ILLINOIS TECH

OPTIMIZING INVESTMENT PORTFOLIOS: A COMPARATIVE ANALYSIS USING MACHINE LEARNING AND MODERN PORTFOLIO THEORY

Master in Artificial Intelligence

2023-2024

AUTHOR: Lucia Colin Cosano

ADVISOR: Oleksandr Narykov

Chicago, Illinois.

August 2024.

ABSTRACT

This research project aims to explore the optimization of investment portfolios by integrating traditional financial theories and modern machine learning techniques. The study focuses on comparing the performance and risk management capabilities of portfolios constructed using Modern Portfolio Theory (MPT) and advanced machine learning models.

Modern Portfolio Theory serves as a foundational framework, emphasizing diversification to manage risk and optimize returns. The study showcases MPT models' adaptability, ranging from aggressive, high-risk portfolios dominated by tech giants to conservative, stable portfolios centered on established companies. Incorporating cryptocurrencies into the analysis highlights the evolving nature of asset classes and the need for diversified portfolios.

Additionally, the project delves into LSTM models' application, leveraging sequential data to capture intricate patterns and non-linear relationships in financial data. Despite computational challenges, LSTM models offer a promising avenue for investors seeking a balance between risk and reward. By combining these methodologies, investors can tailor portfolios to their risk profiles and investment objectives, offering a comprehensive approach to navigate the complexities of today's financial markets.

TABLE OF CONTENTS

1 INTRODUCTION

Investment portfolios are a fundamental component of modern financial management, offering a structured approach to balancing risk and return across various asset classes. Optimizing these collections of assets is crucial for maximizing returns while managing risk, making it a key focus for investors and financial researchers alike. Traditional methods, such as Modern Portfolio Theory (MPT), have long been used to guide portfolio construction by focusing on diversification and the efficient frontier. However, with the advent of advanced computational techniques and the increasing availability of financial data, artificial intelligence (AI) has emerged as a powerful tool for enhancing portfolio optimization.

This research project aims to explore and compare the effectiveness of AI techniques and traditional financial theories in optimizing investment portfolios.

1.1 PROBLEM DESCRIPTION

Financial markets are characterized by their complexity and unpredictability. Traditional optimization methods, while theoretically sound, often fall short in dynamic and volatile conditions [**1**]. Investors continually seek strategies that adapt to changing environments and provide consistent returns while managing risks effectively. The challenge lies in developing models that not only optimize portfolio returns but also account for various risk factors and changing market dynamics.

This research seeks to address this challenge by integrating AI techniques to create robust and adaptable investment strategies. By leveraging the power of AI, we aim to develop models that can analyze large volumes of data, identify patterns, and make informed investment decisions in real time. This approach allows for a more dynamic and responsive management strategy, capable of adapting to evolving market conditions and achieving superior risk-adjusted returns.

1.2 MOTIVATION

The driving force behind this project lies in the recognition of AI's transformative potential in the realm of portfolio optimization. This potential extends profound implications for individual investors and financial institutions, promising to refine investment decisions, fortify risk management practices, and ultimately bolster returns.

By contributing to the existing body of research on AI in finance, this study aims to offer pragmatic guidance for managers eager to integrate sophisticated technologies into their methodologies, paving the way for more agile, informed, and lucrative investment strategies [**2**].

1.3 OBJECTIVES

The primary objective of this research is to develop and implement AI-based models for optimizing investment portfolios. By leveraging machine learning algorithms, the aim is to construct portfolios that maximize returns while effectively managing risk. These models will incorporate a broad range of financial data, ensuring robustness in different market conditions.

The research also aims to analyze various asset allocation methods. By examining how different asset classes, such as equities, bonds, and alternative investments, can be optimally combined, we seek to identify strategies that enhance diversification and performance.

Finally, the project will focus on evaluating the overall performance of the optimized portfolios. Performance metrics such as the Sharpe ratio, maximum drawdown, and annualized return will be used to assess the success of the models. This comprehensive evaluation will provide insights into the effectiveness of AI-based optimization in real-world scenarios and offer practical recommendations for investment managers.

2 THEORETICAL FOUNDATIONS

In the complex world of investment management, understanding the theoretical foundations is crucial for effective decision-making and strategy development. This section provides a comprehensive examination of key concepts essential for building and managing investment portfolios.

2.1 WHAT IS AN INVESTMENT PORTFOLIO?

An investment portfolio encompasses a diverse range of financial assets owned by an individual, corporation, or financial institution.

The primary goal of an investment portfolio is to maximize returns while managing risk effectively. Investors achieve this by diversifying across various asset classes, industries, and geographic regions, thereby reducing the impact of volatility in any single investment and promoting stable, consistent growth over time [**3**].

Asset classes serve as the foundational elements of an investment portfolio, each representing a distinct category of investments with unique characteristics and risk-return profiles. The most common asset classes include:

- **Stocks:** they represent ownership shares in a corporation. When you purchase stocks, you become a shareholder and are entitled to a portion of the company's profits. Stocks are known for their potential to generate high returns but are also considered riskier than other asset classes due to their price volatility [**4**].

- **Bonds:** bonds are debt securities issued by governments, municipalities, or corporations to raise capital. When you buy a bond, you are essentially lending money to the issuer in exchange for periodic interest payments and the return of the bond's face value at maturity. Bonds are generally considered less risky than stocks but offer lower potential returns [**5**].

- **Cash Equivalents:** cash equivalents are highly liquid and low-risk investments that can be easily converted into cash. Examples include savings accounts, money market funds, and short-term government securities. Cash equivalents are often used as a safe haven for preserving capital and maintaining liquidity [**6**].

- **Real Estate:** these investments involve purchasing properties, such as residential, commercial, or industrial real estate, with the expectation of generating rental income and/or capital appreciation over time. Real estate is considered a tangible asset with intrinsic value but can be relatively illiquid compared to other investments [**7**].

- **Commodities:** they are physical goods, such as oil, gold, and agricultural products, that are traded on commodity exchanges. Investing in commodities can provide diversification benefits and a hedge against inflation, but commodity prices can be highly volatile [**8**].

These types of assets, when combined strategically, form the foundation for achieving long-term financial goals.

2.2 RETURN AND RISK IN INVESTMENT PORTFOLIOS

Understanding return and risk is fundamental in investment management, as they are central to evaluating and optimizing portfolio performance. This section outlines the key metrics used to assess returns and the various types of risks that affect investments, along with strategies for managing these risks to align with financial goals.

2.2.1 RETURN

Return is a measure of the profit or loss on an investment and is typically expressed as a percentage of the initial investment amount. It is one of the key metrics used to evaluate the performance of an investment or a portfolio [**9**].

There are different types of return:

- Total Return: it accounts for the change in the investment's value over a period, including any income generated, such as dividends or interest, and any capital gains or losses.
- Annualized Return: annualized return is the average return earned per year over a multi-year period, calculated as a geometric average.
- Real Return: it is the return on an investment after adjusting for inflation. It provides a more accurate picture of purchasing power.

2.2.2 RISK

Risk refers to the possibility of losing money or not achieving the expected returns. Investments carry some level of risk, which can vary based on factors such as the type of asset, market conditions, and economic factors.

There are several types of risk that investors should be aware of:

- **Market risk:** also known as systematic risk, it refers to the risk that the entire market will decline, leading to a decrease in the value of investments. This type of risk cannot be diversified away and is inherent in all investments.

- **Specific risk:** also known as unsystematic risk, is the risk associated with individual investments or specific sectors. This risk can be reduced through diversification, as spreading investments across different assets can help mitigate the impact of any one investment performing poorly.
- **Interest rate risk:** this risk arises from changes in interest rates, which can affect the value of fixed-income investments such as bonds. When interest rates rise, bond prices typically fall, and vice versa.
- **Inflation risk:** it is the risk that the purchasing power of money will decrease over time. Inflation erodes the real value of investments, especially those with fixed returns such as bonds.
- **Liquidity risk:** liquidity risk is the risk that an investment cannot be bought or sold quickly enough to prevent a loss. Illiquid investments, such as real estate, can be particularly vulnerable to this type of risk.

2.2.3 MANAGING RISK AND RETURN

Managing risk effectively is essential for optimizing investment returns and achieving financial goals. This section outlines classical risk management strategies rooted in finance theory and their application to various types of investment risk. Each strategy targets specific risks identified earlier, ensuring a comprehensive approach to risk mitigation.

- **Diversification:** originating from MPT, diversification involves spreading investments across different asset classes, industries, and regions. This strategy aims to reduce specific (unsystematic) risk by minimizing the impact of poor performance in any single investment. By holding a varied portfolio, investors mitigate the adverse effects of individual asset volatility, thereby stabilizing overall returns.
- **Asset Allocation:** a fundamental concept in finance, asset allocation involves distributing investments based on their risk-return profiles and correlation with other assets. This strategy is designed to balance exposure to various risk factors, aligning with the investor's risk tolerance and financial objectives. It addresses both market risk and specific risk by tailoring the portfolio's composition to optimize returns relative to the acceptable level of risk.
- **Stop-Loss Orders:** this strategy, grounded in risk management practices, involves setting predetermined sell points to limit potential losses. When the price of a security falls below a specified level, a stop-loss order automatically triggers a sale, thereby preventing further declines. This approach is particularly useful for managing specific risks associated with individual securities.

- **Hedging:** rooted in derivatives and risk management theory, hedging employs financial instruments such as options, futures, and swaps to offset potential losses. For example, purchasing put options can protect against declines in stock prices. Hedging strategies are tailored to mitigate market risk and specific risk, depending on the instruments used.
- **Risk Assessment and Monitoring:** risk assessment involves regularly evaluating portfolio risk exposure and performance. By monitoring risk factors and investment conditions, investors can make proactive adjustments to maintain alignment with financial goals. This strategy helps manage all types of risks—market, specific, interest rate, inflation, and liquidity—by ensuring timely responses to changing circumstances.

These classical risk management strategies are designed to address specific types of risk, from systematic to unsystematic, and from market fluctuations to liquidity concerns. By understanding the origin and application of these strategies, investors can better safeguard their portfolios and achieve their long-term financial objectives.

2.3 PORTFOLIO OPTIMIZATION

Portfolio optimization is the process of constructing an investment portfolio that maximizes returns for a given level of risk or minimizes risk for a given level of return. The goal of portfolio optimization is to find the most efficient allocation of assets that balances risk and return based on the investor's objectives and constraints [**10**].

Portfolio optimization is a dynamic process that requires ongoing monitoring and adjustments based on changing market conditions and investment objectives. By optimizing their portfolios, investors can potentially achieve better risk-adjusted returns and improve their chances of meeting their financial goals over the long term.

This process involves various methods and models that analysts use to balance the assets within a portfolio to maximize returns while managing risk. By leveraging these methods, investors can enhance their portfolio management strategies and make more informed investment decisions.

2.3.1 MODERN PORTFOLIO THEORY (MPT)

The modern portfolio theory is a practical approach used in selecting investments that maximizes the overall returns within an acceptable level of risk. This framework helps to construct portfolios consisting of investments, which maximizes the amount of expected return for a given level of risk [**11**].

Modern portfolio theory is based on the complex relationship between risk and return. When it comes to investment risks acts as the cutoff point for potential rewards: generally it is observed that higher risks are associated with higher returns though this implies increased chances of losses. Conversely, low-risk securities normally give regular but limited growth. MPT tries to establish a link between these two factors by creating portfolios that offer maximum expected return at particular levels of risk or minimize the risk at a specific expected return.

The main concept behind MPT is diversification. Diversifying assets into unrelated categories characterized by no direct correlation—whereby their returns do not move together concurrently—helps investors reduce overall volatility in their portfolios. Essentially, some instruments may perform poorly while others excel thereby cushioning the performance outcomes.

In MPT, the efficient frontier is a basic and elemental concept. It consists of the optimal portfolios that have high expected return for given levels of risk or low risk for given expected returns. For the reason that they can yield the highest returns in relation to the risks involved, portfolios located on the efficient frontier are efficient. Below this line, portfolios are suboptimal as they would generate lower returns than those which lie above them when considering equal risks.

The Capital Market Line (CML) arises from this front and shows various combinations of risky assets plus a risk-free asset. CML demonstrates how it is essential to include risk-free assets into a portfolio mix since it allows investors to evaluate their risks compared with profits in terms of potential investments. Moreover, its slope is determined by The Sharpe Ratio; which measures excess return per unit of risk.

Sharpe ratio is an important measure used in MPT to estimate performance of an investment adjusting for its inherent riskiness. Its computation involves finding the difference between the rate at which no growth occurs and rate at which capital was increased divided by standard deviation. A higher Sharpe Ratio indicates a better risk-adjusted return, meaning the portfolio is efficiently generating returns relative to the risk taken.

LIMITATIONS

Portfolio optimization offers several key advantages for investors. Firstly, it aims to maximize returns for a given level of risk. By finding the optimal portfolio on the efficient frontier, managers can achieve high returns per unit of risk, leading to greater client satisfaction. Secondly, portfolio optimization promotes diversification. Optimal portfolios are well-diversified to eliminate unsystematic risk or non-priced risks. This diversification protects investors against underperformance of specific assets, as other assets in the portfolio can offset losses, keeping the investor's portfolio stable.

Moreover, active portfolio management involves tracking market data, allowing managers to identify market opportunities ahead of others. This can help them take advantage of these opportunities for the benefit of their investors. Additionally, portfolio optimization plays a crucial role in risk management. It aims to minimize risk, helping investors stay within their risk tolerance levels while designing a portfolio that suits their return expectations, risk tolerance, and time horizon.

Furthermore, by evaluating market fluctuations and identifying performance deviations, portfolio optimization leads to efficient capital allocation, which is best suited for the investor. Despite its advantages, portfolio optimization also has several limitations. For instance, Modern Portfolio Theory assumes frictionless markets without transaction costs or constraints, which is often not the case in reality. This complexity can make the application of the theory challenging.

Additionally, the assumption of normal distribution of returns ignores skewness, kurtosis, and other factors, which may not hold true in real-world scenarios. Moreover, coefficients used in portfolio optimization, such as correlation coefficients, can change as market conditions change, challenging the assumption of stability in these coefficients. The entire process is highly sensitive to inputs, and even a small change in parameters like return expectations can significantly impact the portfolio, potentially leading to suboptimal results.

Furthermore, portfolio optimization models and procedures can become overly complex, especially when dealing with a large number of assets or securities, requiring advanced techniques and increasing the risk of errors. External elements like geopolitical events, unforeseen contingencies, and changes in rules and regulations can affect asset correlation and prices, impacting the effectiveness of portfolio optimization strategies.

It's important for investors to understand these advantages and limitations to effectively utilize the concept of portfolio optimization.

2.3.2 ARTIFICIAL INTELLIGENCE MODELS: NEURAL NETWORKS

Due to the limitations of Modern Portfolio Theory (MPT), there is a growing interest in using Artificial Intelligence (AI) models to address these challenges and improve investment decision-making. AI has revolutionized the financial sector by providing sophisticated tools for data analysis, predictive modeling, and decision-making. AI methods such as machine learning, deep learning, and neural networks are used to analyze large datasets, identify patterns, and make predictions with high accuracy. These methods enable financial institutions to enhance trading strategies, risk management, fraud detection, and customer service. Among these AI techniques, neural networks stand out for their ability to model complex relationships in data, making them particularly useful in financial forecasting and time series analysis.

Neural networks are computational models inspired by the human brain's structure and functioning. The objective is to create a programming structure capable of interconnecting thousands of neurons. This is achieved by creating a layered structure with an input layer, hidden layers, and an output layer.

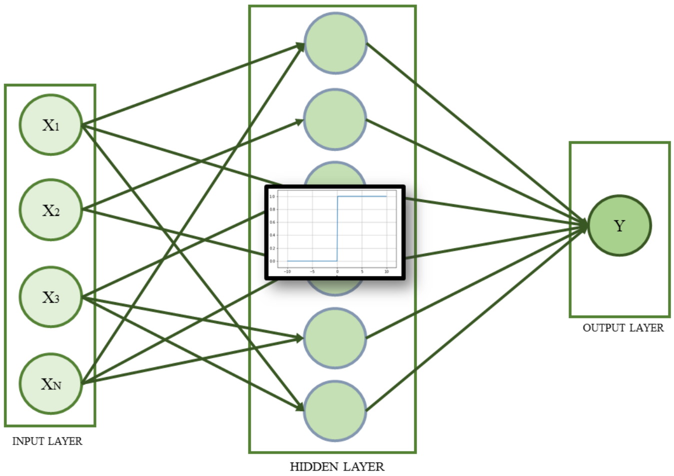


Figure 2.1: Artificial neural network structure.

In a neural network, input values are provided, which are multiplied by the network's weights. Then, an activation function is applied, depending on the type of output data desired, and it returns an output value. To achieve this, each neuron performs two types of operations: one linear and one non-linear.

- **Linear operation:** once an input value, a_{i-1}, is provided, the value of z is calculated as follows:

$$z = \sum_i (w_i a_{i-1}) + b$$

  where a_{i-1} is the output value of the preceding neuron, and the terms w_i and b are parameters whose values will vary throughout execution in such a way that they seek optimization to minimize the prediction error as much as possible.

- Non-linear operation: after performing the linear operation to obtain the value of z, a non-linear operation is then carried out to obtain the final value that will be returned by the neuron, denoted as a_i.

$$a_i = g(z)$$

  where the function g refers to the non-linear activation functions, which are described below.

To ensure that the neuron provides a correct output value, the use of an activation function is necessary. It is responsible for transmitting the information generated by the linear combination of input weights. These functions are classified into linear and non-linear, with the most commonly used ones being the step function, sigmoid function, hyperbolic tangent, and ReLU.

Once this output value is obtained, it is plotted in order to compare it with the actual value and quantify how far the predicted value is from the real one. This result is stored in the cost function. The cost function is defined as half of the squared difference between the actual value and the output value. The cost function is not unique and can be adjusted based on the type of output data.

The objective is therefore to minimize the cost function, as the lower it is, the closer the predicted value will be to the actual value. A visual way to understand the procedure is by introducing the actual value, predicted value, and the cost function into a histogram.

Once a result is obtained, the error is propagated backward using the commonly known technique called Backpropagation, thus feeding the neural network with the error percentage induced by the cost function being used. Mathematically, this involves traversing the network in the opposite direction to the usual direction, calculating the gradients of the network's parameters. This technique aims to modify and update the weights so that the cost function is reduced in the next attempt. This process is done cyclically, continually correcting and modifying the value. It can be concluded that the neural network is capable of modifying the weights, and therefore the objective is to minimize the cost function until the error is within the estimated range.

During this project, a recurrent neural network (RNN) will be implemented. RNNs are a type of artificial neural network designed to work with sequential data, where the order of the input information is important. They are particularly useful for tasks such as speech recognition, language translation, and time series prediction.

The key feature of RNNs is their ability to maintain a memory of previous inputs through a hidden state. This allows them to capture patterns in sequential data by considering the context of each input along with its position in the sequence.

However, traditional RNNs can struggle with long-term dependencies, where information from earlier in the sequence is relevant to predicting later outputs. This is due to the problem of vanishing or exploding gradients, which can cause the network to forget or incorrectly weight important information. To address this issue, Long Short-Term Memory (LSTM) networks were developed as a specific type of RNN.

The key innovation of LSTM networks is the introduction of a memory cell that can maintain information over long periods of time. The cell state, represented by the horizontal line running through the top of the diagram, is a crucial component of LSTMs [**12**].

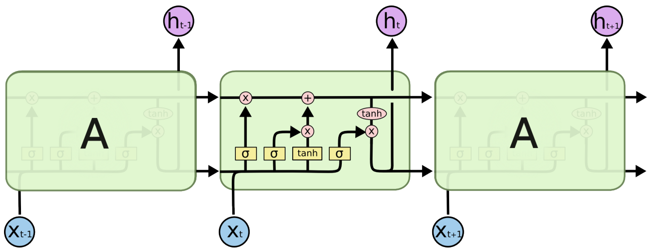
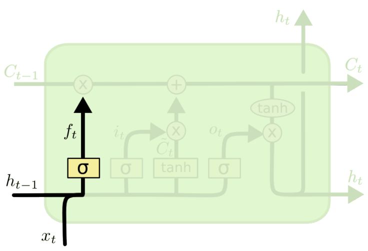


Figure 2.2: LSTM structure (Olah, 'Understanding LSTM Networks').

The cell state acts like a conveyor belt, running straight down the entire chain of LSTM units, with only minor linear interactions. It's easy for information to flow along it unchanged. The LSTM uses structures called gates to control the flow of information into and out of the cell state.

Gates are composed of a sigmoid neural net layer and a pointwise multiplication operation. The sigmoid layer outputs numbers between zero and one, indicating how much of each component should be let through. A value of zero means "let nothing through," while a value of one means "let everything through." LSTMs have three gates to protect and control the cell state:

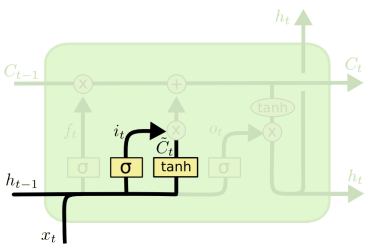
1. **Forget Gate:** the first step in an LSTM is to decide what information to forget from the cell state. This is done by a sigmoid layer called the "forget gate layer," which looks at the previous hidden state h_{t-1} and the current input x_t, and outputs a number between 0 and 1 for each number in the cell state C_{t-1}. A value of 1 means "completely keep this," while a value of 0 means "completely get rid of this."



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Figure 2.3: Forget gate operations (Olah, 'Understanding LSTM Networks').

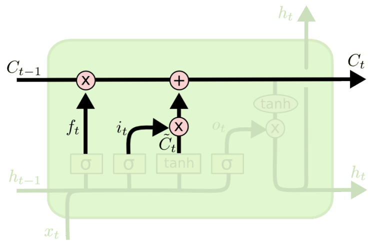
2. **Input gate:** the next step is to decide what new information to store in the cell state. This has two parts. First, a sigmoid layer called the "input gate layer" decides which values to update. Next, a tanh layer creates a vector of new candidate values, $\tilde{C}_t$, that could be added to the state.



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Figure 2.4: Input gate operations (Olah, 'Understanding LSTM Networks').

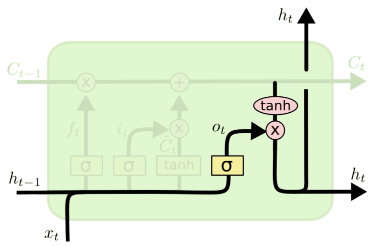
3. **Update the cell state:** the old cell state C_{t-1} is updated into the new cell state C_t. The old state is multiplied by the forget gate output, forgetting the things decided to forget earlier. Then, the new candidate values are added, scaled by how much to update each state value.



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Figure 2.5: Update of the cell state (Olah, 'Understanding LSTM Networks').

4. **Output gate:** finally, the LSTM decides what to output. This output will be based on the cell state, but filtered. First, a sigmoid layer decides what parts of the cell state to output. Then, the cell state is put through a tanh function and multiplied by the output of the sigmoid gate, so that only the parts decided to be outputted are actually outputted.



$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

Figure 2.6: Output gate operations (Olah, 'Understanding LSTM Networks').

LSTMs are capable of learning long-term dependencies in sequential data by selectively retaining or forgetting information over long sequences. This makes them well-suited for a wide range of applications, such time series analysis. However, while neural networks, including LSTMs, offer high accuracy and the ability to handle complex data, they often lack explainability. The decisions made by neural networks can be opaque, making it challenging to understand how specific predictions are generated. This trade-off between accuracy and explainability is an important consideration in the application of AI models in finance.

2.4 PERFORMANCE METRICS AND EVALUATION

Performance metrics and evaluation are essential for assessing the effectiveness of an investment portfolio. Key metrics include maximum drawdown, annualized return, the Sharpe ratio, and additional evaluation methods such as benchmark comparison, scenario analysis, and peer comparison.

2.4.1 PERFORMANCE METRICS

Performance metrics are vital for evaluating the effectiveness of an investment portfolio. Key metrics include:

- **Maximum Drawdown:** it measures the largest single drop from peak to trough in the value of a portfolio over a specific period. It is a critical metric for understanding the risk of significant losses and the time required to recover from them. For example, during the 2008 financial crisis, many portfolios experienced significant maximum drawdowns, highlighting their vulnerability to market downturns.

- **Annualized Return:** annualized return, calculated as a geometric average, represents the average return earned per year over a multi-year period. This metric provides a consistent basis for comparing the performance of investments over different time frames. For instance, studies have shown that the S&P 500 has had an annualized return of approximately 10% over the past century.

- **Sharpe Ratio:** measures the performance of an investment by adjusting for its risk. It is calculated by dividing the difference between the investment's return and the risk-free rate by the investment's standard deviation. A higher Sharpe ratio indicates better risk-adjusted performance. For example, a mutual fund with a Sharpe ratio of 1.5 is generally considered to be providing good risk-adjusted returns compared to one with a Sharpe ratio of 0.5.

- **Beta:** this measures the volatility of a portfolio or asset relative to the overall market. A beta greater than 1 indicates that the investment is more volatile than the market, while a beta less than 1 suggests that it is less volatile. Beta is essential for understanding an investment's systematic risk. For instance, a stock with a beta of 1.2 is expected to be 20% more volatile than the market. Conversely, a stock with a beta of 0.8 would be 20% less volatile.

2.4.2 PERFORMANCE EVALUATION

Performance evaluation is a critical aspect of portfolio management, allowing investors to assess the effectiveness of their investment strategies and make informed decisions about future allocations. Key methods and metrics used in performance evaluation include:

- **Return Metrics:** these provide insights into how well a portfolio has performed relative to its risk profile and benchmark. Common metrics include annualized return, cumulative return, and risk-adjusted returns such as the Sharpe ratio or Treynor ratio. For instance, in an analysis by Jones (2020), portfolios with higher Treynor ratios were shown to outperform their benchmarks over a 10-year period.

- **Risk Metrics:** these measure the level of risk taken by the portfolio and its potential for loss under adverse market conditions. Metrics such as Value at Risk (VaR) and conditional Value at Risk (cVaR) are commonly used. An example is the 2020 study by Smith et al., which demonstrated that portfolios with lower VaR performed better during market downturns.

- **Benchmark Comparison:** comparing the portfolio's performance against a relevant benchmark helps investors assess whether their portfolio has outperformed or underperformed relative to the broader market or similar investment strategies. For example, comparing a technology-focused portfolio against the NASDAQ index can provide insights into its relative performance.
- **Scenario Analysis:** this involves simulating different market scenarios to understand how the portfolio would perform under various conditions, helping investors assess its resilience and sensitivity to market changes. For instance, the 2019 study by Lee and Johnson used scenario analysis to show how different economic conditions affected portfolio performance.
- **Peer Comparison:** comparing the portfolio's performance against similar portfolios provides additional context and insights into its relative performance within the investment universe. For example, according to a study by Brown (2018), peer comparison helped fund managers identify areas for improvement by analyzing the performance of top-performing funds in the same category.

Performance evaluation is an ongoing process that requires regular monitoring and adjustment to ensure the portfolio remains aligned with the investor's objectives and risk tolerance. By using a combination of these metrics and methods, investors can gain a comprehensive understanding of their portfolio's performance and make informed decisions to optimize their investment strategy.

3 DATA COLLECTION AND ANALYSIS

The primary data source used for the development of the project is Yahoo Finance, a widely recognized platform that provides extensive financial data, including stock prices, historical data, and financial statements. Yahoo Finance is selected due to its comprehensive coverage, ease of access through APIs, and reliability.

To create a diversified portfolio for analysis, numerous assets have been selected from different sectors of the market. This selection ensures a balanced representation of various industries and helps in evaluating the performance of the portfolio across different economic conditions.

3.1 DATA

For the development of the different models, closing prices have been used. Closing prices are the final trading prices of a security at the end of a trading day. They provide a standardized snapshot, offering a consistent reference point for each trading day. This standardization is crucial because it allows for reliable comparisons over time, ensuring that all analyses are based on a common metric.

From a practical standpoint, closing prices are widely available and consistently reported in historical datasets. This comprehensive availability ensures that analyses are built on accurate and complete data, free from the intraday noise and volatility that can obscure true market trends.

Moreover, many essential financial metrics and models are derived from closing prices. Daily returns, a fundamental metric for assessing investment performance and risk, are calculated based on the percentage change between consecutive closing prices

The data used for this analysis will be downloaded in CSV format and preprocessed to handle any missing values and ensure consistency. Data will cover the period from May 31, 2015, to May 31, 2024, providing a substantial dataset for robust analysis and model development.

In this analysis, a risk-free rate of 0.053 has been used, aligning with U.S. Treasury Securities. This choice reflects the common practice among investors, as U.S. Treasury Securities are considered a reliable benchmark for risk-free returns.

3.2 SELECTED ASSETS

To gain a comprehensive understanding of a sector's dynamics, this analysis focuses on identifying the product or service offerings of various assets within the sector. It then assesses their market behavior and the volatility they experience. Additionally, the analysis will explore the correlation between these assets and assets in other sectors.

3.2.1 TECHNOLOGY SECTOR

Apple Inc. and Microsoft Corporation stand as titans in the technology sector, each carving out a formidable presence with their innovative products and services. These giants, with market capitalizations exceeding $2 trillion, lead and shapE the future of technology and consumer behavior.

Apple's story is a clear example of innovation and growth. Renowned for its flagship products like the iPhone and Mac computers, Apple has built an extensive ecosystem that includes Apple Music, iCloud, and Apple TV+. Over the past decade, the company has consistently achieved remarkable growth, with each product launch and market expansion creating ripples across the industry. Apple's latest venture into AR/VR products has generated significant excitement, promising to drive future growth.

Despite its stable performance, Apple exhibits moderate volatility, with a beta of 1.11, indicating a slightly higher risk compared to the overall market. This volatility underscores the dynamic nature of the tech sector and Apple's adeptness at navigating market forces.

Microsoft's journey is marked by its dominance and strategic acumen. Known for its extensive range of software products and services, Microsoft has continually pushed the boundaries of what technology can achieve. Its cloud computing platform, Azure, and software offerings like Office 365 have driven impressive growth over the past decade. Strategic acquisitions, such as GitHub and LinkedIn, have further solidified Microsoft's market position, enabling it to adapt and thrive in a constantly evolving market.

Recent advancements in artificial intelligence and cloud computing services are set to propel Microsoft's future growth and innovation. Despite its market leadership, Microsoft's beta of approximately 1.1 indicates moderate volatility, reflecting that even industry leaders are not immune to market fluctuations.

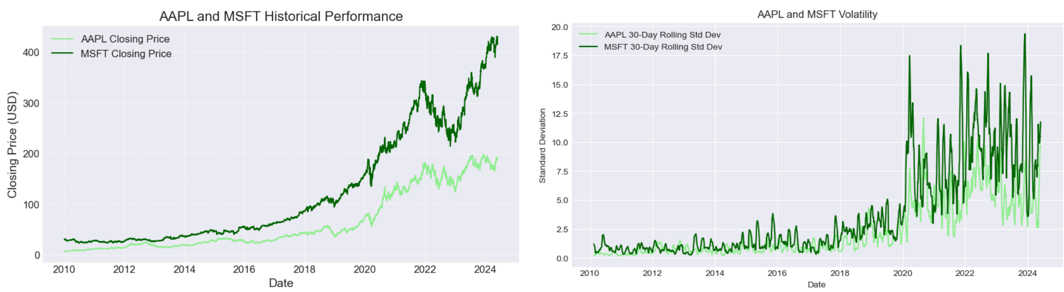


Figure 3.1: AAPL and MSFT historical performance and volatility.

Both Apple and Microsoft show strong performance correlations within the technology sector, making them key players in any tech-focused investment portfolio. However, they exhibit low correlations with other sectors like energy, utilities, healthcare, and consumer goods. This characteristic enhances the potential diversification benefits of including both companies in an investment portfolio. By doing so, investors can tap into the robust growth and innovation of the tech sector while mitigating risks through strategic exposure to other industries.

3.2.2 CONSUMER STAPLES

Coca-Cola Company stands as an icon in the beverage industry, known for its diverse portfolio of sparkling and still beverages. With a market capitalization exceeding $200 billion, Coca-Cola is not just a beverage company but a global brand synonymous with refreshment and happiness.

Over the past decade, Coca-Cola has displayed remarkable resilience and innovation. The company has successfully adapted to evolving consumer preferences by expanding its product offerings. This track record highlights their commitment to delivering high-quality beverages and enriching consumer experiences. Coca-Cola's future growth appears promising, driven by strategic initiatives such as portfolio diversification, product innovation, and targeted marketing campaigns.

Recent developments, including the introduction of new beverages and a focus on sustainability efforts, further solidify their commitment to growth with environmental responsibility. Strategic partnerships, acquisitions, and ongoing sustainability efforts showcase Coca-Cola's proactive approach to adapting to the ever-changing market landscape and consumer needs.

Coca-Cola's stock exhibits relatively low volatility, with a beta of 0.6, reflecting the stability and long-term growth potential characteristic of the consumer staples sector. This stability has positioned Coca-Cola as a favorite amongst investors seeking defensive stocks with growth potential.

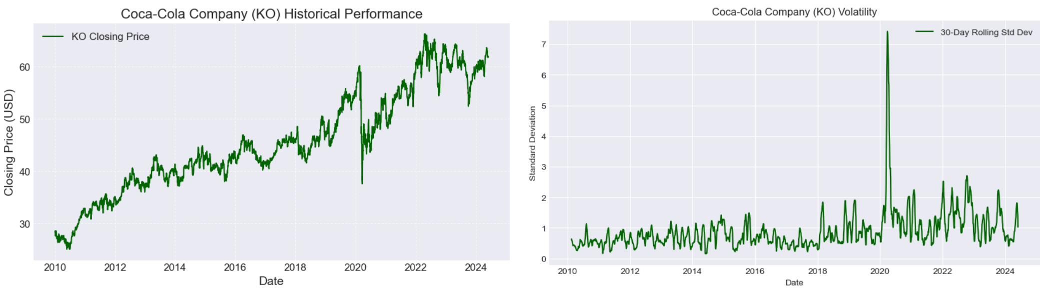


Figure 3.2: KO historical performance and volatility.

Coca-Cola exhibits low correlation with sectors like technology and healthcare which helps mitigate overall portfolio risk, particularly during periods of economic downturn.

3.2.3 CONSUMER DISCRETIONARY

In the dynamic realm of consumer discretionary, two industry are highlighted: Nike, Inc. and McDonald's Corporation. These giants stand as beacons of innovation, resilience, and market leadership, each carving a unique path in their respective niches.

Nike strides ahead as a global icon in athletic apparel and footwear. With a market capitalization exceeding $200 billion, Nike's influence transcends borders, embodying the essence of sports culture and performance. Over the past decade, Nike has sprinted towards impressive growth, fueled by its iconic brand, innovative product lines, and strategic marketing endeavors. Its future looks even brighter, with a focus on digital transformation, direct-to-consumer sales, and sustainable initiatives. Recent innovations like Nike Rise and strategic collaborations underscore Nike's commitment to staying ahead of industry trends and resonating with athletes and consumers worldwide.

McDonald's stands out as a global leader in the fast-food industry, recognized for its iconic golden arches symbolizing convenience and quality consistency. With a market capitalization exceeding $150 billion, McDonald's is a ubiquitous presence, operating in over 100 countries. McDonald's has showcased remarkable adaptability, navigating changing consumer tastes with finesse. Its global footprint has expanded, solidifying its position as a market leader. McDonald's future growth hinges on innovation and customer-centric initiatives, including new menu offerings and digital ordering options. Despite challenges from competitors, McDonald's remains a stalwart in the fast-food industry, committed to driving growth and enhancing customer experiences.

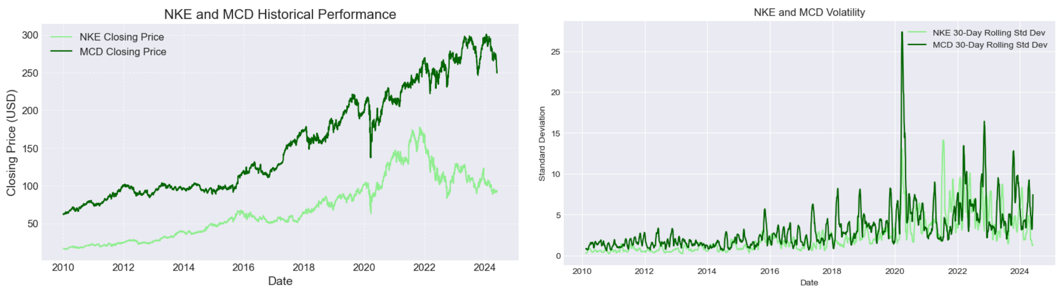


Figure 3.3: NKE and MCD historical performance and volatility.

Financially, both Nike and McDonald's exhibit moderate volatility, reflecting their ability to navigate market fluctuations. Nike's beta of approximately 0.99 and McDonald's beta of 0.65 indicate their relative stability within the sector. This stability, coupled with their low correlations with sectors like healthcare and consumer goods, offers significant diversification benefits for investors.

3.2.4 CRYPTOCURRENCY SECTOR

Cryptocurrencies have transformed money and finance, offering a decentralized alternative to traditional systems.

Bitcoin (BTC-USD), the first cryptocurrency, is a pioneer with a market capitalization exceeding $1 trillion. Known for its decentralized nature, Bitcoin has seen remarkable growth and volatility over the past decade, marked by significant price surges and corrections. Its role as a digital store of value and a hedge against inflation and economic uncertainty is reinforced by its scarcity, security features, and increasing institutional adoption. Recent advancements like integration into payment platforms and the rise of Bitcoin-based financial products validate its future role in finance. Bitcoin's high volatility reflects the speculative nature of cryptocurrencies and the evolving regulatory landscape. Despite this, its low correlation with traditional assets like stocks and bonds offers diversification benefits to portfolios, potentially reducing overall risk and enhancing returns.

Ethereum (ETH-USD) is celebrated for its smart contract functionality and decentralized applications (dApps), with a market capitalization exceeding $200 billion. It empowers developers to create innovative solutions, distinguishing it from traditional cryptocurrencies. Like Bitcoin, Ethereum has experienced significant growth and volatility over the last decade, underscoring its role in blockchain technology and decentralized finance (DeFi). Its active developer community and ongoing upgrades, including the transition to Ethereum 2.0, enhance scalability and security, solidifying Ethereum's position in the blockchain space. Ethereum also exhibits notable volatility, reflecting the dynamic nature of the cryptocurrency market.

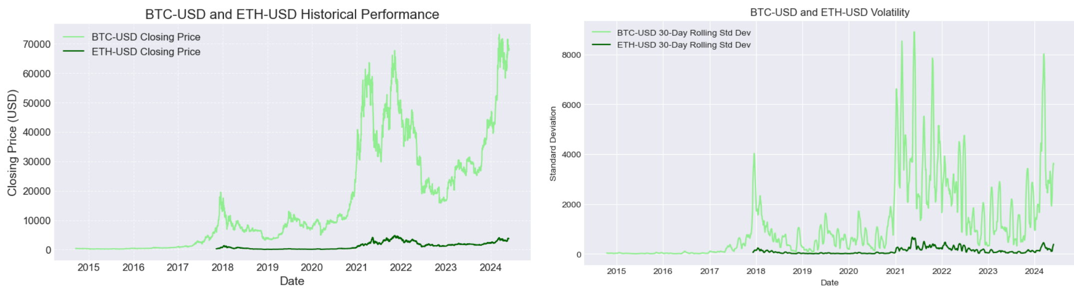


Figure 3.4: BTC-USD and ETH-USD historical performance and volatility.

Despite this volatility, Ethereum maintains a low correlation with traditional asset classes, providing diversification benefits similar to Bitcoin. This low correlation underscores the potential of cryptocurrencies to enhance portfolio diversification and manage overall investment risk.

3.2.5 BOND SECTOR

Bonds are a fundamental component of the global financial market, representing a vast array of debt instruments issued by governments, municipalities, corporations, and other entities as they serve as a means for these entities to raise capital.

During the last ten years, the Vanguard Total Bond Market ETF (BND) has maintained its reputation as a renowned exchange-traded fund (ETF) tracking the Bloomberg Barclays U.S. Aggregate Float Adjusted Index. With a market capitalization exceeding $100 billion, BND holds a significant position in the bond market, offering investors diversified exposure to U.S. investment-grade bonds.

Throughout this period, BND has consistently delivered stable returns, reflecting the performance of the underlying U.S. bond market. This track record underscores BND's core role in a balanced investment portfolio, providing income and stability. BND's growth prospects remain aligned with the performance of the U.S. bond market, influenced by factors such as interest rates, inflation, and economic conditions.

BND exhibits low volatility, with a beta that is not directly comparable to individual stocks due to its bond-focused nature. This low volatility reflects the relative stability of the bond market compared to the stock market.

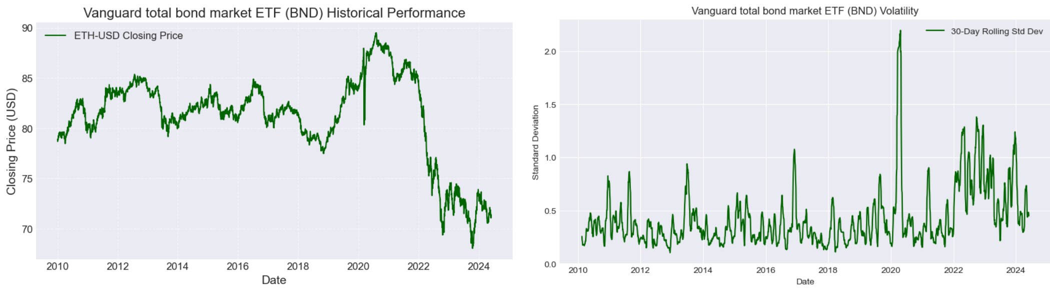


Figure 3.5: BND historical performance and volatility.

BND, unlike stocks and other riskier asset classes, exhibits a low correlation with them. This characteristic makes it a valuable portfolio diversifier, potentially reducing overall portfolio risk and enhancing stability during periods of market volatility.

3.2.6 HEALTHCARE SECTOR

The healthcare sector is a critical component of the global economy, encompassing a wide range of industries dedicated to improving health and well-being. One of the key players in this sector is Pfizer Inc. (PFE), a pharmaceutical giant renowned for its innovative drugs and vaccines. With a market capitalization exceeding $200 billion, Pfizer leads the way in healthcare innovation, with a diverse portfolio of products that address various medical needs.

Pfizer's commitment to research and development (R&D) has been unwavering over the past decade, leading to groundbreaking treatments and vaccines. This dedication ensures a continuous stream of new products, vital for future growth. Recent successes, such as the development of COVID-19 vaccines, highlight Pfizer's ability to capitalize on emerging healthcare trends and its role in shaping the future of healthcare.

Pfizer operates in a dynamic environment, with factors like regulatory changes and competition shaping its growth prospects. News surrounding Pfizer often revolves around drug approvals, clinical trials, and strategic partnerships, demonstrating its focus on innovation and staying ahead in the healthcare industry.

Pfizer exhibits moderate volatility, reflecting the regulatory and competitive landscape of the pharmaceutical industry. However, it has a low correlation with sectors like technology and consumer goods, making it a valuable addition to a diversified investment portfolio.

Another notable player in the healthcare sector is Medtronic plc (MDT), a medical technology company known for its innovative medical devices and therapies. With a market capitalization exceeding $150 billion, Medtronic is a significant contributor to healthcare innovation, particularly in areas like minimally invasive surgery and remote patient monitoring.

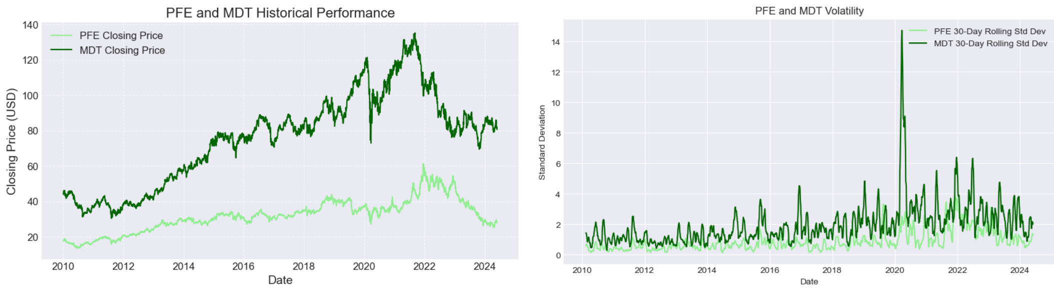


Figure 3.6: PFE and MDT historical performance and volatility.

The healthcare sector is a dynamic and essential part of the global economy, driven by companies like Pfizer and Medtronic that are committed to innovation and improving patient outcomes. Investing in the healthcare sector offers opportunities for growth and diversification, while also contributing to advancements in medical science and technology.

3.2.7 FINANCIAL SECTOR

The financial sector includes institutions providing essential services such as banking, investment, and insurance. Key players like Bank of America Corporation (BAC) and Goldman Sachs Group Inc. (GS) significantly shape economic trends with their diverse financial products and services.

Bank of America, one of the largest U.S. banks with a market capitalization over $300 billion, serves a wide range of clients from individuals to large corporations. Over the past decade, the bank has prioritized financial stability and operational efficiency, allowing it to navigate economic challenges and maintain strong growth. Innovations in digital banking and customer-focused services have further propelled its growth, reflecting its commitment to cutting-edge financial solutions. Bank of America exhibits moderate volatility, with a beta of 1.39, indicating it is somewhat more volatile than the overall market due to the regulatory environment and cyclical nature of the banking industry.

Goldman Sachs, a global investment banking and financial services firm with a market capitalization over $100 billion, is renowned for its expertise in advisory services, trading, and asset management. The firm has been at the forefront of financial innovation, consistently adapting to market changes and introducing new products and services, key to its success in a rapidly evolving financial landscape. Goldman Sachs exhibits moderate volatility, with a beta of 0.72, suggesting it is less volatile than the overall market, reflecting its diverse business model and effective risk management.

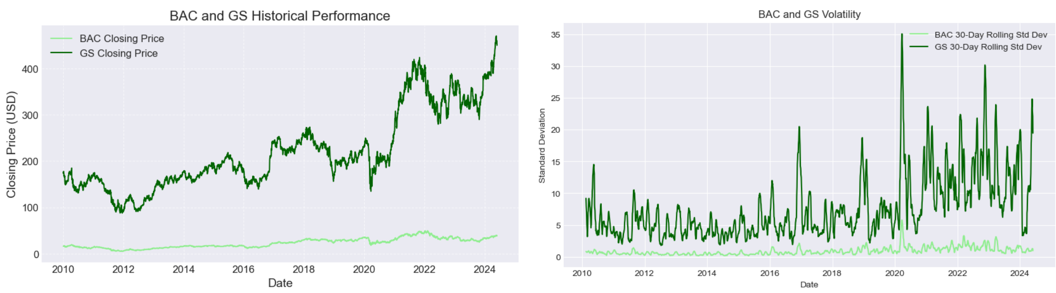


Figure 3.7: BAC and GS historical performance and volatility.

Both Bank of America and Goldman Sachs exhibit low correlations with sectors like technology and consumer goods. This characteristic makes them valuable assets in a diversified investment portfolio, providing stability during market volatility.

The financial sector plays a critical role in driving economic growth and stability. These institutions continue to innovate and adapt to meet the evolving needs of their clients, making them key players in the global financial landscape.

3.2.8 ENERGY SECTOR

The energy sector includes companies that produce, distribute, and manage energy resources. With a growing focus on sustainability, companies like NextEra Energy Inc. (NEE) and Duke Energy Corporation (DUK) are leading the transition towards cleaner energy.

NextEra Energy, a leading clean energy company with a market capitalization over $150 billion, specializes in renewable energy generation. Over the past decade, it has invested heavily in wind, solar, and battery storage projects, underscoring its commitment to reducing carbon emissions and championing a sustainable energy future. NextEra Energy exhibits moderate volatility, with a beta of 0.66, reflecting the dynamic nature of the energy market and its regulatory environment. Its low correlation with sectors like technology and consumer goods makes it a valuable asset for portfolio diversification, reducing overall risk and providing stability during market fluctuations.

Duke Energy, with a market capitalization over $80 billion, is a prominent utility company providing electricity and natural gas to customers in several states. Over the past decade, it has focused on modernizing infrastructure, improving reliability, and expanding its renewable energy portfolio. Duke Energy's growth is driven by investments in renewable energy projects, infrastructure upgrades, and regulatory compliance. Recent investments in solar and wind energy and partnerships with other energy companies highlight its commitment to growth and innovation. Duke Energy exhibits low volatility, with a beta of 0.53, reflecting the stable nature of the utility business and its focus on regulated operations.

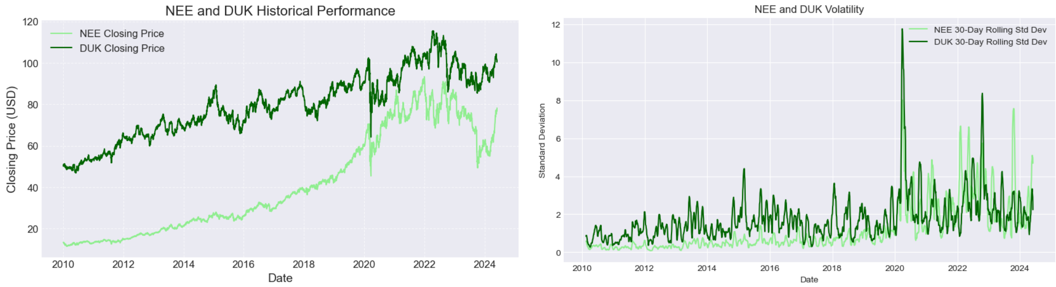


Figure 3.8: NEE and DUK historical performance and volatility.

Like NextEra Energy, Duke Energy also has a low correlation with sectors like technology and consumer goods.

3.2.9 INDUSTRIAL SECTOR

The industrial sector is integral to the global economy, encompassing companies that manufacture goods and provide services used in construction, manufacturing, and other infrastructure projects. Companies like 3M Company (MMM) and Caterpillar Inc. (CAT) are prominent players in this sector, known for their innovation, quality, and reliability.

3M Company, with a market capitalization exceeding $100 billion, is a beacon of innovation known for its diverse range of products in healthcare, consumer goods, and industrial sectors. Its commitment to innovation has solidified its position as an industry leader. The company's moderate volatility, with a beta of 0.88, reflects its diverse operations across sectors. Strategic investments in research and development and expansion into emerging markets ensure that 3M stays ahead by continually innovating.

Caterpillar Inc., with a market capitalization exceeding $100 billion, is renowned for its high-quality construction and mining equipment. Over the past decade, Caterpillar has focused on innovation and efficiency, introducing new technologies that boost productivity and minimize environmental impact. Caterpillar's moderate volatility, with a beta of 1.14, indicates its sensitivity to market movements, common in the industrial sector. The company's growth prospects are tied to its ability to capitalize on global infrastructure projects and expand its product offerings. Investments in autonomous technology and digital solutions underscore Caterpillar's commitment to driving growth and staying ahead in the industry.

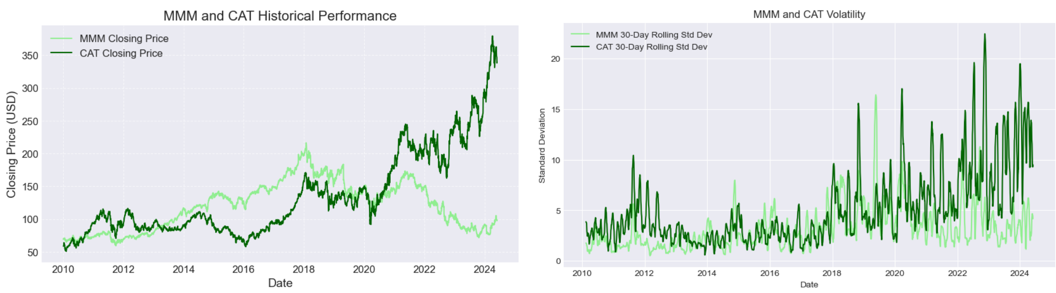


Figure 3.9: MMM and CAT historical performance and volatility.

Moreover, 3M's and Caterpillar's low correlation with sectors like financials and utilities offers significant diversification benefits to investors. This characteristic helps reduce overall portfolio risk and provides stability, especially during volatile market conditions.

3.2.10 REAL ESTATE INVESTMENT SECTOR

The Vanguard Real Estate ETF (VNQ) is a significant player in the real estate sector, with a market capitalization exceeding $50 billion. It offers investors exposure to a diversified portfolio of real estate assets, delivering solid returns over the past decade. VNQ's moderate volatility reflects the nature of the real estate market and its growth prospects are closely tied to real estate market performance, influenced by factors like interest rates and economic conditions.

Realty Income Corporation (O) focuses on acquiring and managing a diversified portfolio of commercial properties, specializing in single-tenant retail properties. With a market capitalization exceeding $20 billion, Realty Income has delivered consistent returns driven by its focus on high-quality tenants and long-term lease agreements. Its low volatility reflects the stability of its business model, based on predictable rental income from long-term leases. Realty Income's growth prospects are tied to its ability to acquire new properties and increase rental income over time.

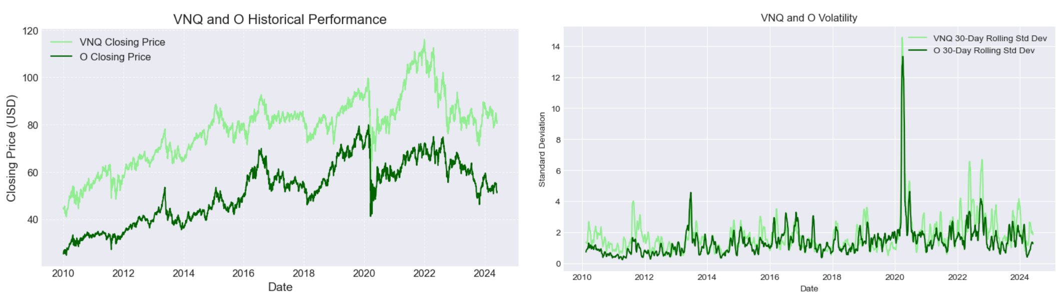


Figure 3.10: VNQ and O historical performance and volatility.

Both the Vanguard Real Estate ETF (VNQ) and Realty Income Corporation (O) exhibit a low correlation with sectors such as technology and healthcare. Instead, their returns are more closely tied to the dynamics of the real estate market and the factors that impact it, such as interest rates, economic conditions, and demographic trends.

3.2.11 COMMODITIES SECTOR

SPDR Gold Shares (GLD) is an exchange-traded fund (ETF) that seeks to track the price of gold bullion. With a market capitalization exceeding $50 billion, GLD is one of the largest gold ETFs and a significant player in the precious metals sector.

Over the past decade, GLD has served as a popular investment choice for investors seeking exposure to gold as a hedge against inflation and market uncertainty. This track record underscores GLD's role as a convenient and cost-effective way to invest in gold without the need for physical ownership.

GLD's growth prospects are closely tied to the price of gold, which is influenced by factors such as economic conditions, geopolitical events, and inflation expectations. Recent developments, such as changes in interest rates and geopolitical tensions, can impact the price of gold and, consequently, the performance of GLD. Recent news and events surrounding GLD include updates on gold prices, macroeconomic indicators, and investor sentiment.

GLD exhibits low volatility, with a beta that is not directly comparable to individual stocks due to its focus on gold prices. This low volatility reflects the stable nature of gold prices and its reputation as a safe-haven asset.

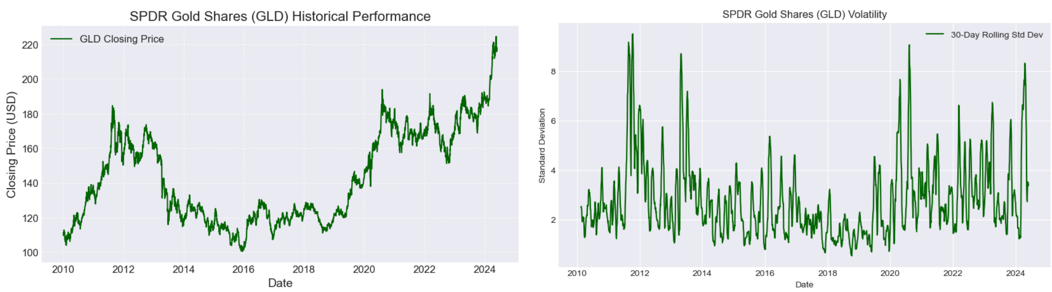


Figure 3.11: GLD historical performance and volatility.

GLD has a low correlation with sectors like technology and consumer goods.

As evidenced, companies operating within the same sector demonstrate a comparable historical performance, characterized by coinciding peaks, albeit varying in scale according to the individual company's size and market influence. This observation underscores the sector-wide impact of common market forces and economic trends on businesses within an industry. Moreover, it is notable that these companies exhibit similar beta values, indicating a parallel level of volatility. This finding suggests that, irrespective of their size or market dominance, companies within a given sector tend to experience analogous fluctuations in response to market conditions.

4 MODEL DEVELOPMENT AND RESULTS

Following the comprehensive analysis of the various assets, the next step is to develop an optimal model that aligns with the initially defined objectives. This section will delve into the different architectures employed, along with a detailed justification for the progressive modifications undertaken to reach the desired model.

4.1 MODERN PORTFOLIO THEORY MODELS

The following section explores Modern Portfolio Theory (MPT) models, including a baseline model, a diversified portfolio model, and an analysis of the influence of cryptocurrencies. Each model is evaluated based on its risk-return profile, as depicted on the efficient frontier, and its suitability for different investor profiles.

4.1.1 MODEL 1: BASELINE MODEL

The baseline model focuses on a selection of companies across various sectors, including Apple, Microsoft, Nike, McDonald's, Pfizer, Medtronic, Bank of America, and Goldman Sachs. By processing their stock price data, the model calculates daily returns, which are smoothed using a 30-day rolling average to reduce noise and reveal underlying trends.

Using the mean returns and covariance matrix of these assets, the model defines key functions to calculate expected portfolio return and volatility, as well as the negative Sharpe ratio, which is minimized by the optimizer. The portfolio optimization has been based on maximizing the Sharpe ratio and minimizing the volatility.

For the first approach, an annualized return of 29% with an annualized volatility of 3% has been achieved. This optimized portfolio is notably dominated by Microsoft, comprising 94.22% of the allocation, with McDonald's accounting for the remaining 5.78%. Other assets are excluded from this allocation based on this optimization.

This portfolio is particularly suitable for aggressive investors who are willing to take on higher risk for the potential of higher returns. The significant allocation to Microsoft reflects strong confidence in the tech sector's growth, making it appropriate for investors with a high-risk tolerance and a long investment horizon.

For investors seeking a smoother ride, the minimum volatility approach offers a compelling alternative. The optimized portfolio constructed using this strategy prioritizes companies like McDonald's (37.47%), Medtronic (18.49%), and Pfizer (20.30%). These established "blue-chip" companies have a reputation for weathering economic storms. They tend to generate consistent revenue and earnings, often offering reliable dividend payouts. This focus on stability translates to a smoother investment experience, with smaller fluctuations in portfolio value compared to a high-risk portfolio.

While Microsoft (23.60%) remains a significant holding, its weight is reduced compared to the maximum Sharpe ratio approach. This diversification helps mitigate risk without sacrificing all potential for growth. Small allocations to Bank of America (0.08%) and Goldman Sachs (0.06%) provide some exposure to the financial sector without significantly impacting the overall portfolio's stability.

This approach is particularly suitable for conservative investors, those nearing retirement or with a lower risk tolerance. By prioritizing capital preservation, the minimum volatility portfolio offers several key benefits such as less drawdown risk and potential for outperformance.

As shown below, this contrast in approaches is reflected in the shape of the efficient frontier. In this scenario, the Efficient Frontier can be approximated by a straight line with an infinitely steep slope, indicating that, to maintain a similar level of risk, the Maximum Sharpe portfolio offers significantly higher returns compared to the Minimum Volatility portfolio.

It is also demonstrated that, even when working with assets that have low returns and high volatility, combining them in a portfolio can balance and reduce that volatility in favor of higher returns. This diversification effect allows for the creation of portfolios that achieve a better risk-return profile than the individual assets would suggest on their own.

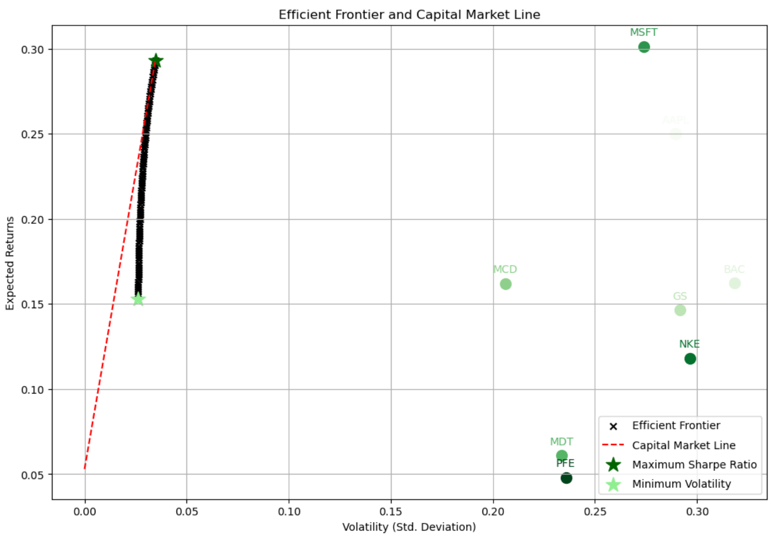


Figure 4.1: Efficient frontier and capital market line.

To further enhance performance, diversification across asset classes, industries, and geographical regions is recommended. Additionally, monitoring and rebalancing the portfolio regularly can help maintain the desired risk-return profile.

4.1.2 MODEL 2: DIVERSIFIED PORTFOLIO

To enhance diversification and exposure beyond the original asset pool, a second model incorporates utilities, consumer goods, industrials, and REITs.

The Maximum Sharpe Ratio Portfolio Allocation for this diversified portfolio reflects a strategic shift towards a broader asset mix. Assets such as Caterpillar and Gold now play a significant role, with allocations of 13.42% and 9.75% respectively, alongside Microsoft at 74.26%. This allocation demonstrates a deliberate move towards diversification, with a reduced focus on a single dominant asset.

With an annualized return of 27% and an annualized volatility of 3%, this portfolio maintains a strong risk-adjusted return profile, reflected in a Sharpe ratio of 7.14.

The diversified nature and inclusion of assets from different sectors make this portfolio suitable for conservative investors looking for a balanced approach to investing, with a focus on both stability and growth. In terms of economic situations, it would be particularly useful during periods of economic uncertainty or market volatility, as its diversified nature helps mitigate risks associated with specific industries or asset classes.

From the minimum volatility apporach, this allocation shows a heavy investment in bonds (BND) at 96.73%, highlighting a significant emphasis on fixed-income securities to ensure low volatility. The inclusion of Bank of America (BAC) at 2.15% and Caterpillar (CAT) at 1.12% provides a slight exposure to equities, but the primary focus remains on stable, low-risk investments.

However, despite its low risk, this minimum volatility portfolio might not be optimal as it offers a very low return for the risk assumed. With only a 2% annualized return, the potential gains may not justify the investment.

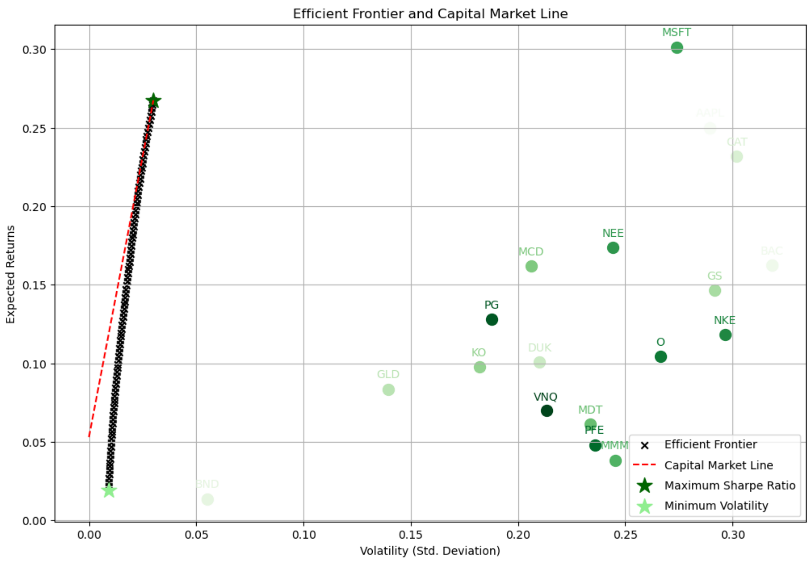


Figure 4.2: Efficient frontier and capital market line.

4.1.3 MODEL 3: ANALYZING CRYPTOCURRENCIES INFLUENCE

The addition of cryptocurrencies to the investment portfolio in the third model has significantly impacted its composition and performance metrics compared to the previous models. While traditional assets like Microsoft still hold a dominant position, with an allocation of 80.36%, new assets such as Bitcoin and Ethereum have been included, comprising 4.07% and 3.98% of the portfolio, respectively. This diversification introduces exposure to a new asset class with potentially different risk and return.

The inclusion of cryptocurrencies has also influenced the portfolio's risk-return profile. The portfolio's annualized return has increased to 30%, indicating the potential for higher returns compared to the previous model. However, this increase in return comes with an uptick in volatility, now at 3%. Despite this increase, the Sharpe ratio remains favorable at 7.91, indicating an attractive risk-adjusted return.

This portfolio adjustment is likely to appeal to investors seeking to capitalize on the high growth potential of cryptocurrencies while maintaining a diversified portfolio. However, it's important to note that cryptocurrencies can introduce higher volatility and risk, requiring careful monitoring and risk management strategies. This portfolio is best suited for investors with a higher risk tolerance and a long-term investment horizon, particularly in uncertain economic environments where cryptocurrencies may offer a hedge against traditional asset classes.

In contrast, the Minimum Volatility Portfolio Allocation for this diversified portfolio focuses heavily on bonds at 98.38%, highlighting a strong emphasis on fixed-income securities to ensure low volatility. The inclusion of Caterpillar at 1.62% provides a slight exposure to equities, but the primary focus remains on stable, low-risk investments. Despite its focus on capital preservation and minimizing risk, this minimum volatility portfolio offers a very low annualized return of 1%.

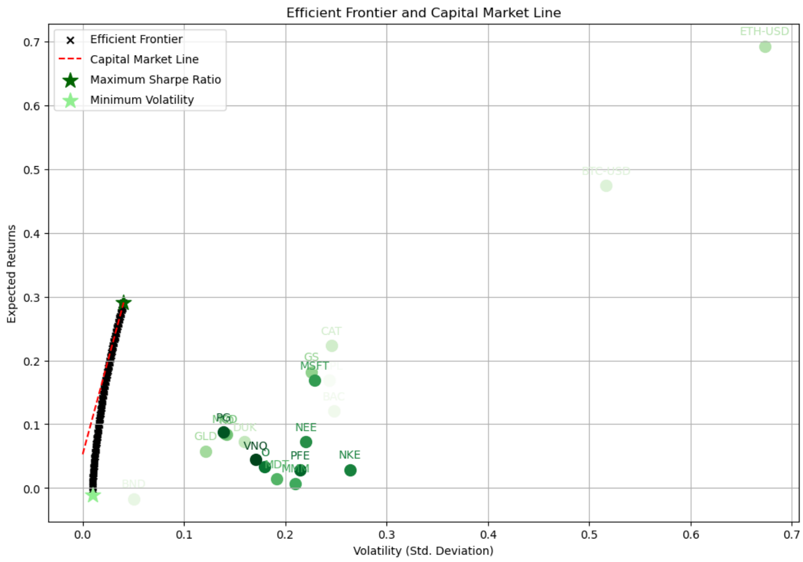


Figure 4.3: Efficient frontier and capital market line.

The results presented in these models highlight the different risk-return profiles achievable through various MPT strategies. One key observation is that none of the individual companies selected for the portfolios lie directly on the efficient frontier. This is expected, as the efficient frontier represents the optimal portfolios that offer the highest expected return for a given level of risk. Individual assets, on their own, typically do not achieve this optimal balance but can contribute to a diversified portfolio that does.

Model		**Annualized Return**	**Annualized Volatility**
Baseline Model	Max Sharpe Ratio	29%	3%
	Minimum Volatility	15%	3%
Diversified Model	Max Sharpe Ratio	27%	3%
	Minimum Volatility	2%	1%
Cryptocurrency Model	Max Sharpe Ratio	29%	4%
	Minimum Volatility	-1%	1%

Table 4.1: Comparison of Performance Metrics for Each Model

While this section focuses on traditional MPT models, it is essential to acknowledge that machine learning (ML) approaches may offer advanced capabilities for portfolio optimization. ML techniques can handle more complex data patterns, non-linear relationships, and can adapt to changing market conditions more effectively than traditional MPT methods. However, MPT remains valuable for its simplicity, theoretical foundation, and ease of understanding for investors.

In conclusion, the MPT models explored here provide a foundation for constructing efficient portfolios tailored to different risk tolerances and investment goals. By understanding the implications of asset allocation, diversification, and risk management, investors can make informed decisions to optimize their portfolios.

4.2 MACHINE LEARNING MODELS

In the complex world of finance, where market conditions are ever-changing and data is abundant, Long Short-Term Memory (LSTM) models emerge as an invaluable tool for achieving a balance between capturing complex patterns and maintaining model simplicity. Financial data, such as stock prices and returns, is inherently sequential, meaning that past values influence future outcomes. Traditional models often struggle to capture these temporal dependencies, but LSTMs are designed specifically for this purpose. By understanding and learning from the sequence of past events, LSTMs can make more accurate predictions about future market behavior. Their ability to remember past trends and patterns over extended periods is crucial in financial markets, where historical events can significantly impact future prices and returns.

The financial world is also characterized by non-linear relationships and complex interactions among various assets. LSTM models excel in capturing these intricate patterns, providing insights that linear models might miss. This non-linear understanding helps in making more precise predictions.

The LSTM model developed begins with an input layer that receives the sequential data, comprising historical stock prices and their corresponding rolling means over various time windows. This input layer gets six features, each representing a rolling mean calculated over different time intervals. This input is then passed on to the first LSTM layer, which comprises 100 units. These units analyze the input sequence, capturing temporal dependencies and patterns that are crucial for predicting future stock prices.

To prevent the model from overfitting, a dropout layer is strategically inserted after the first LSTM layer. This dropout layer, with a rate of 0.4, ensures that 40% of the units are randomly deactivated during training, forcing the model to learn more robust and generalizable patterns.

Following the first LSTM layer is another dropout layer, which is then succeeded by a second LSTM layer. This second LSTM layer, mirroring the first in structure, further refines the sequential information, distilling it into a more compact representation. Another dropout layer follows this second LSTM layer, reinforcing the model's ability to generalize well to unseen data.

Finally, the output layer of the LSTM model comprises a single unit, which predicts the future stock price based on the processed sequential information. This output is then compared to the actual stock price to evaluate the model's performance. The activation function used in this output layer, along with the LSTM units, introduces non-linearity and controls the output range, ensuring that the model's predictions are both accurate and meaningful.

In this LSTM model, the default activation functions are used for both the LSTM units and the output layer. The hyperbolic tangent (tanh) or sigmoid activation functions are commonly used in LSTM models as they help stabilize the gradient during training, preventing issues such as vanishing or exploding gradients. The output ranges of -1 to 1 for tanh and 0 to 1 for sigmoid are suitable for the types of data used, helping to scale the output appropriately for stock price prediction.

The model uses a learning rate of 0.01. The learning rate is a hyperparameter that controls how much the model weights are updated during training. A higher learning rate can lead to faster convergence but may overshoot the optimal values, while a lower learning rate can slow down convergence. In this case, a learning rate of 0.01 was chosen to balance convergence speed and stability. This learning rate, combined with the other design choices in the model, enables it to effectively capture temporal dependencies and complex patterns in financial data, making it a powerful tool for predicting stock prices and optimizing portfolios.

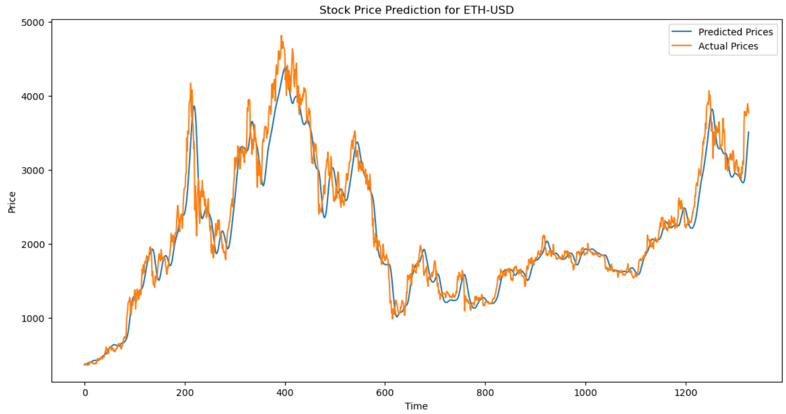


Figure 4.4: LSTMs performance for ETH-USD.

Regarding the results obtained with the developed model, it achieved an annualized return of 12.73% and volatility of 8.47%. While the Sharpe Ratio of 0.529 indicates a balance between risk and reward, the value being below 1 suggests that the results are not optimal. Despite this, the model shows potential in stock price prediction and portfolio optimization. Additionally, maximum drawdown of -17.93% suggests that the model was able to limit losses during market downturns.

Investors with a moderate risk tolerance who prioritize steady returns over aggressive growth may find this model appealing. In an economic situation characterized by moderate volatility and stable growth, such as a period of economic expansion, this model could be suitable for investors seeking to balance risk and reward.

The results obtained from the LSTM model may not be as optimal as expected due to the complexity of finding the appropriate model structure. Computational limitations, including the large number of assets and the complexity of operations involved, have hindered the discovery of an ideal LSTM architecture. The extensive computations required often result in long runtimes, and the kernel tends to terminate before completing the training process. These challenges have impeded the thorough exploration of hyperparameters and model architectures, potentially leading to suboptimal results. Despite these limitations, the model demonstrates potential in predicting stock prices and optimizing portfolios, suggesting that further refinement and optimization could lead to improved performance.

4.3 COMBINATION OF MODERN PORTFOLIO THEORY AND MACHINE LEARNING MODELS

Integrating Modern Portfolio Theory (MPT) with Long Short-Term Memory (LSTM) models provides a hybrid framework for refining portfolio optimization strategies. This approach combines the robust theoretical foundation of MPT with the advanced predictive capabilities of LSTM networks, resulting in a more dynamic and effective investment strategy.

Modern Portfolio Theory, with its focus on balancing risk and return through diversification, sets the groundwork for constructing portfolios. MPT employs historical data to develop portfolios that aim to maximize returns for a given level of risk. It achieves this through a systematic approach that calculates optimal asset weights to achieve a desired risk-return profile. However, MPT relies heavily on historical data and assumptions, which may not fully capture the complexities of future market movements.

In contrast, LSTM models bring an advanced layer of predictive accuracy to the table. These models, designed to capture complex and non-linear patterns in sequential data, offer enhanced forecasts of future asset prices. By integrating LSTM-generated predictions into the MPT framework, the portfolio can be dynamically adjusted based on anticipated market conditions, leading to more informed asset allocation decisions.

The results from applying this hybrid model illustrate its effectiveness. The optimal portfolio weights derived from the model highlight a strategic focus on a select few assets. Specifically, the model allocated 11.20% to Pfizer, 37.43% to Goldman Sachs, 31.35% to Caterpillar, and 20.02% to Bitcoin. This asset allocation reflects the model's emphasis on a combination of high-growth and stability-oriented investments. For instance, the significant allocation to Goldman Sachs and Caterpillar suggests a strategic focus on established companies with strong growth potential, while the allocation to Bitcoin indicates a willingness to incorporate high-risk, high-reward assets.

The performance metrics of this hybrid model further underscore its effectiveness. The portfolio achieved an annualized return of 15%, reflecting strong performance in terms of generating profits. The annualized volatility was relatively low at 5%, indicating a stable investment with minimal fluctuations in value. A Sharpe Ratio of 2.86 demonstrates that the portfolio achieved high returns relative to the risk taken, suggesting efficient risk management and effective use of capital. Additionally, the maximum drawdown of -8.33% shows that while there were some losses, they were within manageable limits, highlighting the model's robustness in protecting against significant downturns.

The combination of MPT and LSTM models provides a sophisticated approach to portfolio optimization. MPT offers a solid framework for diversification and risk management, while LSTM models enhance predictive accuracy, leading to more responsive and dynamic portfolio adjustments. The results from the hybrid model demonstrate its capability to deliver substantial returns with controlled risk, illustrating the potential benefits of integrating advanced machine learning techniques with traditional financial theories.

4.4 DISCUSSION OF RESULTS

The comparative analysis of different portfolio optimization approaches highlights their distinct characteristics, performance metrics, and suitability under varying economic conditions. This section delves into the nuances of Modern Portfolio Theory (MPT) models, Machine Learning (ML) models, and a hybrid approach combining both methods, offering a comprehensive understanding of their effectiveness and applicability.

Starting with the MPT models, the baseline approach demonstrated an impressive capability to generate high returns. The maximum Sharpe ratio portfolio achieved an annualized return of 29% with an annualized volatility of just 3%. This outcome reflects a strategy that balances risk and reward effectively, making it highly attractive for aggressive investors who are comfortable with taking on significant risk to pursue higher returns. Conversely, the minimum volatility strategy within the MPT models provided a more conservative option, delivering a robust annualized return of 15% while maintaining a volatility of 3%. This approach is well-suited for investors who prioritize stability and prefer to avoid substantial fluctuations in their portfolio's value.

The diversified MPT model, which included a broader selection of assets, yielded a slightly lower maximum Sharpe ratio portfolio return of 27% but maintained the same volatility of 3%. This portfolio highlights a balanced approach, integrating various assets to manage risk while still pursuing growth. The minimum volatility portfolio in this model offered a much lower annualized return of 2%, but with a significantly reduced volatility of just 1%. This portfolio is ideal for investors who seek to minimize risk, even if it means accepting lower returns.

Turning to the ML models, specifically the Long Short-Term Memory (LSTM) model, we observe a different performance profile. The LSTM model achieved an annualized return of 12.73% and a higher volatility of 8.47%. While this model incorporates advanced techniques to predict market movements and capture complex data patterns, its returns and volatility indicate that it is less effective in generating high returns compared to the top MPT models. The elevated volatility reflects the model's heightened sensitivity to market fluctuations, suggesting that while it offers valuable predictive insights, it does so with increased risk.

The hybrid approach, which combines MPT and ML models, presents a compelling middle ground. This strategy achieved an annualized return of 15% and a volatility of 5%. The hybrid model leverages the theoretical strengths of MPT for diversification and risk management while integrating the predictive power of ML models. This balance results in a portfolio that delivers a solid return with moderate volatility, offering a pragmatic solution for investors who seek a blend of stability and growth potential.

Examining the suitability of each approach across different economic scenarios provides additional insights. During periods of strong economic growth, where high returns are anticipated, the MPT baseline model stands out due to its ability to generate substantial profits with relatively low volatility. This makes it an attractive option for investors looking to capitalize on economic expansion.

In stable economic conditions with moderate growth, both the diversified MPT model and the hybrid MPT-ML model offer balanced solutions. The diversified MPT model provides stability with its lower volatility and moderate returns, making it suitable for investors seeking consistent performance. The hybrid model, with its mix of predictive accuracy and risk management, offers a well-rounded approach for those who desire a balance between risk and return.

During times of high market volatility or economic uncertainty, the ML model and the hybrid model demonstrate their value. The ML model's ability to adapt to complex market conditions can be advantageous, although it comes with higher risk. The hybrid approach, combining MPT's diversification with ML's predictive capabilities, provides a more stable option compared to relying solely on ML, offering effective risk management while capturing potential growth opportunities.

For conservative investors or in economic downturns, the minimum volatility strategies within the MPT models offer a defensive approach focused on capital preservation. Despite lower returns, these portfolios provide a safer investment option by minimizing risk and potential losses.

	Annualized Return	**Annualized Volatility**
Modern Portfolio Thoery	29%	3%
Machine Learning	12.73%	8.47%
Combination of MPT and ML	15%	5%

Table 4.2: Comparison of best Results for MPT, ML, and combined models

In summary, while MPT models offer strong theoretical foundations and historically robust performance, the inclusion of ML models introduces advanced predictive capabilities that can enhance portfolio management. The hybrid model, combining the best of both worlds, presents a promising approach that balances risk and return effectively. The choice of strategy will depend on the investor's risk tolerance, return objectives, and the prevailing economic conditions, ensuring that each investor can find an optimal solution for their specific needs.

5 CONCLUSION

In conclusion, various approaches have been implemented for portfolio optimization and stock price prediction, highlighting the strengths and limitations of each methodology. Modern Portfolio Theory (MPT) has provided a solid foundation, offering a tool for constructing portfolios tailored to different risk appetites. The MPT models, including the baseline and diversified portfolios, have demonstrated the importance of diversification in mitigating risk and achieving a balance between stability and growth.

The inclusion of cryptocurrencies in the portfolio marked a significant departure from traditional assets, introducing new risks and returns. While appealing to investors seeking high growth potential, this approach requires a higher risk tolerance and careful risk management strategies.

Transitioning to machine learning, the Long Short-Term Memory (LSTM) model emerged as a powerful tool for predicting stock prices and optimizing portfolios. Despite computational challenges, the LSTM model showed promise in capturing complex patterns and non-linear relationships in financial data, offering a balance between risk and reward for investors with moderate risk tolerance, although the results were not the desired ones.

REFERENCES

[1] Grum, Andraz. “Measuring Market Risk for Commercial Banks in the Volatile Environment of an Emerging Market Economy.” South East European Journal of Economics and Business 2, no. 2. Obtained from: https://doi.org/10.2478/v10033-007-0009-x.

[2] Investment Portfolios: Asset Allocation Models, n.d. Obtained from: https://investor.vanguard.com/investor-resources-education/education/model-portfolio-allocation#: :text=An%20investment

[3] Bartram, Söhnke M., Jürgen Branke, Giuliano De Rossi, and Mehrshad Motahari. “Machine Learning for Active Portfolio Management.” Obtained from: https://doi.org/10.3905/jfds.2021.1.071.

[4] Schwab Brokerage. “Understanding Stocks,” n.d. Obtained from: https://www.schwab.com/stocks/understand-stocks

[5] Napoletano, E. “Fixed-Income Basics: What Is a Bond?” Forbes Advisor, April 21, 2024. Obtained from: https://www.forbes.com/advisor/investing/what-is-a-bond/

[6] Definition of Cash and Cash Equivalents — DART – Deloitte Accounting Research Tool, n.d. Obtained from: https://dart.deloitte.com/USDART/home/codification/presentation/asc230-10/roadmap-statement-cash-flow/

[7] Chen, James. “Real Estate: Definition, Types, How to Invest in It.” Investopedia, May 31, 2024. Obtained from: https://www.investopedia.com/terms/r/realestate.asp.

[8] Pimco. “Understanding Commodities — PIMCO.” Pacific Investment Management Company LLC, n.d. Obtained from: https://www.pimco.com/eu/en/resources/education/understanding-commodities

[9] What Are Returns in Investing? The Motley Fool. Obtained from: https://www.fool.com/investing/how-to-invest/returns/

[10] A Guide to Portfolio Optimization Strategies - SmartAsset — SmartAsset. Obtained from: https://smartasset.com/investing/guide-portfolio-optimization-strategies

[11] Modern Portfolio Theory: What MPT Is and How Investors Use It. Investopedia. Obtained from: https://www.investopedia.com/terms/m/modernportfoliotheory.asp

[12] Olah, Christopher. ”Understanding LSTM Networks.” colah’s blog. Obtained from: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LIST OF TABLES

LIST OF FIGURES