Logistics

- TA Office Hours:
  - Mon & Wed 18:30-19:30pm
  - In person (Uris 465) or by Zoom (same link as before).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 Two-sector Model[1]

1.1 Important theorems:

Under diversification:

- **Factor price equalization theorem:** inside the cone of diversification, factor prices of capital r and labor w don't change when factor endowments K and L change.
- **Stolper-Samuelson theorem:** when the price of a good A goes up, the price of the factor in which good A is intensive in goes up, and the price of the other factor (the one in which good A is not intensive in) goes down.
- **Rybcszynsk theorem:** in a 2x2 model with no factor intensity reversion, if the endowment of a factor increases, the production of the good that uses this factor intensively increases, and the production of the other good decreases.

Under diversification or specialization:

- **The Hecksher-Ohlin Theorem:** The higher capital/labor ratio country will produce relatively more of the capital-intensive good, and country with the lower capital/labor ratio will produce relatively more of the labor-intensive good.

1.2 Questions

1. A small open economy provides 2 goods, A and B, using 2 inputs, capital k and labor l. The production function for the two goods are:

$$f_A(l_A, k_A) = min\{\alpha_A l_A, \beta_A k_A\}$$
$$f_A(l_A, k_A) = min\{\alpha_B l_B, \beta_B k_B\}$$

[1] These notes borrow from Jaden Chen's notes from 2020.

Let p_A and p_B be the world output prices of good A and B respectively. Also, let the country's stock of capital and labor be $(K, L) \gg 0$. Denote the prices of capital and labor by r and w respectively. Suppose $\frac{K}{L} \in (1/4, 1/2)$.

(a) Let $p_A = \alpha_A = \alpha_B = 1$ and $\beta_A = 4$, $\beta_B = 2$. Suppose we are in the situation where both A and B are produced in positive quantities. Solve for the competitive equilibrium when both wage rate and rental rate are positive.

(b) Suppose the economy specified above is in a diversified competitive equilibrium with $w, r > 0$. What can you say regarding the factor intensity of industry A compared to industry B? What is the effect of an increase in p_B on the equilibrium input prices?

2. For a country with an endowment in the interior of the cone of diversification, conjecture and prove a result on the effects of a small increase in the quantity of a factor on output.

Q2. 2 goods $\to$ A, B   2 inputs $\to$ k, ℓ

$$F_A(k_A, \ell_A) = \min\{\alpha_A \ell_A ; \beta_A k_A\}$$
$$F_B(k_B, \ell_B) = \min\{\alpha_B \ell_B ; \beta_B k_B\}$$

p_A, p_B $(K, L) \gg 0$ r, w

$$\frac{K}{L} \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

1. Suppose diversification (both goods are produced). Solve for CE.

$p_A = \alpha_A = \alpha_B = 1$; $\beta_A = 4$; $\beta_B = 2$.

- First, we have to find the cost function. Notice that to produce a level of output 1, we must have:

- $\alpha_i \ell_i = 1 \longrightarrow \ell_i = 1/\alpha_i \longrightarrow \ell_A = 1$, $\ell_B = 1$
- $\beta_i k_i = 1 \longrightarrow k_i = 1/\beta_i \longrightarrow k_A = 1/4$, $k_B = 1/2$

$$\Rightarrow c_i(r,w) = r \frac{1}{\beta_i} + w \cdot \frac{1}{\alpha_i}$$

- $C_A(r,w) = r \cdot \frac{1}{4} + w$.
- $C_B(r,w) = r \cdot \frac{1}{2} + w$

[interior sol]

$$\begin{cases} 1 = p_A = C_A(r,w) \\ p_B = C_B(r,w) \end{cases}$$

$$1 = \frac{r}{4} + w \longrightarrow r = 4(1-w)$$

$$p_B = \frac{r}{2} + w \longrightarrow p_B = \frac{4(1-w)}{2} + w$$

$$w = 2 - p_B$$

$$r = -4 + 4p_B$$

- We also know the system balance condition (market clearing):

$$\begin{cases} \frac{\partial C_A}{\partial w} x_A + \frac{\partial C_B}{\partial w} \cdot x_B = L \\ \frac{\partial C_A}{\partial r} x_A + \frac{\partial C_B}{\partial r} x_B = K \end{cases}$$

$$\frac{\partial C_A}{\partial w} = 1 \qquad \frac{\partial C_A}{\partial r} = \frac{1}{4}$$

$$\frac{\partial C_B}{\partial w} = 1 \qquad \frac{\partial C_B}{\partial r} = \frac{1}{2}$$

$$\begin{cases} x_A + x_B = L \longrightarrow x_A = L - x_B \\ \frac{1}{4} x_A + \frac{1}{2} x_B = K \end{cases}$$

$$\longrightarrow \frac{1}{4}(L - x_B) + \frac{1}{2} x_B = K$$

$$x_B^* = 4K - L$$

$$x_A^* = 2L - 4K$$

- $$l_A^* = \frac{x_A^*}{\alpha_A} = \frac{2L-4K}{1} \qquad k_A^* = \frac{x_A^*}{\beta_A} = \frac{2L-4K}{4}$$

$$l_B^* = \frac{x_B^*}{\alpha_B} = \frac{4K-L}{1} \qquad k_B^* = \frac{x_B^*}{\beta_B} = \frac{4K-L}{2}$$

2. Diversified CE $w, r > 0$

we have:

$$\frac{\partial C_A / \partial r}{\partial C_A / \partial w} = \frac{1}{4} < \frac{1}{2} = \frac{\partial C_B / \partial r}{\partial C_B / \partial w}$$

$\longrightarrow$ Industry B is relatively more intensive in capital.

Given $r^* = -4 + 4p_B$
$w^* = 2 - p_B$

If $p_B \uparrow \longrightarrow r^* \uparrow$ and $w^* \downarrow$

When the price of good B goes up, the price of the factor which is intensive in (capital) goes up and the price of the other factor goes down. Stolper-Samuelson theorem.

Q3. Investigate $(K, L) \longrightarrow (x_A, x_B)$

Inside the cone of diversification:

$$\begin{bmatrix} \dfrac{\partial c_A}{\partial w} & \dfrac{\partial c_B}{\partial w} \\ \dfrac{\partial c_A}{\partial r} & \dfrac{\partial c_B}{\partial r} \end{bmatrix} \cdot \begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix}$$

Suppose there is an increase in L:

differentiate the system wrt L:

$$\underbrace{\begin{bmatrix} \dfrac{\partial c_A}{\partial w} & \dfrac{\partial c_B}{\partial w} \\ \dfrac{\partial c_A}{\partial r} & \dfrac{\partial c_B}{\partial r} \end{bmatrix}}_{D} \cdot \begin{bmatrix} \partial x_A / \partial L \\ \partial x_B / \partial L \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Since diversification $\longrightarrow$ $\text{Det}(D) \neq 0$, then apply Cramer's rule:

$$\frac{\partial x_A}{\partial L} = \begin{vmatrix} 1 & \partial c_B / \partial w \\ 0 & \partial c_B / \partial r \end{vmatrix} \frac{1}{D} = \frac{\partial c_B / \partial r}{D} \quad (1)$$

$$\frac{\partial x_B}{\partial L} = \begin{vmatrix} \partial c_A / \partial w & 1 \\ \partial c_A / \partial r & 0 \end{vmatrix} \frac{1}{D} = \frac{-\partial c_A / \partial r}{D} \quad (2)$$

with $\det(D) = \partial c_A / \partial w \cdot \partial c_B / \partial r - \partial c_A / \partial r \cdot \partial c_B / \partial w$

Now assume that sector A is capital intensive; ie; : (B is L-intensive)

$$\frac{K_A}{L_A} > \frac{K_B}{L_B}$$

Therefore: $\quad \dfrac{\partial C_A/\partial r}{\partial C_A/\partial w} > \dfrac{\partial C_B/\partial r}{\partial C_B/\partial w}$

which implies:

$$\partial C_A/\partial r \cdot \partial C_B/\partial w - \partial C_B/\partial r \cdot \partial C_A/\partial w > 0$$

$$\Longrightarrow \qquad D < 0$$

Since by Sheppards' lemma we know that $\dfrac{\partial C_i}{\partial r} = k_i \geq 0$ for $i = A, B$.

and $\dfrac{\partial C_i}{\partial w} = L_i \geq 0$

Back to ① and ②:

- $$\frac{\partial x_A}{\partial L} = \frac{\overbrace{\partial C_B/\partial r}^{\geq 0}}{\underbrace{D}_{<0}} \leq 0$$

- $$\frac{\partial x_B}{\partial L} = -\frac{\overbrace{\partial C_A/\partial r}^{\geq 0}}{\underbrace{D}_{<0}} \geq 0$$

Rybczynski Theorem: In a 2×2 model, with no factor intensity and with diversification (interior solution!) reversion (only one intersection) if the endowment of a factor increases, then the production of the good that uses this factor relatively more intensive increases, and the production of the other good decreases.