Lock-In and Productive Innovations:

Implications for Firm-to-Firm Innovation Pass-Through *

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Abstract

Firms innovate to improve efficiency and reduce their costs of production (*productive* innovations) and to increase customer dependency through product customization (*lock-in* innovations). In this paper, I quantitatively study the macroeconomic implications of lock-in innovations for aggregate productivity and market power. I develop a theoretical framework that allows firms to invest in lock-in innovations by reducing product substitutability, while also nesting standard macroeconomic models of productive innovations. A key prediction of the model is that productive innovations by suppliers increase customer firms' sales by lowering input costs, while lock-in innovations decrease customer firms' sales by making the suppliers' products harder to substitute. I use this theoretical insight to identify the nature of innovation in the data and calibrate the model to the U.S. economy. Informed by the observed changes in the response of customer firms' sales to their suppliers' innovations, I find that the incidence of lock-in innovations among high-markup firms has increased significantly in the post-2000 period. Moreover, had incidence of lock-in innovations remained at pre-2000 levels, observed aggregate productivity would have been 3% higher, median markups would have stayed at pre-2000 levels, and markup dispersion would have been 9% lower.

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1 Introduction

Firms invest in innovations to enhance their productivity as well as to customize their products, making it more difficult for customers to switch to competitors. Productive innovations reduce the marginal cost of production or improve product quality (Aghion and Howitt, 1992). In contrast, innovations aimed at customization seek to create customer dependency, or a "lock-in" effect, making products more dissimilar or influencing their compatibility with other products (Farrell and Klemperer, 2007).

Lock-in strategies are particularly common in markets for technological products, where follow-on purchases of complementary products and services are necessary to maintain or improve the initial investment. Companies often derive significant profits from these aftermarket sales. The reliance on proprietary systems and product compatibility make it expensive for customers to adopt new alternative technologies. The original supplier then holds considerable market power, and can charge high prices for upgrades and related products. A notable example is Bell Atlantic's experience with AT&T. In the mid-1980s, Bell Atlantic invested \$3 billion in AT&T's state-of-the-art 5ESS digital switches to modernize its telephone network, choosing AT&T over rivals like Northern Telecom and Siemens. However, this investment locked Bell Atlantic into AT&T's proprietary system, forcing them to rely on AT&T for costly software upgrades and enhancements.[1] In this example, a productive innovation by AT&T (i.e., 5ESS digital switch) was followed up by successive lock-in innovations.

Both being highly productive and offering specialized products are important sources of market power (Pellegrino, 2023), and this accumulation of market power is central to firms' incentives to invest in innovation (Peters, 2020). While much of the literature has focused on productivity-enhancing innovations, the macroeconomic implications of lock-in innovations have been largely overlooked. In this paper, I study the macro implications of lock-in and productive innovations for aggregate productivity and aggregate market power. I first develop a new macroeconomic model that incorporates both productive and lock-in innovations. Next, I combine the theory with novel evidence on firm-to-firm innovation pass-through to identify the nature of innovations in the data. Finally, I calibrate the model to the U.S. economy to analyze the role of lock-in innovations for aggregate productivity and market power over recent decades.

The economy is populated by a continuum of customer firms, each of which purchases inputs from a finite number of *supplier firms* and produces with a CRESH (Constant Ratio Elasticity of Substitution with Homotheticity) technology (Hanoch (1971)), i.e., a non-CES homothetic production function that allows for supplier-specific product substitutability. Customer firms imperfectly substitute across suppliers, and the degree of product substitutability varies across

[1] Similarly, Apple first established the iPhone as a market leader and then introduced an ecosystem—including the App Store and iCloud—that locks users into their platform. Microsoft followed a similar strategy by integrating its Office suite with Windows, creating a seamless user experience that makes switching to other platforms difficult and costly.

suppliers. Each supplier firm produces with a linear technology in labor and heterogeneous labor productivity. Suppliers compete *à la* Bertrand, and choose prices to maximize profits each period. They also make two type of dynamic innovation decisions: they can invest in *productive* innovations, that increase their labor productivity; or they can invest in *lock-in* innovations, that reduce their product substitutability; or both.

An oligopolistic competition market structure induces endogenous markups by supplier firms. Markups are a function of a firms' market share —shaped by their productivity and substitutability—as well as their own substitutability, their competitors' substitutability, and the elasticity of substitution between customer firms. Consequently, there are two sources of market power: suppliers can charge high markups either because they are highly productive and/or because customers find it difficult to substitute away from them. Since markups directly influence profits, the model has a rich interplay between productivity and product substitutability in shaping the market value of the firm.

The model generalizes the workhorse model of heterogeneous firms and innovation by Atkeson and Burstein (2010) in three ways. First, I introduce production linkages, with supplier firms that are heterogeneous in productivity and in their firm-specific degree of product substitutability. Second, these suppliers compete oligopolistically to sell their products to other firms (Aghion, Harris, Howitt and Vickers, 2001).[2] Third, suppliers can invest in two types of innovations: productive innovations and lock-in innovations. I use the model to analyze firms' incentives to invest in these alternative innovations and to characterize the nature of innovation pass through from suppliers to their customer firms. The model nests different market structures and technology classes. This nesting ensures that all the mechanisms present in the canonical model of innovation with oligopolistic competition where suppliers differ in productivity, (Aghion *et al.* (2001)), are also present in the framework. A relevant particular case is when substitutability is identical for all suppliers that provide inputs to a given customer, in which case the model simplifies to the standard CES framework with oligopolistic competition and a non-unitary elasticity of substitution between customer firms, as in Atkeson and Burstein (2008).[3]

The model provides a key prediction on how innovation affects customer firms depending on the type of innovation undertaken by supplier firms. If suppliers invest in productive innovations, the sales of their customer firms increase. Productive innovations reduce the supplier's marginal costs of their products, resulting in lower input prices for the customer firm and higher sales. In contrast, if suppliers engage in lock-in innovations, the model predicts a decline in customer firms' sales. Lock-in innovations reduce product substitutability, enabling

[2]Customers could either be final consumers or other firms to which suppliers sell their products. In the former case, lock-in innovations would have direct implications for consumer welfare, while in the latter case, they would impact aggregate productivity. This paper focuses on a firm-to-firm context, which allows for a clear measurement of customer firms' responses to supplier innovations using firm-level balance sheet data. In contrast, measuring changes in consumer utility would be substantially more challenging.

[3]In Atkeson and Burstein (2008), *supplier firms* correspond to within-industry firms, while *customer firms* correspond to industry-level firms.

suppliers to charge higher prices. Customer firms suffer a decline in their total sales because they are unable to pass on these increased input costs to their products due to intense competition in their own markets.

I use these model testable predictions to characterize the nature of suppliers' innovations in the data. I combine data on firm-to-firm linkages and firm financials from US Compustat Fundamentals, together with measures of product differentiation from the Hoberg and Phillips (2016) Index of Product Similarity. This index measures similarity of a product's firm compared to other firms based on text analysis of firm's product's descriptions. I further combine this information with innovation shocks from Kogan, Papanikolaou, Seru and Stoffman (2017), defined as the excess stock market return of patents assigned to a given firm. I begin by documenting that high-markup suppliers produce more differentiated products, consistent with Pellegrino (2023), and that this correlation has become stronger in the years after 2000. I also show that innovations by high-markup suppliers lead to a significant increase in product differentiation after 2000, but were associated with a significant decline in differentiation prior to 2000, while innovations by low-markup suppliers have no significant impact on differentiation in either period. Last, innovations by low-markup suppliers increase customer firms' sales, while innovations by high-markup suppliers lead to a decline in customer firms' sales after 2000 and an increase in customer firms' sales before 2000. These facts are novel to the literature. Through the lens of the model, my findings indicate that high-markup firms are more inclined to pursue lock-in innovations, particularly in recent years. In contrast, low-markup firms tend to invest in productive innovations.

I then study the implications of this shift in the prevalence of lock-in innovations for aggregate TFP and market power. I calibrate the model by simulating a panel of firms and running local projection regressions on the pass-through of innovation on customer sales in both the model and the data. This approach helps discipline key parameters related to the cost structures of lock-in and productive innovations, including the relationship between a firm's productivity gap relative to its competitors and the cost of each type of innovation. Using pre- and post-2000 data, I calibrate the model for two steady states: one for the post-2000 period and another for the pre-2000 period. Comparing these steady states, the model predicts a greater prevalence of lock-in innovations after 2000, driven largely by high-markup firms shifting their investment towards lock-in innovations.

Finally, I answer the question: How much of the observed changes in aggregate TFP, markup levels, and markup dispersion between the pre- and post-2000 periods can be explained by shifts in the composition of innovation? To answer this, I construct a counterfactual post-2000 economy that retains the lock-in innovation cost structure of the pre-2000 period. The results indicate that observed aggregate productivity would have been 3% higher, median markups would have remained at pre-2000 levels, and markup dispersion would have been 9% lower than observed.

Related Literature. This paper relates to several strands of literature.

First, this paper contributes to the literature that emphasizes the diverse nature of innovation. Akcigit and Kerr (2018) differentiate between internal and external innovations, where multi-product incumbents focus on internal innovations to improve existing products, while both new entrants and incumbents pursue external innovations to acquire new product lines. In both cases, innovation drives changes in productivity—either through new products or by enhancing the productivity of existing ones. Argente, Baslandze, Hanley and Moreira (2020) introduce the concept of protective versus productive innovations, with protective innovations being patents that never materialize into products. They examine how firms exploit the patent system by patenting without commercialization. My paper introduces a new type of innovation that is well-established in industrial organization literature: firms' ability to create customized products that raise switching costs for customers. Moreover, I explore the strategic behavior of firms investing in both productive and lock-in innovations. In contrast, Argente *et al.* (2020) abstract from strategic behavior, and there is no effect on product substitution.

Second, this paper contributes to the literature on market power and innovation. Aghion *et al.* (2001) developed a seminal model of step-by-step innovations, where firms' markups are endogenously determined by their investments in productivity. Peters (2020) built a theory of creative destruction with an endogenous distribution of markups to quantitatively examine the aggregate effects of market power on resource misallocation and Cavenaile, Celik and Tian (2019) constructed a Schumpeterian growth model with oligopolistic competition to explore the welfare implications of market power. In all these models, the sole endogenous driver of market power accumulation is firms' investments in productivity. In contrast, my framework introduces a new source of market power accumulation: product substitutability, while maintaining the key features of these existing models. I generalize the model in Atkeson and Burstein (2010) to allow for an oligopolistic market structure that endogenously determines the distribution of markups. I show that without lock-in innovations, existing models cannot replicate the observed empirical patterns of innovation pass-through between supplier and customer firms. This highlights the importance of incorporating product substitutability to align the model predictions with empirical evidence. Productive innovations that lower marginal production costs naturally lead to reduced supplier prices, decreasing input costs for customer firms and boosting their sales. My quantitative model further predicts that high-markup suppliers will pass through a larger share of these cost reductions to their customers compared to low-markup suppliers, amplifying the positive impact on customer sales. In contrast, lock-in innovations enable suppliers to raise prices by creating dependencies that prevent customers from switching to alternatives. Since these higher input costs cannot be passed on to final goods producers, customer firms face increased costs, ultimately reducing their sales.[4]

Third, this paper introduces a new mechanism to the literature exploring the links between market concentration, productivity growth, and business dynamism in models of endogenous

[4] A model where suppliers' productive innovations enhance product quality would also lead to higher sales for customer firms. While higher-quality products come with higher prices, they attract greater demand, ultimately increasing the customer firm's sales.

growth. Akcigit and Ates (2023) propose that declining imitation rates between leaders and followers have contributed to these trends, while Olmstead-Rumsey (2019) documents a fall in innovation efficiency among laggard firms over time. Other studies emphasize the role of intangible assets and information and communications technology (ICT) (Aghion, Bergeaud, Boppart, Klenow and Li (2023); De Ridder (2024)), the decline in the growth rate of the labor force (Peters and Walsh (2021)) and the decline in the interest rate (Liu, Mian and Sufi (2022)). This paper identifies the rise of lock-in innovations as a contributor to observed trends in markup dispersion and total factor productivity level.

Fourth, my paper contributes to the literature on the aggregate implications of customer capital accumulation and its relationship with market concentration, including studies that examine the role of investments in advertising (Cavenaile and Roldan-Blanco, 2021; Cavenaile, Celik, Perla and Roldan-Blanco, 2023; Cavenaile, Celik, Roldan-Blanco and Tian, 2024; Shen, 2023), customer acquisition (Ignaszak and Sedláček, 2022) and brand reallocation (Pearce and Wu, 2024). In my framework, the reduction in product substitutability creates customer dependency, which can be interpreted as an alternative form of customer capital accumulation. My paper adds to this literature by introducing a new mechanism for generating customer dependency and analyzing the incentives firms have to invest in both lock-in and productive innovations in a framework where both types of innovations endogenously determine the firms' markups.

Lastly, my paper connects with industrial organization theories on switching costs and business strategies related to lock-in products (Shapiro and Varian (2000)). For example, Farrell and Klemperer (2007) and Klemperer (1987) propose microeconomic theories of optimal firm behavior in the presence of switching costs and explore their effects on market competition. However, these theories do not address the macroeconomic implications of lock-in innovations or how such investment decisions influence the endogenous accumulation of market power in an oligopolistic setting, which is the core focus of this paper.[5]

Organization. The paper is organized as follows: Section 2 presents the model of lock-in and productive innovations; Section 3 describes the data and empirical results; Section 4 presents the model calibration and quantitative analysis on the aggregate implications of lock-in innovations, Section 5 presents policy experiments that simulate antitrust practices, Section 6 discusses a microfoundation of lock-in strategies in the model using existing industrial organization theories, and Section 7 concludes.

[5] Kleshchelski and Vincent (2009) examine the relationship between price rigidity and market share in a general equilibrium framework where customers face switching costs, and Pellegrino (2023) proposes a general equilibrium model of hedonic demand and studies the aggregate welfare implications of product differentiation in oligopolistic competition markets.

2 Model

Time is continuous. There is a representative household with preferences over final consumption who owns the firms in the market. Perfectly competitive firms produce the final good using inputs from a continuum of firms. Each of these firms produces using intermediate inputs purchased from two *supplier firms* that are imperfect substitutes and engage in oligopolistic competition to sell their products to the *customer firm*. Supplier firms are characterized by how productive they are, and also by how substitutable they are for the customer firm. Suppliers can invest in productivity-enhancing innovations or in "lock-in" innovations that make them less substitutable for the customer firm.

I use the model to (i) analyze supplier firms' incentives for productive and lock-in innovations, (ii) characterize how productive and lock-in innovation pass-through from supplier to customer firms and (iii) quantify the aggregate implications of lock-in innovations on aggregate productivity and market power.

2.1 Preferences and Technology

There is a representative household that consumes the final good, saves and supplies labor inelastically to maximize utility from consumption:

$$U_t = \int_0^\infty \exp(-\rho t)) \ln C_t dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, C_t represents consumption at time t. The household faces a budget constraint:

$$P_t C_t + \dot{A}_t = W_t L_t + r_t A_t, \tag{2}$$

where L_t denotes labor and A_t denotes total assets at time t. Prices are given by P_t the price of final consumption good, r_t the interest rate, and W_t the wage rate, which I take to be the numeraire.

Each period t, the problem of the final good firm consists of choosing how much inputs to by from each customer firm X_{ct} to maximize profits, taking prices as given:

$$\max_{X_{ct}} P_t Y_t - \int_0^1 P_{ct} X_{ct} dc \quad \text{s.t. equation (4).} \tag{3}$$

Profit maximization yields the demand of customer firm c's variety: $X_{ct} = \left(\frac{P_{ct}}{P_t}\right)^{-\eta} Y_t$ with aggregate price index given by $P_t = \left(\int_0^1 P_{ct}^{1-\eta} dc\right)^{\frac{1}{1-\eta}}$.

Perfectly competitive firms produce the final good Y_t combining differentiated varieties X_{ct} according to:

$$Y_t = \int_0^1 \left(X_{ct}^{\frac{\eta-1}{\eta}} dc\right)^{\frac{\eta}{\eta-1}}, \tag{4}$$

where $\eta > 1$ represents the elasticity of substitution between varieties.

Each variety c is produced combining intermediate inputs from two *supplier* firms s using a CRESH production technology, implicitly given by the relative size of each supplier:

$$\sum_s \left(\frac{x_{st}}{X_{ct}}\right)^{\frac{\gamma_{st}-1}{\gamma_{st}}} = 1, \tag{5}$$

where x_{st} denotes the output of supplier firm s at time t, and X_{ct} is the output of variety producer c at time t.[6] The variable γ_{st} represents supplier-specific substitutability, capturing the degree of product differentiation—i.e., the lower γ_{st}, the harder it is for variety producer c to substitute away from a supplier's product, indicating greater customer dependency to the supplier's product. Supplier substitutability evolves over time as a result of lock-in innovations.[7] Going forward, I will refer to the variety producers who buy inputs from supplier firms and sell their output to the final good producer as *customer* firms.

Each customer firm c decides the quantity of intermediate inputs to purchase from its two suppliers to maximize profits:

$$\max_{x_{st}, X_{ct}} P_{ct}X_{ct} - \sum_s p_{st}x_{st} \quad \text{s.t. equation (5)} \tag{6}$$

where p_{st} denotes the price charged by each supplier s at time t. The first order conditions for (6) yield a demand for each supplier firm s, $x_{st} = \left(\frac{p_{st}}{P_{ct}D_{ct}}\frac{\gamma_{st}}{\gamma_{st}-1}\right)^{-\gamma_{st}} X_{ct}$ with $D_{ct} \equiv \left(\sum_s \frac{\gamma_{st}-1}{\gamma_{st}}\left(\frac{x_{st}}{X_{ct}}\right)^{\frac{\gamma_{st}-1}{\gamma_{st}}}\right)^{-1}$ a demand index.[8] A detailed derivation of the customer firm problem can be found in Appendix A.1.

For each customer firm, there are two imperfectly substitutable supplier firms that provide intermediate inputs to the customer firm, and produce according to a technology that is linear in labor l_{st}:

$$x_{st} = \exp(a_{st}) l_{st}, \tag{7}$$

where a_{st} denotes the labor log-productivity of firm s at time t. Supplier firms are heterogeneous in productivity a_{st} and substitutability γ_{st}, and engage in oligopolistic competition *à la* Bertrand. Supplier firms solve two problems: First, conditional on their productivity a_{st} and substitutability γ_{st}, they choose prices to maximize static profits each period t; Second, given the profits realized in period t, they make productive and lock-in investment decisions to solve the dynamic problem of maximizing the firm's present discounted value. I first outline the static problem and then provide a detailed description of the innovation decisions.

[6]More generally, this production technology belongs to the Homothetic Demand with Implicit Additivity (HDIA) class (see Matsuyama (2017)) , which can be written as $\sum_s \Upsilon\left(\frac{x_{st}}{X_{ct}}\right) = 1$, with $\Upsilon(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ strictly increasing, strictly concave function, that is twice continuously differentiable with $\Upsilon(0) = 0$ and $\Upsilon(1) = 1$.

[7]In Hanoch (1971), substitutability parameters are factor-specific but do not vary with time.

[8]It is a property of this class of non-CES homothetic technologies to have the demand for a good depending on two relative prices , (p_{st}/P_{ct}) and (p_{st}/D_{ct}). In the limiting case of CES technology where $\gamma_{st} = \gamma$ for all s, $D_c = \frac{\gamma}{\gamma-1}$, which implies $p_{st} = \left(\frac{x_{st}}{X_{ct}}\right)^{\frac{-1}{\gamma_{st}}} P_{ct}$,i.e., there is only one relevant aggregate price given by the CES ideal price index P_{ct}. See Matsuyama (2017) for more details.

Static Pricing Decisions. Each period t, suppliers set prices to maximize profits, subject to the demand from customer firm c. Since suppliers compete oligopolistically, they internalize how their allocation decisions affect the customer firm's production and prices. The resulting profit maximization problem is then:

$$\begin{aligned} \pi_{st} &= \max_{p_{st}} \left\{ p_{st} x_{st} - W \frac{x_{st}}{a_{st}} \right\} \\ \text{s.t.} \quad x_{st} &= \left(\frac{p_{st}}{P_{ct}(p_{st}) D_{ct}(p_{st})} \frac{\gamma_{st}}{\gamma_{st} - 1} \right)^{-\gamma_{st}} X_{ct}(p_{st}), \end{aligned} \tag{8}$$

where the prices and quantities of the customer firm $P_{ct}(p_{st}), D_{ct}(p_{st})$ and $X_{ct}(p_{st})$ as a function of the supplier's pricing decisions p_{st} reflects the strategic behavior given by the oligopolistic market structure. The resulting optimal pricing decision by each supplier firm s is given by a markup m_{st} over marginal cost $p_{st} = \frac{\vartheta_{st}}{\vartheta_{st}-1} \frac{W_t}{\exp a_{st}}$, where ϑ_{st} denotes the firm's s elasticity of demand in period t, characterized later in section 2.2.1.

Innovation

Suppliers can invest in *productive* innovations to increase productivity a_{st} or *lock-in* innovations to reduce their product substitutability γ_{st}.

Productive Innovations. Suppliers undertake *productive* innovations to retain or achieve market leadership by increasing their productivity. When a supplier invests in productive innovations in period t, there is a probability $i_{s,t}$ that her productivity increases in period $t + \triangle$ by a proportional factor $\lambda > 0$, such that $a_{st+\triangle t} = a_{st} + \lambda$, and a probability $(1 - i_{st})$, that her productivity decreases, so that $a_{st+\triangle t} = a_{st} - \lambda$.

A supplier generates a Poisson arrival rate of productive innovations of i_{st} by employing h^i_{st} innovation workers, according to the function $i_{st} = \left(\frac{1}{\exp(a_{st}^{\psi_s} - a_{-st}^{\psi_{-s}})} \right) \left(\phi \frac{h^i_{st}}{\alpha} \right)^{\frac{1}{\phi}}$, where $\phi > 1$ represents the inverse elasticity of productive innovations with respect to innovation workers, ψ_s and ψ_{-s} govern the cost-elasticity of productive innovations to a supplier's own productivity and the productivity of its competitor, and $\alpha > 0$ is a scale parameter. Given the wage rate in the economy W_t, the cost of productive innovations is

$$C(i_{st}) \equiv \alpha \frac{(\exp(a_{st}^{\psi_s} - a_{-st}^{\psi_{-s}}) i_{st})^{\phi}}{\phi} W_t. \tag{9}$$

For $\psi_s > 0$, the cost of productive innovation is increasing in the supplier's productivity a_{st}, reflecting the idea that more advanced technologies are more costly or difficult to improve (Akcigit and Kerr (2018), Atkeson and Burstein (2010)).

Lock-In Innovations. Suppliers can also choose to invest in *lock-in* innovations to reduce their product substitutability, making it more difficult for customers to switch to other suppliers, i.e. locking them in. A successful lock-in innovation in period t decreases the supplier's substitutability in period $t + \triangle$ by a proportional factor $\delta > 0$, such that $\gamma_{s,t+\triangle} = (1 - \delta)\gamma_{s,t}$. A supplier generates a Poisson arrival rate of productive innovations of z_{st} by employing h^z_{st}

innovation workers, according to the function $z_{st} = \left(\frac{1}{\exp(a_{st}^{\tilde{\psi}_s} - a_{-st}^{\tilde{\psi}_{-s}})}\right)\left(\tilde{\phi}\frac{h_{st}^z}{\tilde{\alpha}}\right)^{\frac{1}{\tilde{\phi}}}$, with $\tilde{\phi} > 1$ the inverse elasticity of lock-in innovations with respect to innovation workers and $\tilde{\alpha} > 0$ a scale parameter. Parameters $\tilde{\psi}_s$ and $\tilde{\psi}_{-s}$ govern the elasticity of the cost of lock-in innovations with respect to a supplier's productivity gap relative to its competitor $\exp(a_{st}^{\tilde{\psi}_s} - a_{-st}^{\tilde{\psi}_{-s}})$, which in the quantitative application will play an important role in matching the empirical facts on the relationship between market power and lock-in innovations. The cost of lock-in innovations is therefore given by

$$\mathcal{C}(z_{st}) \equiv \tilde{\alpha}\frac{(\exp(a_{st}^{\tilde{\psi}_s} - a_{-st}^{\tilde{\psi}_{-s}})z_{st})^{\tilde{\phi}}}{\tilde{\phi}}W_t. \tag{10}$$

Dynamic Innovation Decisions. The payoff-relevant state variables for a supplier firm s are its current productivity level a_s, its current substitutability level γ_s, and the productivity and substitutability levels of its competitor, denoted by a_{-s} and γ_{-s}. The stock market value $V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})$ of supplier s at state $(a_s, a_{-s}, \gamma_s, \gamma_{-s})$ is given by:

$$\begin{aligned}
&\rho V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s}) = \pi_s(a_s, a_{-s}, \gamma_s, \gamma_{-s}) \\
&+ \max_{i_s}\{\underbrace{i_{st}\left[V_s(a_s + \lambda, a_{-s}, \gamma_s, \gamma_{-s}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{successful productive innovation}} \\
&+ \underbrace{(1 - i_s)\left[V_s(a_s - \lambda, a_{-s}, \gamma_s, \gamma_{-s}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{unsuccessful productive innovation}} - \underbrace{\mathcal{C}(i_s)}_{\text{productive innov. cost}} \} \\
&+ \max_{z_s}\{\underbrace{z_s\left[V_s(a_s, a_{-s}, \gamma_s(1-\delta), \gamma_{-s}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{successful lock-in innovation}} - \underbrace{\mathcal{C}(z_s)}_{\text{lock-in innov. cost}} \} \\
&+ \underbrace{i_{-s}\left[V_s(a_s, a_{-s} + \lambda, \gamma_s, \gamma_{-s}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{competitor's successful productive innovation}} \\
&+ \underbrace{(1 - i_{-s})\left[V_s(a_s, a_{-s} - \lambda, \gamma_s, \gamma_{-s}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{competitor's unsuccessful productive innovation}} \\
&+ \underbrace{z_{-s}\left[V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s}(1-\delta)) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{competitor's successful lock-in innovation}} \\
&+ \underbrace{\kappa\left[V_s(a_s, a_{-s}, \bar{\gamma}, \bar{\gamma}) - V_s(a_s, a_{-s}, \gamma_s, \gamma_{-s})\right]}_{\text{market restart}}.
\end{aligned} \tag{11}$$

The first line on the right-hand side of expression (11) represents the operating profits in period t. The second line captures the increase in the value of the firm as a result of a successful productive innovation that enhances its productivity by proportional factor λ. The third line accounts for the decrease in firm value if the productive innovation fails, reducing productivity by the same factor λ, net of the cost of investing in productive innovations given by equation 9. The forth line reflects changes in the value of the firm given by a successful lock-in innovation that reduces the firm's product substitutability by a proportional factor δ, net of the cost of investing in lock-in innovations given by equation 10. Given that supplier firms act strategically, they internalize how competitors' actions influence their own value. Accordingly, the fifth and sixth lines capture the impact on firm value from a competitor's successful or unsuccessful productive innovation, respectively, while the seventh line reflects changes in value due to a competitor's successful lock-in innovation. Finally, with exogenous probability κ, the market resets in terms of substitutability, returning all suppliers to the highest possible level of substitutability. This reset mechanism captures external shocks that push firms into a neck-and-neck position in terms of substitutability (e.g., the entry of new firms that induce competitive pressure over incumbents) and ensures the existence of a stationary distribution of firms.

2.2 Equilibrium

Market clearing. The labor market clearing condition requires that the aggregate supply of labor $L = 1$ equalizes the sum of supplier firms' production labor demand and productive and lock-in innovations labor demand, such that $\int_0^1 [l_{st} + l_{-st} + h^i_{st} + h^i_{-st} + h^z_{st} + h^z_{-st}]ds = 1$, with optimal demand of innovation labor given by $h^i_{st} = \alpha \frac{(\exp(a_{st})^{\psi} i_{st})^{\phi}}{\phi}$ and $h^z_{st} = \tilde{\alpha} \frac{(\exp(a_{st}-a_{-st})^{\tilde{\psi}} z_{st})^{\tilde{\phi}}}{\tilde{\phi}}$. The goods market clearing requires that aggregate output equalizes aggregate consumption, i.e., $Y_t = C_t$.

Stationary Distribution of Firms. Denote by $\mu_t(a_s, a_{-s}, \gamma_s, \gamma_{-s})$ the measure of firms in period t and state $(a_s, a_{-s}, \gamma_s, \gamma_{-s})$. The transition path of μ_t for an interior state in which $\underline{\gamma} < (\gamma_s, \gamma_{-s}) < \overline{\gamma}$ and $\underline{a} < (a_s, a_{-s}) < \overline{a}$ is given by:

$$
\begin{aligned}
\frac{\mu_{t+\Delta}(a_s, a_{-s}, \gamma_s, \gamma_{-s}) - \mu_t(a_s, a_{-s}, \gamma_s, \gamma_{-s})}{\Delta t} &= \underbrace{i_{st}\mu_t(a_s - \lambda, a_{-s}, \gamma_s, \gamma_{-s}) + i_{-st}\mu_t(a_s, a_{-s} - \lambda, \gamma_s, \gamma_{-s})}_{\text{inflows from successful productive innovations}} \\
&+ \underbrace{(1 - i_{st})\mu_t(a_s + \lambda, a_{-s}, \gamma_s, \gamma_{-s}) + (1 - i_{-st})\mu_t(a_s, a_{-s} + \lambda, \gamma_s, \gamma_{-s})}_{\text{inflows from unsuccessful productive innovations}} \\
&+ \underbrace{z_{st}\mu_t(a_s, a_{-s}, \gamma_s(1+\delta), \gamma_{-s}) + z_{-st}\mu_t(a_s, a_{-s}, \gamma_s, \gamma_{-s}(1+\delta))}_{\text{inflows from successful lock-in innovations}} \\
&- \underbrace{(2 + z_{st} + z_{-st} + \kappa)\,\mu_t(a_s, a_{-s}, \gamma_s, \gamma_{-s})}_{\text{outflows}} + o(\Delta t)/\Delta t.
\end{aligned}
\tag{12}
$$

The first line on the right-hand side represents inflows to the state $(a_s, a_{-s}, \gamma_s, \gamma_{-s})$ resulting from successful productive innovations by firms that are one λ step below in productivity. In contrast, the second line corresponds to inflows from unsuccessful productive innovations by firms that are one λ step above in productivity. The third line captures inflows from successful lock-in innovations by firms that are one δ step above in product substitutability. Outflows occur either due to successful or unsuccessful productive innovations, or from successful lock-in innovations or from the market reset experienced by firms in the state $(a_s, a_{-s}, \gamma_s, \gamma_{-s})$. The term $o(\Delta t)/\Delta t$ represents second-order moments that capture the probability of two or more innovations happening within the interval Δ, and satisfies $lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$. In a stationary equilibrium, the mass of supplier firms at each state must be time invariant. This implies that the measure of firms entering and leaving each state must be equal at every instant, ensuring $\mu_{t+\Delta}(a_s, a_{-s}, \gamma_s, \gamma_{-s}) = \mu_t(a_s, a_{-s}, \gamma_s, \gamma_{-s})$.

Definition 1. ***Equilibrium.*** *A dynamic general equilibrium in this economy is a sequence of allocations* $\{r_t, P_t, P_{ct}, p_{jt}, x_{jt}, l_{jt}, h^i_{jt}, h^z_{jt}, i_{jt}, z_{jt}, X_{ct}, L_t, Y_t, C_t, \mu_t\}^{t\in[0,\infty)}_{j\in\{s,-s\};\, c\in[0,1]}$ *such that (i) Supplier firms* *prices p_{st} solve the static profit maximization 8, (ii) innovation decisions i_{st}, z_{st} solve the dynamic problem 11; (iii) Customer firms quantities X_{ct} and prices P_{ct} solve the profit maximization problem 6; (iv) aggregate output Y_t is derived from the profit maximization problem of the Final Good producer; (iv) The real interest rate r_t is given by the Euler equation of the household; (v) Labor market clears, equation 2.2; (vii) Final goods aggregate price index P_t clears the good markets, $Y_t = C_t$ and (viii) the measure of firms μ_t evolve according to 12 consistent with firms' innovation decisions.*

2.2.1 Properties

In this section, I discuss the equilibrium implications of the model, with a focus on its novel insights about the trade-off between firm productivity and product substitutability across key dimensions including markups, market share, and profits.

Trade-off between Productivity and Product Substitutability

Proposition 1. ***Equilibrium elasticity of demand.***
Let $\epsilon_{X_{ct},p_{st}} \equiv \frac{d\ln X_{ct}}{d l p_{st}}$ be the elasticity of customer's quantities X_{ct} with respect to changes in supplier's price p_{st}, and $\epsilon_{P_{ct}D_c,p_{st}} \equiv \frac{d\ln P_{ct}D_{ct}}{d\ln p_{st}}$ the elasticity of customer's adjusted price index $P_{ct}D_{ct}$ with respect to changes in supplier's price. Under Bertrand oligopolistic competition between suppliers, supplier's elasticity of demand ϑ_{st} is given by:

$$\vartheta_{st}^{Bertrand} = \underbrace{\underbrace{\gamma_{st}}_{\text{monopolistic competition}} \left(1 - \varepsilon_{P_{ct}D_{ct},p_{st}}\right) + \varepsilon_{X_{ct},p_{st}}}_{\text{oligopolistic competition}}.$$

See Proof in Appendix A.2.1.[9]

Proposition 1 characterizes the elasticity of demand for supplier firms in equilibrium when they compete on prices in an oligopolistic market and customer firms use CRESH production technology. In equilibrium, this elasticity of demand depends on the slope of the demand curve, which is captured by the time-varying, supplier-specific substitutability $\gamma_{s,t}$ in the CRESH framework. This would represent the elasticity of demand if suppliers were competing in a monopolistic market, as discussed further below. However, in oligopolistic competition, suppliers internalize the effect of their pricing decisions on customer allocations. As a result, a supplier's elasticity of demand also depends on two key factors: the elasticity $\epsilon_{P_cD_c,p_s}$ of the customer's adjusted price index P_cD_c with respect to the supplier's price p_s, and the elasticity ϵ_{X_c,p_s} of the customer's production X_c with respect to the supplier's price. [10]

The elasticity of customer allocations to supplier prices, which determine the supplier's equilibrium elasticity of demand (see Proposition 1), , can be expressed as functions of the supplier's market share, as demonstrated in the following corollary.

[9] The Cournot competition version of Proposition 1 is stated in Appendix A.2.1.

[10] If suppliers compete in quantities (Cournot), the elasticity of demand is a function of the elasticity ϵ_{X_c,x_s}, of customer's production X_c to supplier's quantities x_s, the elasticity ϵ_{P_c,x_s} of customer's price P_c to supplier's quantities, and the elasticity ϵ_{D_c,x_s} of customer's demand aggregator D_c to supplier quantities. See Appendix A.2.1 for details.

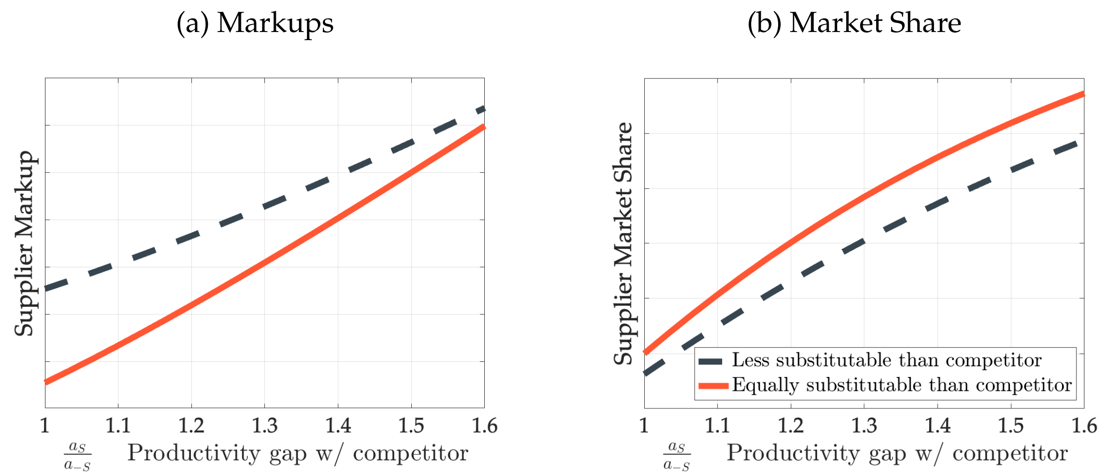
Corollary 1. *The equilibrium elasticity of demand of supplier firms s in each period t can be expressed as a function of the market share of supplier s, $S_{st}(a_{st}, \gamma_{st}) \equiv \frac{p_{st}x_{st}}{P_{ct}X_{ct}}$, according to:*

$$\vartheta_{st}^{Bertrand} = \gamma_{st}\left(1 - \frac{\gamma_{st}S_{st}(a_{st},\gamma_s)}{\sum_s \gamma_{st}S_{st}(a_{st},\gamma_{st})}\right) + \eta S_{st}(a_{st}, \gamma_{st}) \tag{13}$$

See Proof in Appendix A.2.2.

Corollary 1 establishes that the supplier's elasticity of demand is influenced by its product substitutability (γ_s), the elasticity of substitution between customer firms (η), its market share ($S(a_s, \gamma_s)$), and the overall distribution of market shares and product differentiation among suppliers ($\sum_s \gamma_s S_s(a_s, \gamma_s)$). In this model, a supplier's market share is a function of both its productivity and substitutability, as detailed below.

Figure 1: Supplier Firm: Markups and Market Share

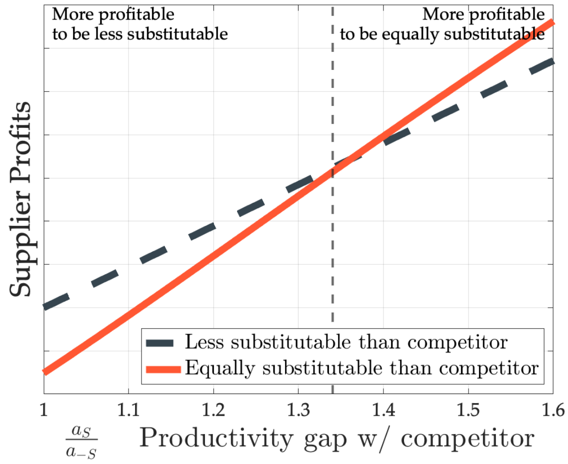


Notes: markups and market shares for when supplier firms are equally substitutable ($\gamma_s = \gamma_{-s}$) and for when the supplier s is half as substitutable as its competitor ($\gamma_s = 0.5\gamma_{-s}$). Markups are derived using the elasticity of demand from Corollary 1.

Figure 1 panel (a) illustrates the relationship between a supplier's markup and its productivity gap relative to its competitor. The green line represents the case where both suppliers are equally substitutable, corresponding to a customer CES technology where $\gamma_s = \gamma_{-s}$. In contrast, the pink line shows the case where the supplier is less substitutable than its competitor, reflecting a customer CRESH technology with $\gamma_s < \gamma_{-s}$. There are two key insights. First, the solid line indicates that with equal substitutability, a larger productivity gap between a firm and its competitor leads to a higher markup. This prediction aligns with standard models of oligopolistic competition where firms differ in productivity. However, when product substitutability is introduced as a second dimension of heterogeneity, a firm that is less substitutable relative to its competitor can charge a higher markup for any given productivity gap (dashed line). Thus, the model features two sources to market power: firms can secure high markups either by outperforming their competitors in productivity or by being less substitutable. Consequently, firms' dynamic decisions regarding productive and lock-in innovations, which I describe in the next section, will shape their accumulation of market power over time.

Although higher productivity and lower substitutability both lead to higher markups, they differ in how they affect a firm's market share, as illustrated in Figure 1 panel (b). When the two suppliers are equally substitutable, a larger productivity gap results in a higher market share, consistent with Aghion *et al.* (2001). However, when a supplier is less substitutable than its competitor, it captures a smaller market share for any given level of the productivity gap. The relationship between profits, productivity, and product substitutability, as discussed next, will be shaped by these trade-offs.

Figure 2: Supplier Firm: Profits



Notes: equilibrium supplier profits for when supplier firms are equally substitutable ($\gamma_s = \gamma_{-s}$) and for when the supplier s is half as substitutable as its competitor ($\gamma_s = 0.5\gamma_{-s}$).

Figure 2 illustrates the relationship between a supplier's profits, its productivity gap relative to its competitor, and its product substitutability. The profits of a supplier with productivity a_s, product substitutability γ_s, that competes with another supplier with productivity a_{-s} and product substitutability γ_{-s} are given by $\pi_t(a_s, a_{-s}, \gamma_s, \gamma_{-s}) = p_{st}x_{st} - W_t l_{st}$. As before, the solid line represents the case where the firm is equally substitutable as its competitor, while the dashed line represents the scenario where the firm is less substitutable. The figure highlights the trade-off between product substitutability and productivity in determining profits. When a firm is moderately more productive than its competitor, capturing a niche market by being less substitutable is more profitable than being equally substitutable. However, once the firm becomes significantly more productive, to the point where the competitor no longer poses a threat, it becomes more profitable to be equally substitutable and capture a larger share of the market.

Mapping to Standard Models

A key feature of the model is its ability to encompass both monopolistic and oligopolistic competition, as well as CES and non-CES homothetic demand systems. Table 1 illustrates this versatility by mapping the CRESH version of the model, with oligopolistic competition used in this paper, to the canonical CES models with oligopolistic competition and to a monopolistic competition structure. The first column of the table outlines the demand or technology class, either CRESH or CES. The second column presents the functional form of the homothetic aggregator that determines the customer firm's production technology, described in the third column. The definition of the aggregator remains independent of the market structure in which firms operate. In the CRESH case, as explained before when describing the problem of the customer firm, the homothetic aggregator is $(\frac{x_s}{X_c})^{\frac{\gamma_{s,t}-1}{\gamma_{s,t}}}$, leading to a customer production technology implicitly defined by condition $\sum_s (\frac{x_s}{X_c})^{\frac{\gamma_{s,t}-1}{\gamma_{s,t}}} = 1$. In the CES case, the homothetic aggregator is $(\frac{x_{s,t}}{X_{c,t}})^{\frac{\gamma-1}{\gamma}}$, which results in a customer firm technology analytically derived as the standard CES production function: $X_{c,t} = (\sum_s x_{s,t}^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}$.

The last two columns of the table use Proposition 1 to outline the equilibrium elasticity of demand for a supplier firm s under both monopolistic (fourth column) and oligopolistic (last column) competition. In a monopolistic market structure with CRESH technology, the elasticity of demand is determined by the supplier-specific, time-varying substitutability $\gamma_{s,t}$. For monopolistic competition with CES technology, the elasticity of demand is constant and equal to the common elasticity of substitution between suppliers, denoted by γ. Under oligopolistic competition with CRESH technology, the elasticity of demand follows Corollary 1. In this case, the larger a supplier's market share, the more its demand elasticity—and therefore its prices and profits—are influenced by the elasticity of substitution between customers. Conversely, the smaller the market share, the more its demand elasticity is driven by its own product substitutability. The key distinction between this model and the canonical CES models with oligopolistic competition lies in the supplier-specific, time-varying substitutability, as opposed to the common substitutability across firms in the CES case. In fact, when $\gamma_{s,t} = \gamma$ for all s, the model reverts to Atkeson and Burstein (2008), where demand elasticity smoothly adjusts with market share, weighted by the elasticity of substitution across customers and the common elasticity of substitution across suppliers.

Table 1: Model Applications

Technology class	**Homothetic aggregator**	**Customer technology**	**Supplier elasticity of demand ϑ_s**	
			Monopolistic	Oligopolistic
CRESH	$\left(\frac{x_s}{X_c}\right)^{\frac{\gamma_{st}-1}{\gamma_{st}}}$	$\sum_s \left(\frac{x_s}{X_c}\right)^{\frac{\gamma_{st}-1}{\gamma_{st}}} = 1$	γ_{st}	This paper $\gamma_s\left(1 - \frac{\gamma_s S_{st}(a_{st},\gamma_{st})}{\sum_s \gamma_s S_{st}(a_{st},\gamma_{st})}\right) + \eta S_{st}(a_{st}, \gamma_{st})$
CES	$\left(\frac{x_{st}}{X_{ct}}\right)^{\frac{\gamma-1}{\gamma}}$	$X_{ct} = \left(\sum_s x_{st}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$	γ	Atkeson and Burstein (2008) $\gamma\left(1 - S_{st}(a_{st})\right) + \eta S_{st}(a_{st})$

Notes: Model application to CRESH and CES technology, under monopolistic and oligopolistic competition between supplier firms. See Cournot competition version and Kimball demand application in Appendix Table 10.

Innovation Pass-Through from Supplier to Customer Firms

An advantage of this setup is that it allows for the analysis of how changes in the productivity or substitutability of supplier firms differently impact customer firms. I define *innovation pass-*

through as the transmission of changes in a supplier's productivity (a_s) or substitutability (γ_s) to the sales of customer firms. This definition is particularly useful because, in the data, I have access to customer firms' balance sheets, including their sales, but I do not observe prices and quantities separately. In the model, a total differentiation of the customer firm's sales with respect to changes in the supplier's productivity and substitutability yields:

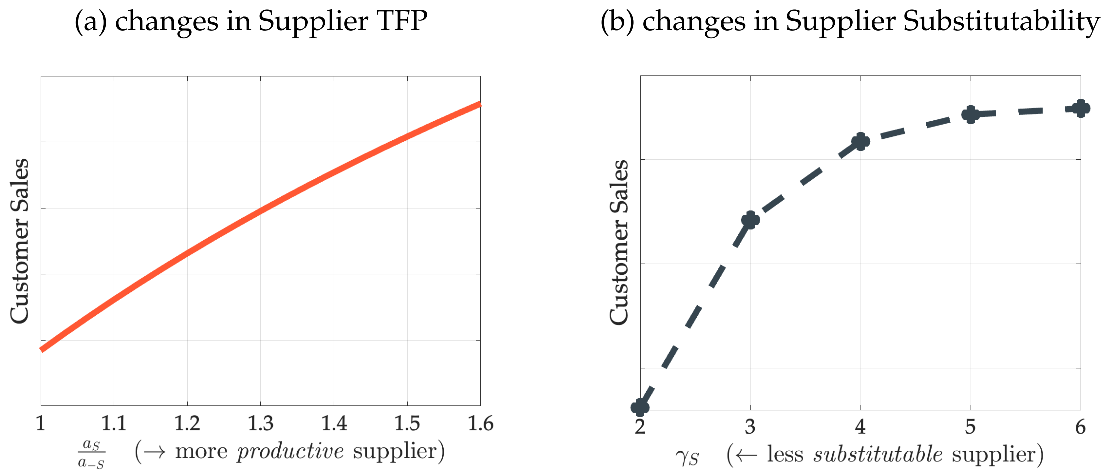
$$dlogP_cX_c = \underbrace{(1-\eta)}_{<0}\left[\underbrace{\frac{\partial logp_s}{\partial a_s}}_{<0} S_s da_s + \underbrace{\frac{\partial logp_s}{\partial(1/\gamma_s)}}_{>0} S_s d(1/\gamma_s)\right] \tag{14}$$

Equation 14 shows that the impact of changes in a supplier firm's productivity or substitutability on its customer's sales is driven by how the supplier's price adjusts in response to these changes, weighted by the supplier's market share in the customer's sales.

On one hand, an increase in the supplier's productivity reduces its marginal cost, leading to a lower price charged to the customer firm. This decrease in the supplier's price, in turn, reduces the marginal cost for the customer firm, resulting in an increase in the customer firm's sales (see Figure 3, panel a). On the other hand, when substitutability decreases, the supplier can charge higher prices for a given level of productivity. The resulting increase in the supplier's price raises the customer firm's marginal cost, which it cannot pass on to final good producers due to strong competition from other customer firms. The higher production costs and limited ability to pass them on lead to a reduction in the customer firm's sales (see Figure 3, panel b).

These opposing effects of changes in supplier productivity and substitutability on customer sales provide a testable implication, which I will apply in the empirical analysis to infer the types of innovations that suppliers are implementing.

Figure 3: Comparative Statics: Changes in Customer Sales after...



Notes: change in customer sales after changes in suppliers' productivity a_s, keeping competitor's productivity a_{-s} fixed (panel a), or after changes suppliers' product substitutability γ_s (panel b).

3 Lock-in and Productive Innovations in the Data

This section presents empirical findings on innovation, market power and product differentiation, and innovation pass-through from supplier to customer firms. I start by describing the data sources, followed by the empirical strategy and results.

3.1 Data description

I combine firm-level estimates of markups, product differentiation and innovation shocks, together with supplier-customer firm linkages and balance sheet information.

Firm's balance sheet data. I obtain firm-level financial data from Compustat Fundamentals, a panel of publicly listed U.S. firms, which I access through the Wharton Research Data Services (WRDS) platform. Compustat offers two main advantages for this study: (i) it includes rich financial data for a long panel of firms, starting in 1978, which allows to use within-firm variation, and also to exploit variation before and after year 2000s when the U.S. economy experienced remarkable changes in market power and business dynamism, as discussed below; (ii) it allows to match firm-identifiers to the firm linkages dataset described below, obtaining balance-sheet information for both supplier and customer firms.[11]

Markups estimation. I estimate markups at the firm level by production function estimation as in De Loecker and Warzynski (2012) and De Loecker, Eeckhout and Unger (2020), using data on sales and variable input expenditures from Compustat, together with estimates of output elasticity.[12] For the rest of the analysis, I define *high markup suppliers* as those supplier firms whose markups lie within the 80th or higher percentiles of the markup distribution, and define *low markup suppliers* as the rest of supplier firms. However, results are robust to alternative thresholds (60th, 70th, or 90th percentile or higher).[13]

Innovation shocks. I use the market value of patents issued by a public firms in U.S., estimated by Kogan *et al.* (2017) as firm-level measure of innovation shocks. They estimate the excess stock market return of patents assigned to a given firm in a window around patent approval dates. The main advantage of this measure for my study is that it provides a private dollar-value of patents which can be mapped to the stock market value of a firm, which drives firms' innovation decisions in the model. Moreover, the excess stock market return of patents capture unexpected shocks.

[11] The main disadvantage of Compustat dataset is that it does not include privately held firms. In the Data Appendix, I provide robustness checks for the empirical patterns presented below using a broader sample of firms from FACTSET dataset, which includes privately held ones.

[12] An alternative method to estimate markups, referred to as the demand approach, estimates marginal costs using data on prices and quantities. However, because Compustat Fundamentals lack firm-level price data, this approach is not applicable. See Nevo (2001), Berry, Levinsohn and Pakes (1995) and Goeree (2008) for well-known industry studies.

[13] Bond, Hashemi, Kaplan and Zoch (2021) examine the challenges of identifying and estimating markups when firm-level output prices are not available. They suggest that a viable alternative is to compare mean markups across groups of firms, as long as one is willing to assume that production function elasticities do not vary systematically with firm characteristics (in this context, markup status).

Product differentiation. I use the product similarity measure developed by Hoberg and Phillips (2016) as an estimate of firm-level product differentiation. They created a publicly available database of product similarity scores for nearly all publicly traded U.S. firms, which has become a widely used resource in both finance and industrial organization research. Their methodology employs natural language processing techniques to analyze the content of annual 10-K filings submitted to the U.S. Securities and Exchange Commission (SEC), producing product similarity scores that vary annually over time. The 10-K is an annual regulatory report required of publicly traded companies in the U.S., and Item 1 of the report contains detailed descriptions of the firm's products and services. Hoberg and Phillips (2016) utilize these textual descriptions to construct a dataset of product cosine similarities, capturing the extent of similarity in product characteristics across firms. For my analysis, I use the *Total Similarity* index, which is calculated as the sum of the pairwise similarities between a given firm and all other firms in the sample within a given year. Henceforth, I will refere to this measure as *HP Similarity Score*. Intuitively, a lower total similarity score for a firm in a given year indicates higher product differentiation or greater uniqueness of its products relative to other firms.

Supplier-customer firm linkages. I use the dataset constructed by Barrot and Sauvagnat (2016) to obtain production linkages between supplier and customer firms. To identify the linkages, they rely on the obligation that publicly listed U.S. firms have to report the identity of any customer representing more than 10% of their total sales, under regulation Statement of Financial Accounting Standards (SFAS) No.131. Customers firms in this dataset are representative of the U.S. economy, covering approximately 75% of the total sales in Compustat over the sample period between 1978 and 2013. Following Barrot and Sauvagnat (2016), I consider that a supplier and customer firms are linked all quarters from the first to the last quarter that the customer is reported by the supplier.

3.2 Empirical Facts

In this section, I present suggestive evidence on the prevalence of lock-in strategies before and after 2000. This includes facts on the relationship between market power and product differentiation across the two periods, and new findings on market power and the impact of innovation on product differentiation, and on how innovation pass-through from supplier firms to customer firms' sales varies with suppliers' market power.

1. Higher markups are correlated with greater product differentiation, and this relationship has strengthened in the years following the 2000s.

2. After 2000, innovations by high-markup suppliers significantly increase product differentiation, while innovations by low-markup suppliers lead to non-significant changes in product differentiation. In contrast, before 2000, innovations by high-markup firms reduced product differentiation.

3. Post year 2000, innovations by high markup suppliers lead to a decline in customer firms' sales, while innovations by low markup suppliers lead to an increase in customer firms' sales. However, prior to 2000, innovations by high-markup firms led to an increase in customer firms' sales.

When combined when the model's key predictions on innovation pass-through, these results inform the nature of innovation in the data. Through the lens of the model, the findings indicate that low-markup suppliers tend to invest in productive innovations that positively affect their customer firms' sales. In contrast, high-markup firms are more likely to pursue lock-in innovations that negatively impact customer firms' sales after 2000, but they were investing mostly in productive innovations during the pre-2000 period. I now present each empirical finding in detail.

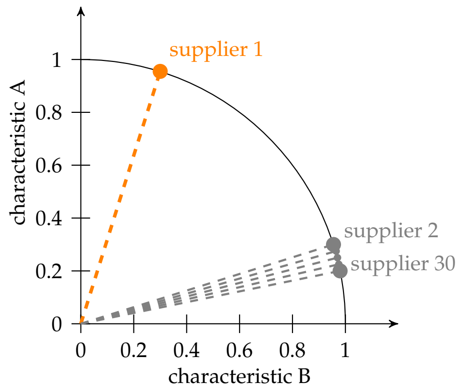
Fact 1: Market Power and Product Differentiation

Table 2: Markups and Product Similarity

	Pre-2000s	Post-2000s
Log Markups	-0.139***	-0.199***
	(0.0247)	(0.0126)
R^2	0.408	0.571
Sector & Year FE	yes	yes

Notes: Results from regressing the standardized firm-level HP product similarity score on the log of supplier firms' markups, controlling for sector and year fixed effects. Pre-2000 indicates the estimation for years previous to 2000, and Post-2000 the estimation for years after 2000.

Figure 4: Firms in the Product Space



Notes: Graphical representation of the firm's unit circle product space. The example illustrates a product space where firms are defined by two characteristics, A and B, and their position is determined by their products' content along these dimensions. In the figure, Supplier 1 is more differentiated than Suppliers 2 to 30, as it has a higher content of characteristic A and a lower content of characteristic B,.

I document a negative correlation between supplier firms' markups and the HP product similarity score, as shown in Table 2. Before 2000, a 1% increase in a firm's markup is associated with a 0.14 standard deviation decrease in the cosine similarity index of its products relative to those of competitors (first column). After 2000, this correlation becomes stronger, with a 1% increase in markup corresponding to a 0.18 standard deviation decrease in the cosine similarity index (second column). These findings suggest that higher markups are linked to greater product differentiation, as firms position their products strategically within a space where distinctive features set them apart, while still retaining some similarities with competing offerings (Rosen, 1974; Lancaster, 1975). Greater product differentiation is associated with higher market power, as it reduces competitive pressures. Figure 4 illustrates the product space.

I provide additional evidence on the relationship between firms' markups and various measures of product differentiation in Appendix C. First, Table 11 presents the results of a regression of the log of a firm's R&D-to-sales ratio on a dummy variable indicating whether the firm belongs to the high-markup group. The findings show that high-markup firms, on average, invest 82% more in R&D as a share of sales compared to other firms in the economy. I use the R&D expenditure share as a proxy for product specificity, following Barrot and Sauvagnat (2016).

Fact 2: Innovation, Market Power and Product Differentiation

While the relationship between product differentiation and market power is well documented, evidence on how product differentiation changes after firms innovate—and how this relates to firms' market power—remains elusive. I provide new empirical evidence on innovation, market power, and product differentiation. I estimate a local projection that analyzes how changes in a firm's HP similarity score after firm's innovation shocks depends on the firm's markups. This analysis combines HP similarity score data with measures of innovation shocks, defined as the excess stock market returns of patents assigned to the firm, alongside firm-level balance sheet data to estimate the following local projection:

$$\begin{aligned}\Delta \log HP_{s,t+h} = \beta_{H,h} & \sum_s Innov_{s,t} * \mathbf{1}_{\{s \in \text{high markup}\}} \\ & + \beta_{L,h} \sum_s Innov_{s,t} * [1 - \mathbf{1}_{\{s \in \text{high markup}\}}] \\ & + m_h \mathbf{1}_{\{s \in \text{high markup}\}} + \alpha_s + \alpha_{ith} + \mathbf{\Gamma}'_h Z_{st-1} + e_{th}. \end{aligned} \tag{15}$$

The dependent variable, $\Delta \log HP_{s,t+1}$, represents the log change in the HP similarity score of supplier firm s from period t to $t+1$. The variable $\sum_s Innov_{s,t} * \mathbf{1}_{\{s \in \text{high markup}\}}$ captures the sum of innovation shocks $Innov_{s,t}$ to firm s in year t, interacted with the indicator $\mathbf{1}_{\{s \in \text{high markup}\}}$, which takes the value of one if the supplier is in the top distribution of markups in period $t-1$, prior to the innovation shock. The term $\sum_s Innov_{s,t} * [1 - \mathbf{1}_{\{s \in \text{high markup}\}}]$ captures innovation shocks in year t for the remaining suppliers (i.e., the *low-markup* suppliers). The main coefficients of interest, $\beta_{H,1}$ and $\beta_{L,1}$, measure the semi-elasticity of firm-level product differentiation to innovation shocks for high-markup and low-markup firms, respectively. I control for firm-specific factors $Z_{s,t-1}$ that may influence product similarity, including firm size (total assets, total sales, and capital stock) and firm-level volatility. I also include the indicator $\mathbf{1}_{\{s \in \text{high markup}\}}$ to control for permanent differences between high- and low-markup firms, firm fixed effects α_s to capture time-invariant differences across firms, and industry-by-year fixed effects $\alpha_{i,t}$ to account for sectoral heterogeneity, time trends, and their interaction. I cluster the standard errors by year.

I estimate regression 15 separately for the pre- and post-2000 periods, and present the results in Table 3. Before 2000, a one-dollar increase in innovation spending by a supplier firm led to non-significant changes in product similarity for low-markup firms. In contrast, high-markup firms experienced a significant 36% increase in product similarity during the first year, followed by a slightly negative but non-significant change in the second year after the innovation (panel a). The patterns shift notably in the post-2000 period. During this time, a one-dollar increase in innovation spending by a high-markup firm led to a statistically significant decrease in the product similarity score in the two years following the innovation, with reductions of 9% and 16%, respectively (panel b). In comparison, innovations by low-markup firms resulted in non-significant decreases in product similarity, with effect sizes two to five times smaller. These findings suggest that post-2000, high-markup firms are more likely to pursue lock-in innovations that increase product differentiation (or reduce product substitutability). In contrast, innovations by low-markup firms and high-markup firms before 2000 are less likely to take this form. This highlights the importance of a firm's position in the markup distribution when examining the nature of innovations. In the following section, I test the model's main predictions on the pass-through effects of innovations from suppliers to customers, conditioning on firms' position in the markup distribution.

Table 3: Markups and Changes in Product Similarity after Innovation

(a) Pre-2000s

	Year=1	Year=2
Low Markups	0.07	-0.12
	(0.17)	(0.14)
High Markups	0.36**	-0.02
	(0.15)	(0.24)
R^2	0.615	0.682
Firm, Sector & Year FE	yes	yes

(b) Post-2000s

	Year=1	Year=2
Low Markups	0.05	-0.03
	(0.08)	(0.09)
High Markups	-0.09*	-0.17**
	(0.05)	(0.07)
R^2	0.453	0.619
Firm, Sector & Year FE	yes	yes

Notes: estimation results of the semi-elasticity of changes in HP Similarity Score to firm-level innovation shocks, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups). See equation 15 for specification details.

Fact 3: Market Power and Innovation pass-through from Supplier to Customer Firms

I combine data on supplier-customer linkages with firm-level financials, innovation shocks, and supplier markup estimates to estimate how innovation pass-through from supplier to customer firms. Table 4 presents summary statistics of the sample used in the analysis. I winsorize the sample at the top and bottom 0.5% of observations to ensure the results are not driven by outliers. Panel (a) presents statistics for the supplier firm's sample, divided into those with high-markups and those with low-markups. I define *high-markup supplier firms* as those whose markups are in the 80th percentile of markup distribution or higher, and categorized the rest of supplier firms as *low-markup supplier firms*. There are 490 high-markup and 831 low-markup suppliers in the sample, with 5729 and 14107 firm-quarters observations respectively, from 1984 to 2010. High-markup suppliers on average receive larger innovation shocks (1.98 vs 0.65) [14], have 1.8 times higher markups on average, have slightly smaller size in terms of sales, but have higher profits and assets than low-markup supplier firms. High-markup firms have 1.38

[14] Consistent with Kogan *et al.* (2017), the distribution of innovations across firms is highly-skewed.

customers and low-markup firms have 1.48 customers on average. [15] Panel (b) presents the summary statistics for the customer firms in the sample. There are 367 customer firms in the sample, with 9132 firm-quarter observations from 1984 to 2010. Customer firms have an average markup of 1.33 and exhibit greater size (both in terms of sales and assets) and profits compared to supplier firms. On average, they are connected to 3.15 suppliers.

Table 4: Summary Statistics

(a) Supplier firms

	High-Markups Suppliers					Low-Markups Suppliers				
	mean	sd	p25	p50	p90	mean	sd	p25	p50	p90
innovation shocks	1.98	4.53	0.00	0.00	6.17	0.65	1.99	0.00	0.00	1.84
markups	1.85	0.63	1.43	1.63	3.24	1.03	0.29	0.91	1.07	1.31
log sales	4.05	2.01	2.65	3.88	7.00	4.17	1.98	2.76	4.07	6.99
log profits	3.41	2.03	1.95	3.24	6.43	2.96	1.87	1.71	2.79	5.53
log assets	5.52	2.11	3.99	5.34	8.44	5.49	1.91	4.17	5.27	8.19
nr of customers	1.38	0.70	1.00	1.00	2.00	1.48	0.78	1.00	1.00	3.00
Observations	5729					14107				

(b) Customer firms

	All Customers				
	mean	sd	p25	p50	p90
markups	1.34	0.72	0.99	1.16	1.90
log sales	7.64	1.44	6.81	7.85	9.41
log profits	6.44	1.41	5.57	6.59	8.25
log assets	8.86	1.51	7.98	9.01	10.80
nr of suppliers	3.15	4.96	1.00	1.00	7.00
Observations	9132				

Notes: Panel (a) shows summary statistics for Supplier firms in the sample, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups). Panel (b) shows summary statistics for Customer firms in the sample.

I analyze how customer firms respond to innovations by high market power suppliers using local projection methods following Jordà (2005). To test the model's predictions on innovation pass-through, I focus on customer firms' sales as the primary outcome, while considering additional outcomes to explore alternative mechanisms and robustness checks. The empirical specification is given by:

$$\begin{aligned}\Delta \log Sales_{ct+h} = \beta_{H,h} & \sum_s \omega_{sct} Innov_{st} * \mathbf{1}_{\{s \in \text{top markup}\}} \\ & + \beta_{L,h} \sum_s \omega_{sct} Innov_{st} * [1 - \mathbf{1}_{\{s \in \text{top markup}\}}] \\ & + m_h \mathbf{1}_{\{s \in \text{high markup}\}} + \alpha_c + \alpha_{ith} + \boldsymbol{\Gamma}_h' Z_{ct-1} + e_{th} \end{aligned} \tag{16}$$

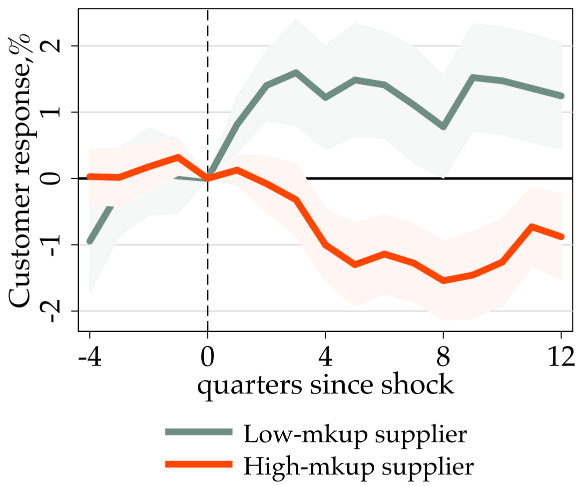
In line with the local projection specification 15, the variable $Innov_{s,t} * \mathbf{1}_{\{s \in \text{high markup}\}}$ represents innovation shocks from supplier firms interacted with an indicator that equals one if

[15] Notice that given the structure of SFAS No.131, a firm in the sample can have at most 10 customers.

the supplier is in the top markup distribution. However, since the outcome of interest is now customer firm sales, I weight the sum of innovation shocks from all high-markup suppliers s within a quarter by supplier s's share of total sales to customer firm c in period t, denoted by ω_{sct}, resulting in the term $\sum_s \omega_{sct} Innov_{st} * \mathbf{1}_{\{s \in \text{top markup}\}}$. I apply the same approach for innovation shocks from low-markup suppliers, yielding the term $\sum_s \omega_{sct} Innov_{st} * [1 - \mathbf{1}_{\{s \in \text{top markup}\}}]$. The coefficients of interest, $\beta_{H,h}$ and $\beta_{L,h}$ measure the cumulative response of customer firm sales in quarter $t+h$ to a 1 standard-deviation increase in innovation by high-markup and low-markup supplier firms, respectively.[16] I control for customer firm characteristics $Z_{c,t-1}$ that may influence sales, including size (total assets and capital stock), volatility, and the value of the firm's own innovation shocks (measured as the excess stock market return of innovations attributed to the customer firm). I also include the indicator $\mathbf{1}_{\{s \in \text{high markup}\}}$ to control for permanent differences between high- and low-markup suppliers, customer firm fixed effects α_c to capture time-invariant differences, and customer industry-by-quarter fixed effects $\alpha_{i,th}$ to account for sectoral heterogeneity, time trends, and their interactions. Standard errors are clustered by quarter.

I first estimate specification 16 for the years after 2000. Figure 5 shows the cumulative differential response of customer firms to innovation shocks by supplier firms with high markups (coefficient $\beta_{H,h}$ in equation 16) and to innovation shocks by supplier firms with low markups (coefficient $\beta_{L,h}$ in equation 16) from the quarter since the innovation shock and until three years later. A one standard deviation increase in innovation by low-markup suppliers leads to an average increase of up to 1.3% in the sales growth of customer firms. A similar increase in innovation by high-markup suppliers results in an average decline of up to 1.4% in customer firm sales growth two years after the innovation, with the effect gradually reversing four years post-shock.

Figure 5: Δ Customer Sales after Supplier Innovation



Notes: estimation results of the semi-elasticity of changes Customer sales after Supplier's innovation shocks, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups). See equation 16 for specification details.

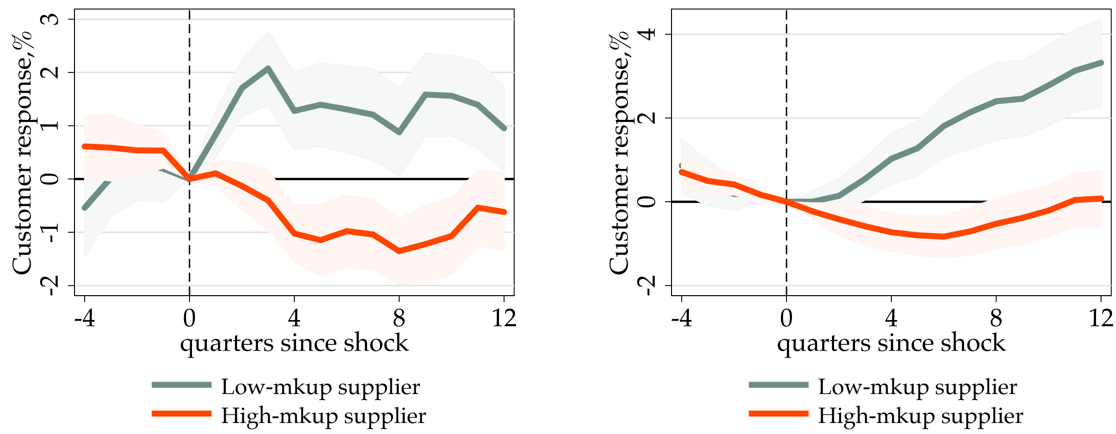
[16] The measure of innovation shocks is standardized to facilitate comparisons with alternative measures considered in robustness checks and to align with estimates in Kogan *et al.* (2017), which examine the response of firms' outcomes to their own innovation shocks.

In summary, the results show that, in the post-2000s period, innovations by low-markup suppliers lead to an increase in customer firms' sales growth, whereas innovations by high-markup suppliers result in a decline in customer sales growth. This pattern is consistent with the model's predictions on the pass-through effects of productive versus lock-in innovations. High-markup firms are more likely to pursue lock-in innovations that allow them to raise prices, which their customer firms cannot pass on to final good producers. As a result, customers bear the higher input costs, ultimately harming their own sales growth. In contrast, low-markup firms tend to invest in productive innovations that lower the customer firm's marginal costs, boosting their sales growth. In Appendix C, I document a similar pattern in the response of customer profits to innovations by low-markup versus high-markup suppliers (Figure 18). The findings for both customer sales and profits remain robust when controlling for the number of citations received by the supplier's patents, which serves as a proxy for invention quality (Figure 19, panel a and b).

Figure 6: Business Stealing? Technology Adoption Costs?

(a) Customer Sales, Control for Same Industry

(b) Customer Investment

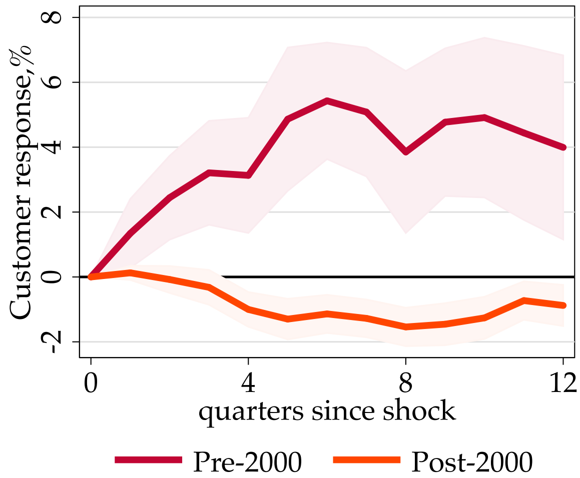


Notes: Panel (a) shows estimation results of the semi-elasticity of changes Customer sales after Supplier's innovation shocks, controlling for the supplier being in the sample industry as the customer firm, and conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups). Panel (b) shows estimation results of the semi-elasticity of changes Customer investment after Supplier's innovation shocks, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups).

I explore and rule out other potential explanations for the empirical patterns observed in Figure 5 that are not related to changes in product differentiation driven by innovation. First, I consider the possibility that high-markup supplier firms may be "stealing business" from their customer firms, thereby contributing to the decline in customer sales observed in the data. To test this, I re-estimate specification 16 while controlling for the differential response of customer firms that operate in the same industry as their suppliers, using both 4-digit and 2-digit SIC industry classifications. The results remain robust, as shown in Figure 6, panel (a), suggesting that the decline in customer sales is not due to high-markup suppliers taking business away from their customers. Second, I examine whether the relative decline in customer firms' real output and profits could be explained by short-term technology adoption costs that arise after their suppliers innovate. To investigate this, I re-estimate specification 16 using customer firms' investment as the outcome variable. As shown in Figure 6 , panel (b), customer firms experience a short-run decline in investment following innovation shocks from high-markup suppliers, which is inconsistent with the hypothesis that customers are incurring additional costs to adapt their production processes to new technologies introduced by their suppliers.

Given the evidence from Fact 1 and Fact 2 on the increasing prevalence of lock-in innovations in the post-2000 period—both in terms of the correlation between markups and product differentiation, and the changes in product differentiation following innovations by high-markup firms—a natural next step is to compare the pass-through of innovations from high-markup supplier firms to their customer firms across the two periods. Figure 7 presents the estimated coefficient $\beta_{H,h}$ from the specification 16, estimated separately for the pre- and post-2000 periods. Notably, before 2000, innovations by high-markup firms led to an increase in customer firms' sales of up to 5.8%, suggesting that these firms primarily invested in productive innovations during this period. In contrast, post-2000 innovations by high-markup firms resulted in a negative response in customer sales, indicating a shift toward a higher prevalence of lock-in innovations.

Figure 7: Δ Customer Sales after High-Markup Supplier Innovation



Notes: estimation results of the semi-elasticity of changes Customer sales after Supplier's innovation shocks for years previous and after 2000, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups).

Finally, I present additional evidence on the prevalence of lock-in innovations by analyzing the relationship between the private and social values of innovation. The private value is measured by the stock-market dollar returns of patents from Kogan *et al.* (2017), while the social value is assessed through the number of citations these patents receive. A strong correlation between citation counts and private value suggests that firms are deriving higher private value from inventions that also yield substantial societal benefits. Conversely, a weaker correlation implies that firms may be capturing more private value from inventions that primarily serve market-protection or strategic motives (Abrams, Akcigit and Grennan, 2013). In Appendix Table 12 , I show that a 1% increase in private value among high-markup firms is associated with a 0.7% rise in social value pre-2000 (panel a), declining to a 0.4% rise post-2000 (panel b). For low-markup firms, the correlation between private and social value remains steady at 0.6, both before and after 2000.

Overall, the empirical results indicate a rise in the prevalence of lock-in innovations in the years following 2000, specially among high-markup firms. I use these findings to calibrate key parameters of the model and quantify the aggregate impact of both lock-in and productive innovations on aggregate total factor productivity (TFP) and market power.

4 Quantitative Analysis

This section quantifies the implications of lock-in and productive innovations for aggregate TFP and the dispersion of markups. To do this, I first calibrate the model from Section 2 to match the empirical patterns from Section 3.2 along with other variables of interest for the post-2000 period. Using this baseline calibration, I describe firms' investment strategies in lock-in and productive innovations to provide intuition about the model. I then re-calibrate the model to match the empirical patterns from Section 3.2 for the pre-2000 period. Next, I conduct a counterfactual analysis to quantify what aggregate TFP and markup dispersion in the post-2000 period would have been if the cost structure of lock-in innovations had remained the same as in the pre-2000 period.

4.1 Baseline Calibration: Post-2000s

The model has 13 structural parameters, described in Table 5, which identification happens in three ways. First, two parameters are externally calibrated to match existing results in the literature (ρ, ψ), (Table 5, *Panel A*). I set the households' discount rate parameter to $\rho = 6\%$. For the curvature of the cost function of productive innovations, I consider a quadratic function with $\psi = 2$, in line with previous results in the literature that estimate the elasticity of patenting to R&D expenditures (Acemoglu and Akcigit, 2012; Acemoglu, Akcigit, Hanley and Kerr, 2016). Second, two other parameters are directly matched to microdata (Table 5, *Panel B*): I set the lock-in innovation step-size, δ, to match the empirical fact presented in Section 3.2 on the change in product similarity after high-markup firms —which are the ones most likely to invest in lock-in— innovate. I take the probability of market reset, κ, to be given by the average firm entry rate, since in the model it reflects situations in which new firms enter to compete with incumbents, leveling the substitutability between firms. I use estimates for the entry rate in US by Akcigit and Ates (2021), which are based on U.S. Census Bureau's Business Dynamics Statistics. For the remaining nine parameters $(\eta, \lambda, \tilde{\psi}, \phi_s, \phi_{-s}, \tilde{\phi}_s, \tilde{\phi}_{-s}, \alpha, \tilde{\alpha})$, I perform an internal calibration in two steps (Table 5, *Panel C*): I replicate the empirical facts on innovation pass-through between suppliers and customer firms from Section 3.2 using data generated by the model, and I target data moments that are informative about relevant features of the model. Next, I describe these two identification steps in detail.

Table 5: Parameter Values: Post-2000 Period

Parameter	Description	Value
	— *Panel A. External Calibration* —	
ρ	Rate of time preference	6%
ψ	Productive innovation cost curvature	2
	— *Panel B. Direct Match to Data* —	
δ	Lock-in innovation step size	17%
κ	Market reset probability	10%
	— *Panel C. Internal Calibration* —	
η	Elasticity of substitution between customers	1.5
λ	Productive innovation step size	3.3%
$\tilde{\psi}$	Lock-in innovation cost curvature	2.8
ϕ_s	Productive innovation cost relation w/ productivity level	2.8
ϕ_{-s}	Productive innovation cost relation w/ productivity level	0
$\tilde{\phi}_s$	Lock-in innovation cost relation w/ productivity gap	-2.8
$\tilde{\phi}_{-s}$	Lock-in innovation cost relation w/ productivity gap	-2.8
α	Productive innovation scale	1
$\tilde{\alpha}$	Lock-in innovation scale	0.5

Notes: List of model parameters and calibrated values for the Post-2000 economy.

Replicating the empirical facts from Section 3

A crucial identification step involves replicating the regressions of Section 3 using data simulated from the model. I simulate a panel of 1000 sectors for 200 years, taking into account all the possible events that can happen in the economy, which probabilities are determined by the firms' lock-in and productive policy functions and the probability of market reset, κ.[17] Using the model-generated data, I run the local projection from equation (16), regressing the change in the customer sales after innovations (both lock-in and productive) performed by the supplier firm. To define the low- and high-markup suppliers, I use the same criteria that I used in the empirical section, and consider the high-markup firms as those who belonged to the 80-th percentile or higher of the markup distribution ex ante the innovation happened, and low-markup firms as the remaining ones. Figure 8 shows the dynamic response of customer sales to innovations by low- and high- markup suppliers, both in the data and in the model.

When combined with the predictions of the model on the positive response of customer sales after productive innovations by suppliers, and the negative response of customer sales to lock-in innovations by suppliers, the dynamic response of customer firms' sales to innovations by

[17] I consider a sector as a group of two (supplier) firms that provide their inputs to the customer (sector) firms. Since suppliers are ex-ante homogeneous, this is equivalent to simulating one sector for 100,000 years.

low- and high-markup suppliers observed in the data are informative of the parameters governing the relationship between the cost of productive and lock-in innovations and the markup of the firm, i.e., $\phi_s, \phi_{-s}, \tilde{\phi}_s, \tilde{\phi}_{-s}$.[18] The dynamics of innovation pas-through are also informative of the overall prevalence of each type of innovation, therefore are informative the convexity of lock-in innovations, $\tilde{\psi}$ (given that the convexity of productive innovations is externally calibrated).

Targeted moments

The second step of the internal calibration involves targeting some moments of interest: the average markup and moments of the markup distribution, the average annual ratio of R&D spending to GDP, sourced from the National Science Foundation data, and the profit share of GDP, which I take from estimates in Akcigit and Ates (2021). Table 6 presents the list of targeted moments for the Post-2000 calibration and its comparison with the data. One target is the average markup rate and the distribution of markups (75th and 90th percentile). As described in Section 2.2.1, in the model markups are endogenous to the suppliers' productivity and substitutability, therefore the average markup level and the markup dispersion are informative of the step-size of productive innovations, λ, as well as the innovation scale parameters, α and $\tilde{\alpha}$. Another targeted moment is the the Since in the model innovations are driven by both productive and lock-in incentives, the R&D share of GDP disciplines the scale parameters of productive and lock-in innovations as well.

Figure 8: Δ Customer Sales to Supplier Innovation

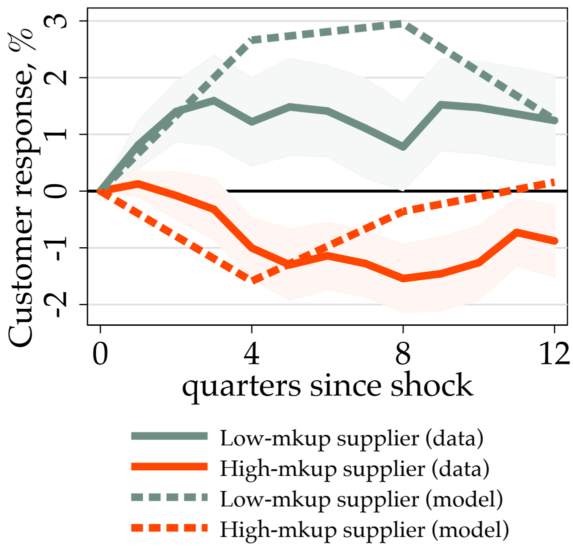


Table 6: Model Fit

Moment	Model	Data
Average markup	1.46	1.34
Markup 75th percentile	1.57	1.54
Markup 90th percentile	1.71	1.92
R&D share of GDP, %	1.59	1.65
Profit share of GDP,%	14	17

Notes: Figure 8 shows data and model estimation results of the semi-elasticity of changes in Customer sales after Supplier's innovation shocks for post-2000 period, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups). Table 6 presents the value of moments in the data and in the calibrated model for the post-2000 period.

[18] Since in the model markups are endogenous and increasing on the firms' productivity gap relative to its competitor, these four parameters discipline the relationship between the cost of innovation and the markup of the firm.

4.2 Properties of the Baseline Estimation

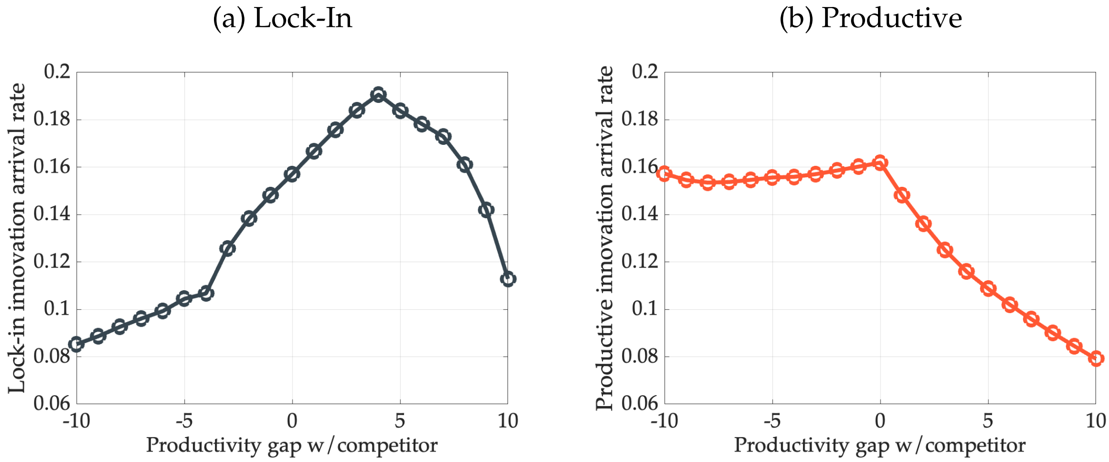
The first order conditions of problem 11 with respect to the two types of innovation yield the optimal productive and lock-in innovation decisions:

$$i_{st} = \frac{1}{\exp(a_{st}^{\psi_s} - a_{-st}^{\psi_{-s}})} \left[\frac{1}{\alpha W_t} (V_t(a_s + \lambda, a_{-s}, \gamma_s, \gamma_{-s}) - V_t(a_s - \lambda, a_{-s}, \gamma_s, \gamma_{-s})) \right]^{\frac{1}{\phi - 1}}$$

$$z_{st} = \frac{1}{\exp(a_{st}^{\tilde{\psi}_s} - a_{-st}^{\tilde{\psi}_{-s}})} \left[\frac{1}{\tilde{\alpha} W_t} (V_t(a_s, a_{-s}, \gamma_s - \delta, \gamma_{-s}) - V_t(a_s, a_{-s}, \gamma_s, \gamma_{-s})) \right]^{\frac{1}{\tilde{\phi} - 1}}.$$

Figure 9 presents the policy functions for lock-in and productive innovations across different values of the log-productivity gap between a firm and its competitor, based on the baseline estimation described in Section 4.1. Panel (a) displays the arrival rate of lock-in innovations. The investment intensity in lock-in innovations follows a hump-shaped pattern relative to the productivity gap. For firms lagging behind their competitors in productivity, the intensity of lock-in innovation increases with the productivity gap, peaking at a moderate positive gap. Beyond this point, the intensity declines sharply for firms that are significantly more productive than their competitors. This pattern aligns with the relationship between productivity gaps, profits, and product substitutability discussed in Section 2.2.1. Specifically, profits increase as products become less substitutable, up to a point where the firm gains a substantial productivity advantage. Beyond this threshold, firms benefit more from offering more substitutable products, enabling them to capture a larger share of the market and further increase profits. This highlights one of the main contributions of the paper: characterizing the incentives for firms to pursue lock-in innovations—an area previously unexplored. By linking the model's predictions with the empirical evidence on innovation pass-through, I provide a disciplined framework to better understand these incentives. Panel (b) displays the arrival rate of productive innovations, which also follows a hump-shaped pattern but peaks when the firm and its competitor have equal productivity levels. This result is well-established in the literature that studies the relationship between competition and innovation (Aghion *et al.*, 2001; Aghion, Bloom, Blundell, Griffith and Howitt, 2005).

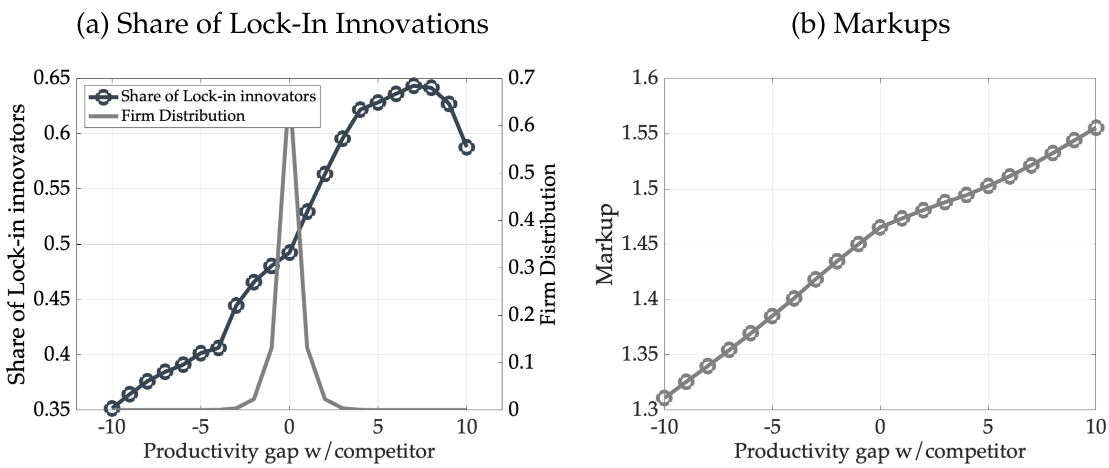
Figure 9: Innovation Policy Functions



Notes: calibrated model lock-in (panel a) and productive (panel b) innovations' policy functions, against the supplier's productivity gap (in terms of number of steps) with respect to their competitor.

How does the composition of lock-in and productive investments within the innovation portfolio change with the productivity gap? Figure 10 illustrates the share of lock-in innovations in the total innovation portfolio, measured as $\frac{z_s}{z_s+i_s}$. The share of lock-in innovations rises sharply with the productivity gap, starting at 35% when the firm is lagging by ten steps behind its competitor and reaching 65% when the firm is ahead by seven steps. Beyond this point, the share of lock-in innovations declines. The figure also includes the distribution of firms across productivity gaps as implied by the baseline model. Panel (b) shows the increasing relationship between the productivity gap of the firm and the markups in the model. Together, the figures suggest that, in equilibrium, there are no firms in regions where high-markup firms do not pursue lock-in innovations. Consequently, in areas with a positive density of firms, the model exhibits a positive relationship between markups and the share of investment in lock-in innovations.

Figure 10: Share of Lock In Innovations, Markups, and Productivity Gap



Notes: Panel (a) shows the share of lock-in innovations in the total innovation portfolio, $\frac{z_s}{z_s+i_s}$ together with the calibrated model firm's distribution, against the supplier's productivity gap (in terms of number of steps) with respect to their competitor. Panel (b) shows the calibrated model relationship between a supplier's markups and its productivity gap (in terms of number of steps) with respect to their competitor.

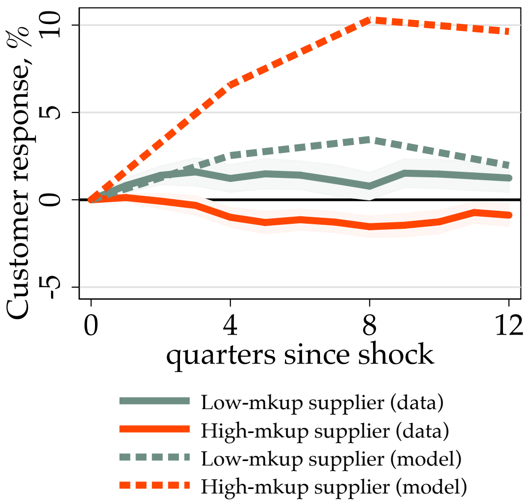
4.3 Innovation Pass-Through Without Lock-In

In this section, I explore a counterfactual scenario in which the cost-scale parameter of lock-in innovations, $\tilde{\alpha}$, is set to infinity. This exercise provides intuition for why the inclusion of lock-in innovations in the model is essential to replicate the decline in customer sales after innovations by high-markup suppliers observed in the data. I simulate a panel of firms under this counterfactual scenario, where the cost of lock-in innovations becomes prohibitively high, and estimate the response of customer sales to innovations by supplier firms.

Figure 11 presents the results, comparing the local projections from Section 3.2—capturing the response of customer sales to innovations by high- and low-markup suppliers—with model-based local projections in the absence of lock-in innovations. Without lock-in innovations, the response of customer sales would always be positive. Moreover, innovations by high-markup suppliers would generate a stronger response in customer sales than those by low-markup suppliers, implying that high-markup firms pass-through more of the productivity improvements to their customers.[19] This counterfactual highlights that lock-in innovations are necessary for the model to align with the empirical evidence presented in Section 3.2.

Appendix Figure 15 compares the policy functions for both lock-in and productive innovations under the counterfactual scenario, where the lock-in innovation scale parameter is set to infinity, with those in the baseline (Post-2000) economy. The comparison reveals that lock-in investments crowd out productive investments.

Figure 11: Δ Customer Sales under No Lock-in Scenario



Notes: data estimation results of the semi-elasticity of changes in Customer sales after Supplier's innovation shocks, conditioning on the firm belonging to the top 80th percentile distribution of markups (High Markups), or not (Low Markups), compared with the model estimation results for the counterfactual scenario in which lock-in innovations are infinitely costly.

[19]This result aligns with the findings of Amiti, Itskhoki and Konings (2019), which show that high-markup firms pass through a smaller share of negative cost shocks to their customers.

4.4 Pre-2000s Calibration

I re-estimate the model for the Pre-2000 economy, applying the same identification strategy used for the Post-2000 estimation described in Section 4.1. The estimated parameters for both steady states are presented in Table 7, and the list of targeted moments for both periods is presented in Table 8. Beside the moments described before, I also target the aggregate TFP level ratio between Post-2000 and Pre-2000 period, sourced from the Bureau of Economic Analysis.[20]

I account for changes in parameters between the Pre- and Post-2000 periods that align with other explanations for the slowdown in business dynamism and the rise in market power observed after 2000. Compared to the Pre-2000 period, the cost-elasticity of productive innovations with respect to the firms' productivity, ϕ_s, is higher in the Post-2000 period, reflecting the hypothesis from Bloom, Jones, Van Reenen and Webb (2020) that coming up with new ideas has become increasingly difficult. The step-size of productive innovations, λ, declines in the Post-2000 period, indicating an average decrease in patent quality, as studied by Olmstead-Rumsey (2019). Furthermore, the firm's entry rate, captured by the market reset probability κ, declines in the Post-2000 period, in line with the findings of Akcigit and Ates (2021).

As it has been widely documented, in the Post-2000 period there has been a significant increase in the markup level and also in the markup dispersion, which disciplines the differences in productive innovation step-size and the innovation scale parameters. Crucially, the parameters governing the elasticity of lock-in and productive innovations' costs with respect to the productivity gap, $\phi_s, \phi_{-s}, \tilde{\phi}_s, \tilde{\phi}_{-s}$ are informed by changes in the empirical response of customer sales to supplier innovations. Specifically, in the Pre-2000 period, customer sales exhibit a positive response to innovations by high-markup suppliers, contrasting with the negative response observed in the Post-2000 period. Figure 12 shows the dynamic response of customer sales to innovations by high-markup suppliers in the Pre- and Post-2000 periods, both in the data and in the local projections simulated in the model.

Table 7: Parameter Values: Pre-2000 vs Post-2000 Periods

Parameter	Description	Pre-2000 Value	Post-2000 Value
ρ	Rate of time preference	6%	6%
η	Elasticity of substitution between customers	1.5	1.5
λ	Productive innovation step size	3.5%	3.3%
δ	Lock-in innovation step size	17%	17%
ψ	Productive innovation cost curvature	2	2
$\tilde{\psi}$	Lock-in innovation cost curvature	3	2.8
ϕ	Productive innov. cost relation w/ productivity level	2.5	2.8
$\tilde{\phi}$	Lock-in innov. cost relation w/ productivity level	-2	-2.8
α	Productive innovation scale	0.5	1
$\tilde{\alpha}$	Lock-in innovation scale	2	0.5
κ	Market reset probability	12%	10%

Notes: List of model parameters and calibrated values for the Post-2000 economy.

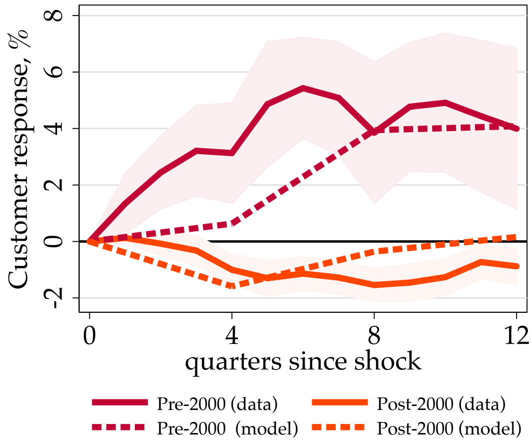
[20] I target the aggregate TFP ratio to reflect observed changes in aggregate trends, which I achieve by lowering the lower bound of productivity levels in the pre-2000 period to capture shifts in the technological frontier over time.

Table 8: Model Fit

Moment	Model Pre-2000	Data Pre-2000	Model Post-2000	Data Post-2000
Average markup	1.3	1.2	1.46	1.34
Markup 75th percentile	1.4	1.3	1.57	1.54
Markup 90th percentile	1.5	1.6	1.71	2.20
R&D share of GDP, %	3.7	1.5	1.59	1. 65
Profit share of GDP,%	10	16	17	14
Aggregate TFP ratio	1	1	1.10	1.10

Notes: Table 6 presents the value of moments in the data and in the calibrated model for the Pre-2000 and the Post-2000 periods. "Aggregate TFP ratio" refers to the ratio of aggregate TFP between Post-2000 and Pre-2000 periods.

Figure 12: Δ Customer Sales after Innovation by High-Markup Suppliers

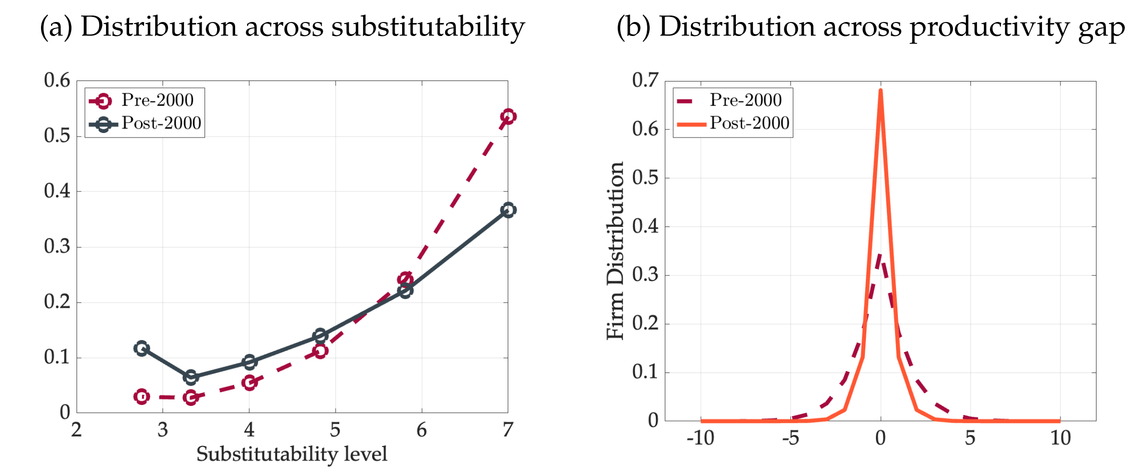


Notes: data and model estimation results of the semi-elasticity of changes in Customer sales after High-markup Supplier's innovation shocks for the pre-2000 and post-2000 period. High markup firms refer to suppliers that belong to the top 80th percentile distribution of markups ex-ante the innovation happened.

4.5 Comparison Between the Pre-2000 and the Post-2000 Economies

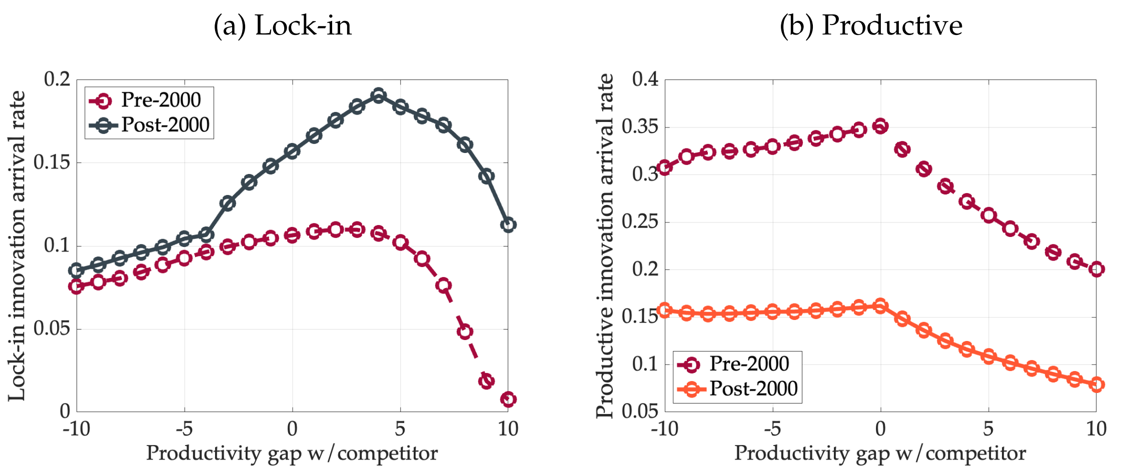
From the pre-2000 to the post-2000 period, the economy experienced a noticeable shift toward lower product substitutability, with firms producing less standardized products (Figure 13, panel a). At the same time, the distribution of productivity gaps became more compressed, reflecting a reduction in the dispersion of productivity across firms (Figure 13, panel b). When combined with the observed increase in both average markup levels and markup dispersion, these trends suggest that, in the post-2000 economy, firms increasingly derive market power from lower product substitutability rather than from large productivity advantages over their competitors. Figure 14 illustrates the changes in both the level and composition of innovation intensity by comparing the lock-in and productive innovation policy functions across the two periods. Panel (a) reveals that the intensity of lock-in investments nearly doubled in the post-2000 period among firms with higher markups, i.e., larger productivity gap. Panel (b) shows a decline in overall productive innovation intensity during the post-2000 period.

Figure 13: Firm Distribution: Pre-2000 vs Post-2000



Notes: Pre-2000 and Post-2000 distributions. Panel (a) shows the calibrated firm distribution across product substitutability γ's. Panel (b) shows the firm distribution across firm's productivity gaps (in terms of number of steps) with respect to their competitor.

Figure 14: Innovation Policy Functions: Pre-2000 vs Post-2000



Notes: Panel (a) shows the lock-in policy functions Pre- and Post-2000, and Panel (b) shows the productive innovations' intensity.

4.6 What are the Aggregate Implications of Lock-In innovations?

I construct a counterfactual scenario to assess what aggregate TFP, markup levels, and markup dispersion would have been if the post-2000 economy had retained the lock-in innovation cost structure of the pre-2000 period. Specifically, I re-estimate the post-2000 economy by resetting the cost scale parameter $\tilde{\alpha}$, the cost curvature parameter $\tilde{\phi}$, and the elasticity of lock-in costs with respect to the productivity gap $\tilde{\psi}_s$ and $\tilde{\psi}_{-s}$ to their pre-2000 values. The results are presented in Table 9. The first two columns display the ratio of Post-2000 to Pre-2000 aggregate TFP, median markups, and markup dispersion, both in the data (first column) and from the two steady-state model estimations presented in the previous section (second column). In both the data and the model, aggregate TFP in the Post-2000 period is 10% higher than in the Pre-2000 period. The median markup is 7% higher in the Post-2000 period compared to the Pre-2000 period in the data and 4% higher in the model. Markup dispersion, measured as the ratio of the 75th to the 25th percentile of markups, is 15% higher Post-2000 in the data and 12% higher in the model. Overall, the model closely matches the observed data, , capturing the changes in aggregate productivity and markup moments across the two periods.

The third column of Table 9 presents the results of the counterfactual scenario, where I reset the cost structure of lock-in innovations in the post-2000 economy to pre-2000 values, holding all other parameters constant. If lock-in innovations had remained as costly as they were in the pre-2000 period, aggregate TFP post-2000 would have been 3% higher than observed, the median markup would have remained at the pre-2000 levels, and the observed level of markup dispersion would have been 9% lower.

Table 9: Counterfactual: Post-2000 Economy with Pre-2000 Lock-in

Ratio: Post-2000 / Pre-2000	**Data**	**Model**	**Counterfactual**
Aggregate TFP	1.10	1.10	1.13
Median markup	1.07	1.04	1.00
Markup dispersion	1.15	1.12	1.02

Notes: *Counterfactual* refers to the scenario in which the Post-2000 economy is assigned the Pre-2000 lock-in innovation structure by setting $\tilde{\alpha}_{\text{pre-2000}} = \tilde{\alpha}$, $\tilde{\phi}_{\text{pre-2000}} = \tilde{\phi}$, and $\tilde{\psi}_{\text{pre-2000}} = \tilde{\psi}$."Markup dispersion" refers to the 75th to 25th percentiles markups ratio.

5 Policy Experiments

In this section, I analyze the aggregate effects of antitrust policy experiments implemented to foster competition and restrict firms' market power. These policies aim to address lock-in strategies that restrict consumer choice, such as penalizing firms for excessive product customization or practices that hinder customers from switching to competitors. Notable recent cases include NVIDIA, scrutinized in France for anti-competitive practices due to dependency on its CUDA software; Apple, sued by the U.S. Department of Justice for alleged lock-in practices in the smartphone market; and Microsoft, required by the European Union to unbundle Teams from its Office 365 package and establish clearer interoperability with competing products. In practice, identifying lock-in strategies systematically presents challenges. Therefore, I evaluate the effects of both a targeted policy and broader untargeted policy intervention aimed at reducing these lock-in practices.

Lock-in targeted regulation

I consider the impact of a regulation that increases the cost of innovation, in a hypothetical scenario in which an hypothetical scenario in which the government can implement a lock-in targeted regulation that increases the cost of lock-in innovation by a proportion τ, such that:

$$\mathcal{C}(z_{st}) \equiv (1+\tau)\tilde{\alpha}\frac{((exp(a_{st}-a_{-st}))^{\tilde{\psi}}z_{st})^{\tilde{\phi}}}{\tilde{\phi}}W_t. \tag{17}$$

I evaluate the impact of this lock-in-targeted regulation on aggregate productivity, markup dispersion, and the GDP shares of productive and lock-in innovations across the pre-2000 and post-2000 periods. For simplicity, I assume that the resources collected through this regulation are lost. Appendix Figure 16 presents the results of implementing this regulation at various tax levels. This targeted regulation results in a non-linear increase in aggregate productivity (panel a), ranging from a 0.08% rise under a 10% tax to a 0.56% increase under a 40% tax, with more pronounced effects in the post-2000 period, especially at higher tax levels. The regulation also reduces both markup levels and markup dispersion (panels b and c), with a stronger impact in the post-2000 period. For instance, while a 20% tax barely affects dispersion before 2000, it decreases dispersion by nearly 25% in the post-2000 period. These results are driven by the higher incidence of lock-in innovations and the stronger crowding-out effect between lock-in and productive innovations in the post-2000 period (panel d). In practice, however, it is unlikely that the government could directly target lock-in innovations. Therefore, I next consider a progressive markup tax designed to imperfectly target lock-in investments.

Progressive tax on markups

In a second experiment, I examine the effects of a progressive tax on firms' markups. Given that lock-in strategies are more common among high-markup firms, this tax scheme could serve as an imperfect means of targeting lock-in strategies, particularly in the post-2000 period. I leverage on models of progressive income tax (Heathcote, Storesletten and Violante, 2014) and consider a tax function of firm's markups m_s of the form:

$$T_t(m_s) = m_{st} - \varsigma m_{st}^{1-\tau}. \tag{18}$$

The parameter τ determines the degree of progressivity of the tax system, and ς represents a scale parameter.[21]I assume that the government's tax collection is distributed back to the households in a lump sum[22] The government balanced budget then reads:

$$G_t = \int T_t(m_{st}(a_s, \gamma_s, a_{-s}, \gamma_{-s}))d(a_s, \gamma_s, a_{-s}, \gamma_{-s}).$$

Ex ante, the aggregate implications of incrementally taxing firms with higher markups are ambiguous, since firms in the model are accumulating market power through lock-in innovations and also from productive innovations. Thus, aggregate implications will depend on whether the tax affects the innovation incentives of firms that are sourcing market power mostly from producing products that are less substitutable or from being more efficient than their competitors. I solve the model outlined in Section 2, incorporating the progressive tax on markups. The following corollary characterizes the equilibrium markups under this progressive tax scheme.

Corollary 2. *Let $\varepsilon_{P_cD_c,m_s}$ be the elasticity of adjusted price index P_cD_c with respect to suppliers' markups, and ε_{P_c,m_s} be the elasticity of customer price to suppliers' markups. Under a progressive*

[21]A tax scheme is usually defined as progressive (regressive) if the ratio or marginal to average tax rates is larger (smaller) than one for every level of income (or, in this case, markups). In this class of tax system this ratio is defined as: $\frac{1-T'(m_s)}{1-T(m_s)/m_s} = 1-\tau$. When $\tau > 0$,marginal rates are higher than average rates, and the tax system is therefore progressive. Conversely, the tax system is regressive when $\tau < 0$. See Heathcote *et al.* (2014) for details.

[22]The household budget constraint is thus given by $P_tC_t + \dot{A}_t = W_tL_t +_r {}_tA_t + G_t$.

markups tax scheme given by equation 18, supplier firms' equilibrium markups m_{st} are given by:

$$m_{st} = \left\{ \frac{1}{\varsigma} \left[\frac{\gamma_{st} \left((1-\tau) - \varepsilon_{(P_{ct}D_{ct}),m_{st}} \right) + \eta \varepsilon_{P_{ct},m_{st}}}{\left[\gamma_{st} \left((1-\tau) - \varepsilon_{(P_{ct}D_{ct}),m_{st}} \right) + \eta \varepsilon_{P_{ct},m_{st}} - (1-\tau) \right]} \right] \right\}^{\frac{1}{(1-\tau)}} .$$

See Proof in Appendix A.3.

Figure 17 illustrates the effects of a progressive tax on markups for various levels of τ. This tax structure leads to an increase in aggregate productivity (panel a) while also widening the dispersion of markups (panel b). These two outcomes occur together because, under the calibrated post-2000 economy, the policy decreases the share of lock-in investments (panel d) and encourages more productive investments (panel f). As a result, the policy intensifies the dispersion in productivity gaps across firms, which further drives the observed increase in markup dispersion (panel e).

6 Discussion on Lock-In Microfoundation

In the framework presented in Section 2, lock-in innovations reduce the substitutability of a firms' products, represented by γ_{st}. This reduction in product substitutability, which deters competition and amplifies a firm's market power, is central to industrial organization theories that characterize lock-in strategies. One compelling type of lock-in strategy consists of inducing or even "forcing" the customer to purchase a bundle of compatible products (Shapiro and Varian, 2000; Carlton and Waldman, 1998). Examples of these strategies include follow-on products that render the initial product obsolete if not purchased (e.g., costly software updates) and product compatibility tactics (e.g., NVIDIA's CUDA software, compatible only with their GPUs, or Microsoft leveraging Office product interoperability with limited compatibility for alternative interfaces). In the first case, the bundle includes the initial purchase and follow-on products, while in the second, it consists of compatible products essential for full functionality. Thus, lock-in strategies in the model could be microfounded with suppliers' bundling strategies. Each time a firm invests in a lock-in innovation in the model, it's as if it were introducing a new essential complementary product, bundling it with existing offerings to deepen customer reliance and making the supplier less substitutable for the customer.

The empirical evidence presented in Section 3 suggests a rise in lock-in strategies after 2000, particularly among firms with high market power. This prompts a key question: what factors drove the rise in lock-in (or bundling) incentives for these firms? Nalebuff (2004) software firms, unlike others, face near-zero marginal production costs, making product bundling an optimal equilibrium strategy to deter competition. He further demonstrates that the incentive to bundle is especially strong when products are complementary. But what about durable manufacturing firms, which were more prevalent before 2000 and faced higher marginal costs? As Nalebuff (2004) notes, higher marginal costs make bundling less attractive due to inefficiencies; some consumers may still buy the bundle even if they value one component below its production cost. Indeed, Adams and Yellen (1976) show that bundling may not be an optimal strategy for firms with higher marginal costs. This may explain why, in the post-2000 era, firms' incentives to invest in product bundling rose as the cost of lock-in innovations decreased for firms with high market power. This is precisely how the quantitative model in Section 2 interprets the increased prevalence of lock-in strategies in the years after 2000.

7 Conclusion

This paper introduces a new framework to study the macroeconomic implications of firms' investments in productive and lock-in innovations. While productive innovations lower production costs or enhance product quality, ultimately benefiting customer firms by boosting their sales, lock-in innovations create dependencies that increase switching costs, reduce product substitutability, and diminish customer sales. My model bridges standard approaches in the literature by incorporating both forms of innovation, offering novel insights into how firms strategically balance productivity improvements with efforts to increase customer dependency.

The results highlight a significant shift in the nature of innovation since the 2000s, driven by a growing prevalence of lock-in innovations among high-markup firms. Quantitatively, the findings suggest that if the cost of lock-in innovations had remained at pre-2000 levels, the observed aggregate productivity in the post-2000 period would have been 3% higher, median markups would have remained unchanged from pre-2000 levels, and the markup dispersion would have been 9% lower than observed. These results underscore the macroeconomic importance of understanding not just how firms innovate to improve efficiency but also how they leverage lock-in strategies to increase market power.

The analysis contributes to the literature by documenting the rising role of lock-in innovations in shaping market outcomes, particularly in industries reliant on technological compatibility and proprietary ecosystems. The findings suggest that the composition of innovation matters for long-run growth, productivity, and market competition, with the rise in lock-in innovations playing a critical role in shaping the economic landscape of the post-2000s economy.

This paper lays the groundwork for a new research agenda on the macroeconomic implications of lock-in innovations. It highlights pressing policy questions, as the increasing reliance on lock-in innovations suggests that policies focused solely on promoting productivity-enhancing innovations may be insufficient to foster competition and sustainable economic growth. Recent antitrust cases involving major technology firms underscore the challenges faced by regulators, with companies accused of developing products so customized or integrated that they limit competitors' market access. Future research could focus on identifying optimal policies, particularly regarding how lock-in and productive strategies should vary among firms with different levels of market power or market size to maximize welfare. A deeper understanding of these dynamics will be essential for designing policies that balance the promotion of innovation with the maintenance of competitive markets.

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A Theory Appendix

A.1 Customer firms problem

For ease of exposition, time subscripts have been omitted where they do not cause ambiguity. The customer firms solves the following static profit maximization problem:

$$\underset{x_s, X_c}{max} P_c X_c - \sum_{s\in\Omega_c} p_s x_s \qquad s.t. \qquad \sum_{s\in\Omega_c} \Upsilon\left(\frac{x_s}{X_c}\right) = 1,$$

with $\Upsilon\left(\frac{x_s}{X_c}\right) \equiv \left(\frac{x_s}{X_c}\right)^{\frac{\gamma_s-1}{\gamma_s}}$ in the case of CRESH technology, but more generally for any $\Upsilon\left(\frac{x_s}{X_c}\right)$, with $\Upsilon(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ strictly increasing, strictly concave function, that is twice continuously differentiable with $\Upsilon(0) = 0$ and $\Upsilon(1) = 1$.

The Lagrangian of the customer firms' profit maximization problem is given by:

$$\mathcal{L} = P_c X_c - \sum_{s\in\Omega_c} p_s x_s + \lambda \left[\sum_{s\in\Omega_c} \Upsilon\left(\frac{x_s}{X_c}\right) - 1\right]$$

$$[x_s] \qquad p_s = \lambda \Upsilon'\left(\frac{x_s}{X_c}\right)\frac{1}{X_c}$$

$$[X_c] \qquad P_c = \lambda \sum_{s\in\Omega_c} \Upsilon'\left(\frac{x_s}{X_c}\right)\left(\frac{x_s}{X_c^2}\right)$$

Combine the two first order conditions to obtain the inverse demand function:

$$p_s = \Upsilon'\left(\frac{x_s}{X_c}\right) P_c D_c, \tag{19}$$

and the demand function:

$$x_s = \Upsilon'^{-1}\left(\frac{p_s}{P_c D_c}\right) X_c, \tag{20}$$

with demand index $D_c \equiv \left[\sum_{s\in\Omega_c} \Upsilon'\left(\frac{x_s}{X_c}\right)\left(\frac{x_s}{X_c}\right)\right]^{-1}$. Finally, for the CRESH application, substitute $\Upsilon\left(\frac{x_s}{X_c}\right) = \frac{\gamma_{st}-1}{\gamma_{st}}\left(\frac{x_{st}}{X_{ct}}\right)^{\frac{-1}{\gamma_s}}$ to obtain the demand function presented in section 2.2.

A.2 Supplier firms problem

A.2.1 Proof of Proposition 1.

Proof. **Cournot Competition.** The profit maximization problem of leader supplier firm s is given by:

$$\underset{x_s}{max}\left\{\Upsilon'\left(\frac{x_s}{X_c(x_s)}\right) P_c(x_s) D_c(x_s) x_s - \frac{W}{a_s} x_s\right\}$$

With first order condition with respect to x_s:

$$[x_s] \quad \Upsilon''\left(\frac{x_s}{X_c}\right)\frac{\partial \frac{x_s}{X_c}}{\partial x_s} P_c D_c x_s + \Upsilon'\left(\frac{x_s}{X_c}\right)\left[P_c D_c + x_s \frac{\partial P_c D_c}{\partial x_s}\right] = \frac{W}{a_s}$$

$$\Upsilon''\left(\frac{x_s}{X_c}\right)\frac{\partial \frac{x_s}{X_c}}{\partial x_s} P_c D_c x_s + \Upsilon'\left(\frac{x_s}{X_c}\right)\left[P_c D_c + x_s \frac{\partial P_c}{\partial X_c}\frac{\partial X_c}{\partial x_s} D_c + x_s P_c \frac{\partial D_c}{\partial x_s}\right] = \frac{W}{a_s}$$

$$Y''\left(\frac{x_s}{X_c}\right)\frac{\partial\frac{x_s}{X_c}}{\partial x_s}x_s\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+Y'\left(\frac{x_s}{X_c}\right)\left[\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+x_s\frac{\partial P_c}{\partial X_c}\frac{\partial X_c}{\partial x_s}\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)P_c}+x_s\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)D_c}\frac{\partial D_c}{\partial x_s}\right]=\frac{W}{a_s}$$

$$Y''\left(\frac{x_s}{X_c}\right)\frac{\partial\frac{x_s}{X_c}}{\partial x_s}x_s\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+Y'\left(\frac{x_s}{X_c}\right)\left[\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\underbrace{\frac{\partial P_c}{\partial X_c}\frac{\partial X_c}{\partial x_s}\frac{x_s}{P_c}}_{\varepsilon_{P_c,x_s}}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\underbrace{\frac{\partial D_c}{\partial x_s}\frac{x_s}{D_c}}_{\varepsilon_{D,xs}}\right]=\frac{W}{a_s} \tag{21}$$

Separately solve:

$$\frac{\partial\frac{x_s}{X_c}}{\partial x_s}x_s=\frac{X_c-x_s\frac{\partial X_c}{\partial x_s}}{X_c^2}x_s=\frac{X_c-x_s\frac{\partial X_c}{\partial x_s}}{X_c}\frac{x_s}{X_c}=\left(1-\underbrace{\frac{\partial X_c}{\partial x_s}\frac{x_s}{X_c}}_{\varepsilon_{X_c,x_s}}\right)\frac{x_s}{X_c}$$

Substituing it into (21):

$$Y''\left(\frac{x_s}{X_c}\right)\left(1-\underbrace{\frac{\partial X_c}{\partial x_s}\frac{x_s}{X_c}}_{\varepsilon_{X_c,x_s}}\right)\frac{x_s}{X_c}\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+Y'\left(\frac{x_s}{X_c}\right)\left[\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\underbrace{\frac{\partial P_c}{\partial X_c}\frac{\partial X_c}{\partial x_s}\frac{x_s}{P_c}}_{\varepsilon_{P_c,x_s}}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\underbrace{\frac{\partial D_c}{\partial x_s}\frac{x_s}{D_c}}_{\varepsilon_{D,xs}}\right]=\frac{W}{a_s}$$

$$Y''\left(\frac{x_s}{X_c}\right)(1-\varepsilon_{X_c,x_s})\frac{x_s}{X_c}\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+Y'\left(\frac{x_s}{X_c}\right)\left[\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\varepsilon_{P_c,x_s}+\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\varepsilon_{D_c,xs}\right]=\frac{W}{a_s}$$

$$\frac{p_s}{Y'\left(\frac{x_s}{X_c}\right)}\left\{Y''\left(\frac{x_s}{X_c}\right)(1-\varepsilon_{X_c,x_s})\frac{x_s}{X_c}+Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]\right\}=\frac{W}{a_s}$$

Which leads to the expression for p_s in equilibrium:

$$p_s=\frac{W}{a_s}\underbrace{\frac{Y'\left(\frac{x_s}{X_c}\right)}{\left\{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})+Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]\right\}}}_{markup\ \mu_s} \tag{22}$$

Now solve for the implied elasticity of demand:

$$\mu_s=\frac{\vartheta}{\vartheta-1}=\frac{1}{1-\frac{1}{\vartheta}}=\frac{Y'\left(\frac{x_s}{X_c}\right)}{\left\{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})+Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]\right\}}$$

$$\frac{1}{\vartheta}=1-\frac{\left\{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})+Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]\right\}}{Y'\left(\frac{x_s}{X_c}\right)}$$

$$\frac{1}{\vartheta}Y'\left(\frac{x_s}{X_c}\right)=Y'\left(\frac{x_s}{X_c}\right)-\left\{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})+Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]\right\}$$

$$\vartheta=\frac{Y'\left(\frac{x_s}{X_c}\right)}{Y'\left(\frac{x_s}{X_c}\right)-Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})-Y'\left(\frac{x_s}{X_c}\right)[1+\varepsilon_{P_c,x_s}+\varepsilon_{D_c,xs}]}$$

$$\vartheta=\frac{Y'\left(\frac{x_s}{X_c}\right)}{Y'\left(\frac{x_s}{X_c}\right)[1-1-\varepsilon_{P_c,x_s}-\varepsilon_{D,xs}]-Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}(1-\varepsilon_{X_c,x_s})}$$

$$\vartheta = \frac{Y'\left(\frac{x_s}{X_c}\right)}{Y'\left(\frac{x_s}{X_c}\right)\left[-\varepsilon_{P_c,x_s} - \varepsilon_{D,xs}\right] - Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}\left(1 - \varepsilon_{X_c,x_s}\right)}$$

$$\vartheta = \frac{Y'\left(\frac{x_s}{X_c}\right)}{-Y'\left(\frac{x_s}{X_c}\right)\left[\varepsilon_{P_c,x_s} + \varepsilon_{D,xs}\right] - Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}\left(1 - \varepsilon_{X_c,x_s}\right)}$$

$$\vartheta = -\frac{Y'\left(\frac{x_s}{X_c}\right)}{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}\left(1 - \varepsilon_{X_c,x_s}\right) + Y'\left(\frac{x_s}{X_c}\right)\left[\varepsilon_{P_c,x_s} + \varepsilon_{D_c,xs}\right]}$$

Rearranging terms, we obtain the equations in Proposition 1:

$$\vartheta = -\left[\frac{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}}{Y'\left(\frac{x_s}{X_c}\right)}\left[1 - \varepsilon_{X_c,x_s}\right] + \left[\varepsilon_{P_c,x_s} + \varepsilon_{D_c,x_s}\right]\right]^{-1}$$

$$\vartheta = \left[-\frac{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}}{Y'\left(\frac{x_s}{X_c}\right)}\left[1 - \varepsilon_{X_c,x_s}\right] + \frac{1}{\eta}\varepsilon_{X_c,x_s} - \varepsilon_{D_c,x_s}\right]^{-1} \tag{23}$$

where $\varepsilon_{D_c,x_s} \equiv \frac{\partial D_c}{\partial x_s}\frac{x_s}{D_c}$ and in the last row I used the fact that $\frac{\partial P_c}{\partial X_c} = -\frac{1}{\eta}X_c^{-\frac{1}{\eta}-1}Y^{\frac{1}{\eta}}P = -\frac{1}{\eta}\frac{P_c}{X_c}$, and therefore $\varepsilon_{P_c,x_s} \equiv \frac{\partial P_c}{\partial X_c}\frac{\partial X_c}{\partial x_s}\frac{x_s}{P_c} = -\frac{1}{\eta}\frac{\partial X_c}{\partial x_s}\frac{x_s}{X_c} \equiv -\frac{1}{\eta}\varepsilon_{X_c,x_s}$.

Bertrand Competition. Following the same logic as with Cournot competition, one can prove that the equilibrium elasticity of demand under Bertrand competition is given by:

$$\vartheta = \left[-\frac{Y'\left(\frac{x_s}{X_c}\right)}{Y''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}}\left(1 - \varepsilon_{(P_cD_c),p_s}\right) + \eta\varepsilon_{P_c,p_s}\right] \tag{24}$$

with $\varepsilon_{(P_cD_c),p_s} \equiv \frac{\partial P_cD_c}{\partial p_s}\frac{p_s}{P_cD_c}$ and $\varepsilon_{P_c,p_s} \equiv \frac{\partial P_c}{\partial p_s}\frac{p_s}{P_c}$. □

Lemma 1. *Market share of supplier firms.*

Substituing the customer firm demand for supplier firms goods (20) and the customer firm inverse demand (19) in the definition of supplier firm market share we obtain:

$$S_s \equiv \frac{p_s x_s}{P_c X_c} = \frac{Y'\left(\frac{x_s}{X_c}\right)P_cD_cx_s}{P_cX_c} = \frac{x_s}{X_c}D_cY'\left(\frac{x_s}{X_c}\right) \tag{25}$$

$$S_s \equiv \frac{p_s x_s}{P_c X_c} = \frac{p_sY'^{-1}\left(\frac{p_s}{P_cD_c}\right)X_c}{P_cX_c} = \frac{p_s}{P_c}Y'^{-1}\left(\frac{p_s}{P_cD_c}\right) \tag{26}$$

A.2.2 CRESH application.

Lemma 2. *Elasticities as functions of market share of supplier firms.*

Elasticities $\varepsilon_{X_c,x_s}, \varepsilon_{D_c,x_s}, \varepsilon_{(P_cD_c),p_s}$ and ε_{P_c,p_s} can be expressed as a function of model parameters and the market share of supplier firms over customer firms:

$$\varepsilon_{X_c,x_s} \equiv \frac{\partial X_c}{\partial x_s}\frac{x_s}{X_c} \equiv \frac{dlogX_c}{dlogx_s} = S_s$$

$$\varepsilon_{D_c,x_s} \equiv \frac{\partial D_c}{\partial x_s}\frac{x_s}{D_c} \equiv \frac{dlogD_c}{dlogx_s} = \frac{1}{\gamma_s}S_s - \left(\sum_{j\in\Omega_c} S_j \frac{1}{\gamma_j}\right) S_s$$

$$\varepsilon_{(P_cD_c),p_s} \equiv \frac{\partial P_c D_c}{\partial p_s}\frac{p_s}{P_c D_c} \equiv \frac{dlogP_cD_c}{dlogp_s} = \frac{\gamma_s S_s}{\sum_{s\in\Omega_c}\gamma_s S_s}$$

$$\varepsilon_{P_c,p_s} \equiv \frac{\partial P_c}{\partial p_s}\frac{p_s}{P_c} \equiv \frac{dlogP_c}{dlogp_s} = S_s$$

Proof. **Cournot competition.**

1. ε_{X_c,x_s}:
Differentiating condition $\sum_{s\in\Omega_c} \Upsilon\left(\frac{x_s}{X_c}\right) = 1$:

$$\sum_{s\in\Omega_c} d\Upsilon\left(\frac{x_s}{X_c}\right) = 0$$

$$\begin{aligned} d\Upsilon\left(\frac{x_s}{X_c}\right) &= \Upsilon'\left(\frac{x_s}{X_c}\right) d\frac{x_s}{X_c} \\ &= \Upsilon'\left(\frac{x_s}{X_c}\right)\left[\frac{\partial \frac{x_s}{X_c}}{\partial x_s}dx_s + \frac{\partial \frac{x_s}{X_c}}{\partial X_c}dX_c\right] \\ &= \Upsilon'\left(\frac{x_s}{X_c}\right)\left[\frac{1}{X_c}dx_s + \left(-\frac{1}{X_c^2}\right)x_s dX_c\right] \\ &= \Upsilon'\left(\frac{x_s}{X_c}\right)\left[\frac{1}{X_c}dx_s - \frac{x_s}{X_c}dlogX_c\right] \\ &= \Upsilon'\left(\frac{x_s}{X_c}\right)\frac{1}{X_c}dx_s - \Upsilon'\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}dlogX_c \end{aligned}$$

Substituing from the definition of market share (25):

$$\begin{aligned} d\Upsilon\left(\frac{x_s}{X_c}\right) &= \frac{S_s}{D_c x_s}dx_s - \frac{S_s}{D_c}dlogX_c \\ &= \frac{S_s}{D_c}dlogx_s - \frac{S_s}{D_c}dlogX_c \end{aligned}$$

Summing across suppliers:

$$\begin{aligned} 0 &= \sum_{s\in\Omega_c}\left(\frac{S_s}{D_c}dlogx_s - \frac{S_s}{D_c}dlogX_c\right) \\ 0 &= \frac{1}{D_c}\sum_{s\in\Omega_c}(S_s dlogx_s - S_s dlogX_c) \\ dlogX_c &= \sum_{s\in\Omega_c} S_s dlogx_s \end{aligned} \tag{27}$$

Which gives the result:

$$\varepsilon_{X_c,x_s} \equiv \frac{\partial X_c}{\partial x_s}\frac{x_s}{X_c} \equiv \frac{dlogX_c}{dlogx_s} = S_s$$

2. ε_{D_c,x_s}:
The sum of market shares across suppliers for each customer has to be one:$\sum_{s\in\Omega_s} S_s = \sum_s \frac{x_s}{X_c} D_c \Upsilon'\left(\frac{x_s}{X_c}\right) = 1$. Differentiating this condition:

$$\sum_s d\frac{x_s}{X_c} D_c \Upsilon'\left(\frac{x_s}{X_c}\right) = 0$$

$$\underbrace{d\frac{x_s}{X_c}D_c\Upsilon'\left(\frac{x_s}{X_c}\right)}_{\equiv G} = \frac{\partial G}{\partial x_s}dx_s + \frac{\partial G}{\partial X_c}dX_c + \frac{\partial G}{\partial D_c}dD_c \tag{28}$$

I now separately derive each term of equation (28). The first term is given by:

$$\frac{\partial G}{\partial x_s}dx_s = \left[\frac{D_c}{X_c}\Upsilon'\left(\frac{x_s}{X_c}\right) + \frac{x_s D_c}{X_c}\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\frac{\partial \frac{x_s}{X_c}}{\partial x_s}\right]dx_s.$$

Substituting from the definition of market share (25):

$$\frac{\partial G}{\partial x_s}dx_s = \left[\frac{S_s}{x_s} + \frac{S_s}{\Upsilon'}\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\frac{x_s}{X_c}\frac{1}{x_s}\right]dx_s$$

$$\frac{\partial G}{\partial x_s}dx_s = \left[S_s + S_s\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\frac{\frac{x_s}{X_c}}{\Upsilon'}\right]dlogx_s$$

$$\frac{\partial G}{\partial x_s}dx_s = S_s\left(1 - \frac{1}{\gamma_s}\right)dlogx_s \tag{29}$$

where in the last row I have substituted $\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\frac{\frac{x_s}{X_c}}{\Upsilon'} = -\frac{1}{\gamma_s}$.
The second term in equation (28) is given by:

$$\frac{\partial G}{\partial X_c}dX_c = \left[-\frac{1}{X_c^2}x_s D_c\Upsilon'\left(\frac{x_s}{X_c}\right) + \frac{x_s D_c}{X_c}\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\left(-\frac{x_s}{X_c^2}\right)\right]dX_c$$

$$= \left[-S_s + S_s\frac{\partial \Upsilon'\left(\frac{x_s}{X_c}\right)}{\partial \frac{x_s}{X_c}}\frac{-\frac{x_s}{X_c}}{\Upsilon'}\right]dlogX_c$$

$$= -S_s\left[1 - \frac{1}{\gamma_s}\right]dlogX_c \tag{30}$$

The last term in equation (28) is given by:

$$\frac{\partial G}{\partial D_c}dD_c = \Upsilon'\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}dD_c = S_s dlogD_c \tag{31}$$

Substituting (29), (30) and (31) in equation (28) and using the previous result from equation (27), $dlogX_c = \sum_{s\in\Omega_c} S_s dlogx_s$:

$$0 = \frac{\partial G}{\partial x_s}dx_s + \frac{\partial G}{\partial X_c}dX_c + \frac{\partial G}{\partial D_c}dD_c$$

$$0 = \sum_{s\in\Omega_c}\left[S_s\left(1 - \frac{1}{\gamma_s}\right)dlogx_s - S_s\left[1 - \frac{1}{\gamma_s}\right]dlogX_c + S_s dlogD_c\right]$$

$$dlogD_c = \sum_{s\in\Omega_c}\left[-S_s\left(1 - \frac{1}{\gamma_s}\right)dlogx_s + S_s\left[1 - \frac{1}{\gamma_s}\right]dlogX_c\right]$$

$$= -\sum_{s\in\Omega_c} S_s dlogx_s + \sum_{s\in\Omega_c} S_s\frac{1}{\gamma_s}dlogx_s + \sum_{s\in\Omega_c} S_s dlogX_c - \sum_{s\in\Omega_c} S_s\frac{1}{\gamma_s}dlogX_c$$

$$= -dlogX_c + dlogX_c + \sum_{s\in\Omega_c} S_s\frac{1}{\gamma_s}dlogx_s - \sum_{s\in\Omega_c} S_s\frac{1}{\gamma_s}dlogX_c$$

$$= \sum_{s\in\Omega_c} S_s \frac{1}{\gamma_s} dlogx_s - \sum_{s\in\Omega_c} S_s \frac{1}{\gamma_s} dlogX_c$$

It then follows that:

$$\varepsilon_{D_c,x_s} \equiv \frac{\partial D_c}{\partial x_s}\frac{x_s}{D_c} \equiv \frac{dlogD_c}{dlogx_s} = \frac{1}{\gamma_s}S_s - \left(\sum_{j\in\Omega_c} S_j \frac{1}{\gamma_j}\right) S_s.$$

Bertrand competition

1. $\varepsilon_{(P_cD_c),p_s}$:

Using supplier demand (20) and differentiating condition $\sum_{s\in\Omega_c} \Upsilon\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) = 1$:

$$\sum_{s\in\Omega_c} d\Upsilon\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) = 0$$

$$\begin{aligned} d\Upsilon\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) &= \Upsilon'\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) d\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right) \\ &= \underbrace{\frac{p_s}{P_cD_c}}_{\equiv g_s} d\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right) \\ &= g_s \frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{\partial g_s}{\partial p_s}dp_s + \frac{\partial g_s}{\partial P_cD_c}dP_cD_c\right] \\ &= g_s \frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{1}{P_cD_c}dp_s - \frac{p_s}{(P_cD_c)^2}dP_cD_c\right] \\ &= g_s \frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{1}{P_cD_c}dp_s - \frac{p_s}{P_cD_c}dlogP_cD_c\right] \end{aligned}$$

Substituing from the definition of market share (26):

$$\begin{aligned} d\Upsilon\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) &= g_s \frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{S_s}{p_s\Upsilon'^{-1}}\frac{1}{D_c}dp_s - \frac{S_s}{\Upsilon'^{-1}}\frac{1}{D_c}dlogP_cD_c\right] \\ &= \frac{\partial \Upsilon'^{-1}}{\partial g_s}\frac{g_s}{\Upsilon'^{-1}}S_s\frac{1}{D_c}dlogp_s - \frac{\partial \Upsilon'^{-1}}{\partial g_s}\frac{g_s}{\Upsilon'^{-1}}S_s\frac{1}{D_c}dlogP_cD_c \\ &= -\gamma_s S_s \frac{1}{D_c}dlogp_s + \gamma_s S_s \frac{1}{D_c}dlogP_cD_c, \end{aligned}$$

where in the last row I have substituted $\frac{\partial \Upsilon'^{-1}}{\partial g_s}\frac{g_s}{\Upsilon'^{-1}} = -\gamma_s$.

Summing across suppliers:

$$\sum_{s\in\Omega_c} d\Upsilon\left(\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right) = \sum_{s\in\Omega_c} -\gamma_s S_s \frac{1}{D_c}dlogp_s + \gamma_s S_s \frac{1}{D_c}dlogP_cD_c$$

$$dlogP_cD_c = \frac{\sum_{s\in\Omega_c}\gamma_s S_s dlogp_s}{\sum_{s\in\Omega_c}\gamma_s S_s} \tag{32}$$

Therefore:

$$\varepsilon_{(P_cD_c),p_s} \equiv \frac{\partial P_cD_c}{\partial p_s}\frac{p_s}{P_cD_c} \equiv \frac{dlogP_cD_c}{dlogp_s} = \frac{\gamma_s S_s}{\sum_{s\in\Omega_c}\gamma_s S_s}.$$

2. ε_{P_c,p_s} :

The sum of market shares across suppliers for each customer has to be one, that is, $\sum_{s\in\Omega_s} S_s = \sum_s \frac{p_s}{P_c}\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right) = 1$. Differentiating this condition:

$$\sum_s d\frac{p_s}{P_c}\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right) = 0$$

$$d\underbrace{\frac{p_s}{P_c}\Upsilon'^{-1}\underbrace{\left(\frac{p_s}{P_cD_c}\right)}_{\equiv g_s}}_{\equiv G_s} = \frac{\partial G_s}{\partial p_s}dp_s + \frac{\partial G_s}{\partial P_c}dP_c + \frac{\partial G_s}{\partial \Upsilon'^{-1}}\frac{\partial \Upsilon'^{-1}}{\partial g_s}dg_s$$

$$dG_s = \frac{1}{P_c}\Upsilon'^{-1}(g_s)\,dp_s + \left(-\frac{1}{P_c^2}\right)p_s\Upsilon'^{-1}(g_s)\,dP_c + \frac{p_s}{P_c}\frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{1}{P_cD_c}dp_s - \frac{p_s}{P_cD_c}dlogP_cD_c\right]$$

Substituting from the definition of market share (26):

$$d\underbrace{\frac{p_s}{P_c}\Upsilon'^{-1}\underbrace{\left(\frac{p_s}{P_cD_c}\right)}_{\equiv g_s}}_{\equiv G_s} = S_sdlogp_s - S_sdlogP_c + \frac{S_s}{\Upsilon'^{-1}}\frac{\partial \Upsilon'^{-1}}{\partial g_s}\left[\frac{p_s}{P_cD_c}dlogp_s - \frac{p_s}{P_cD_c}dlogP_cD_c\right]$$

$$= S_sdlogp_s - S_sdlogP_c - S_s\gamma_sdlogp_s + S_s\gamma_sdlogP_cD_c$$

where in the last row I have substituted $\frac{\partial \Upsilon'^{-1}}{\partial g_s}\frac{g_s}{\Upsilon'^{-1}} = -\gamma_s$.

Summing across suppliers and substituting for $dlogP_cD_c$ from (32):

$$\begin{aligned}
0 &= \sum_{s\in\Omega_c} S_sdlogp_s - \sum_{s\in\Omega_c} S_sdlogP_c - \sum_{s\in\Omega_c} S_s\gamma_sdlogp_s + \sum_{s\in\Omega_c} S_s\gamma_sdlogP_cD_c \\
dlogP_c &= \sum_{s\in\Omega_c} S_sdlogp_s - \sum_{s\in\Omega_c} S_s\gamma_sdlogp_s + \sum_{s\in\Omega_c} S_s\gamma_s\frac{\sum_{s\in\Omega_c}\gamma_sS_cdlogp_s}{\sum_{s\in\Omega_c}\gamma_sS_s} \\
&= \sum_{s\in\Omega_c} S_sdlogp_s
\end{aligned}$$

It then follows that:

$$\varepsilon_{P_c,p_s} \equiv \frac{\partial P_c}{\partial p_s}\frac{p_s}{P_c} \equiv \frac{dlogP_c}{dlogp_s} = S_s.$$

□

Corollary 3. *When customer firm c produce with a CRESH production function* $\Upsilon\left(\frac{x_s}{X_c}\right) = \left(\frac{x_s}{X_c}\right)^{\frac{\gamma_s-1}{\gamma_s}}$, *the elasticity of demand of a supplier firm s in equilibrium is given by:*

$$\begin{aligned}
\vartheta_s^C &= \left[\frac{1}{\gamma_s}(1-S_s) + \frac{1}{\eta}S_s + \left(\sum_{j\in\Omega_c} S_j\frac{1}{\gamma_j} - \frac{1}{\gamma_s}\right)S_s\right]^{-1} \textit{ if Cournot competition,} \\
\vartheta^B &= \left[\gamma_s\left(1 - \frac{\gamma_sS_s}{\sum_{s\in\Omega_c}\gamma_sS_s}\right) + \eta S_s\right] \textit{ if Bertrand competition.}
\end{aligned}$$

Proof. The demand elasticities in equilibrium are obtained from combining equations (24) and (23) with the result that under CRESH $\frac{\Upsilon'\left(\frac{x_s}{X_c}\right)}{\Upsilon''\left(\frac{x_s}{X_c}\right)\frac{x_s}{X_c}} = \gamma_s$, and with the elasticities derived in Lemma 2. □

Table 10: General Framework Applications

	monopolistic competition		oligopolistic competition (Bertrand)	
	Kimball Klenow & Willis (2016)	CES	CES Atkeson & Burstein (2008)	CRESH This paper
$Y\left(\frac{x_s}{X_c}\right)$ function	$Y' = \frac{\gamma-1}{\gamma} exp \frac{1-\frac{x_s}{X_c}^{\frac{\xi}{\gamma}}}{\xi}$	$Y = \left(\frac{x_s}{X_c}\right)^{\frac{\gamma-1}{\gamma}}$	$Y = \left(\frac{x_s}{X_c}\right)^{\frac{\gamma-1}{\gamma}}$	$Y = \left(\frac{x_s}{X_c}\right)^{\frac{\gamma_s-1}{\gamma_s}}$
X_c production	$\int_{S_c} Y\left(\frac{x_s}{X_c}\right) ds = 1$	$X_c = \left(\sum_s x_s^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$	$X_c = \left(\sum_s x_s^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$	$\sum_s Y\left(\frac{x_s}{X_c}\right) = 1$
ϑ_s elasticity of demand	$\gamma \frac{x_s}{X_c}^{\frac{-\xi}{\gamma}}$	γ	$\gamma(1-S_s) + \eta S_s$	$\gamma_s\left(1 - \frac{\gamma_s S_s}{\sum_{s\in\Omega_c} \gamma_s S_s}\right) + \eta S_s$

Notes: General model applications to non-CES and CES demand, under monopolistic and oligopolistic competition between supplier firms.

A.3 Progressive tax on markups

Proof of Corollary 2

Proof. For the static maximization problem of each supplier firm, under Bertrand competition, chosing prices to maximize profits is akin to chosing markups to maximize profits, given wages W and the labor productivity of the firm a_s. Ignoring the time subscript t for simplicity, one can then write the profit maximization problem of the supplier firm as:

$$\max_{m_s} (\varsigma m_s^{(1-\tau)} - 1)\frac{W}{a_s}\left(\frac{\lambda m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s} X_c(m_s)$$

The first order condition with respect to markups reads:

$$\varsigma(1-\tau)m_s^{-\tau}\frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s} X_c(m_s)$$
$$+(\varsigma m_s^{(1-\tau)} - 1)\frac{\partial \frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s} X_c(m_s)}{\partial m_s} = 0 \tag{33}$$

Let's first solve separately for $\frac{\partial \frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s} X_c(m_s)}{\partial m_s}$:

$$\frac{\partial \frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}}{\partial m_s} X_c(m_s) + \frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}\frac{\partial X_c(m_s)}{\partial m_s} \tag{34}$$

Which in turn implies solving for $\frac{\partial \frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}}{\partial m_s}$:

$$
\begin{aligned}
&\frac{W}{a_s}\left(-\gamma_s\right)\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s-1}\frac{\partial\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}}{\partial m_s}\\
&=\frac{W}{a_s}\left(-\gamma_s\right)\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s-1}\frac{\gamma_s}{\gamma_s-1}\\
&\times\left(\frac{\varsigma(1-\tau)m_s^{-\tau}\frac{W}{a_s}P_c(m_s)D_c(m_s)-\varsigma m_s^{(1-\tau)}\frac{W}{a_s}\frac{\partial P_c(m_s)D_c(m_s)}{\partial m_s}}{P_c(m_s)^2D_c(m_s)^2}\right)\\
&=\frac{W}{a_s}\left(-\gamma_s\right)m_s^{-\gamma_s-1}\left(\frac{\varsigma m_s^{-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s-1}\\
&\times\frac{\gamma_s}{\gamma_s-1}\frac{\varsigma m_s^{-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\left((1-\tau)-\frac{\partial P_c(m_s)D_c(m_s)}{\partial m_s}\frac{m_s}{P_c(m_s)D_c(m_s)}\right)\\
&=\frac{W}{a_s}\left(-\gamma_s\right)m_s^{-\gamma_s-1}\left(\frac{\varsigma m_s^{-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)
\end{aligned}\tag{35}
$$

Substitute equation 35 back into equation 34:

$$
\begin{aligned}
&\frac{W}{a_s}\left(-\gamma_s\right)m_s^{-\gamma_s-1}\left(\frac{\varsigma m_s^{-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)X_c(m_s)\\
&+\frac{W}{a_s}m_s^{-\gamma_s}\left(\frac{\varsigma m_s^{-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}\frac{\partial X_c(m_s)}{\partial m_s}\\
&=\frac{W}{a_s}\left(\frac{\varsigma m_s^{1-\tau}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}\{m_s^{-1}\left(-\gamma_s\right)\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)X_c(m_s)+\frac{\partial X_c(m_s)}{\partial m_s}\}
\end{aligned}\tag{36}
$$

Next, substitute equation 36 into equation 33:

$$
\begin{aligned}
\frac{W}{a_s}\left(\frac{\varsigma m_s^{(1-\tau)}\frac{W}{a_s}}{P_c(m_s)D_c(m_s)}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}&\{\varsigma(1-\tau)m_s^{-\tau}X_c(m_s)\\
+(\varsigma m_s^{(1-\tau)}-1)\left[m_s^{-1}\left(-\gamma_s\right)\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)X_c(m_s)+\frac{\partial X_c(m_s)}{\partial m_s}\right]\}&=0\\
\varsigma(1-\tau)m_s^{-\tau}X_c(m_s)m_s^{-1}m_s+(\varsigma m_s^{(1-\tau)}-1)&\\
\times\{m_s^{-1}\left(-\gamma_s\right)\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)X_c(m_s)+\frac{\partial X_c(m_s)}{\partial m_s}m_s^{-1}m_s\}&=0\\
\frac{\partial X_c(m_s)}{\partial m_s}\frac{m_s}{X_c(m_s)}-\varsigma(1-\tau)m_s^{1-\tau}+\varsigma m_s^{(1-\tau)}\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)&\\
-\varsigma m_s^{(1-\tau)}\frac{\partial X_c(m_s)}{\partial m_s}\frac{m_s}{X_c(m_s)}&=\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)\\
\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)-\varepsilon_{X_c,m_s}&=\varsigma m_s^{(1-\tau)}
\end{aligned}
$$

Reorganizing terms we obtain::

$$
\begin{aligned}
\varsigma m_s^{(1-\tau)} &= \frac{\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)-\varepsilon_{X_c,m_s}}{\left[\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)-\varepsilon_{X_c,m_s}-(1-\tau)\right]} \\
m_s &= \left\{\frac{1}{\varsigma}\left[\frac{\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)-\varepsilon_{X_c,m_s}}{\left[\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)-\varepsilon_{X_c,m_s}-(1-\tau)\right]}\right]\right\}^{\frac{1}{(1-\tau)}}
\end{aligned}
$$

From the demand of the customer firm, we know that $X_c = \left(\frac{P_c}{P}\right)^{-\eta} Y \rightarrow \frac{\partial X_c}{\partial P_c} = -\eta P_c^{-1} X_c$. It then follows that $\frac{\partial X_c}{\partial m_s}\frac{m_s}{X_c} = \frac{\partial X_c}{\partial P_c}\frac{\partial P_c}{\partial m_s}\frac{m_s}{X_c} = -\eta P_c^{-1} X_c \frac{\partial P_c}{\partial m_s}\frac{m_s}{X_c} = -\eta\varepsilon_{P_c,m_s}$, which leads to the final expression for markups under progressiv tax:

$$
m_s = \left\{\frac{1}{\varsigma}\left[\frac{\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)+\eta\varepsilon_{P_c,m_s}}{\left[\gamma_s\left((1-\tau)-\varepsilon_{(P_cD_c),m_s}\right)+\eta\varepsilon_{P_c,m_s}-(1-\tau)\right]}\right]\right\}^{\frac{1}{(1-\tau)}}.
$$

□

B Quantitative Appendix

B.1 Algorithm to compute Static Profits

1. First, make a guess of the initial value of firm's prices that is equal to a constant markup over marginal cost:
$$
p_s^0 = \frac{\gamma_s}{\gamma_s - 1}\frac{W}{a_s}.
$$

2. Given p_s^0 , obtain the initial values of equilibrium values of customer firm price P_c^0 and demand aggregator D_c^0, by solving the system of equations given by condition (37) that states that market shares across suppliers of a given customer have to sum to one, and condition (38) that states that sum of Y function across suppliers of the same customer is one:
$$
\sum_s S_s = \sum_s \frac{p_s}{P_c} \Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right) = 1 \tag{37}
$$
$$
\sum_s \Upsilon\left(\frac{x_s}{X_c}\right) = \sum_s \Upsilon\left[\Upsilon'^{-1}\left(\frac{p_s}{P_cD_c}\right)\right] = 1 \tag{38}
$$
   (a) For the application of this paper, in which $\Upsilon\left(\frac{x_s}{X_c}\right) = \left(\frac{x_s}{X_c}\right)^{\frac{\gamma_s-1}{\gamma_s}}$, these conditions are:
$$
\sum_s \frac{p_s}{P_c}\left(\frac{p_s}{P_cD_c}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s} = 1
$$
$$
\sum_s \left(\frac{p_s}{P_cD_c}\frac{\gamma_s}{\gamma_s-1}\right)^{1-\gamma_s} = 1
$$

3. Given p_s^0, P_c^0 and D_c^0 we can compute the initial market share of each supplier firm s in equilibrium S_s^0 from:
$$
S_s = \frac{p_s}{P_c}\left(\frac{p_s}{P_cD_c}\frac{\gamma_s}{\gamma_s-1}\right)^{-\gamma_s}
$$

4. Now we are ready to iterate over values the market share with initial values p_s^0, P_c^0 and D_c^0, S_s^0. The iteration steps are:

5. For a given competition type (Cournot or Bertrand), compute the elasticity of demand in equilibrium given by:

$$\vartheta_s^C = \left[\frac{1}{\gamma_s}(1-S_s) + \frac{1}{\eta}S_s + \left(\sum_{j\in\Omega_c} S_j\frac{1}{\gamma_j} - \frac{1}{\gamma_s}\right)S_s\right]^{-1} \text{ if Cournot competition}$$
$$\vartheta^B = \left[\gamma_s\left(1 - \frac{\gamma_s S_s}{\sum_{s\in\Omega_c}\gamma_s S_s}\right) + \eta S_s\right] \text{ if Bertrand competition}$$

6. Having computed ϑ, we can now compute the markups in equilibrium as:

$$m_s = \frac{\vartheta_s}{\vartheta_s - 1}$$

7. Update the new value of the markup to be $m_s^{new} = m_s^0 + 0.5 * (m_s - m_s^0)$

8. Compute the new value of the firm's price as $p_s^{new} = m_s^{new}\frac{W}{a_s}$

9. Given p_s^{new}, repeat step 2. to compute the new values of P_c^{new} and D_c^{new} in equilibrium.

10. Given p_s^{new}, P_c^{new} and D_c^{new} repeat step 3. to compute the new market shares S_s^{new}.

11. Update $m^0 = m^{new}$ and $S_s^0 = S_s^{new}$, and iterate until convergence.

12. Once converged, we have the values of m_s, S_s, p_s, P_c, D_c in equilibrium. Given values of aggregate prices and GDP P and Y, now we can compute the customer firm production Y_c from the demand function of the customer firm given by:

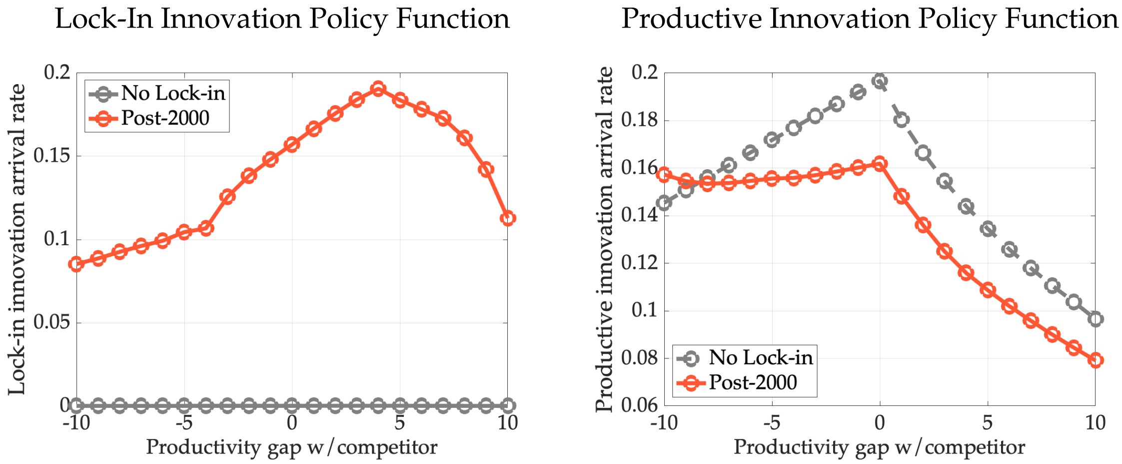
$$X_c = \left(\frac{P_c}{P}\right)^{-\eta} Y$$

13. The production of the supplier firm x_s is given by the demand function:

$$x_s = \Upsilon'^{-1}\left(\frac{p_s}{P_c D_c}\right) X_c = \left(\frac{p_s}{P_c D_c}\frac{\gamma_s}{\gamma_s - 1}\right)^{-\gamma_s} X_c.$$

B.2 Counterfactual Exercises

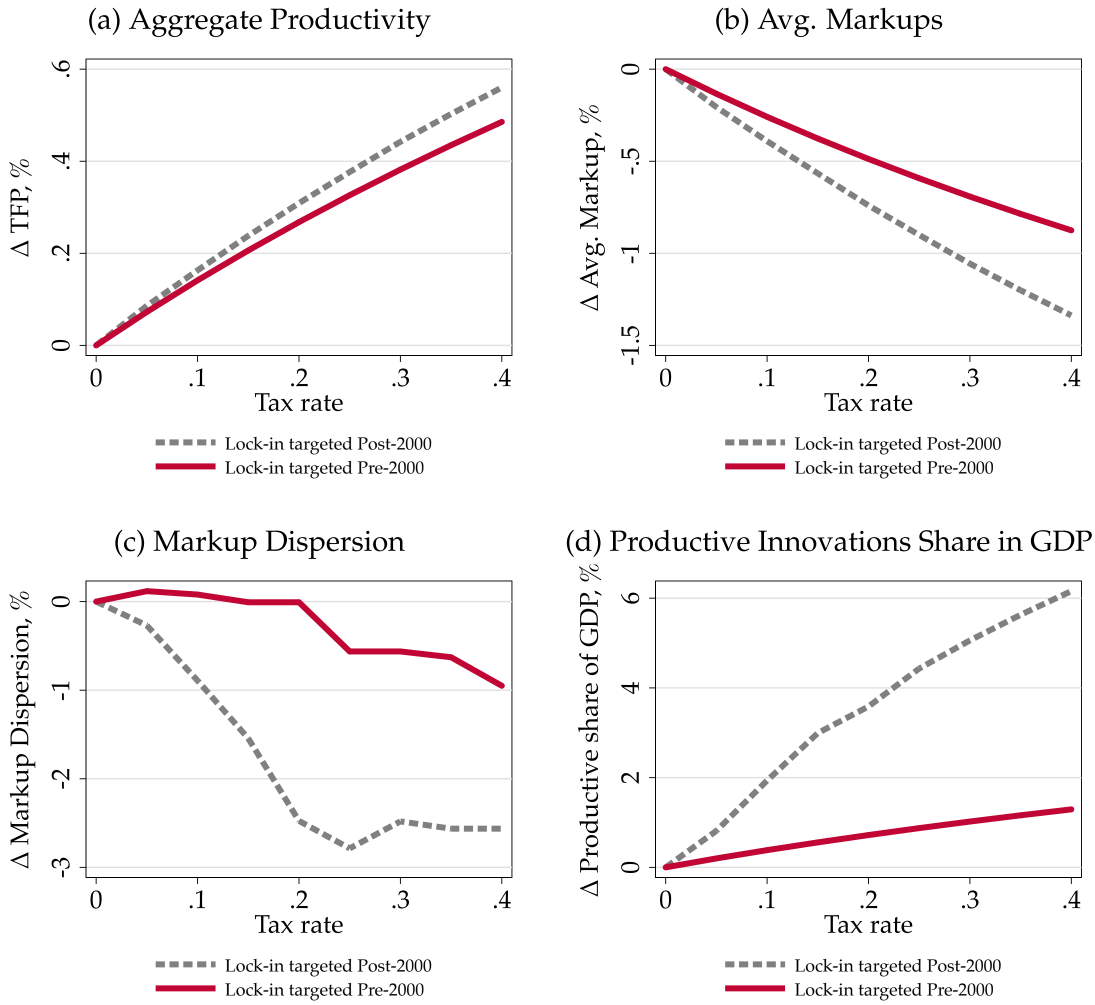
Figure 15: Innovation Policy Functions: No Lock-in Scenario



Notes: calibrated model lock-in (panel (a)) and productive (panel (b)) innovations' policy functions, against the supplier's productivity gap (in terms of number of steps) with respect to their competitor, for the *Baseline* Post-2000 calibrated economy, compared to the *Without Lock-in* counterfactual economy with infinitely costly lock-in innovations.

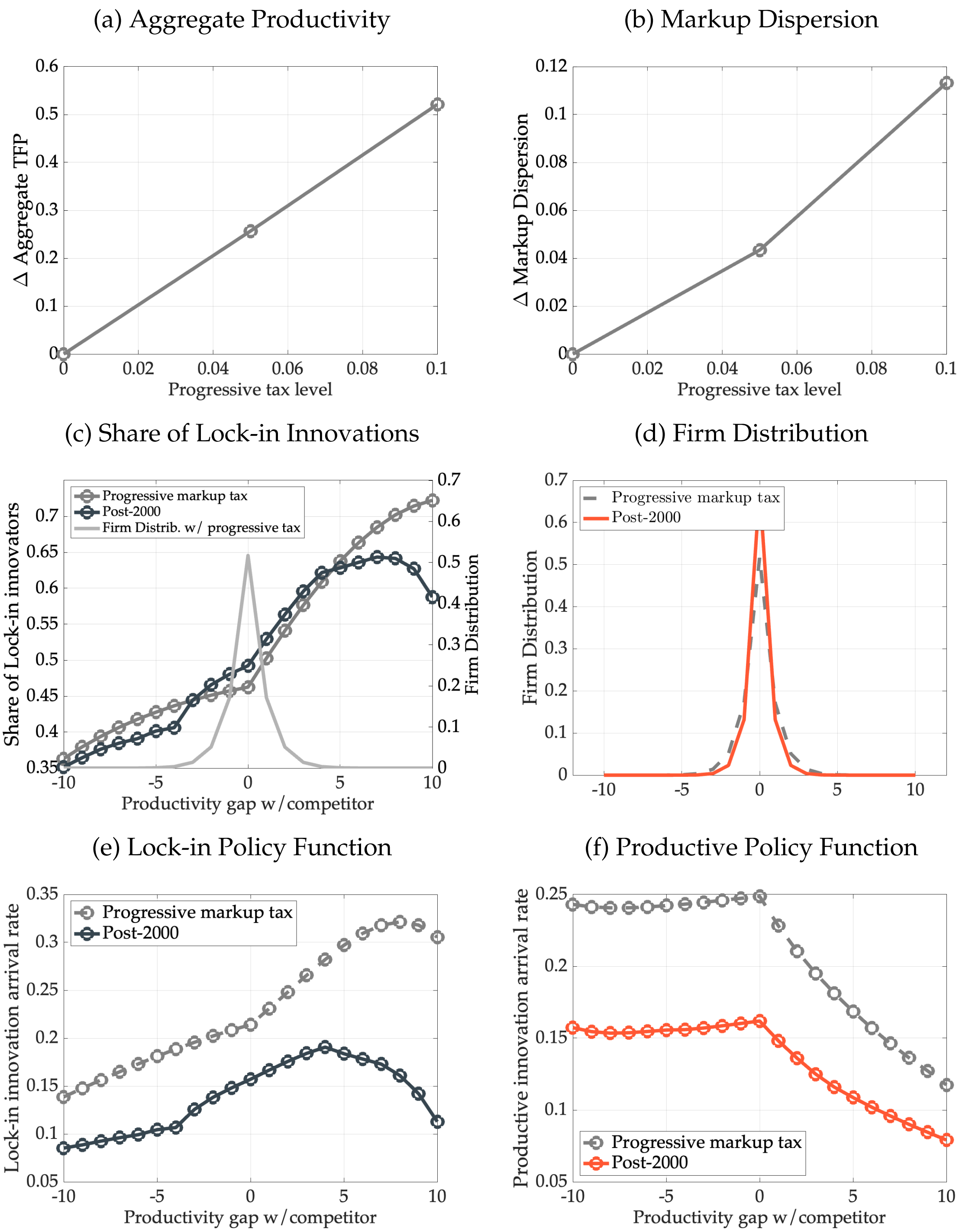
B.3 Policy Experiments

Figure 16: Lock-in targeted regulation



Notes: results from calibrating the economy Post-2000 under a lock-in targeted regulation that increases the cost of lock-in innovations (see Section 5 for details). The figure shows the change in aggregate productivity (panel a), average markup level (panel b), markup dispersion (panel c) and productive innovation share in GDP (panel d), relative to the non-tax scenario for different levels of the tax rate τ.

Figure 17: Progressive tax on markups



Notes: the figure show the results of calibrating the Post-2000 economy under a progressive tax scheme on suppliers' markups (see Section 5 for details). Panel (a) and Panel (b) shows the change —relative to non-tax scenario— in aggregate productivity and markup dispersion for different levels with level of tax progressivity $\tau = 10\%$, and for scale parameter $\varsigma = 1$. Panel (c) shows the share of Lock-In innovations for different values of the productivity gap between the supplier and its competitors (in number of steps), under both the baseline calibrated economy Post-2000 and the calibrated economy with the progressive markup tax, when $\tau = 10\%$ and $\varsigma = 1$. The panel also shows the distribution of supplier firms across productivity gaps, for the progressive markup tax economy. Panel (d) shows the comparison of the distribution of firms across productivity gaps for both the baseline Post-2000 and the progressive markup tax economy, when $\tau = 10\%$ and $\varsigma = 1$. Panel (e) and Panel (f) shows the lock-in and productive policy functions for both economies.

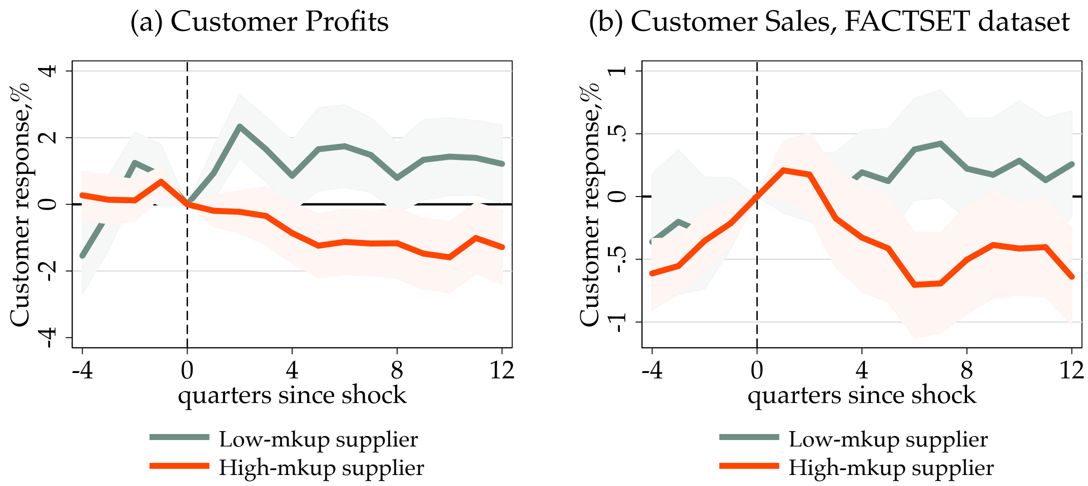
C Empirical Appendix

Table 11: Markups and R&D Sales Share

	R&D Sales Share
High Markup	0.823***
	(0.0299)
R^2	0.564
Sector & quarter FE	yes
Control for size	yes

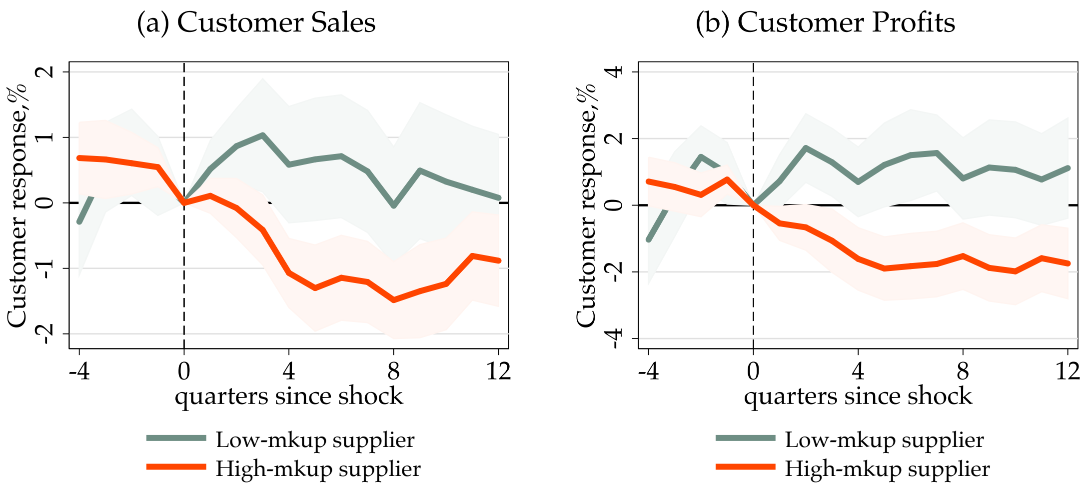
Notes: the table shows the correlation between the R&D expenditures and the markup of the supplier firm. It presents estimation results from regressing the firms' R&D expenditures as a share of its sales, against a dummy variable that takes the value of one if the supplier is in the 80th percentile of the markup distribution, controlling for sector and quarter fixed effects, as well as the size of the firm in terms of sales.

Figure 18: Δ Customer after innovation by suppliers



Notes: *High-mkup supplier* refers to supplier firms in the sample that belong to the top 80th percentile of the markup distribution, while *Low-mkup supplier* refers to the rest of supplier firms. Panel (a) shows the estimated coefficients β_H and β_L for each quarter, obtained when estimating local projection equation 16 when considering customer sales' profits as dependent variable. Panel (b) shows the estimated coefficients β_H and β_L for each quarter, obtained when estimating local projection equation 16 when using a data sample from FACTSET dataset, which includes private firms.

Figure 19: Δ Customer after innovation by suppliers, controlling for citations



Notes: *High-mkup supplier* refers to supplier firms in the sample that belong to the top 80th percentile of the markup distribution, while *Low-mkup supplier* refers to the rest of supplier firms. The figures show the estimated coefficients β_H and β_L for each quarter, obtained when estimating local projection equation 16 including a control variable with the number of citations received by the patents granted to the supplier firm. Panel (a) shows the cumulative response of customer firms' sales to innovations by High-markup and Low-markup suppliers. Panel (b) shows the cumulative response of customer firms' profits to innovations by High-markup and Low-markup suppliers.

Table 12: Private vs Social Value of Innovation, and Firm's Markups

(a) Pre-2000s

	Social value (Cit) High Markup	Social value (Cit) Low Markup
Private value (SM)	0.692***	0.614***
	(0.0446)	(0.0256)
R^2	0.719	0.801
Sector & Quarter FE	yes	yes

(b) Post-2000s

	Social value (Cit) High Markup	Social value (Cit) Low Markup
Private value (SM)	0.412***	0.591***
	(0.0253)	(0.0160)
R^2	0.573	0.645
Sector & Quarter FE	yes	yes

Notes: *Private value (SM)* refers to the dollar value of innovation at the firm level taken from Kogan *et al.* (2017), and *Social value (Cit)* refers to the social value of innovation measured as the number of citations received by the patents granted to a firm. *High markup* refers to supplier firms in the sample that belong to the top 80th percentile of the markup distribution, while *Low markup* refers to the rest of supplier firms. The table shows estimation results of regressing the log of private value of innovation against the log of social value of innovation for high-markup (first column) and low-markup (second column) firms, for both the pre-2000s period (panel a) and the post-2000s period (panel b).