On the Investment Network and Development *

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Abstract

Capital accumulation and the systematic reallocation of economic activity across sectors are two of the most salient features of economic development. These two features are interconnected through the production of various types of capital and heterogeneous usage intensity across sectors, which is summarized by the investment network. Our paper introduces the first harmonized measures of the investment network across the development spectrum and documents novel empirical regularities. We propose a simple theory linking disparities in this network to disparities in income per capita across countries. We show that Domar weights and the elasticity of output to sectorial productivity are nontrivial functions of the investment network and equilibrium sectorial investment rates. For our sample of 58 countries, we show that 33% of cross-country differences in income per capita can be accounted for by disparities in the investment network. These differences are twice as large as the role of capital in income disparities estimated through standard development accounting.

JEL Codes: E23; E21; O41.

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1 Introduction

Capital accumulation and the systematic reallocation of economic activity across sectors are two of the most salient features of economic development. Different sectors utilize various investment goods for production, which are either produced by other sectors of the economy or imported. As economic activity shifts across sectors, the economy's ability to produce new capital—or to export goods in exchange for this new capital—changes, facilitating further capital accumulation. The study of the nature of this continuous feedback is crucial for understanding the mechanics of economic development (Hirschman, 1958). Such a study requires measures of sectorial links in both the production and use of new capital, i.e., the investment network. This paper provides the first harmonized measures of investment networks across the development spectrum. We document novel facts about the characteristics of these networks and construct a theory to evaluate the impact of these differences on observed income disparities across countries.

Recent studies have documented systematic changes in the sectorial composition of inputs used for investment as countries develop, Garcia-Santana, Pijoan-Mas and Villacorta (2021); Herrendorf, Rogerson and Valentinyi (2021). Work has also examined how disparities in the bundles of capital goods used for sectorial production lead to structural change (Caunedo and Keller, 2023). We combine these two approaches and characterize the link between production and uses of capital, i.e., the investment network, and aggregate output. Through the lens of a neoclassical multisector open economy model, we show that the investment network is a key component of the elasticity of GDP to sectorial productivity. In the seminal work of Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), this elasticity is called "*sectorial influence*", or the direct and indirect impact of changes in sectorial productivity on aggregate economic activity.

Sectional influence is a function of the input–output structure, the investment network and sectorial investment rates through the augmented Leontief inverse, a matrix that summarizes the extent of roundabout effects in the economy. Our economy has two distinct features relative to other economies with sectorial linkages. The first is that the magnitude of roundabout effects depends on sectorial investment rates. This feature is a consequence of the durable nature of capital and a novel result in the production networks literature that, for the most part, focuses on nondurable inputs. The second is that welfare and GDP differ, but along the balanced growth path (BGP), both are proportional to the augmented Leontief inverse. Since welfare is a Domar-weighted sum of sectorial productivities, an implication of the previous result is that Domar weights are also functions of sectorial investment rates along the equilibrium path.

To bring empirical content to the augmented Leontief inverse and therefore *sectorial influ-*

ence, one needs measures of intermediate and investment networks, as well as factor shares and investment rates. While estimates of the input–output structure have become increasingly available across countries, estimates of the investment network are only available for the US (vom Lehn and Winberry, 2022) and a handful of years across OECD economies; see Ding (2023).[1] We advance previous measurement efforts by providing cross-country and time-series harmonized estimates of the investment network for 58 countries with income per capita levels between $428 and $81,599 constant 2015 PPP dollars and time spans that date back to the 1960s for a subset of countries. In our analysis, capital is disaggregated into multiple equipment types, including ICT, electronics, machinery, transportation and other durables, as well as structures, measured through construction investment.[2]

To create our harmonized measures, we exploit a methodology similar to that of the Bureau of Economic Analysis (BEA) in the US. The BEA combines the occupational composition of each industry and an allocation rule for capital to workers to estimate investment by capital type and sector (Meade, Rzeznik and Robinson-Smith, 2003). Unfortunately, the apportioning of stocks to workers is not publicly available. Hence, to ensure replicability, we opt for an allocation of capital across sectors that follows Caunedo, Jaume and Keller (2023) for equipment sectors and an allocation that follows intermediate inputs for construction and other sectors with positive investment in the national accounts. While the allocation of investment may seem arbitrary, it is reassuring that our own estimates of the investment network for the US closely follow those published by the BEA.

With this newly constructed measure, we document two novel facts. First, the properties of the investment network and the input–output tables are substantially different. For example, the degree of homophily of the network, summarized by the weight of its diagonal terms, is stronger in the input–output network than in the investment network. This means that sectors are more important providers of intermediate inputs for themselves than they are of investment goods. Hence, sectorial productivity shocks have differential impacts on aggregate outcomes depending on whether the sector produces mostly for intermediate or investment uses. Second, the investment network in richer economies is more diversified than that of poor economies. We measure this feature through sectors' outdegrees, which summarize the row sum of the entries in the network and therefore measure the relevance of a sector in producing investment inputs for other sectors in the economy. In poor economies, the outdegree of construction is

[1]These measures are self-reported by country offices to the OECD Statistics office, and it is unclear whether measurement is comparable across countries. Ding (2023) exploits these investment flows to estimate capital services in each sector from different sectors and countries. To do so, he uses bilateral import flows to input cross-country linkages and estimates user costs along a BGP. In other words, he treats the investment network as a primitive.

[2]Our benchmark estimates include 10 sectors, but estimates for as many as 19 sectors consistently defined across countries and time can be made readily available.

much larger than that of any other sector in the economy, while in rich economies, outdegrees are more distributed across sectors.

A natural question to ask is whether sectors with high *outdegrees* in the investment network are also sectors where changes in productivity have the strongest impact on aggregate activity. The answer is no. Our theory predicts that their role is mediated by other features of the economy, including value added and capital shares in production, as well as the full input–output network. Hence, to assess the role of the investment network for income disparities, we exploit the characterization of *sectorial influence*.

Given an estimate of the augmented Leontief inverse and valued-added expenditure shares, we can calibrate sectorial productivity differences to match disparities in value added across sectors within countries and across countries for a sector. By construction, our model exactly matches the empirical variance of output per capita at baseline. We then study the role of the investment network in driving these disparities through counterfactual exercises. First, we drive the capital share in value added to zero, so that the investment network drops out from *sectorial influence*. Second, we only include the investment network in computing the augmented Leontief inverse, so roundabout effects from intermediate inputs are eliminated. In the former exercise, the role of the investment network is the difference between the variance at baseline and the counterfactual. In the latter exercise, the role of the network is the portion of baseline variance that is not explained by the intermediate input linkages. Our main finding is that, averaging across these counterfactuals, the investment network accounts for 33% of the observed disparities in income per worker. To place the magnitude of our finding in perspective, a standard development accounting exercise imputes approximately 13% of observed income differences to disparities in measured capital in our sample of countries. We conclude that accounting for roundabout effects in the technology for producing capital doubles its role for income disparities.

Our model rationalizes differences in the investment network through disparities in the frontier technology facing poor and rich countries. This frontier is characterized by a height, which intuitively summarizes the availability of better technologies across all capital types, and a gradient, which intuitively summarizes the availability of technologies that are more or less intensive in certain capital types relative to others. We can then account for the role of disparities in the height and in the relative intensity across capital types in driving income disparities. We find that the role of height of the frontier for the contribution of the investment network to cross-country income disparities varies across benchmark economies and can be as high as 2/5th of the induced variation from the network. The rest stems from the relative intensities of use of capital. We can also ask how different income levels would be if all countries operated some benchmark technology. For example, we explore the use of a technology consistent with the investment network of Korea in 1965, before the country entered a period of sustained economic growth. We show that poorer economies benefit relatively more from producing with this investment network, given their current input–output structures, sectorial productivities, and patterns of final expenditure shares. In contrast, richer economies are negatively affected by employing this technology.

Finally, since most of the capital in the world is produced by a handful of countries, it is likely that much of the investment used in an economy is imported from elsewhere. The imported nature of capital is potentially relevant to our accounting because the magnitude of the roundabout effect is sensitive to how much capital is locally produced and because countries may buy productive capital by generating exports. Our theory accommodates these channels by modeling investment goods as a combination of locally and foreign-sourced investment and by explicitly modeling exports of final goods as a source to finance investment imports. We run counterfactual exercises where we construct the loadings of the investment network including only the domestically produced share of investment and shut down the productivity amplification through the production of tradable goods. We find that the counterfactual economy induces a 44% decline in steady-state income per capita on average across countries and that the variance of income falls by 12% relative to baseline. In other words, trade accounts for slightly more than one-third of the variance in income accounted for by the investment network.

Contribution to the literature. There is a growing literature studying the relevance of sectorial linkages, mostly through intermediate input use, for differences in income per capita across countries; see Ciccone (2002); Jones (2011). This role is quantified in Fadinger, Ghiglino and Teteryatnikova (2022), who show that differences in the input–output structure amplify the role of sectorial productivity for differences in income per capita. We document that the input–output and investment networks are empirically different, leading to differential roles in driving income disparities across countries. The investment network, which summarizes the technology for producing new capital in the economy, can double the effect of measured disparities in capital on income disparities. This finding brings renewed attention to the ability to produce or source capital as a driver of income disparities.

Garcia-Santana *et al.*, 2021; Herrendorf *et al.*, 2021 document systematic shifts in the inputs used for investment as economies develop. In the US time series, Gaggl, Gorry and vom Lehn (2023) document disparate shifts in the composition of inputs for investment and consumption, while Caunedo and Keller (2023) show how differences in the investment bundles used by different sectors drive sectorial reallocation. Our paper completes the puzzle by documenting disparities in the investment bundle and the composition of investment along the development spectrum. The study of the role of the investment network in determining GDP is relatively recent. In the US, vom Lehn and Winberry (2022) focus on short-run fluctuations, while Foerster, Hornstein, Sarte and Watson (2022) study implications for long term growth. Our main contribution relative to Foerster *et al.* (2022) is that we model the endogenous allocation of labor and allow for equipment imports. Trade is an important driver of the level of output in steady state and the magnitude of *sectorial influence*.

A key novel finding of our theory is that sectorial investment rates mediate the impact of sectorial productivity on aggregate GDP and affect the rate of convergence of the economy to its BGP. As in Liu (2019), output elasticities to sectorial productivity are different from Domar weights. In our model, this difference arises from the presence of investment, i.e., sectorial value-added shares and consumption shares do not equalize. Indeed, because our economy is efficient, welfare is a Domar-aggregated measure of sectorial productivities. These Domar weights are a function of equilibrium investment rates, a channel that is absent in economies where inputs are nondurable, as in Acemoglu *et al.* (2012) and the extensive literature that follows. Buera and Trachter (2024) is the closest paper to our study, characterizing differential GDP responses to sectorial productivity. Their work emphasizes the role of distortions and nonconvexities for endogenous technology choices. Our paper instead emphasizes the intensive margin of the adoption of capital-embodied technology, i.e., investment, and documents systematic disparities in its nature along the development spectrum.

The remainder of the paper is organized as follows: Section 2 presents the model, Section 3 discusses the methodology to construct estimates of the investment networks and empirical regularities of the investment network and *sectorial influence* along the development spectrum; Section 4 presents the main result from income accounting exercises; and Section 5 concludes the paper.

2 A Model of the Investment Network and Economic Development

We build a framework to study the roles of the investment network and tradable equipment in determining aggregate GDP. We write an open economy version of Long and Plosser (1983) augmented to include a choice of technologies in investment-producing sectors that follows Caselli and Coleman (2006).

The economy consists of N sectors that combine capital, labor and intermediate inputs to produce output:

$$y_{nt} = \left(\frac{v_{nt}}{\gamma_n}\right)^{\gamma_n} \left(\frac{m_{nt}}{1-\gamma_n}\right)^{1-\gamma_n}, \qquad \text{for } \gamma_n \in [0,1],$$

with a measure of value added $v_{nt} = \exp(z_{nt}) \left(\frac{k_{nt}}{\alpha_n}\right)^{\alpha_n} \left(\frac{l_{nt}}{1-\alpha_n}\right)^{1-\alpha_n}$ that depends on TFP, z_{nt} and capital and labor allocations, k_{nt}, l_{nt}. The intermediate input aggregator is constant returns to scale (CRS) , $m_{nt} = \prod_{i=1}^{N} \left(\frac{m_{int}}{\mu_{in}}\right)^{\mu_{in}}$, and intermediate inputs from sector i used in sector n are m_{int}. This flow of intermediate inputs is summarized by an input–output matrix, M_t, with typical element μ_{in}. The rows of M_t sum to an indicator of the importance of a sector as an intermediate inputs provider to the rest of the economy, the columns describe the input composition of the intermediate input bundle in a sector, and $\sum_i \mu_{in} = 1$.

The capital stock used in each sector evolves according to the following law of motion:

$$k_{nt+1} = x_{nt} + (1-\delta_n)k_{nt},$$

for a composite of investment from different sectors.

There is a continuum of firms that produce sector-specific investment goods by optimally choosing the intensity of use of different capital types given a menu of technologies available at a point in time. Technologies are summarized by the height of the production possibility frontier for investment in a sector, B_n, and the shape of the frontier, summarized by an elasticity ν_n and its loadings ξ_{in}, as in Caselli and Coleman (2006).[3] Firms maximize profits by simultaneously choosing the amount of investment in each capital type and its intensity of use:

$$\max_{\omega_{int}, \chi_{int}} p^x_{nt} x_{nt} - \sum_i p_{it} \chi_{int}$$

subject to

$$x_{nt} = \prod_{i=1}^{N} \left(\frac{\chi_{int}}{\omega_{int}}\right)^{\omega_{int}}, \tag{1}$$

$$\sum_i \xi_{in} \omega_{int}^{\nu_n} = B_n \tag{2}$$

for $\sum_{i=1}^{N} \xi_{in}^{\frac{1}{1-\nu_n}} = 1$ and ω_{int}, the expenditure share in investment from sector i in sector n. The flow of investment across sectors is summarized by the investment network, Ω_t, with typical element ω_{int}. The rows of the investment network describe the production of investment

[3] A key difference from their environment is that firms here choose from capital services produced within the economy, rather than endowment goods.

in each sector, while the columns represent the use of investment by each sector such that $\sum_{i=1}^{N} \omega_{int} = 1$. We assume that $\nu_n > 1$, which ensures an interior solution to the technology choice problem. Finally, inputs from sector i into the production of investment in other sectors, χ_{it}, can be domestically produced or imported, $\chi_{int} = (\frac{\chi^d_{int}}{1-\phi_i})^{1-\phi_i}(\frac{\chi^f_{int}}{\phi_i})^{\phi_i}$, where ϕ_i is the expenditure share in foreign inputs for capital type i.

Each sector's output can be used for final goods production, c, intermediate uses, m, or domestic investment, χ^d:

$$y_{nt} = c_{nt} + \sum_i m_{nit} + \sum_i \chi^d_{nit}.$$

Sectorial output allocated to the production of final goods is combined with a homothetic aggregator, Y_t, and can be used for exports, ϵ, or for consumption by the representative household:

$$Y_t = \prod_{n=1}^{N} \left(\frac{c_{nt}}{\theta_n}\right)^{\theta_n}, \qquad \sum_{n=1}^{N} \theta_n = 1 \text{ and } \quad \theta_n > 0;$$

$$Y_t = C_t + \epsilon_t.$$ [4]

The representative household derives utility $U(C_t)$ that satisfies usual regularity conditions and discounts the future at rate β.

We set up a small open economy that exports final goods in exchange for capital goods of different types, as in Jones (2011). We define the value of net exports in the economy as the difference in the value of exports and imports:

$$NX_t = p_{Yt}\epsilon_t - p_{\epsilon^f t}\epsilon^f_t.$$

The value of imports is the product of the price index of imports and a composite import value $\epsilon^f_t = \prod_{i=1}^{N} \frac{\chi^f_{it}}{\phi^f_i}^{\phi^f_i}$, as in Basu, Fernald, Fisher and Kimball (2005).[5] The terms of trade are given by the ratio between the price of exports and the price of imports $\tau \equiv \frac{p_{Yt}}{p_{\epsilon^f t}}$, where the price of imported goods is a CRS aggregator of the (exogenous) prices of imported investment for production.

2.1 Balanced Growth Path

Definition *A balanced growth path (BGP) is an allocation such that sectorial output, consumption, investment and capital each grows at a constant (possibly different) rate.*

[4] Our findings are robust to having two different aggregators for exports and consumption. When the price of consumption is defined in units of exports, the results carry through. Alternatively, the amount of exports could be formulated as a constant fraction of final uses, Y_t.

[5] Any unitary elasticity aggregator preserves the BGP properties discussed in the Appendix.

Proposition .1. *There exists a BGP of this economy where the vector of gross output and consumption growth in each sector satisfies*

$$g^y = g^c = g^m = g^x = b_z^y \gamma_z + b_\tau^y \gamma_\tau.$$

where b_z^y, b_τ^y are parameters that depend on technology, namely, the investment network, the input–output network, capital expenditure and value-added shares in gross output.

The growth rate of final output is a weighted average of the growth rates of sectorial gross output:

$$g^Y = \boldsymbol{\theta}' g^y$$

with weights equal to the final output elasticities to sectorial inputs. The vector of capital and total investment growth in each sector satisfies

$$g^k = g^x = b_z^k \gamma_z + b_\tau^k \gamma_\tau,$$

where b_z^k, b_τ^k are functions of the technology in the economy.

The proof of this proposition can be found in Appendix 6.1.1. We use Proposition .1 to detrend the economy and characterize equilibrium allocations.

2.2 Equilibrium Characterization, Detrended Economy

Most choices in this problem are standard, except perhaps for the choice of technology, which we describe next.

Technology choice. The optimal loadings on the sectorial investment aggregators satisfies

$$\frac{\omega_{i'nt}}{\omega_{int}} = \left(\frac{\xi_{i'n}}{\xi_{in}}\right)^{\frac{1}{1-\nu_n}}.$$

Hence, the optimal (relative) use intensity of each capital types reflects the shape of the production possibility frontier and, through it, the menu of technologies available in each country for a given sector.[6]

The level of the intensity is pinned down by the height of the productivity possibility fron-

[6]Appendix 6.2.1 presents a more general version of this problem with an arbitrary CRS aggregator for investment, while Appendix 6.2.2 presents a version with wedges in the cost of capital. We choose not to use the general CRS aggregator as a benchmark because there are no readily available cross-country harmonized price data that would allow us to discipline the path of those relative prices at our level of disaggregation. Studying the nature of changes in the network is beyond the scope of this accounting exercise but a natural next research step.

tier, given a normalization of the shape parameters, i.e., $\zeta_{1n} = 1$,

$$\omega_{1nt} = B_n^{\frac{1}{\nu_n}}.$$

The remainder of the analysis focuses on optimal factor demand and the allocation of gross output to different uses, describing equilibrium aggregate GDP and welfare in our economy. We emphasize how these key variables depend on the features of the investment network.[7]

Domar weights. Since our economy is efficient, the envelope theorem dictates that welfare is a Domar-weighted average of sectorial productivities. Let the Domar weight of sector n be $\eta_n \equiv \frac{p_n y_n}{p\nu}$, the share of value added allocated to the production of final goods be $\zeta_n \equiv \frac{p_n c_n}{p\nu}$ and the value-added share of each sector be $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \chi_n^d}{p\nu}$. We also define the adjusted depreciation rate in the detrended economy $\hat{\delta}_i \equiv 1 - \frac{1-\delta}{1+g_i^k}$, the (diagonal) matrixes of value-added shares $\Gamma_t = \text{diag}\{\gamma_n\}$, and sectorial capital expenditure shares in value added, $\alpha = \text{diag}\{\alpha_n\}$.

Proposition .2. *The equilibrium Domar weights satisfy*

$$\left[I - \tilde{\beta}^{-1}\hat{\delta}\Gamma\boldsymbol{\alpha}(1-\boldsymbol{\phi})\Omega - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{3}$$

for $\tilde{\beta}_i \equiv \frac{1}{\beta} - (1-\hat{\delta}_i)$. In vector form, we have

$$\eta_n = \zeta_n + \sum_{i=1}^{N} \alpha_i \gamma_i \omega_{ni}(1-\phi_i)\eta_i + \sum_{i=1}^{N}(1-\gamma_i)\mu_{ni}\eta_i.$$

Along the transition to the steady state, Domar weights are functions of their full equilibrium path:

$$\left[I - \tilde{\beta}_{t+1}^{-1}\frac{x_{t+1}}{k_{t+1}}\Gamma\boldsymbol{\alpha}(1-\boldsymbol{\phi})\Omega\frac{g_{\eta_{t+1}}}{g_{x_{t+1}}} - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta}_t \equiv \boldsymbol{\eta}_t,$$

with elements of the discount factor $\tilde{\beta}_{it+1} \equiv \frac{1}{R_t} - (1-\hat{\delta}_{it})\frac{p_{it+1}^x}{p_{it}^x}$.

Proofs to all propositions can be found in Appendix 6.1.2.

Note that the role of the investment network for the Domar weight scales with the importance of domestic investment across sectors, $(1-\phi) \in (0,1)$. The lower the importance of domestic investment, the less relevant the investment network is for the roundabout effects on equilibrium Domar weights.[8] In our economy, equilibrium Domar weights are nontrivial

[7] We present closed-economy versions of these results where $\phi_i = 0$, or $\chi_{it}^f = 0$ in all sectors i and there are no exports $\epsilon_t = 0$ in the Online Appendix.

[8] This is driven by our modeling of exports from a composite of final uses. If all equipment is tradable, then roundabout effects from the investment network will appear directly in GDP through trade amplification; see Φ in Proposition .4. If instead we model each sector's allocation of gross output to exports, these flows appear in the equilibrium representation of the Domar weight.

functions of the investment rates along the BGP. Indeed, along the transition to the BGP, Domar weights are functions of the entire path of future Domar weights.

Welfare. We start by describing how aggregate welfare, W, in the economy depends on the investment network.

Proposition .3. *Along the equilibrium path, welfare satisfies*

$$W \approx \boldsymbol{\eta}\Gamma z,$$

where $\boldsymbol{\eta}$ are the equilibrium Domar weights.

The proof of this result follows from the envelope theorem and is analogous to the extensive literature on production networks studying the implications of roundabout effects on the aggregate economy. Value-added shares scale productivity levels because of how productivity is defined within the production technology. Proposition .3 is consistent with results in vom Lehn and Winberry (2022) for short-run fluctuations: Domar weights are scaled by the ratio between the value of GDP and final consumption. Along the BGP, this ratio is a constant. In the transition, the welfare effect of sectorial productivity shocks can be amplified or dampened depending on the relative allocation of value added between consumption and investment uses.

Aggregate GDP. In an economy with investment, aggregate consumption and GDP differ. Next, we study the effect of the investment network on aggregate GDP.

Proposition .4. *The equilibrium level of value added in the economy satisfies*

$$\ln(\nu) = \Phi\tilde{\boldsymbol{\eta}}'\Gamma(z + \alpha\boldsymbol{\phi}\Omega'\tau) + \boldsymbol{\epsilon},$$

where $\tilde{\boldsymbol{\eta}}$ is the vector of sectorial influence; $\Phi \equiv (I - \tilde{\boldsymbol{\eta}}\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}')^{-1}$ is an adjustment factor for the tradable nature of investment; and $\boldsymbol{\epsilon}$ is an adjustment factor that depends on the equilibrium Domar weights.[9] *Sectorial influence is the product of sectorial value-added shares, $\tilde{\boldsymbol{\zeta}}'$, and an adjusted Leontief inverse $\Xi \equiv (I - \tilde{\beta}^{-1}\Gamma\alpha(1-\phi)\Omega - (1-\Gamma)M)^{-1}$, i.e., $\tilde{\boldsymbol{\eta}} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$.*

In vector form, GDP can be described as

$$\begin{aligned} \ln(\nu)(1 - \sum_n \tilde{\eta}_n\gamma_n\alpha_n \sum_i \omega_{in}\phi_i) \quad &= \quad \sum_n \tilde{\eta}_n\gamma_n z_n + \sum_n \tilde{\eta}_n\gamma_n\alpha_n \sum_i \omega_{in}\phi_i \ln(\tau) - \\ &\quad\ \ln(\sum_n \gamma_n(1-\alpha_n)\eta_n) \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n). \end{aligned}$$

[9] The term $\boldsymbol{\epsilon} \equiv -\Phi\tilde{\boldsymbol{\eta}}'\Gamma(1-\alpha)\ln(\Gamma(1-\alpha)\boldsymbol{\eta})$ maps onto the equilibrium distribution of employment and is quantitatively small, so for most of the analysis, it can be omitted.

Value added is therefore a function of sectorial productivities, z, the terms of trade, τ, and a constant, ϵ. The first term showcases the impact of productivity on value added and the vector of sectorial influence $\tilde{\eta}$, similarly to Acemoglu *et al.* (2012).[10] This effect is augmented by the tradable nature of investment, $\Phi \equiv \left(I - \tilde{\boldsymbol{\eta}}\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\right)^{-1}$, as in Jones (2013) for tradable intermediate inputs. The reason is that when productivity increases within the economy, its export capacity also increases, and due to the trade balance, this implies higher imports of investment. The greater the dependence on imported equipment, ϕ, and the intensity of capital use in gross output, $\Gamma\alpha$, are, the stronger this amplification channel.

Sectorial influence differs from Domar weights because in an economy with investment, the GDP deflator is not necessarily the deflator for consumption. Indeed, welfare in our economy is characterized through Domar weights as described above. The main difference between influence on welfare and influence on GDP is whether sectors are loaded by their relevance to consumption, ζ, or their relevance as producers of value added, $\tilde{\zeta}$.

Second, the terms of trade enter as a channel directly affecting value added in the economy. Once adjusted for the role of imported investment, the capital share and the investment network, the terms of trade affect the economy similarly to a TFP shock. Note that as $\phi \to 0$, the economy loses its dependence on tradable investment, and Propositions .2 and .4 reduce to their closed-economy counterparts, which we describe in the Online Appendix.

Our quantitative analysis will focus on assessing the role of the investment network in driving cross-country disparities in GDP per capita along the BGP. However, the model economy is rich enough to have implications for the distribution of capital, consumption and output across sectors; see Appendix 6.2.3 for an illustration of these outcomes in a two-sector, two-capital-type economy.[11]

3 Investment Network

We are now prepared to outline the methods used to create measures of the investment network across countries. We describe data sources, explain our methodology and finally characterize the properties of the investment network at different stages of development.

We group sectors into ten categories: five equipment types — information and communication technology (ICT), electronics, machinery, transportation equipment and other durables —

[10] Value-added shares also mediate this effect because productivity enters into the value added expression. If modeled through gross output, the factor Γ drops out.

[11] The main model with 10 sectors entails 10 (capital types) + 10*9 (shares of gross output allocated to each capital type)=100 state variables. The study of the full transition dynamics of the system is challenging but a natural next step.

Table 1: The Investment Network

Investment Production	Investment Expenditures									
	Agri	**Constr**	**Dur**	**Elec**	**ICT**	**Mach**	**Non-Dur**	**Serv**	**Trpt**	**Trpt Serv**
Agriculture										
Construction								$\omega_{Cons,Serv}$		
Durables										
Electronics				$\omega_{Elec,Elec}$						
ICT					$\omega_{ICT,ICT}$		$\omega_{ICT,Ser}$			
Machinery	$\omega_{Mach,Agri}$									
Non-Durables										
Services										
Transportation					$\omega_{Tra,ICT}$			$\omega_{Tra,Serv}$		
Trpt Services										

Notes: Illustrative example of the investment network. Columns indicate consuming sectors, while rows indicate production sectors. Entries are expenditure shares by consuming sectors in different investment types, ω.

along with construction, agriculture, nondurables, transportation services and other services (see Appendix Table 8 for details).[12]

3.1 Methodology

Table 1 presents an example investment network table. Each entry (i, i') in the table indicates the total investment expenditures by column–sector i' purchased from row–sector i. Summing across columns yields the total production of investment by each sector, while summing across rows yields the total investment expenditures for each sector. For instance, each element of the ICT row indicates how significant ICT is as a provider of investment for each sector i', whereas each element of the agriculture column represents the amount of investment that agriculture purchases from each other sector i. To express the investment network in terms of expenditure shares ω_{ij}, we simply divide each entry of column–sector i' by total expenditures in that sector, so the sum across rows for each column is equal to one, $\sum_i \omega_{ij} = 1$.

Estimates of investment produced by each sector are readily available from *Use tables*, which record the uses of sectorial output between intermediate and final uses, including consumption and investment. Our contribution is to estimate how much of the investment produced (or imported and allocated to investment) by each sector is purchased by other sectors of the economy.

US investment flow tables are widely available. Meade *et al.* (2003) describes the procedure for inputting flows across sector uses. The process consists of three stages:

[12]Computers are generally classified under electronics. Software is included under ICT equipment. Investment data from ICT for years previous to 2000 and for countries in sub-Saharan Africa include professional services. For time and cross-country consistency, we include those services under the ICT category; see Table 8. We exclude the Mining sector throughout to avoid including variation in value added coming purely from commodity rich economies in our accounting exercise, see Section 4.1.

1. direct assignment
2. proportional to occupational composition of the sector.
3. proportional to sectorial capital expenditures (only used for structures).

Direct assignment entails simply assigning all investment from a given category to the most likely use, i.e., nuclear plant investment to the utilities sector. The second method assumes that there are certain occupations that are good predictors of the type of capital that will be used in a given sector.

Our methodology follows the BEA's to the extent that the procedure is replicable and information is widely available across countries and over time. Prioritizing replicability implies that manual assignment of categories to sectors of use is avoided (1). The reason is that such a manual assignment requires information on investment by narrow sectors of disaggregation. This information is not consistently available across countries. Following such a rule would imply different assignment in countries with disaggregated sectorial data and those that lack it. We therefore favor (2) as our benchmark assignment and propose a version of (3) whenever (2) is not feasible. Since capital expenditures by sector are not available (the BEA uses Census micro-data from firms to impute it), we opt for a different measure of proportionality, namely, proportional to the demand for intermediate inputs from each sector. We describe these procedures in detail next.

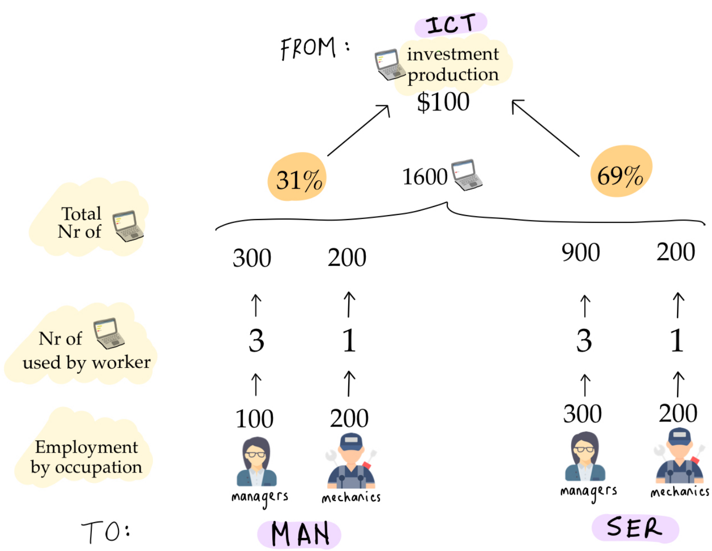
Allocation of equipment investment flows. Equipment-producing sectors include electronics, ICT, machinery and transportation. We allocate their investment flows following the methodology of the BEA for the investment network in the US. This methodology exploits the occupational composition of the labor force in each sector and the types of capital that these occupations likely use.[13]

Figure 1 provides an illustrative example of how the production of ICT equipment investment is allocated across purchasing sectors following our methodology. Suppose that there are three sectors in the economy, ICT, manufacturing and services, and that the ICT sector produces \$100 worth of new capital goods, i.e., computers. Our goal is to determine how much of the \$100 of ICT investment was purchased by the manufacturing sector and how much was purchased by the services sector. As noted above, we leverage the occupational composition of workers in the purchasing sectors and the type of capital that these occupations are more likely to use. In the example, both manufacturing and services employ 200 mechanics each,

[13] The BEA's allocation is as outlined in their publicly available documentation, but details of the exact assignment to workers and sector are not available.

along with 100 managers in the manufacturing sector and 300 managers in services sector. We normalize the use of computers by mechanics in any industry to 1 and, using data from the allocation of tools to workers (Caunedo *et al.*, 2023), assign three times as many computers to each manager in any industry. Hence, the total demand for computers (in units of the normalized usage for mechanics) in the manufacturing sector is 500, with 300 of them being used by managers and 200 being used by mechanics. The total demand for computers in the services sector is 1100, with 900 of them being used by managers and 200 being used by mechanics. Of the 1600 computers used in the economy, 31% are used in the manufacturing sector and 69% are used in the services sector. Accordingly, of the $100 worth of computers produced by the ICT sector, 31% are purchased by the manufacturing sector, and 69% are purchased by the services sector.

Figure 1: Example: Allocation of ICT Investment Flows



Notes: Illustrative example of the allocation of investment flows across consuming sectors obtained by exploiting the sectorial occupational composition and the equipment intensity across occupations from Caunedo *et al.* (2023).

Our assignment follows the tools utilized in each occupation in the US, as described by O*NET. We implement the methodology introduced by Caunedo *et al.* (2023) to assign equipment investment (and therefore stocks) to workers in different occupations.[14] Following the example in Figure 1, the underlying identification assumption is that the number of computers used by a mechanic relative to that used by a manager is the same across countries and equal to the US value.[15] The amount of investment assigned to each purchasing sector still differs

[14]The methodology in Caunedo *et al.* (2023) crosswalks equipment categories to the tools used within each SOC occupation. We use Dingel and Neiman (2020)'s crosswalk between SOC and ISCO to map these tools to harmonized cross-country occupational definitions.

[15]This identification restriction can be relaxed projecting tool usage in each occupation to the tasks performed

across countries because the aggregate investment flow of ICT is different across countries and because the numbers of mechanics and managers that work in manufacturing and services are different across countries, i.e., the occupational composition of the industry varies with development.

Formally, we first compute the share of total production of equipment capital type j purchased by industry i in country c at time t, $\tilde{\omega}^c_{ijt}$, as:

$$\tilde{\omega}^c_{ijt} = \sum_o \frac{\tau_j^{o,US} n_{it}^{oc}}{\sum_{o,i} \tau_j^{oUS} n_{it}^{oc}}, \tag{4}$$

where n_{it}^{oc} is the number of workers in occupation o and industry i in country c at time t and τ_j^{oUS} is the number of tools of capital type j used by a worker in occupation o in the US. Since $\tilde{\omega}^c_{ijt}$ represent shares of investment of given capital type allocated to different sectors in the economy, they sum to 1.

Next, we compute the product of the production of investment of capital type j by sector i' in country c at time t, $x^c_{j^{i'}t}$, and $\tilde{\omega}^c_{ijt}$ to obtain the dollar value assigned to each industry i, $x^c_{ij^{i'}t}$.

$$x^c_{ij^{i'}t} = \tilde{\omega}^c_{ijt} x^c_{j^{i'}t} \quad \text{if } j \in \text{equipment type.}$$

From sectors to equipment. Notably, in the assignment of flows to different equipment types, the mapping is not one to one. In other words, a sector may produce multiple equipment types, and an equipment type may be produced by different industries. This information is encoded in "bridge tables" that underlie national accounts. For example, using an average bridge table between 2000 and 2018 in the US, one can see that 78% of the investment in computers is produced by the electronics sector, while 22% of it is produced by the ICT sector. Conversely, 28% of the output produced by the electronics sector is computer production, 34% of it is communication equipment, and the rest belong to other equipment categories. To the best of our knowledge, bridge tables are not available across countries. Hence, we use the average allocation of sectorial production to equipment types from the US bridge tables between 2000 and 2018.[16] Appendix Figure 6 provides an illustrative example of the construction of flows of investment from a producing sector i' to equipment type j. Then, we assign this flow

on the job. Then, cross-country variation in tasks for the same occupation, such as that documented in Caunedo, Keller and Shin (2021), can be used to predict tool usage for the same occupation across countries at different stages of development. The task projection is available in slightly more than half of our sample, mostly for middle- and high-income countries.

[16] The total production of a sector allocated to equipment types does not include replacement of used goods or trade margins. Hence, when we impute investment flows from a sector to equipment in other countries (which may include these margins), we are effectively distributing these margins equally across equipment types.

following the imputation procedure described above. Finally, for each demanding sector i, we sum across investment flows from all equipment types that are relevant to the producing sector i',

$$x^c_{ii't} = \sum_{j\in i'} x^c_{ij^{i'}t}.$$

We can then renormalize these investment flows by the total demand for investment in a sector to generate the loadings of the investment network, $\omega_{ii't}$.[17]

Allocation of construction and other sectors' investment flows. There is no information on worker usage of capital goods produced by the construction (i.e., structures), agriculture, nondurables, transportation services and other services sectors. Hence, we use the input–output structure of each country and assign the investment flows from these sectors proportionally to their role as intermediate goods providers for other sectors in the economy.

Denote by $\tilde{\mu}^c_{ii't}$ the share of total intermediate inputs produced by sector i' that are purchased by sector i. For the nonequipment sectors, we compute the dollar value of each entry in the investment table as:

$$x^c_{ii't} = \tilde{\mu}^c_{ii't} x^c_{i't} \quad \text{if } i' \in \text{Non-equipment sector.}$$

The investment network. Finally, to express the investment network in terms of expenditure shares $\omega^c_{ii't}$—such that the columns of the matrix sum to 1— we simply divide each dollar-value entry by its respective column sum, i.e., the total expenses on new capital for any given sector:

$$\omega^c_{ii't} = \frac{x^c_{ii't}}{\sum_{i'} x^c_{ii't}}.$$

3.2 Data Description

Five pieces of data are necessary to construct the investment network: production of investment goods by each sector x^c_{it}, the number of tools per worker in each occupation τ^{oUS}_j, the employment distribution by occupation and sector n^{oc}_{it}, the bridge table to construct equipment–sector flows $x_{ji'}$ and the input–output structure $\tilde{\mu}^c_{ii't}$.Table 2 summarizes the data sources, and a detailed description is provided in Appendix Table 9.

Investment production by sector and input–output tables. We obtain production of gross fixed capital formation (GFCF) by sector from the *use tables* that underlie the measurement of the input-output matrix. The *use tables* contain information on investment production by sector,

[17] Our results are robust to constructing a crosswalk between sectors and equipment types that assigns the total flow from a sector to its most common use. These results are available upon request.

Table 2: Investment Network: Data Sources

Data Description	Source
Investment production by sector	Mensah and de Vries (2023),WIOD, OECD
Sector–commodity bridge	US Bridge tables (BEA)
No. of tools per worker in each occupation	Caunedo *et al.* (2023)
Employment by occupation and sector	IPUMS, ILOSTAT, PIAAC
IO structure	Mensah and de Vries (2023), WIOD, OECD

which is either produced domestically or imported from abroad. For the baseline estimates of the investment network for each country, we consider total production of investment by sector (both domestic and imported) from the *use tables*.[18] We also construct a domestic investment network for each country, which is based on the domestic production of investment by sector. For the 9 countries from sub-Saharan Africa in our sample, we use data provided by Mensah and de Vries (2023). For the remaining countries, we source this information from the World Input Output Dataset (WIOD) and OECD input–output tables.

Employment by occupation and sector. We use the estimates of employment by occupation and sector from the PIAAC survey, IPUMS International and ILOSTAT. For those countries with data available from all sources, we favor PIAAC over IPUMS International and ILOSTAT because the former use more detailed occupational categories.[19]

Country Coverage. Our dataset covers 58 countries at different stages of development, with income levels ranging from \$428 and \$81,599 GDP per capita (PPP). For 20 of these countries, we construct time series of investment networks from 1965 to 2014, and for the 9 countries in the sub-Saharan Africa region, we construct time series of investment networks for from 1990 to 2019. For the remaining 29 countries, the investment network time series covers the period 2000–2014. See Table 9 in the Appendix for a full description.

3.3 Comparison with US Investment Networks

In this section, we compare our estimates of the investment network for the United States to the investment networks constructed by vom Lehn and Winberry (2022) ("*VLW*"), which are based on the capital flows tables from the BEA. We use 2012 as the primary reference year,

[18] GFCF flows are reported in nominal currency, which we deflate using the output PPP prices from Penn World Tables. To abstract from business cycle fluctuations, we hp-filter sectorial GFCF flows.

[19] PIAAC measurement aggregated at the 1-digit level correlates strongly with IPUMS data, Caunedo *et al.* (2021). In IPUMS International, the industry classification does not include disaggregation of equipment sectors within manufacturing. However, detailed industry classifications are available prior to their harmonization procedure. For each country for which we source data from IPUMS, we manually construct crosswalks between the disaggregated (not harmonized) industries and our 10 sectors. In PIAAC and ILOSTAT, sectors are classified according to ISIC Rev.4 or ISIC Rev.3, which we also crosswalk to our 10-sector dataset.

as this is when data on employment distribution by sector and occupation were collected by the PIAAC survey.[20] To control for potential disparities in the sectorial GFCF flows from BEA and WIOD, we apply our methodology using the same estimates of sectorial production of investment following vom Lehn and Winberry (2022). We aggregate their estimates of the investment network for 41 sectors to our 10 sectors, consistently with the ISIC Rev.4 crosswalk presented in Table 8.

Appendix Table 10 compares estimates of (a) the sectorial outdegrees of the investment network, a measure of each sector's relevance as an investment provider to other sectors in the economy, and (b) the homophily of the investment network, a measure of each sector's relevance as a provider of investment for its own sector. The *outdegree* is calculated as the row sum of the entries in the investment network, and the *homophily* is calculated as the diagonal elements of the matrix. Comparing the estimates in the second and third columns of Table 10, Panels (a) and (b), we find that our methodology aligns very well with the estimates from vom Lehn and Winberry (2022), especially considering that the only common inputs are sectorial investment flows.

For further comparison, we regress the elements of our investment network estimates against those from VLW and report the mean squared error (MSE) as a measure of prediction accuracy. The MSE values are 0.005 for 2012, 0.008 for 1992, and 0.017 for 1972, indicating relatively small differences between our estimates and theirs.

3.4 The Investment Network in the Development Spectrum

3.4.1 Investment Network Outdegrees

We begin by documenting the *outdegree* of each sector, which corresponds to the row sum of the investment network. In Table 3, we divide the sample countries into three groups based on their income per capita and report the median sectorial investment network outdegrees for each group.[21]

First, the importance of the construction sector as provider of investment declines with the level of income: The outdegree in low-income countries is 19% higher than that in high-income countries.

Second, the outdegrees of the machinery, transportation (both equipment and services)

[20]The Online Appendix show that the patterns observed in 2012 are largely consistent with those from 1972 and 1992. However, since the occupational composition has changed over time, it is surprising that the assignment works relatively well 20 and 40 years ago.

[21]For cross-country comparisons, we use 2005 as the reference year because this is the year for which PPP deflators are available.

and electronics sectors follow a hump-shaped pattern with respect to income. In contrast, the outdegree of the ICT sector increases 4.5 fold between low- and high-income countries.

Finally, the outdegrees of the other durables and services sectors have a U-shaped relationship with development, while the outdegree of the nondurable sector increases with income.

Table 3: Investment Network Outdegrees

	Low Income	Medium Income	High Income
Agriculture	0.22	0.14	0.08
Construction	3.37	3.11	2.82
Durables	0.49	0.39	0.44
Electronics	0.73	0.91	0.76
ICT	0.27	0.76	1.23
Machinery	1.11	1.34	1.06
Nondurables	0.11	0.18	0.20
Services	1.53	1.39	1.52
Transportation	0.92	1.23	0.96
Trpt Services	0.11	0.15	0.12

Notes: Data for 2005; outdegrees represent the sectorial row sum of the elements of the investment network. Average per capita GDP (PPP): Low income \$2587, medium income \$11,110, high income \$35,056.

3.4.2 Investment Network vs. Input–Output Network

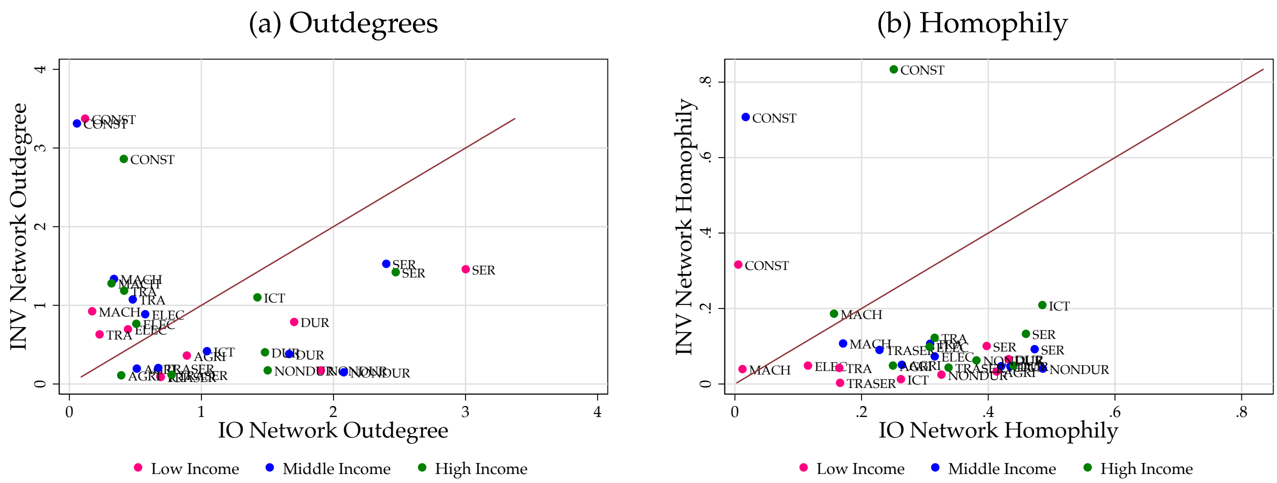
While the characteristics of the investment network were not known prior to this paper, there is considerably more evidence about the features of the input–output structure. Figure 2, Panel (a) documents substantial differences between these two networks, as measured by the median outdegrees across different income levels; see Table 11 in the Appendix for detailed statistics. Figure 2, Panel (b) documents disparities in homophily.

Several patterns emerge. Agriculture is a significant provider of intermediate inputs, but it plays a very minor role as a provider of investment. Construction has the highest outdegree in investment but is consistently among the least important sectors in the production of intermediate inputs for other sectors. Furthermore, the construction sector's investment outdegrees decrease slightly with income, while intermediate input outdegrees increase.

In the electronics sector, both investment and input–output outdegrees exhibit a hump-shaped relation with development, with investment outdegrees being slightly higher. ICT exhibits outdegrees that increase with development, and the qualitative patterns are present in the investment and input–output networks. The magnitudes of the input–output outdegrees are up to 8 times higher than in the investment network for low-income countries and closer to 1.3 times higher for high-income countries. Hence, ICT has a changing role as provider of intermediate inputs and investment across the development spectrum. Most other service sectors (including transportation services) are providers of intermediate inputs rather than of investment in the economy, while the transportation sector is mostly a provider of investment rather than intermediate inputs.

The machinery sector is also generally an important provider of investment, and its investment outdegrees have a hump-shaped relationship with development. In other manufacturing sectors, including durable and nondurable manufacturing goods, the input–output outdegrees are between 2 to 13 times higher than the investment outdegrees, respectively. Interestingly, the outdegrees for durable manufacturing goods fall with development, whereas that of nondurable goods remains stable.

Figure 2: Investment Network vs. Input–Output Network



Notes: Characteristics of the investment network (y-axis) and of the input–output network (x-axis), averages by income group and sector. The *outdegree* of the network corresponds to the row sum of its entries, while *homophily* is the weight of the diagonal term. Countries are grouped according to average per capita GDP (PPP): Low income \$2587, medium Income \$11,110, high income \$35,056.

Next, we compare the homophily of the investment network relative to the input–output network by examining the values of the diagonal entries of each respective matrix. As shown in Figure 2, Panel (b), sectors are significant providers of intermediate inputs for themselves but depend more on other sectors for investment goods. The only exception is the construction sector, which plays little role in intermediate input provision for itself. [22]

[22]These patterns also hold within countries. Appendix Figure 8 compares estimates of the investment network of Korea in 2014 with those of the input–output network. The loadings of the diagonal in the investment network are lower than the loadings of the diagonal in the input–output network.

3.5 Sectorial Influence

Thus far, we have characterized key features of networks across the development spectrum, but to understand the role of different sectors in determining income levels, we describe the sectorial influence, i.e., $\tilde{\eta} \equiv \tilde{\zeta}'\Xi$ in Proposition .4.

Table 4 reports average sectorial influence across income groups. Among the most salient features of influence is the steady decline in the influence of agriculture and a steady increase in the influence of ICT and services as countries develop. This is of course in part driven not only by the characteristics of the network, which map onto the augmented Leontief inverse, Ξ, but also by value added expenditure shares, $\tilde{\zeta}$. Transportation services, electronics, machinery, durable manufacturing goods and construction display a hump-shaped relationship with sectorial influence across the development spectrum. Transportation and nondurable manufacturing goods display declining sectorial influence with income.

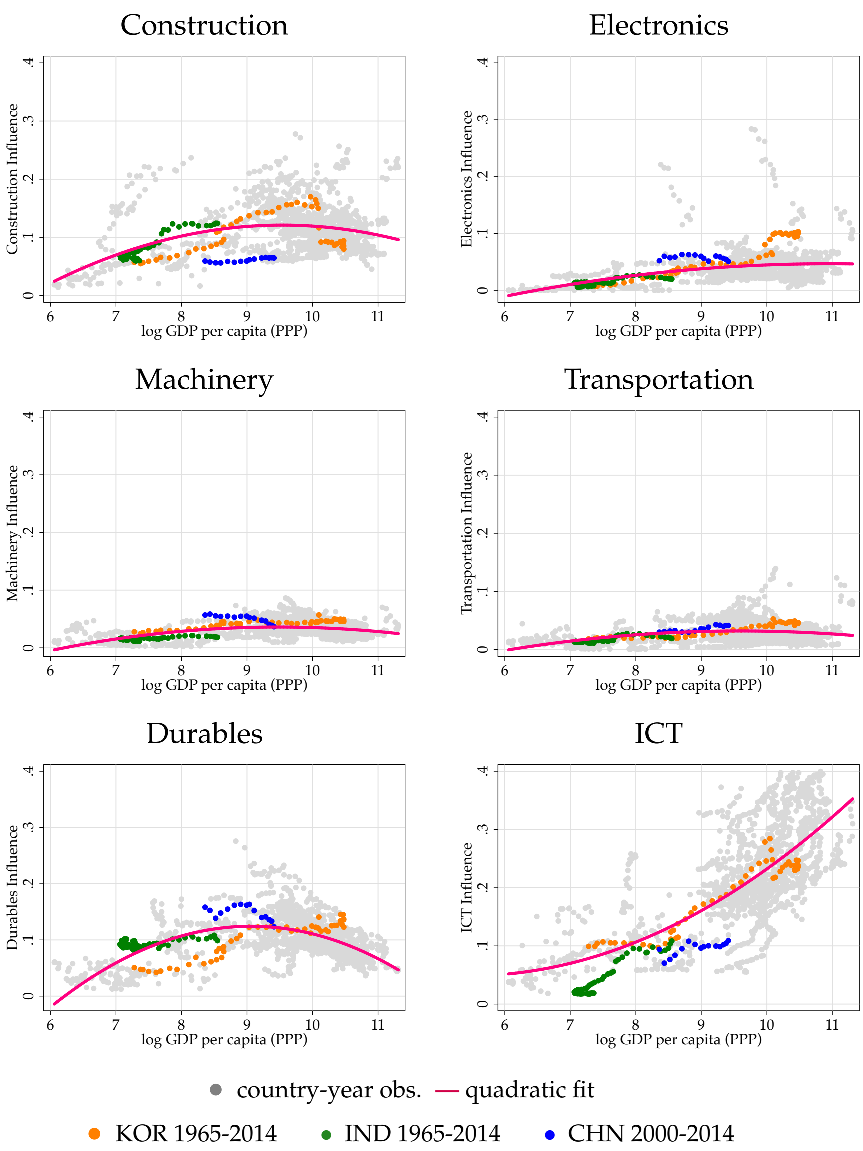
Table 4: Sectorial Influence

	Low Income	Medium Income	High Income
Agriculture	0.23	0.07	0.02
Construction	0.12	0.13	0.11
Durables	0.11	0.12	0.09
Electronics	0.03	0.04	0.03
ICT	0.11	0.21	0.28
Machinery	0.02	0.04	0.02
Nondurables	0.16	0.14	0.08
Services	0.51	0.65	0.68
Transportation	0.03	0.03	0.02
Trpt Services	0.09	0.10	0.07

Notes: This table reports average sectorial influence by income group. Low-income countries have an average per capita GDP (PPP) of 5030, medium-income countries have an average per capita GDP (PPP) of 44,472, and high-income countries have an average per capita GDP (PPP) of 84,671 in 2005.

Prima facie, these patterns could be driven entirely by the sectorial shares of value added, $\tilde{\zeta}$. Hence, we separately report the outdegrees of the augmented Leontief inverse, Ξ (see Table 12 in the Appendix). Comparing these magnitudes to those of sectorial influence, we find for services that influence is mostly driven by sectorial value-added shares. The reason is that the outdegrees of the Leontief inverse for services increase only slightly across the income spectrum. For the remaining sectors, the qualitative patterns of influence correlate with the dynamics of the outdegrees of the Leontief inverse, although the relative magnitudes vary across sectors.

Figure 3: Sectorial Influence along the Development Spectrum



Notes: This table sectorial influence for each country-year observation in our sample. It singles out the paths of sectorial influence for South Korea, India and China. and overlays a quadratic fit of influence on log GDP per capita in the sample.

Figure 3 shows country–year observations (in gray) in our full sample. We highlight in orange the development path of South Korea from 1965 to 2014. The time-series of the path of Korea aligns surprisingly well with the fitted average across the sample. These systematic patterns in the nature of the shifts in the investment network across development levels, albeit beyond the scope of this paper, likely deserve further attention.

Another concern with pooling country–year observations across income levels is that technologies available 50 years ago may not be those available to countries catching up in recent years. Disparities in the path of investment across development experiences could be also further explored with these data. As a first path, we highlight the paths of India and China, two countries that have experienced relative rapid growth since 2000s, when the technologies available for production were arguably different than those available in the 1960s. We find that even for these countries, the path of sectorial influence follows that predicted given their income level. In India, the path of sectorial influence overlays the fitted path across the development spectrum. This is also the case in China, except for construction and ICT, where measured sectorial influence seems to be below the average path for the country's income level.

4 Income Accounting

Differences in the investment network across countries over time or across income levels could prima facie reflect systematic disparities in production technologies. As a first step to highlight the implications of these newly uncovered patterns for income differences across countries, we now combine the structural predictions of the model with our newly constructed measures of the investment network to conduct an income accounting exercise.

4.1 Data Description

To estimate sectorial influence as described in Proposition .4 we need data on sectorial value-added shares in gross output ($\mathbf{\Gamma}$), sectorial value-added shares ($\tilde{\boldsymbol{\zeta}}$), capital shares in value added ($\boldsymbol{\alpha}$), sectorial imported share of investment ($\boldsymbol{\phi}$), sectorial depreciation rates ($\hat{\boldsymbol{\delta}}$), as well as estimates of sectorial productivity for each country in the sample.

To ensure comparability across countries, we rely on data for the year 2005, as it is the only year with available PPP sectorial prices that we use to convert nominal values into real units in the WIOD sample. For those countries not included in this sample, we use GDP PPP price deflators from Penn World Tables (PWT).

Value-added shares in production (Γ) and sectorial value-added shares ($\tilde{\zeta}$). We compute sectorial value-added shares in gross output and sectorial value-added shares using the same data sources as the input-output tables for each country: Mensah and de Vries (2023), WIOD, and OECD.

Capital share in value added (α). We exploit data from PWT version 10.01 to compute the labor expenditure share. We estimate capital shares as residuals from labor expenditure shares, under the assumption of CRS value-added production technologies. The capital expenditure share is computed at the aggregate level and therefore country specific but common across sectors.[23]

[23] Some high- and medium-income countries have data on sectorial capital shares in value added from WIOD. However, we lack this information for more than half of the countries in our sample. For consistency, we use country-specific aggregate capital shares.

Sectorial imported investment shares ($\boldsymbol{\phi}$). For each sector and country, we compute the share of sectorial investment that is sourced from abroad using the *use tables* described in Section 3.2, which contain information on imported and domestic sectorial investment.

Sectorial depreciation rates ($\hat{\delta}$). Estimates of depreciation rates by sector are not available across countries. Given this data limitation, we compute sectorial depreciation rates for the US using data from the Fixed Assets Tables from the BEA. We first compute, for each sector, the associated depreciation rates ($\hat{\delta}_i$) of equipment, structures and intellectual property as the ratio of depreciation over the net stock of each capital type. We then construct a sectorial-level depreciation rate as a weighted average of the sectorial depreciation rate of each capital type, weighted by the share of each type in the total capital stock of the sector. We impose the same depreciation rates for a given sector across countries. Depreciation rates are combined with household discount factors to compute the effective rate of discount in steady state, and we set the household discount factor to $\beta = 0.96$.

"Productivity-like" shifters. As discussed for the model economy, the terms of trade work similarly to a TFP shock across sectors. Both of these can be inferred residually using the structural restrictions of the model. Let this residual be $\boldsymbol{a} \equiv (\mathbf{z} + \alpha\boldsymbol{\phi}\Omega'\tau)$.[24] Then,

$$\boldsymbol{a} = \left(I - \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\right)^{-1} \Xi\Gamma \ln(\boldsymbol{\nu}), \tag{5}$$

with $\ln(\boldsymbol{\nu})$ being a vector of log of sectorial value added. We use this identity to infer relative productivities across sectors and then discipline the level of productivity in an economy (i.e., for a sector) $\boldsymbol{a}$ to match the observed level of income per capita in each country.

4.2 Accounting

Equipped with estimates of sectorial influence and the implied GDP in each country, we answer the following question: How important are cross-country differences in the investment network for observed disparities in income per capita?

Then, we construct a counterfactual estimate of GDP for alternative measures of sectorial influence, driven by alternative assumptions on the investment and intermediate input networks. To facilitate exposition, we repeat the main expression for GDP:

$$\boldsymbol{\eta}^{\text{GDP}}\boldsymbol{a} \equiv \underbrace{\Phi}_{\substack{\text{trade}\\\text{amplification}}} \quad \underbrace{\tilde{\zeta}'}_{\text{exp share in VA}} \quad \underbrace{(I - \tilde{\beta}^{-1}\Gamma\alpha(1-\phi)\Omega - (1-\Gamma)M)^{-1}}_{\substack{\text{augmented}\\\text{Leontief inverse}}}\Gamma\boldsymbol{a}, \tag{6}$$

[24] We could further split these residuals between TFP and the terms of trade effect. Such an exercise requires constructing terms of trade for equipment.

Table 5: Development Accounting

	Income Variance	Contribution of Ω
Baseline	1	
Only Investment Links	0.35	35%
Only Intermediate Inputs Links	0.69	31%
	$\alpha = 1/3$	
Only Investment Links	0.37	37%

Notes: *Baseline* scenario is normalized to 1 and refers to income variances when GDP per capita in each country is given by Equation (6), which matches empirical observations. *Only Investment Links* refers to the scenario with only investment linkages and without intermediate linkages, i.e., $\boldsymbol{\eta}^{\text{GDP}}\boldsymbol{a} \equiv \Phi\tilde{\zeta}'(I - \tilde{\beta}^{-1}\Gamma\alpha(1-\phi)\Omega)\Gamma\boldsymbol{a}$. *Only Intermediate Input Links* refers to the scenario with no investment linkages and only intermediate linkages, i.e., $\boldsymbol{\eta}^{\text{GDP}}\boldsymbol{a} \equiv \Phi\tilde{\zeta}'(I - (1-\Gamma)M)^{-1}\Gamma\boldsymbol{a}$.

Given the non-linear nature of the effect of the investment network on GDP, we study two counterfactual scenarios: One where we only include the investment network and another where we remove the investment network altogether (including its effect through trade amplification Φ). Intuitively, we compute the change in the variance of income to the investment network at two different points. The role of the investment network in explaining income variances can be assessed as an average across these two orderings of the counterfactual exercises.

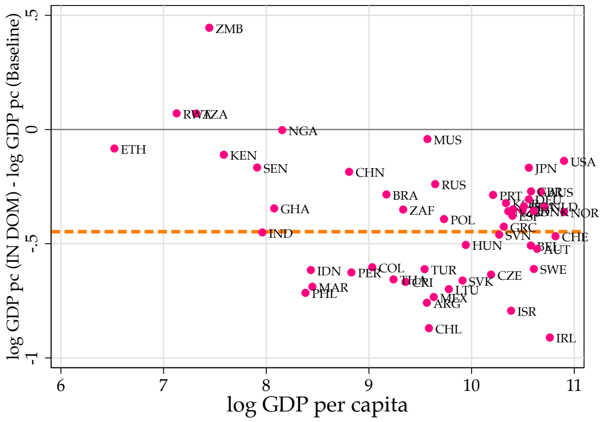
Table 5 presents our results. We normalize the *Baseline* scenario as the model-based income predicted by the model (Equation 6), which matches observed levels of income per capita by construction. When we only include investment links (second row in Table 5), the model predicts 35% of the observed income disparities. This counterfactual summarizes the impact of the network when roundabout effects from intermediate inputs are abstracted away. Part of the variation from investment link is plausibly explained by variation in capital intensity across countries. When we set a common output elasticity across countries to 1/3, we find that the investment network explain an additional 2% of the income variation. This finding suggest a minor role for variation in output elasticities.

When we eliminate the investment network altogether (third row in Table 5), we find that the investment network can account for 31% of the observed disparities. This counterfactual summarizes an economy with no capital, i.e., $\alpha = 0$ and roundabout effects solely from intermediate input linkages. Consistently with Fadinger *et al.* (2022), we find a strong role for these intermediate inputs at baseline.

Averaging across counterfactuals, we conclude that the investment network accounts for 33% of the observed income differences in our sample.

We can benchmark the magnitude of this result relative to the role of measured disparities in capital–output ratios across countries through a standard income accounting exercise, Jones (2016); the Online Appendix presents a full description of these results. Following his method-

Figure 4: Counterfactual: Domestic Investment Links



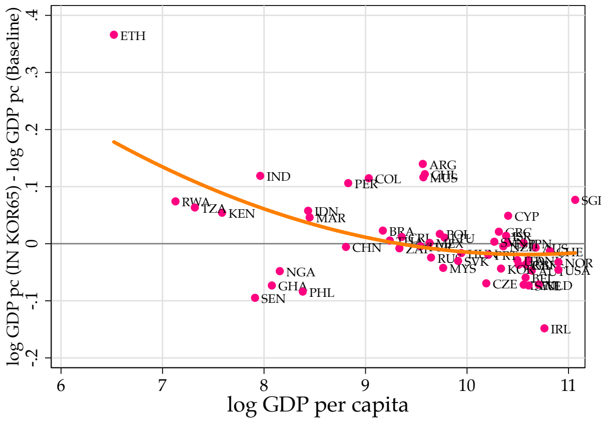
Notes: Changes in income per capita in an economy with only domestic investment links and total intermediate input links, relative to baseline. Countries with outlier income changes were excluded from the graph: Malaysia (log GDP per capita 9.8, change of-179%) and Singapore (log GDP per capita 11.1, change of -141%).

ology, we find that the role of capital in driving income disparities is 13% in our sample. We can alternatively compare the observed variance in output per worker to those generated from measured inputs: following Caselli (2005), we find that the role of measured capital disparities is 7%, while if we also include the covariance between productivity and measured inputs, as in Klenow and Rodríguez-Clare (1997), we find that measured capital disparities explain 17% of income differences. We conclude that including the investment network, i.e., the technology for producing capital, at least doubles the role of the accumulation of capital in determining income disparities.

The role of trade. Given that capital goods (in particular equipment) are produced in a handful of countries (Eaton and Kortum, 2001), it is natural to ask whether trade is a driver of the role of the investment network in determining income disparities. We would expect some role if the incidence of trade differs across countries over the course of development and across equipment types.

Quantitatively, we answer this question by constructing an economy where we eliminate the trade amplification effect, Φ, and the investment network is recomputed to consider only sectorial domestic investment flows. We find that average income per capita falls by 44% in our sample; see Figure 4. The decline in GDP is stronger for rich countries than it is for poor countries, consistent with a decline in the variance of income of 12%; see Appendix Table 13. Hence, trade accounts for one-third of the role of the investment network in determining investment disparities. This relatively muted effect of trade on income variances despite the large flows of imported equipment across the world is in part driven by low heterogeneity in import shares across the income spectrum for many equipment categories; see Appendix Figure 7.

Figure 5: Counterfactual: South Korea's Investment Network in 1965



Notes: Changes in income per capita in an economy with the investment network of Korea in 1965, relative to baseline. Countries with outlier income changes were excluded from the graph: Zambia (log GDP per capita 7.4, change of 74%).

Alternative investment networks. As noted above, investment networks can be interpreted as technologies for the production of new capital goods in the economy. As discussed in Section 2.2, these technologies are the outcome of some endogenous production choice, given endowments, comparative advantage, and (possibly) distortions. One interpretation of the documented disparities in investment networks across countries is that countries differ in the technology frontier over which they choose how to produce investment, i.e., different B_n and ζ_{in} per sector n.

To fix ideas, we focus on network estimates for Korea in 1965 and in 2014; see Appendix Figure 9 . The network becomes more diversified in recent years, while the nondiagonal terms of the network remain important. One can interpret these shifts as an increase in the level of the frontier facing Korea, i.e., using ICT as a base sector, B_n would be rising because ICT shares increase throughout, as well as shifts in the relative intensity of equipment, with movements away from the construction, machinery and services sectors, which were prevalent in 1965.

What would happen to our economies if we were to introduce a technology for investment production that resembles that of Korea at the beginning of its development process in 1965, when its income per capita was \$1450 PPP, more than 30 times lower than it currently is? In terms of the investment network, this exercise is equivalent to giving countries a technological frontier with lower B_n in all sectors and a schedule of ζ_{in} that is relatively concentrated in construction, machinery and services.

Table 6: Counterfactuals: US and Korean Investment Networks

	GDP per capita $\frac{p90}{p10}$
$\Omega_c = \Omega_{USA14}$ for all countries c	0.92
$\Omega_c^{frontier} = \Omega_{USA14}^{frontier}$ for all countries c	1.00
$\Omega_c = \Omega_{KOR65}$ for all countries c	0.85
$\Omega_c^{frontier} = \Omega_{KOR65}^{frontier}$ for all countries c	0.94

Notes: Counterfactual exercises imposing the network of the USA in 2014 on all countries (top row) and the network of Korea in 1965 on all countries (third row). The second and fourth rows show counterfactuals where only the level of the frontier of a given country is imposed on all others and the relative intensities of usage across capital types other than the base group within a sector are kept at baseline. Each entry corresponds to the 90th to 10th percentile ratio of the cross-country income distribution relative to the baseline (year 2015).

Figure 5 shows the differences in counterfactual income levels and the baseline: a positive value indicates an improvement in GDP per capita relative to the observed value, while a negative number indicates a deterioration of GDP per capita relative to the observed value. We find that poorer countries would benefit relatively more from producing with the investment network of Korea in 1965, but the vast majority of economies would suffer from this technology. This finding suggests that economies are shifting investment technologies as they develop, perhaps optimally. Our next section studies the plausibility of these systematic shifts across the development spectrum.

We also run robustness exercises where we impose the composition of domestic and imported shares of capital in Korea in 1965. In that case, the gradient with development is even more negative (see Figure 10 in the Appendix), suggesting that poorer countries are relatively closed and that rich countries' shifts in the investment network are also consistent with their openness in terms of equipment trade.

The role of the frontier. Through the lens of our theory, disparities in the observed networks are driven by both disparities in the frontier technology used in each sector–country and the composition of the investment bundle in each sector. We can isolate the effect of the frontier by constructing counterfactuals where we impose the B_n of a given country and sector in all countries in the sample and then rescale the composition of the bundle of other capital types to maintain their relative composition, i.e., $\zeta_{in}/\zeta_{i'n}$. We do so using two alternative benchmarks: Korea in 1965 for comparison to our previous results and the US in 2014, see Table 6.

We find that when imposing the investment network of Korea on all countries, the ratio of the 90th percentile to the 10th percentile of the GDP per capita distribution falls by 15%, consistent with our previous finding that poor countries benefit relatively more from such a network. If we only impose the frontier associated with each sector (using ICT investment as the baseline sector), we find that the 90/10 ratio falls to 6%. In other words, 2/5th of the decline in cross-country inequality (6%/15%) are due to shifts in the frontier rather than changes in the investment bundle.

Benchmarking results to a given country–year is always arbitrary. One could alternatively imposing the network in the US in 2014, which we interpret as a modern form of production we find similar results. The ratio of the 90th percentile to the 10th percentile of the GDP per capita distribution falls by 8%, partially because poor countries do not benefit as much from this network as they do from that of Korea in 1965. Interestingly, when we only impose the frontier technology associated with the US network in 2014, we find that income disparities remain as in the baseline. Benchmarked this way, the relative intensities of use of capital drive most of the variation in income disparities rather than the level of the frontier.

5 Final Remarks

We have constructed novel measures of the investment network for 58 countries across the development spectrum and time-series estimates that cover the period 1965 to 2014. Our analysis reveals systematic disparities in the sectors' roles as providers of investment goods as economies progress. We also document significant empirical disparities between the investment network and the input–output network for countries at different income levels.

Leveraging our estimates of the investment network across countries at different development stages, we conduct an income accounting exercise, finding that disparities in the investment network can account for 33% of observed differences in income per capita across countries, almost double the effect of capital in the standard income accounting exercise.

We argue that this role relates to systematic shifts in the investment network as countries develop. The extent to which comparative advantage, distortions, or variations in human capital endowments explain shifts in the network is an exciting avenue for future research. Is there a systematic ladder in the type of investment required to transition an economy from low to high income levels? We hope that this work can serve as a foundation for studying this and other critical questions.

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6 Appendix

6.1 Proofs & Derivations

6.1.1 Balanced Growth Path

Proof of Proposition .1. Let us start by defining GDP in the economy, ν, as the value of consumption and investment expenses plus net exports, $p_Y C + \sum p_n x_n + NX = \nu$, in units of consumption.

Definition: *A balanced growth path (BGP) is an allocation where output, consumption, investment and capital in each sector each grow at a constant, possibly different, growth rate.*

Along the BGP

$$g^{\nu} = g^{p_Y} + g^{c} = g^{p^x} + g^{x} = g^{NX},$$

the growth rate of net exports is

$$g^{NX} = g^{p_Y} + g^{\epsilon} = g^{p^f} + g^{\chi^f}.$$

It follows that the growth rate of the terms of trade (considered exogenous) determines the relative growth of real exports and imports whenever trade is balanced.

$$g^{\tau} \equiv g^{p_Y} - g^{p^f} = g^{\chi^f} - g^{\epsilon}. \tag{7}$$

Define g^y as the vector collecting the growth rates of gross output across sectors $g^y = (g^{y_1}, \ldots.., g^{y_N})$. We define g^{ν}, g^m, g^k and g^x analogously. The growth rate of output in each sector grows at a constant rate equal to growth rate of its uses, including consumption, investment and intermediate goods. Feasibility then implies that $g^{m_{in}} = g^{y_i}$, and therefore, given the aggregator of intermediate inputs in sector n, $g^{m_n} = \sum_{i=0}^{N} \mu_{in} g^{y_i}$. In other words, $g^{m_n} = M' g^y$.

Along the BGP, the law of motion for capital requires $g^x = g^k$, where investment includes domestically and foreign sourced investment. Hence,

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi \Omega' g^{\chi^f}$$

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi \Omega' g^{\epsilon} + \phi \Omega' g^{\tau}$$

Note that because of trade balance, the amount of exports in equilibrium equals the amount of imported equipment. The growth rate of exports is in turn equal to the growth rate of final uses along the BGP,

$$g^{\epsilon} = g^{Y} = \boldsymbol{\theta}' g^{y}$$

Finally, the production technology implies that $g^y = \Gamma g^{\tilde{v}} + (1-\Gamma) g^m$, and by definition, $g^{\tilde{v}} = g^z + \alpha g^k + (1-\alpha) g^l$. However, aggregate labor supply is fixed, and along a BGP, the share of labor allocated to each sector is constant (because relative sectorial output is constant). Using the growth rate of capital and collecting the terms with the growth rate of gross output yields $g^y = \Gamma g^z + \Gamma\alpha(1-\phi)\Omega' g^y + \Gamma\alpha\phi\Omega'\theta' g^y + \Gamma\alpha\phi\Omega' g^\tau + (1-\Gamma)M' g^y$. The third term corresponds to the growth rate of exports and is a scalar.[25]

$$g^y = \Xi\Gamma(g^z + \alpha\phi\Omega' g^\tau + \alpha\phi\Omega'\theta' g^y)$$

with a multiplier $\Xi' \equiv (I - \Gamma\alpha(1-\phi)\Omega' - (1-\Gamma)M')^{-1}$, which we call the augmented Leontief inverse throughout the analysis. Solving further,

$$g^y = (I - \Xi\Gamma\alpha\phi\Omega'\theta')^{-1}\Xi\Gamma(g^z + \alpha\phi\Omega' g^\tau).$$

The first inverse on the RHS summarizes the amplifier effect of trade. Defining the vector of loadings into TFP and the terms of trade as $b^y_z = (I - \Xi\Gamma\alpha\phi\Omega'\theta')^{-1}\Xi\Gamma$ and $b^y_\tau = (I - \Xi\Gamma\alpha\phi\Omega'\theta')^{-1}\alpha\phi\Omega'$ proves the result. The loadings on capital are also linear combinations of the growth rate of gross output and the terms of trade.

6.1.2 Equilibrium Outcomes, Open Economy

Proof Proposition (open ec) .2. Use the optimality conditions of the firm, and rewrite the expenses in different intermediate and investment goods as a function of gross output, i.e.,

$$\mu_{ni}(1-\gamma_i) p_{it} y_{it} = p_{nt} m_{nit}$$

$$\alpha_i \gamma_i p_{it} y_{it} = r_{it} k_{it}$$

$$(1-\phi_{jt})\omega_{ji} p^x_{it} x_{it} = p_{jt}\chi^d_{jit}$$

[25] If exports are defined at the sectorial level, instead of from final uses, this term eliminates the aggregator loadings θ. In this case, the terms of trade would need to be defined for tradable goods, instead of the single term we currently have.

Under no arbitrage, the user cost of capital satisfies

$$r_{it} = p^x_{it-1}\left[\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p^x_{it}}{p^x_{it-1}}\right]$$

where $1-\hat{\delta}_i$ corresponds to the adjusted undepreciated value of a unit of capital adjusted along the BGP, i.e., $1-\hat{\delta}_i \equiv \frac{1-\delta_i}{1+g^k_i}$, and $R_t = \beta\frac{U'(c_t)}{U'(c_{t-1})}$ is the interest rate in the economy.

Combining the optimality conditions for capital and investment, as well as the steady-state level of capital

$$\alpha_i\gamma_i p_{it}y_{it} = \left[\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p^x_{it}}{p^x_{it-1}}\right]\frac{p_{jt-1}\chi^d_{jit-1}}{(1-\phi_{jt})\omega_{ji}}\frac{x_{it}}{x_{it-1}}\frac{k_{it}}{x_{it}},$$

which we can use to write the feasibility constraint in each sector n,

$$p_{nt}y_{nt} = p_{nt}c_{nt} + \sum_i p_{nt}\chi^d_{nit} + \sum_j p_{nt}m_{njt}.$$

Then,

$$\zeta_{nt}\frac{y_{nt}}{c_{nt}} = \zeta_{nt} + \sum_i \frac{\alpha_i\gamma_i(1-\phi_{nt})\omega_{ni}}{\frac{1}{R_t}-(1-\hat{\delta}_i)\frac{p^x_{it+1}}{p^x_{it}}}\frac{x_{it+1}}{k_{it+1}}\frac{x_{it}}{x_{it+1}}\frac{p_{it+1}y_{it+1}}{p_{it}y_{it}}\zeta_{it}\frac{y_{it}}{c_{it}} + \sum_j(1-\gamma_j)\mu_{njt}\zeta_{jt}\frac{y_{jt}}{c_{jt}}.$$

This is a system of equations across sectors that can be solved for the Domar weights $\eta_n \equiv \zeta_n\frac{y_n}{c_n}$. Along the BGP, the solution satisfies

$$\left[I - \tilde{\beta}^{-1}\hat{\delta}\Gamma\alpha\Omega(1-\phi) - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{8}$$

where $\tilde{\beta}$ is a vector of effective discount factors, with typical element $\tilde{\beta}_i \equiv \frac{1}{\beta} - (1-\hat{\delta}_i)$, and the vector of depreciation rates contains typical element $\hat{\delta}_i$. ■

Proof Proposition (open ec) .3. The planner's problem associated with our economy is

$$W \equiv \max_{C_t, Y_t, \omega_{int}, \chi_{int}, x_{nt}, k_{nt+1}, m_{int}, \epsilon_t, \epsilon^f_t} \sum_{t=0}^{\infty}\beta^t \ln(C_t)$$

subject to

$$y_{nt} = \left(\frac{\tilde{z}_{nt}\left(\frac{k_{nt}}{\alpha_n}\right)^{\alpha_n}\left(\frac{l_{nt}}{1-\alpha_n}\right)^{1-\alpha_n}}{\gamma_n}\right)^{\gamma_n}\left(\frac{m_{nt}}{1-\gamma_n}\right)^{1-\gamma_n}, \qquad \text{for } \gamma_n \in [0,1],$$

$$k_{nt+1} = x_{nt} + (1-\delta_n)k_{nt},$$

$$x_{nt} = \prod_{i=1}^{N} \left(\frac{\chi_{int}}{\omega_{int}}\right)^{\omega_{int}}, \quad \sum_{in} \xi_{in} \omega_{int}^{\nu_n} = B_n,$$

$$y_{nt} = c_{nt} + \sum_i m_{nit} + \sum_i \chi_{nit}^{d},$$

$$Y_t = \prod_{n=1}^{N} \left(\frac{c_{nt}}{\theta_n}\right)^{\theta_n}, \qquad \sum_{n=1}^{N} \theta_n = 1 \text{ and } \quad \theta_n > 0;$$

$$Y_t = C_t + \epsilon_t, \qquad \epsilon_t - \frac{\epsilon_t^f}{\tau} = 0$$

$$\epsilon_t^f = \prod_{i=1}^{N} \frac{\chi_{it}^{f}}{\phi_i^f}^{\phi_i^f} \qquad \chi_{it}^f = \sum_n \chi_{int}^f,$$

$$\chi_{int} = (\frac{\chi_{int}^d}{1-\phi_i})^{1-\phi_i} (\frac{\chi_{int}^f}{\phi_i})^{\phi_i}.$$

where we have defined $\tilde{z} \equiv \exp z$ for notational convenience.

The envelope condition then yields that

$$\frac{\partial C}{\partial \tilde{z}_{nt}} \tilde{z}_{nt} = \lambda_{nt} y_{nt} \frac{\partial y_{nt}}{\partial \tilde{z}_{nt}} \frac{\tilde{z}_{nt}}{y_{nt}}$$

where λ_n is the Lagrange multiplier associated with the feasibility constraint for good n and the last term in the above equation is simply the elasticity of gross output to productivity, i.e., γ_n. We can rewrite this in terms of the change in welfare, which is proportional to $d \ln(C_t)$ because utility is separable in time.

$$\frac{\partial C_t}{\partial \tilde{z}_{nt}} \frac{\tilde{z}_{nt}}{C_t} = \frac{\nu_t}{C_t} \frac{\lambda_{nt} y_{nt}}{\nu_t} \gamma_n$$

Along the BGP, aggregate consumption is a constant fraction of GDP, ν, and by definition, $\eta_n = \frac{\lambda_{nt} y_{nt}}{\nu_t}$, i.e., the Domar weight.

$$\frac{d \ln C_t}{d \ln \tilde{z}_{nt}} = \frac{\nu_t}{C_t} \eta_{nt} \gamma_n.$$

■

Proof Proposition .4. Use the solution and the definition of ζ_i to solve for relative prices given investment rates.

$$\frac{p_i}{p_j} = \frac{c_j}{c_i} \frac{\zeta_i}{\zeta_j} = \frac{\eta_i}{\eta_j} \frac{y_j}{y_i}$$

These relative prices are useful to define the demand for intermediate inputs, investment and labor as a function of the vector of sectorial gross output. The demand for intermediate inputs follows $(1-\gamma_i)\frac{\eta_i}{\eta_n}y_n = m_{ni}$, while the demand for domestic investment goods is $\frac{1}{\tilde{\beta}_i\hat{\delta}}\frac{x_i}{k_i}(1-\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j = \chi_{ji}$. The demand for imported investment satisfies

$$\frac{1}{\tilde{\beta}_i\hat{\delta}}\frac{x_i}{k_i}(\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{p^f}\nu = \chi^f_{ji}.$$

Total investment in sector i defines the level of the stock of capital as

$$x_i = \prod_j \left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j}\left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{p^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}},$$

, or equivalently, $k_i = \prod_j \left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\left(\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j}\left(\alpha_i\gamma_i\frac{\eta_i}{p^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}}$.

Assume that the supply of labor is inelastic at 1, so the fraction of labor allocated to each sector follows Domar weights adjusted by the sectorial labor expenditure shares in gross output,

$$l_i^\star = \frac{(1-\alpha_i)\gamma_i p_i y_i}{\sum_i(1-\alpha_i)\gamma_i p_i y_i} = \frac{(1-\alpha_i)\gamma_i\eta_i}{\sum_i(1-\alpha_i)\gamma_i\eta_i}.$$

For the purpose of describing final demand, it would be useful to define $\tilde{l}_i = \frac{l_i^\star}{\gamma_i(1-\alpha_i)}$.

Final output in each sector is then

$$y_n = \left[\exp(z_n)\Big(\prod_i\left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\Big(\frac{\eta_n}{\eta_i}y_i\Big)^{1-\phi_i}\Big(\frac{\eta_n}{p^f}\nu\Big)^{\phi_i}\right)^{\omega_{in}}\Big)^{\alpha_n}(\tilde{l}_n)^{1-\alpha_n}\right]^{\gamma_n}\left[\prod_i\left(\frac{\eta_n}{\eta_i}y_i\right)^{\mu_{in}}\right]^{1-\gamma_n}$$

Taking logs and writing output in matrix form, we obtain

$$\ln(\mathbf{y}) = \Gamma\boldsymbol{z} + \boldsymbol{\iota} + \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\ln(\nu) + \Gamma\alpha(1-\boldsymbol{\phi})'\Omega'\ln(\mathbf{y}) + (1-\Gamma)M'\ln(\mathbf{y})$$

where each element of the vector $\boldsymbol{\iota}$ can be described as $\iota_n \equiv \gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \gamma_n\alpha_n\sum_i(1-\phi_i)\omega_{in}\ln(\frac{\eta_n}{\eta_i}) + \gamma_n\alpha_n\sum_i\phi_i\omega_{in}\ln(\frac{\eta_n}{p^f}) - \gamma_n\alpha_n\sum_i\omega_{in}\ln(\tilde{\beta}_i\hat{\delta}_i) + (1-\gamma_n)\sum_i\mu_{in}\ln(\frac{\eta_n}{\eta_i})$.

The solution for gross output is then

$$\ln(\mathbf{y}) = \Xi\Gamma\boldsymbol{z} + \Xi\boldsymbol{\iota} + \Xi\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\ln(\nu) \tag{9}$$

where the multiplier on sectorial productivity is $\Xi \equiv (I - \Gamma\alpha(I-\boldsymbol{\phi})'\Omega' - (1-\Gamma)M')^{-1}$. Let the price level of the economy be normalized to $p=1$; then, aggregate value added is $\nu = \frac{p_n y_n}{\eta_n}$ for any n. We can compute a geometric average of each of the terms using the expenditure shares of consumption and investment $\tilde{\zeta}_n \equiv \frac{p_n \hat{c}_n}{v} + \frac{p_n x_n^d}{v}$ as weights. Note that $p_n \hat{c}_n$ is the value of final uses from sector n that are allocated to aggregate consumption (these values can be split due to the CRS aggregator for final uses). Hence, weights sum to 1 since trade is balanced.

$$\ln(v) = \sum_n \tilde{\zeta}_n \ln(p_n) + \sum_n \tilde{\zeta}_n \ln(y_n) - \sum_n \tilde{\zeta}_n \ln(\eta_n).$$

Given a CRS aggregator of sectorial output, the price index for final goods satisfies $\ln(p) = \sum_n \tilde{\zeta}_n \ln(p_n)$. Because final output is the numeraire, the log of the price index equals zero, and therefore, the first term in the expression for value added drops out. The weighting of the terms in the sum also includes investment shares in value added. Investment shares are proportional to consumption shares in value added whenever sectorial value-added shares are proportional to consumption shares across sectors. This is by construction the assumption in canonical models of input–output linkages without capital, and we assume that feature here.[26]

We have characterized the solution to the last two terms in Equations 8 and 9.

$$\ln(v) = \tilde{\boldsymbol{\zeta}}' \Xi (\Gamma \boldsymbol{z} + \boldsymbol{\iota} + \Gamma \alpha \boldsymbol{\phi}' \boldsymbol{\Omega}' \ln(v)) - \sum_n \tilde{\zeta}_n \ln(\eta_n) \tag{10}$$

where we can define the elasticity of value to sectorial TFP as $\tilde{\eta} \equiv \tilde{\boldsymbol{\zeta}}' \Xi$. Unlike the Domar weight, these elasticities are not adjusted by the investment rate.

Because of the presence of tradable investment goods we obtain an additional amplification (as in Jones (2011) for tradable intermediate inputs). The reason is that as productivity increases within the economy, export capacity improves, and due to trade balance, this implies higher investment imports. The stronger the dependence on imported equipment and the intensity of capital use, the stronger this amplification channel is.

$$\ln(v) = \left(I - \tilde{\boldsymbol{\zeta}}' \Xi \Gamma \alpha \boldsymbol{\phi}' \boldsymbol{\Omega}'\right)^{-1} \left[\tilde{\boldsymbol{\zeta}}' \Xi (\Gamma \boldsymbol{z} + \boldsymbol{\iota}) - \sum_n \tilde{\zeta}_n \ln(\eta_n)\right] \tag{11}$$

Unpacking the vectors, $\tilde{\zeta}_n = \tilde{\eta}_n - \sum_j \gamma_j \alpha_j (1 - \phi_j) \omega_{nj} \tilde{\eta}_j - \sum_j (1 - \gamma_j) \mu_{nj} \tilde{\eta}_j$

$$\sum_n \tilde{\zeta}_n \ln(\mu_n) = \sum_n \tilde{\eta}_n \ln(\eta_n) - \sum_n \sum_j \gamma_n \alpha_n (1 - \phi_n) \omega_{nj} \tilde{\eta}_j \ln(\mu_n) - \sum_n \sum_j (1 - \gamma_n) \mu_{nj} \tilde{\eta}_j \ln(\mu_n)$$

[26] Alternatively, one can set up the economy so that investment in different capital types is produced through the final good. This economy would also allow us to define the price of value added as a function of sectorial prices in a way that they drop out from the expression above while allowing for investment shares that need not be proportional to consumption shares. The undesirable feature of this economy is that sectors producing for final production and intermediate inputs are decoupled from those producing investment.

Now consider the term $\tilde{\eta}\iota$

$$\sum_n \tilde{\eta}_n \iota_n = \sum_n (\tilde{\eta}_n \gamma_n (1-\alpha_n)\ln(\tilde{l}_n) + \tilde{\eta}_n \gamma_n \alpha_n \sum_j (1-\phi_j)\omega_{jn} \ln(\frac{\eta_n}{\eta_j}) + \gamma_n \alpha_n \sum_j \phi_j \omega_{jn} \ln(\frac{\eta_n}{p^f})$$
$$+\tilde{\eta}_n (1-\gamma_n) \sum_j \mu_{jn} \ln(\frac{\eta_n}{\eta_j})) - \gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)$$

which can be rewritten as

$$\sum_n \tilde{\eta}_n \iota_n = \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)\ln(\tilde{l}_n) + \sum_n \tilde{\eta}_n (\gamma_n \alpha_n + 1 - \gamma_n)\ln(\eta_n) - \gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)$$
$$- \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \left((1-\phi_j)\ln(\eta_j) + \phi_j \ln(p^f) \right) - \sum_n \tilde{\eta}_n (1-\gamma_n) \sum_j \mu_{jn} \ln(\eta_j)$$

Therefore, the difference in the last two terms in the expression for value added are

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n)) - \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(p^f)$$
$$-\gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)$$

The last term can be written as a function of the terms of trade for imported equipment j, $\ln(\tau_j) = \ln(p) - \ln(p^f)$. Because the final good is the numeraire, p=1. Hence,

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n)) + \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(\tau_j)$$
$$-\gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)$$

which proves our result. ■

6.2 Alternative Features of the Model Economy

6.2.1 Technology Choices, General Setup.

We populate the economy by a continuum of firms that produce investment goods for each sector. These firms maximize profits by choosing the amount of investment in each equipment type and the intensity of use of each equipment for production following

$$\max_{\omega_{int}, \chi_{int}} r_{nt} x_{nt} - \sum_i p_{it} \chi_{int}$$

subject to

$$x_{nt} = \sum_{i=1}^{N} \left(\omega_{int} \chi_{int}^{\sigma_n} \right)^{\frac{1}{\sigma_n}}, \tag{12}$$

$$\sum_i \xi_{in} \omega_{int}^{\nu_n} = B_n \tag{13}$$

The production technology is a generalization of the investment aggregator described in Equation 1.

The optimal (interior) choices of firms are characterized by two conditions:

$$\left(\frac{\chi_{jnt}}{\chi_{int}}\right)^{1-\sigma_n} = \frac{\omega_{jnt}}{\omega_{int}} \frac{p_{it}}{p_{jt}} \tag{14}$$

$$\left(\frac{\chi_{int}}{\chi_{jnt}}\right)^{\sigma_n} = \frac{\xi_{in}}{\xi_{jnt}} \left(\frac{\omega_{int}}{\omega_{jnt}}\right)^{\nu_n - 1} \tag{15}$$

Substituting 14 into 15, we obtain

$$\frac{\chi_{int}}{\chi_{jnt}} = \left(\frac{\xi_{in}}{\xi_{jnt}} \left(\frac{p_{it}}{p_{jt}}\right)^{\nu_n - 1}\right)^{\frac{1}{\sigma_n \nu_n + 1 - \nu_n}} \tag{16}$$

as well as

$$\frac{\omega_{jnt}}{\omega_{int}} = \left(\frac{\xi_{jnt}}{\xi_{in}}\right)^{\frac{1-\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}} \left(\frac{p_{jt}}{p_{it}}\right)^{\frac{\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}} \tag{17}$$

Hence, if $\sigma_n \nu_n - (\nu_n - 1) < 0$, we obtain an interior solution. This is the same as requiring that $\nu_n > 1/(1 - \sigma_n)$. Such a condition requires more curvature in the technology choice than in the investment aggregator. As in Caselli and Coleman (2006), if $\sigma_n < 0$, firms choose to increase the efficiency of the relatively expensive factor, while if $\sigma_n > 0$, they increase the efficiency of the relatively cheap factor. Furthermore, the relative demand for a particular investment type decreases in its price.

This economy reduces to our benchmark economy as we take the limit when $\sigma_n \to 0$. In that case, expenditure shares in the investment aggregators are simply the parameters characterizing the shape of the production possibility frontier in each economy $\omega_{int} \propto \xi_{in}^{\frac{1}{1-\nu_n}}$ and independent of relative prices.

6.2.2 The Investment Network in an Economy with Distortions.

Consider an economy with distortions, which introduces wedges in the price of investment, $(1 + \tau^i)$. These wedges could be policy distortions, market power, etc. For the purpose of this analysis, we are agnostic about the source of the gap between output prices and marginal costs

$$\max_{\omega_{int}, \chi_{int}} r_{nt} x_{nt} - \sum_i (1+\tau_i) p_{it} \chi_{int}$$

subject to

$$x_{nt} = \prod_{i=1}^{N} \left(\chi_{int}^{\omega_{int}} \right),$$

$$\sum_i \xi_{in} \omega_{int}^{\nu_n} = B_n$$

Note that the choice of loadings onto the production technology are as in the main paper, $\omega_{int} \approx \xi_{in}^{\frac{1}{1-\nu_n}}$. However, the quantities of investment change

$$\frac{\chi_{int}}{\chi_{jnt}} = \frac{\xi_{in}}{\xi_{jnt}}^{\frac{1}{1-\nu_n}} \left(\frac{p_{it}(1+\tau_i)}{p_{jt}(1+\tau_j)} \right) \tag{18}$$

In other words, with a Cobb–Douglas investment aggregator, distortions on the cost of investment affect the quantities demanded of each investment type, but not directly the loadings in the investment network.

It is only when allowing some substitutability or complementarity between investment types into the production of investment that relative prices affect the loadings of the network. From Equation 17 (and with parameters that ensure an interior solution), one can see that the loading of the investment aggregator in a sector is relatively lower for sectors with stronger distortions (higher τ).

6.2.3 Implications of the Investment Network for Steady-State Allocations

To highlight the implications of the investment network for steady-state allocations, we work with a simplified closed economy with only two sectors that produce for consumption and investment; there is no labor or intermediate input. Details of this economy follow in the Online Appendix. The investment network also has implications for the speed of convergence to the BGP. Whereas a full analysis of the transition dynamics is beyond the scope of the current paper, we present comparative statics on that speed of convergence.

Table 7 shows alternative parameterizations of the model economy and its implications for steady-state levels of capital of each type, a measure of aggregate consumption and the share of gross output from each sector devoted to investment in the capital used within the sector, $\kappa_{ii} = \frac{\chi_{ii}}{\chi_{ii}+\chi_{ij}}$. The dynamics of this economy are characterized not only by the capital used in

Table 7: Comparative Statics: Two-Sector, Two-Capital Economy

Baseline	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	0.60	3.42	0.73	0.06	0.75	0.61
$\omega_{11} = 0.5$	1.56	1.56	0.92	0.50	0.50	1.00
$\omega_{11} = 0.85$	2.26	1.63	0.98	0.64	0.76	1.01

$z = [1.1 \;\; 1]$	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	0.54	3.94	0.69	0.05	0.78	0.65
$\omega_{11} = 0.5$	2.41	1.35	1.04	0.64	0.35	0.96
$\omega_{11} = 0.85$	3.44	0.81	1.12	0.90	0.62	0.91

$\alpha = [0.2 \;\; 0.2]$	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	2.66	0.58	0.81	0.45	0.17	0.41
$\omega_{11} = 0.5$	0.89	0.89	0.85	0.50	0.50	0.88
$\omega_{11} = 0.85$	1.24	0.91	0.88	0.63	0.77	1.06

Notes: The baseline economy is parameterized with identical sectors, $\omega_{ii} = 0.5, \alpha = 0.3, z = 1, and \delta = 0.7$. The rate of convergence is computed as the average half-life of the system using the largest eigenvalue of the Jacobian to bound this speed. Convergence rates are relative to the baseline economy with a symmetric investment network, $\omega_{11} = 0.5$.

each sector but also by the share of gross output allocated to investment in new capital goods of each type. The allocation of output across sectors is a state, a result reminiscent of Acemoglu and Guerrieri, 2008 (albeit with a unique capital good). We also report the speed of convergence of the system to the steady state, computed as the average half-life of the process using the largest eigenvalue of the Jacobian to bound this speed. Baseline levels are normalized to 1 so that convergence is a measure of the relative size of the largest eigenvalue across specifications. Our baseline computation sets a homogeneous investment network with loadings equal to 0.5 for each investment type in each sector. We also set an effective depreciation rate of 7% (accounting for capital obsolescence) and a discount factor of 4%. Preferences are assumed to impose equal loadings on each type of consumption. Finally, the production technology is parameterized with a unit productivity z and a capital expenditure share of 0.3.

The row labelled $\omega_{11} = 0.5$ in the top panel of Table 7 shows results for this baseline parameterization. Given that technologies are identical across sectors, the allocation of investment across sectors is endogenously identical, $\kappa_{ii} = 0.5$. We then run comparative statics around the dependence of sector 1 on the production of investment from this same sector. When sector 1's dependence on itself increases $\omega_{11} = 0.85$, the stock of capital in that sector increases (due to its higher marginal product), and through roundabout effects, capital in the other sector also increases. Aggregate consumption and the speed of convergence increase. However, when ω_{11} falls, the capital in steady state in that sector falls while capital in the second sector increases.

This substitution is optimal in attempts to sustain higher steady-state aggregate consumption. This higher steady-state capital in the second sector implies that the share of investment allocated to own production in sector 1 falls substantially. Importantly, aggregate consumption falls by slightly more than 20%, and the speed of convergence is 61% of its baseline level.

The second panel of Table 7 runs identical exercises but with a technology that, at baseline, is 10% more productive in sector 1 than in sector 2. This difference in productivity implies that the steady-state level of capital in the productive sector almost doubles relative to the unproductive sector. The speed of convergence of this system is 4% slower than that when the investment network is homogeneous $\omega_{ii} = 0.5$ and productivities are the same across sectors. Interestingly, increasing the weight of the diagonal in the sector that is more productive can slow the rate of convergence, despite sustaining higher aggregate consumption in steady state.

The third panel of Table 7 analyzes an economy as in Panel 1 but sets the capital share in production to 0.2 instead of 0.3, i.e., reducing the marginal product of capital. Unsurprisingly, the speed of convergence in this economy is 12% slower than in the benchmark, and the steady-state level of capital and aggregate consumption are lower. A noticeable difference from the economy in Panel 1 is that when the capital shares are lower (and identical across sectors), less weight on the diagonal term for sector one increases its stock of capital in steady state. In our first exercise, this same comparative statics lead to a lower stock of capital. These nonlinear responses are associated with relative factor intensities, which are characterized not only by the output elasticity to capital, α_i, but also the investment elasticity to each investment type through the investment network. These intensities in turn affect relative output prices and therefore, the value of the marginal product of capital in each sector.

6.3 Data Appendix

Table 8: Aggregate Sector Definition

ISIC Rev.4 Code	ISIC Rev.4 Description	Aggregate Sector
A01	Crop and animal production, hunting and related service activities	Agriculture
A02	Forestry and logging	
A03	Fishing and aquaculture	
F	Construction	Construction
C26	Manufacture of computer, electronic and optical products	Electronics
C27	Manufacture of electrical equipment	
C18	Printing and reproduction of recorded media	ICT
J58	Publishing activities	
J59_J60	Motion picture, video and television program production, broadcasting activities	
J61	Telecommunications	
J62_J63	Computer programming, consultancy and related activities; information service activities	
M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities	
M71	Architectural and engineering activities; technical testing and analysis	
M72	Scientific research and development	
M73	Advertising and market research	
M74_M75	Other professional, scientific and technical activities; veterinary activities	
N	Administrative and support service activities	
C28	Manufacture of machinery and equipment n.e.c.	Machinery
C33	Repair and installation of machinery and equipment	
C10-C12	Manufacture of food products, beverages and tobacco products	Manufacturing
C13-C15	Manufacture of textiles, wearing apparel and leather products	
C16	Manufacture of wood and of products of wood and cork, except furniture	
C17	Manufacture of paper and paper products	
C19	Manufacture of coke and refined petroleum products	
C20	Manufacture of chemicals and chemical products	
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	
C22	Manufacture of rubber and plastic products	
C23	Manufacture of other non-metallic mineral products	
C24	Manufacture of basic metals	
C25	Manufacture of fabricated metal products, except machinery and equipment	
C31_C32	Manufacture of furniture; other manufacturing	
B	Mining and quarrying	Not included
D35	Electricity, gas, steam and air conditioning supply	Services
E36	Water collection, treatment and supply	
E37-E39	Sewerage; waste collection	
G46	Wholesale trade, except of motor vehicles and motorcycles	
G47	Retail trade, except of motor vehicles and motorcycles	
I	Accommodation and food service activities	
K64	Financial service activities, except insurance and pension funding	
K65	Insurance, reinsurance and pension funding, except compulsory social security	
K66	Activities auxiliary to financial services and insurance activities	
L68	Real estate activities	
O84	Public administration and defense; compulsory social security	
P85	Education	
Q	Human health and social work activities	
R_S	Other service activities	
T	Activities of households as employers	
U	Activities of extraterritorial organizations and bodies	
C29	Manufacture of motor vehicles, trailers and semi-trailers	Transportation
C30	Manufacture of other transport equipment	
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles	
H49	Land transport and transport via pipelines	
H50	Water transport	
H51	Air transport	
H52	Warehousing and support activities for transportation	
H53	Postal and courier activities	

Notes: Crosswalk between ISIC 4 Sectorial classification and our 10-sector disaggregation.

Figure 6: Example: USD value of Computers used by Industry

USD value of computers' investment

(A)	(B)	(C)	(D)	(E)
Commodity	*How much investment (in USD) is produced by Electronics?* [Source: Use Tables WIOD, MDV, OECD]	Equipment type	*Of the total value of production of investment in Electronics, what share are computers, what share are medical equipment?* [Source: Bridge Files US]	USD value of investment by equipment type
Electronics	100	Medical Equipment	30%	30
		Computers	70%	70

Computers by occupation and industry

(1)	(2)	(3)	(4)	(5)	(6)
Industry	Occupation	Nr. of computers used by worker [Source: CJK]	Nr. of workers by Occupation and Industry [Source: ILOSTAT, IPUMS, PIAAC]	Total nr. of computers by Occupation and Industry (3)*(4)	Share of total computers used by Occupation and Industry (5)/Total (5)
Services	Mechanics	1	10	10	4%
	Managers	3	20	60	26%
Transportation	Mechanics	1	40	40	17%
	Managers	3	40	120	52%
Total				230	100%

USD value of computers' investment by industry

Industry	Occupation	USD value of computers used by Occupation and Industry (E)*(6)	**USD value of computers used by Industry**
Services	Mechanics	3	**21**
	Managers	18	
Transportation	Mechanics	12	**49**
	Managers	37	

Notes: Illustrative example for the allocation rule of sectorial investment to equipment types investment, and into sectorial investment demand.

In the example of Figure 6, the starting point is a total investment of USD 100 in the Electronics sector, (from use tables by WIOD, OECD and Mensah and de Vries (2023)). This investment is split between two types of equipment: 30% is allocated to medical equipment and 70% to computers, as indicated by data from "Bridge tables" in the US. This allocation results in USD 30 being invested in medical equipment and USD 70 in computers. The analysis then examines how this USD 70 investment in computers is utilized across different industries and occupations, with data on the relative intensity of computers used by workers in different occupations (Caunedo *et al.*, 2023) and the number of workers per industry and occupation (from ILOSTAT, IPUMS, and PIAAC). In the Services industry, mechanics use 1 computer per worker, with a total of 10 workers, leading to 10 computers being used in this occupation. Managers in the Services industry, who use 3 computers each, total 20 workers, resulting in 60 computers.The allocation of computers to managers and mechanics should be interpreted in relative terms, i.e. managers use 3 times as many computers as mechanics do. Similarly, in the Transportation industry, mechanics use 1 computer per worker with 40 workers, leading to 40 computers, while managers use 3 computers each across 40 workers, resulting in 120 computers. Altogether, these figures yield a total of 230 computers across both industries. The share of total computers used is then calculated for each occupation and industry, with mechanics in the Services industry using 4% of the total, and managers using 26%. Mechanics in the Transportation industry use 17% of the computers, while managers use 52%. These percentages are applied to the total USD 70 investment in computers, yielding a USD 3 investment for Services mechanics, USD 18 for Services managers, USD 12 for Transportation mechanics, and USD 37 for Transportation managers. Consequently, the final USD value of computers used by the Services industry totals USD 21, and by the Transportation industry, USD 49.

Table 9: Country Sample and Data Sources

	Country	GDP per capita 2005 (PPP)	Use-Tables; Input-Output Matrix VA Shares, GFCF imported share	Employment by Occup. and Sector	Investment Network
			Source	*Source*	*Available Years*
1	Ethiopia	679	MDV	ILOSTAT	1990-2019
2	Rwanda	1246	MDV	ILOSTAT	1990-2019
3	Tanzania	1507	MDV	ILOSTAT	1990-2019
4	Zambia	1710	MDV	IPUMS	1990-2019
5	Kenya	1972	MDV	ILOSTAT	1990-2019
6	Cambodia	2048	OECD	ILOSTAT	2005-2015
7	Senegal	2728	MDV	IPUMS	1990-2019
8	India	2872	WIOD	IPUMS	1965-2000; 2000-2014
9	Vietnam	3128	OECD	IPUMS	2005-2015
10	Ghana	3219	MDV	ILOSTAT	1990-2019
11	Nigeria	3481	MDV	IPUMS	1990-2019
12	Philippines	4366	OECD	IPUMS	2005-2015
13	Indonesia	4602	WIOD	IPUMS	2000-2014
14	Morocco	4672	OECD	IPUMS	2005-2015
15	China	6681	WIOD	IPUMS	1965-2000; 2000-2014
16	Peru	6832	OECD	PIAAC	2005-2015
17	Colombia	8367	OECD	ILOSTAT	2005-2015
18	Tunisia	9353	OECD	ILOSTAT	2005-2015
19	Brazil	9610	WIOD	IPUMS	1965-2000; 2000-2014
20	Thailand	10293	OECD	IPUMS	2005-2015
21	South Africa	11311	OECD	IPUMS	2005-2015
22	Costa Rica	11580	OECD	IPUMS	2005-2015
23	Turkey	13941	WIOD	PIAAC	2000-2014
24	Argentina	14247	OECD	ILOSTAT	2005-2015
25	Mauritius	14325	MDV	IPUMS	1990-2019
26	Chile	14534	OECD	PIAAC	2005-2015
27	Mexico	15230	WIOD	PIAAC	1965-2000; 2000-2014
28	Russia	15450	WIOD	PIAAC	2000-2014
29	Poland	16838	WIOD	PIAAC	2000-2014
30	Malaysia	17412	OECD	IPUMS	2005-2015
31	Lithuania	17646	WIOD	ILOSTAT	2000-2014
32	Slovakia	20168	OECD	PIAAC	2000-2014
33	Hungary	20819	WIOD	PIAAC	2000-2014
34	Czechia	26624	WIOD	PIAAC	2000-2014
35	Portugal	27149	WIOD	IPUMS	1965-2000; 2000-2014
36	Slovenia	28821	OECD	PIAAC	2000-2014
37	Greece	30138	WIOD	PIAAC	1965-2000; 2000-2014
38	South Korea	30784	WIOD	PIAAC	1965-2000; 2000-2014
39	New Zealand	31485	OECD	PIAAC	2005-2015
40	Israel	32358	OECD	PIAAC	2005-2015
41	Spain	32769	WIOD	PIAAC	1965-2000; 2000-2014
42	Cyprus	33025	OECD	ILOSTAT	2005-2015
43	Italy	36167	WIOD	PIAAC	1965-2000; 2000-2014
44	France	36651	WIOD	PIAAC	1965-2000; 2000-2014
45	Japan	38466	WIOD	PIAAC	1965-2000; 2000-2014
46	Germany	38475	WIOD	PIAAC	1965-2000; 2000-2014
47	Belgium	39220	WIOD	PIAAC	1965-2000; 2000-2014
48	United Kingdom	39308	WIOD	PIAAC	1965-2000; 2000-2014
49	Denmark	40344	WIOD	PIAAC	1965-2000; 2000-2014
50	Sweden	40381	WIOD	PIAAC	1965-2000; 2000-2014
51	Austria	41678	WIOD	ILOSTAT	1965-2000; 2000-2014
52	Australia	43333	WIOD	ILOSTAT	1965-2000; 2000-2014
53	Netherlands	44662	WIOD	PIAAC	1965-2000; 2000-2014
54	Ireland	47211	WIOD	PIAAC	1965-2000; 2000-2014
55	Switzerland	49859	WIOD	IPUMS	2000-2014
56	Norway	54200	WIOD	PIAAC	2000-2014
57	United States	54210	WIOD	PIAAC	1965-2000; 2000-2014
58	Singapore	63949	OECD	PIAAC	2005-2015

Notes: Country-year coverage and main sources of data.

Table 10: Comparison with vom Lehn and Winberry (2022), 2012

(a) Outdegrees

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	1.65	1.31
Durables	0.30	0.24
Electronics	0.90	0.64
ICT	3.02	3.57
Machinery	1.42	1.94
Nondurables	0.04	0.03
Services	1.16	1.06
Transportation	1.35	1.10
Transportation Services	0.16	0.10

(b) Homophily

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	0.04	0.03
Durables	0.04	0.05
Electronics	0.12	0.07
ICT	0.43	0.61
Machinery	0.18	0.26
Nondurables	0.01	0.005
Services	0.11	0.12
Transportation	0.11	0.06
Transportation Services	0.03	0.01

Notes: Comparison between investment network estimates in year 2012, corresponding to the year in which we fix the occupational composition of the labor force. This table reports the row sum of the networks, *outdegrees*, as well as the weight of the diagonal in the network, *homophily*.

Table 11: Investment Network vs. Input–Output Network Outdegrees

	Low Income		Medium Income		High Income	
	INV	IO	INV	IO	INV	IO
Agriculture	0.36	0.89	0.20	0.51	0.11	0.39
Construction	3.37	0.12	3.31	0.06	2.86	0.41
Durables	0.79	1.70	0.38	1.66	0.40	1.48
Electronics	0.70	0.44	0.89	0.57	0.77	0.51
ICT	0.09	0.69	0.42	1.04	1.10	1.42
Machinery	0.92	0.17	1.34	0.34	1.28	0.32
Nondurables	0.17	1.90	0.15	2.08	0.17	1.50
Services	1.46	3.00	1.53	2.40	1.42	2.47
Transportation	0.63	0.23	1.07	0.48	1.19	0.41
Trpt Services	0.10	0.69	0.20	0.67	0.12	0.77

Notes: Data for 2005; outdegrees represent sectorial row-sum of the elements of the investment network. Average per capita GDP (PPP): Low income \$2587, medium income \$11,110, high income \$35,056.

Table 12: Outdegrees: adjusted-Leontief inverse

	Low Income	Medium Income	High Income
Agriculture	1.64	0.93	0.70
Construction	1.15	1.18	1.06
Durables	1.42	1.52	1.23
Electronics	0.60	0.75	0.71
ICT	1.31	1.64	2.25
Machinery	0.61	0.80	0.61
Nondurables	1.59	1.47	1.00
Services	3.37	3.36	3.56
Transportation	0.55	0.65	0.51
Trpt Services	0.98	1.06	1.07

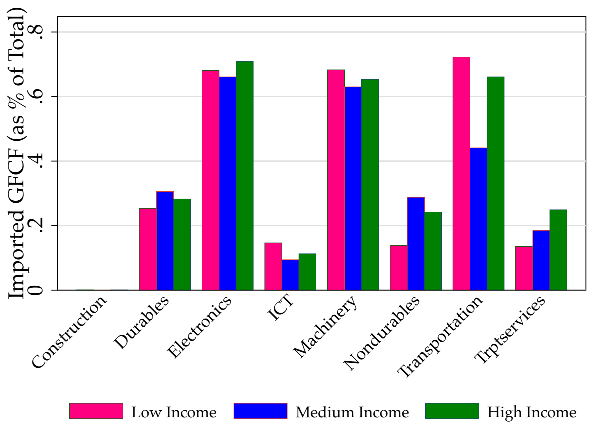
Notes: Data for 2005; outdegrees represent the sectorial row sum of the elements of the investment network. Average per capita GDP (PPP): Low income \$2587 , medium income \$11,110, high income \$35,056.

Table 13: Impact of Trade

	Income Variance	Contribution of Ω
Baseline	1	
Only Domestic Investment Links	0.88	12%

Notes: *Baseline* scenario is normalized to 1 and refers to income variances when GDP per capita in each country is given by Equation (6), which matches the observed value. *Only Domestic Investment Links* refers to the scenario with intermediate inputs linkages and only investment linkages that are sourced domestically, i.e., $\boldsymbol{\eta}^{\text{GDP}}\boldsymbol{a} \equiv \tilde{\zeta}'(I - \tilde{\beta}^{-1}\Gamma\alpha\Omega^{DOM} - (1-\Gamma)M)^{-1}\Gamma\boldsymbol{a}$.

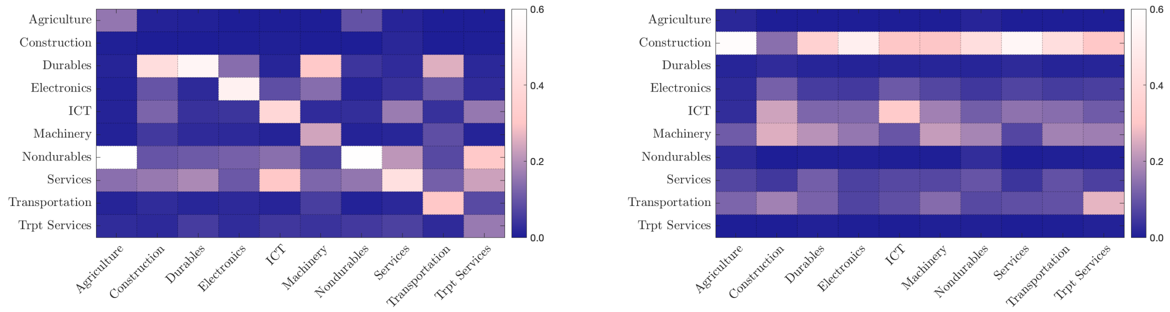
Figure 7: Investment Imported Share



Notes: Share of total GFCF that is sourced from abroad, by sector and income group, 2005. Average per capita GDP (PPP): Low income \$2587, medium income \$11,110, high income \$35,056.

Figure 8: South Korea: Investment Network vs. Input–Output Network, 2014

(a) Input Output Network (b) Investment Network

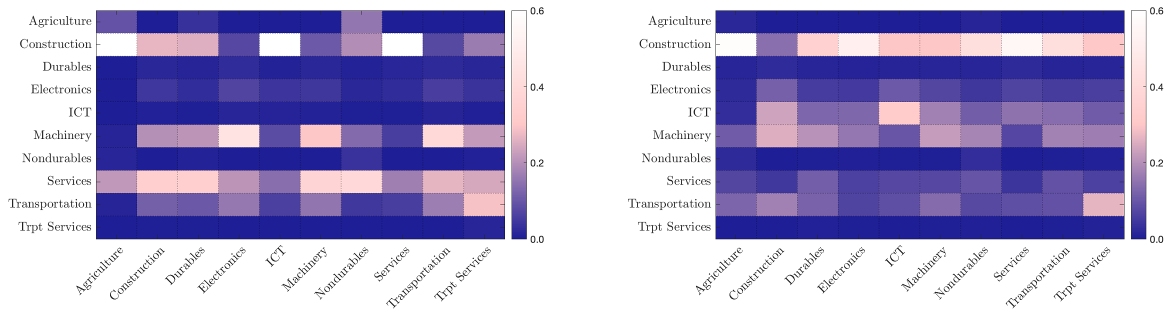


Notes: Estimates of the investment network and the input–output network in Korea, 2014.

Figure 9: South Korea: Investment Network over Time

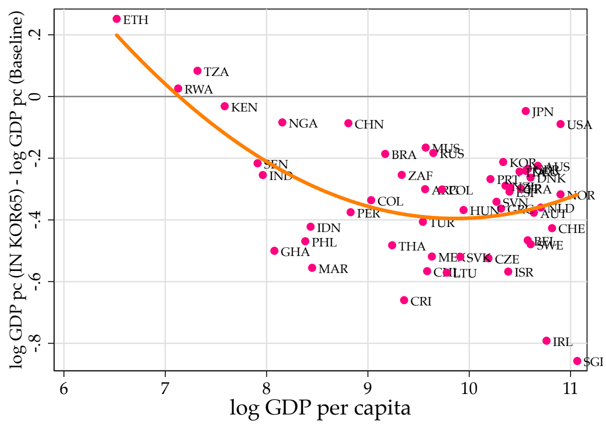
(a) 1965 (b) 2014

GDP per capita (PPP): 1450 GDP per capita (PPP): 35524



Notes: Estimates of the investment network in Korea over time, initial and end year.

Figure 10: Counterfactual: South Korea's domestic investment network (1965)



Notes: This figure presents counterfactual changes in income when countries use the domestic investment network of Korea in 1965. Countries with outlier income changes were excluded from the graph: Zambia (log GDP per capita 7.4, change of 94%) and Malaysia (log GDP per capita 9.8, change of-100%).