

Logistics

- TA Office Hours:
 - Mon & Wed 18:30-19:30pm
 - In person (Uris 465) or by Zoom (same link as before).
- All problem sets are due before Friday section.
- Section's material https://github.com/luciacasal/-Cornell_ECON6100.

1 Two-sector Model¹

1.1 Important theorems:

Under diversification:

- **Factor price equalization theorem:** inside the cone of diversification, factor prices of capital r and labor w don't change when factor endowments K and L change.
- **Stolper-Samuelson theorem:** when the price of a good A goes up, the price of the factor in which good A is intensive in goes up, and the price of the other factor (the one in which good A is not intensive in) goes down.
- **Rybczynski theorem:** in a 2x2 model with no factor intensity reversion, if the endowment of a factor increases, the production of the good that uses this factor intensively increases, and the production of the other good decreases.

Under diversification or specialization:

- **The Heckscher-Ohlin Theorem:** The higher capital/labor ratio country will produce relatively more of the capital-intensive good, and country with the lower capital/labor ratio will produce relatively more of the labor-intensive good.

1.2 Questions

1. A small open economy provides 2 goods, A and B , using 2 inputs, capital k and labor l . The production function for the two goods are:

$$f_A(l_A, k_A) = \min\{\alpha_A l_A, \beta_A k_A\}$$

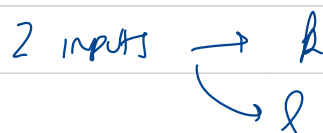
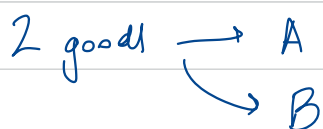
$$f_B(l_B, k_B) = \min\{\alpha_B l_B, \beta_B k_B\}$$

¹These notes borrow from Jaden Chen's notes from 2020.

Let p_A and p_B be the world output prices of good A and B respectively. Also, let the country's stock of capital and labor be $(K, L) \gg 0$. Denote the prices of capital and labor by r and w respectively. Suppose $\frac{K}{L} \in (1/4, 1/2)$.

- (a) Let $p_A = \alpha_A = \alpha_B = 1$ and $\beta_A = 4, \beta_B = 2$. Suppose we are in the situation where both A and B are produced in positive quantities. Solve for the competitive equilibrium when both wage rate and rental rate are positive.
 - (b) Suppose the economy specified above is in a diversified competitive equilibrium with $w, r > 0$. What can you say regarding the factor intensity of industry A compared to industry B ? What is the effect of an increase in p_B on the equilibrium input prices?
2. For a country with an endowment in the interior of the cone of diversification, conjecture and prove a result on the effects of a small increase in the quantity of a factor on output.

Q2.



$$f_A(K_A, L_A) = \min \{ \alpha_A K_A; \beta_A L_A \}$$

$$f_B(K_B, L_B) = \min \{ \alpha_B K_B; \beta_B L_B \}$$

$$P_A, P_B \quad (K, L) \gg 0 \quad r, w$$

$$\frac{K}{L} \in \left(\frac{1}{4}, \frac{1}{2} \right)$$

1. Suppose diversification (both goods are produced).
 Solve for CE.

$$P_A = \alpha_A = \alpha_B = 1; \quad \beta_A = 4; \quad \beta_B = 2.$$

First, we have to find the cost function. Notice that to produce a level of output 1, we must have:

- $\alpha_i l_i = 1 \rightarrow l_i = 1/\alpha_i \rightarrow \begin{matrix} l_A = 1 \\ l_B = 1 \end{matrix}$
- $\beta_i k_i = 1 \rightarrow k_i = 1/\beta_i \rightarrow \begin{matrix} k_A = 1/4 \\ k_B = 1/2 \end{matrix}$

$$\Rightarrow C_i(r, w) = r \frac{1}{\beta_i} + w \cdot \frac{1}{\alpha_i}$$

- $C_A(r, w) = r \cdot \frac{1}{4} + w \cdot$

- $C_B(r, w) = r \cdot \frac{1}{2} + w$

$$[\text{interior sol}] \quad \begin{cases} 1 = p_A = C_A(r, w) \\ p_B = C_B(r, w) \end{cases}$$

$$1 = \frac{r}{4} + w \rightarrow r = 4(1-w)$$

$$p_B = \frac{r}{2} + w \rightarrow p_B = \frac{4(1-w)}{2} + w$$

$$w = 2 - p_B$$

$$r = -4 + 4p_B$$

- We also know the system balance condition (market clearing):

$$\begin{cases} \frac{\partial C_A}{\partial w} x_A + \frac{\partial C_B}{\partial w} x_B = L \\ \frac{\partial C_A}{\partial r} x_A + \frac{\partial C_B}{\partial r} x_B = K \end{cases}$$

$$\left. \begin{array}{ll} \frac{\partial C_A}{\partial w} = 1 & \frac{\partial C_A}{\partial r} = \frac{1}{4} \\ \frac{\partial C_B}{\partial w} = 1 & \frac{\partial C_B}{\partial r} = \frac{1}{2} \end{array} \right\} \begin{array}{l} x_A + x_B = L \rightarrow x_A = L - x_B \\ \frac{1}{4} x_A + \frac{1}{2} x_B = K \\ \rightarrow \frac{1}{4} (L - x_B) + \frac{1}{2} x_B = K \end{array}$$

$$x_B^* = 4K - L$$

$$x_A^* = 2L - 4K$$

$$l_A^* = \frac{x_A^*}{\alpha_A} = \frac{2L - 4K}{1}$$

$$l_B^* = \frac{x_B^*}{\alpha_B} = \frac{4K - L}{1}$$

$$b_A^* = \frac{x_A^*}{\beta_A} = \frac{2L - 4K}{4}$$

$$b_B^* = \frac{x_B^*}{\beta_B} = \frac{4K - L}{2}$$

2. Diversified CE $w, r > 0$

we have:

$$\frac{\frac{\partial C_A}{\partial r}}{\frac{\partial C_A}{\partial w}} = \frac{1}{4} < \frac{1}{2} = \frac{\frac{\partial C_B}{\partial r}}{\frac{\partial C_B}{\partial w}}$$

→ Industry B is relatively more intensive in capital.

Given $r^* = -4 + 4p_B$

$$w^* = 2 - p_B$$

If $p_B \uparrow \rightarrow r^* \uparrow$ and $w^* \downarrow$

When the price of good B goes up, the price of the factor which is intensive in (capital) goes up and the price of the other factor goes down. Stolper-Samuelson theorem.

Q3. Investigate $(K, L) \rightarrow (x_A, x_B)$

Inside the cone of diversification:

$$\begin{bmatrix} \frac{\partial c_A}{\partial w} & \frac{\partial c_B}{\partial w} \\ \frac{\partial c_A}{\partial r} & \frac{\partial c_B}{\partial r} \end{bmatrix} \cdot \begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix}$$

Suppose there is an increase in L :

differentiate the system wrt L :

$$\underbrace{\begin{bmatrix} \frac{\partial c_A}{\partial w} & \frac{\partial c_B}{\partial w} \\ \frac{\partial c_A}{\partial r} & \frac{\partial c_B}{\partial r} \end{bmatrix}}_{D} \cdot \begin{bmatrix} \partial x_A / \partial L \\ \partial x_B / \partial L \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Since diversification $\rightarrow \det(D) \neq 0$, then apply Cramer's rule:

$$\frac{\partial x_A}{\partial L} = \frac{\begin{vmatrix} 1 & \partial c_B / \partial w \\ 0 & \partial c_B / \partial r \end{vmatrix}}{D} = \frac{\partial c_B / \partial r}{D} \quad (1)$$

$$\frac{\partial x_B}{\partial L} = \frac{\begin{vmatrix} \partial c_A / \partial w & 1 \\ \partial c_A / \partial r & 0 \end{vmatrix}}{D} = \frac{-\partial c_A / \partial r}{D} \quad (2)$$

$$\text{with } \det(D) = \partial c_A / \partial w \cdot \partial c_B / \partial r - \partial c_A / \partial r \cdot \partial c_B / \partial w$$

Now assume that sector A is capital intensive; i.e.: (B is L-intensive)

$$\frac{k_A}{L_A} > \frac{k_B}{L_B}$$

Therefore:
$$\frac{\partial C_A / \partial r}{\partial C_A / \partial w} > \frac{\partial C_B / \partial r}{\partial C_B / \partial w}$$

which implies:

$$\partial C_A / \partial r \cdot \partial C_B / \partial w - \partial C_B / \partial r \cdot \partial C_A / \partial w > 0$$

$$\Rightarrow D < 0$$

Since by Sheppard's lemma we know that $\frac{\partial C_i}{\partial r} = k_i \geq 0$ for $i = A, B$.

and $\frac{\partial C_i}{\partial w} = L_i \geq 0$

Back to ① and ②:

- $$\frac{\partial x_A}{\partial L} = \frac{\partial C_B / \partial r}{D} \leq 0$$

- $$\frac{\partial x_B}{\partial L} = - \frac{\partial C_A / \partial r}{D} \geq 0$$

and with diversification
(interior solution!)

Rybczynski Theorem: In a 2x2 model, with no factor intensity
reversion (only one intersection) if the endowment of a factor increases,
then the production of the good that uses this factor relatively more
intensive increases, and the production of the other good decreases.