} & p_{iA} c_{iA} + p_{iB} c_{iB} \leq w_i \end{cases}$$

FOC $[c_{iA}]$ $\frac{\delta}{c_{iA}} = p_{iA}$ $\quad [c_{iB}]$ $\frac{(1-\delta)}{c_{iB}} = p_{iB}$

take the ratio of FOC $\frac{\frac{\delta}{c_{iA}}}{\frac{(1-\delta)}{c_{iB}}} = \frac{p_{iA}}{p_{iB}} \rightarrow \boxed{\frac{c_{iA}}{c_{iB}} = \frac{\delta}{(1-\delta)} \frac{p_{iB}}{p_{iA}}}$

(from Ⓐ and Ⓑ in 7)

recall that $\frac{p_{iB}}{p_{iA}} = \frac{1}{k^2} \frac{p_{jB}}{p_{jA}}$ $j, i = 1, 2$

since the relative prices differ → rel consumption differ.

In ricardo where $k = 1$; then rel prices are the same

↓

rel demands are the same

(trade always happens).