Two-steps Shapley value computation for Smart Grid management

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Abstract

Smart Grid is an electrical grid that allows the management of the resources in an "intelligent" way. This includes the smart allocation of supplies where it is needed, the trading of energy between consumers that are able to produce electricity (prosumer), the exchange of energy at low-level voltage to minimize the power loss and so on. In this research we use a coalitional game theoretical approach to study the trading price and the coalitions formation of a set of prosumers distributed around a Macro Grid. Our approach works through the computation of two set of Shapley values. The first one is used to establish the trading price in a coalition of prosumers, while the second one is used to evaluate the final gain (for sellers) or cost (for buyers), considering the Smart Grid trading price and the Macro Grid selling and buying price. Lastly, we simulate a simplified real-life scenario using the methods previously exposed.

1. Introduction

Smart Grids have been largely considered the future of power grids, mostly because of the positive environmental impact and the possibility to minimize the costs compared to the traditional electric grid system. Nowadays, home electricity is provided by Macro Grids (M-G), usually located far away from city centers: this results in power loss whenever the energy is brought to the houses. Recently, governments and people have been trying to focus the attention on renewable resources, due to climatic issues. This led to an increasing usage of solar panels for the homeowners (consumers), making them becoming the producers for themselves (prosumers). Since the usage of renewable resources is seen to increase in the future, it has been investigated the birth of an internal market inside a coalition of houses. This will eventually lead the prosumers to rely less on the Macro Grid, which means money saving and green choices.

But Smart Grids are not easy to implement, due to the fact that they have to guarantee efficiency, otherwise people are not willing to join the market, preferring an old and efficient method. So, firstly the formation of coalitions is fundamental and it has been studied using game theory concepts, mostly focusing on price models or power loss models. The first ones are based on the idea that players join a group only if, by doing so, their expense will be minimized. The latter focus on the fact that, in these days, power transfers between the Macro Grid and consumers occur with a lot of power loss because of the distance between them, the transformer, the resistance of the cable and other factors. On the other hand, a movement of energy between nearest prosumers would occur at lower voltages, leading to fewer losses. Secondly, a security aspect has to be investigated: hackers could attack the Smart Grid in order to steal information and make profits. This problem could be partially avoided considering energy storing and transfers of chunks of energy. Lastly, prosumers are humans and they are moved by psychological motivations. Since Smart Grids are the results of coalition between people, consumers have to be incentivized to leave their lifelong electricity suppliers in order to approach a new, innovative method.

In this work, we expose a game theoretical approach based on Shapley values computation, evaluated using two different value functions. This method is used to compute the trading price between prosumers inside the Smart Grid, p_{SG}, and to compute the final cost or gain of the individual, considering the electricity price and the power loss.

2. Related Work

There exist different price models inside a Smart Grid in literature, some based on market laws, others on game theory concepts. If a coalitional game is considered, a Shapley value approach seems to be optimal, since each player should get something proportional to her contribution to the group [5]. In this article, a value function that keeps track of sellers and buyers in a coalition is used, in order to show that the grand coalition (i.e. the coalition of all players) is always preferable, and it is used to compute the optimal trading prices between prosumers, using the selling and buying price of the Macro Grid as trade-off.

But price-only schemes are not enough since, when distances are considered, the grand coalition is not always optimal. Regarding this aspect, a nearest neighbors coalition is preferable, since in this way the power loss would be minimized [4].

Lastly, there are studies about the psychological aspect [3]. The change in terms of energy paradigm, from the people prospective, is something that does not vary in a fast way: there exist various models focused on motivating prosumers towards sustainable and green choices.

3. Coalitional game

A *coalitional game* is a game where players join each other to form a coalition in order to maximize their profits. Consider a set $\mathcal{N}$ of n players, each subset of $\mathcal{N}$ is called $\mathcal{S}$. $\mathcal{N}$ is said to be the grand coalition.

There exists a *value function* $v : 2^n \rightarrow \mathbb{R}$ that maps each possible coalition in a real value. Denoting with u_i the payoff of the i-th player belonging to the coalition, it holds that:

$$\sum_{i \in \mathcal{S}} u_i = v(\mathcal{S}) \tag{1}$$

The value function has to satisfy the following property:

$$v(\emptyset) = 0 \tag{2}$$

If it is valid also the superadditive property (equation 3), then the coalitional game is said to be superadditive. In this case, the grand coalition is always preferable.

$$v(\mathcal{A} + \mathcal{B}) \geq v(\mathcal{A}) + v(\mathcal{B}) \tag{3}$$

The *Shapley value*, $\phi(\mathcal{S}, v)$, is a variable that quantifies the reward each player in a coalition $\mathcal{S}$ should receive. It is proportional to the difference between the value of the coalition with player i inside, and the value of the coalition before the entrance of player i. In this way, Shapley value quantifies the collective contribution of the player to the coalition.

$$\phi_i(\mathcal{S}, v) = \frac{1}{|\mathcal{S}|} \sum_{\pi \in \Pi} (v(\mathcal{S}(\pi, i) \cup i) - v(\mathcal{S}(\pi, i))) \tag{4}$$

ϕ_i can be assumed to be the payoff of each player in the coalition. According to equation 1:

$$\sum_{i \in \mathcal{S}} \phi_i(\mathcal{S}, v) = v(\mathcal{S}) \tag{5}$$

The *core* of a coalitional game is defined as a set of allocations (rewards) such that:

$$\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) \qquad \sum_{i \in \mathcal{S}} x_i \geq v(\mathcal{S}) \qquad \forall \mathcal{S} \subseteq \mathcal{N} \tag{6}$$

It is a way to split the rewards of the grand coalition in a way that nobody has the incentive to change alliance. The core is non-empty if and only if the grand coalition is stable. This is a key concept for dynamic coalitional games.

4. Model formulation

Consider a fixed amount of prosumers, each of them connected to a Macro Grid. Each player has a total amount of energy $Q_i = E_i - D_i$, which is the difference between the produced energy E_i and the demand of the house D_i. If $Q_i > 0$, then the prosumer is defined as a "seller", so she wants to sell the surplus of energy to other houses or to the Macro Grid, in order to make profit. If $Q_i < 0$, the prosumer is a "buyer" and she needs to satisfy her demand. We want to understand which are the optimal coalitions and which is the total amount of money a player spends or gains in each configuration.

4.1. Trading price p_{SG} computation

The first step is to evaluate the trading price between players, p_{SG}. We suppose to have two fixed variables: $p_{MG,s}$ and $p_{MG,b}$, which are respectively the selling and buying price of the Macro Grid. Intuitively, $p_{MG,b} \leq p_{SG} \leq p_{MG,s}$. We use the same value function as [7]:

$$v(\mathcal{S}) = p_{MG,b}[Q_+]^+ - p_{MG,s}[Q_-]^+ \tag{7}$$

$$Q_+ = \sum_{s \in \mathcal{N}_s} Q_s - \sum_{b \in \mathcal{N}_b} |Q_b| \tag{8}$$

Where $\mathcal{N}_b$ is the set of buyers, $\mathcal{N}_s$ is the set of sellers and $\mathcal{S}$ is the considered coalition. $Q_- = -Q_+$ and $[\cdot]^+ = max(0, \cdot)$.

For every possible coalition, we generate all the values of $v(\mathcal{S})$ and then we proceed to compute the Shapley values for each the player. This leads to p_{SG}, the trading price inside the coalition between the prosumers, computed as following:

$$p_{SG} = \frac{\sum_{s \in \mathcal{N}_s} \phi_s(\mathcal{S}, v)}{\sum_{s \in \mathcal{N}_s} v(s)} \tag{9}$$

Where the denominator is the sum over all the value function of the coalitions made by the single seller alone (i.e. (1), (2), (3) and so on).

4.2. Final payoff computation

But the previous considerations alone are not enough, since in those cases the grand coalition is always the best one. This is unreal, since for prosumers who are very far away from each other is not convenient to make a coalition, because this would imply huge power losses. So we introduce a second Shapley values computation, which considers both the trading price and the power losses due to distances, materials, transformers and others. We use, as [7] and [4], the following expressions to quantify the power losses:

$$\begin{cases} P_{ij} = \dfrac{R_{ij} P_i(Q_i)^2}{U_1^2} & \text{prosumer} \leftrightarrow \text{prosumer} \\ P_{i0} = \dfrac{R_{i0} P_i(Q_i)^2}{U_0^2} + \beta P_i(Q_i) & \text{prosumer} \leftrightarrow \text{M-G} \end{cases} \tag{10}$$

We have some constant values such as:

- U_0: mean value of the voltage in the process prosumer - M-G
- U_1: mean value of the voltage in the process prosumer - prosumer
- β: coefficient that quantify the power lost in the transformer at the M-G
- R: resistance of the distribution lines
- $R_{ij} = R\, d_{ij}$: resistance of the distribution line prosumer - prosumer
- $R_{i0} = R\, d_{i0}$: resistance of the distribution line prosumer - M-G.

The function $P_i(Q_i)$ is defined as following:

$$P_i(Q_i) = \begin{cases} Q_i & \text{if } Q_i > 0 \\ L_i & \text{if } Q_i < 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

This quantified the total amount of energy transfered. If player i is a seller, then she wants to sell everything she has. On the other hand, the L_i term is due to the fact that, if player i needs an amount of energy Q_i, she needs to buy $Q_i + \delta$ because δ will be lost in the transportation.

If we substitute the term $P_i(Q_i)$ for the $Q_i < 0$ in equation 10 we have, respectively:

$$\begin{cases} \dfrac{R_{ij}}{U_1^2} L_i^2 - L_i - Q_i = 0 & \text{prosumer} \leftrightarrow \text{prosumer} \\ \dfrac{R_{i0}}{U_0^2} L_i^2 - \beta L_i - Q_i = 0 & \text{prosumer} \leftrightarrow \text{M-G} \end{cases} \tag{12}$$

We solve these equations in order to find L_i. If we have two positive and distinct (real) solutions, we assume L_i to be the lowest one (less power loss). In all other cases, we assume the following expressions, which is the maximum amount of energy the buyer can receive:

$$\begin{cases} L_i = \dfrac{U_1^2}{2R_{ij}} & \text{prosumer} \leftrightarrow \text{prosumer} \\ L_i = \dfrac{(1-\beta)U_0^2}{2R_{i0}} & \text{prosumer} \leftrightarrow \text{M-G} \end{cases} \tag{13}$$

At this point, we have to multiply the total amount of energy sold or bought by its price. For sellers:

$$\begin{cases} u_{SG,i}^{seller} = p_{SG} \sum_{b \in \mathcal{N}_b} L_{i,b} & \text{prosumer} \leftrightarrow \text{prosumer} \\ u_{MG,i}^{seller} = p_{MG,b} \left(Q_i - \sum_{b \in \mathcal{N}_b} L_{i,b}\right) & \text{prosumer} \leftrightarrow \text{M-G} \end{cases} \tag{14}$$

While for the buyers:

$$\begin{cases} u_{SG,i}^{buyer} = -p_{SG} \sum_{s \in \mathcal{N}_s} L_{i,s} & \text{prosumer} \leftrightarrow \text{prosumer} \\ u_{MG,i}^{buyer} = -p_{MG,s} \left(Q_i - \sum_{s \in \mathcal{N}_s} L_{i,s}\right) & \text{prosumer} \leftrightarrow \text{M-G} \end{cases} \tag{15}$$

In the equations above, the term $\sum_{b \in \mathcal{N}_b} L_{i,b}$ is the total amount of energy that the seller has to sell to each buyers, whereas $\sum_{s \in \mathcal{N}_s} L_{i,s}$ is the total amount of energy that the buyer has to buy from each sellers.

The final payoffs are:

$$\begin{cases} u_i^{seller} = u_{SG,i}^{seller} + u_{MG,i}^{seller} & \text{if } Q_i > 0 \\ u_i^{buyer} = u_{SG,i}^{buyer} + u_{MG,i}^{buyer} & \text{if } Q_i < 0 \end{cases} \tag{16}$$

At this point it is possible to evaluate the Shapley value, assuming as value function:

$$v_2(\mathcal{S}) = \sum_{i \in \mathcal{S}} (u_i^{seller} + u_i^{buyer}) \tag{17}$$

The comparison between the Shapley values tell us which are the best coalitions. This value function has not the superadditive property, so the grand coalition is not always preferable.

5. Model application

In order to realize a real world application, we consider a small set of houses (players), each of them equipped with solar panels. Every player has different production and different consumption of energy based on multiple factors, such as number of solar panels, season of the year and position of the panels.

Based on these premises, as shown in [2], in table 1 we report the energy production for a 18 solar panels system. According to [1], in 2020 the average annual electricity consumption for a U.S. residential utility customer was about 10.715 kWh with an average of 893 kWh per month. In our model we assume that the monthly consumption of energy does not vary from the values reported. Moreover, we consider just one spatial configuration for the houses and the Macro Grid. What we are going to change is the time span of the analysis: we consider two periods, December and July, and the market of energy during these months.

Month	Value	Unit
December	122	kWh
July	717	kWh

Table 1. Average monthly values computed over 5 years.

6. Numerical results and analysis

For the simulation, we consider some fixed values that can be found in literature [6], these are reported in table 2.

We fix the position of the houses in an imaginary 2 dimensional grid with the Macro Grid in the origin, the number of solar panels for each house and the time window of the analysis. The values and the grid are reported in table 3 and figure 1. Also, since the trading price p_{SG} inside the Smart Grid has to be between $p_{MG,b}$ and $p_{MG,s}$, we fix these arbitrary values to 1 and 2.

U_0 (kV)	U_1(kV)	β	R (Ω)
50	22	0.02	0.2

Table 2. Some fixed values for computations.

Player	Panels	Coordinates (km)
1	14	$(-3.5, -3.5)$
2	10	$(-3.5, -3.7)$
3	20	$(4, 4.3)$
4	36	$(4.1, 4.2)$

Table 3. Coordinates and number of panels

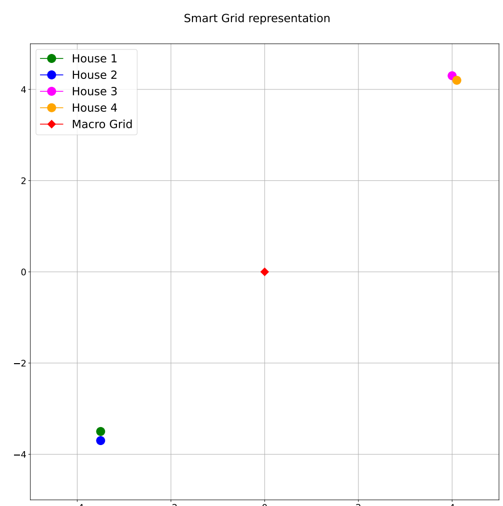

Figure 1. 2 dimensional grid of the network.

Using these premises, we consider the time window corresponding to the month of December and July. In table 4 are reported the numbers of panels and the values of parameter Q.

Player	Panels	Q (kWh) Dec	Q (kWh) July
1	14	-798	-335
2	10	-825	-495
3	20	-757	-96
4	36	-649	541

Table 4. Simulation of Q_i for the month of December and July.

6.1. Winter simulation

For what concern the month of December, we expect all the houses to buy the energy from the Macro Grid. This depends on the low production of solar energy during winter period and so on the lack of sellers between the prosumers.

Since Shapley values are a way to divide the total reward of the coalition, these are assumed to be the final cost or profit for each player, given the amount of energy needed or in surplus. In table 5 we report the Shapley values of the grand coalition. Since there is no trading between prosumers, for every other coalition we have the same values.

Player	Shapley value
1	-2474.74
2	-2406.67
3	-2086.88
4	-2086.88

Table 5. Output of the December simulation.

6.2. Summer simulation

Considering the month of July, the outputs of the different coalitions are reported in table 6. The grand coalition is preferable for every players: each of them receive a final payoff which is equal or greater than any other payoff of any other coalition. Regarding the core of the game, it is non-empty, since the set of allocations given by the Shapley values of the grand coalition, $\mathbf{x} = (-793.90, -1373.18, -151.31, 690.48)$, satisfies equation 6. Because of this, the grand coalition is stable.

Notice that the reward division given by the Shapley values is different from the total amount of costs and profits given by the payoff function of equation 16. For example, the total cost for player 1 in the coalition (1), given the previous equation, is -819.29, while for the grand coalition it is -838.49, which is lower. But the grand coalition is preferable due to the fact that player 1 gives a consistent contribution to the final coalition, so the reward of the other players is partially given to player 1. If the Shapley values split is not considered, the preferable coalitions would change, as in this example.

6.3. Random simulation

We want to investigate the scenario where we have 2 sellers and 2 buyers. We use the same setup as in July but with $Q_1 = 335$ kWh. Outputs of the algorithm are reported in table 7.

For every player, the grand coalition is preferable compared to the all-alone coalitions. From the point of view of the sellers, for player 1 the best coalition is $(1, 2, 3)$, whereas for player 4 it is $(2, 3, 4)$. In both cases, the seller is alone with both buyers, and the trading price inside the

Coalition	Shapley values
(1)	Player 1: -819.29
(2)	Player 2: -1442.50
(3)	Player 3: -206.09
(4)	Player 4: 541.0
(12)	Player 1: -819.29 Player 2: -1442.50
(13)	Player 1: -819.29 Player 3: -206.09
(14)	Player 1: -793.90 Player 4: 566.38
(23)	Player 2: -1442.50 Player 3: -206.09
(24)	Player 2: -1373.18 Player 4: 610.31
(34)	Player 3: -151.31 Player 4: 595.77
(123)	Player 1: -819.29 Player 2: -1442.50 Player 3: -206.09
(124)	Player 1: -793.90 Player 2: -1373.18 Player 4: 635.70
(134)	Player 1: -793.90 Player 3: -151.31 Player 4: 621.16
(234)	Player 2: -1373.18 Player 3: -151.31 Player 4: 665.09
(1234)	Player 1: -793.90 Player 2: -1373.18 Player 3: -151.31 Player 4: 690.48

Table 6. Output of the July simulation.

Coalition	Shapley values
(1)	Player 1: 335.00
(2)	Player 2: -1442.50
(3)	Player 3: -206.09
(4)	Player 4: 541.00
(12)	Player 1: 952.69 Player 2: -824.80
(13)	Player 1: 340.39 Player 3: -200.70
(14)	Player 1: 335.00 Player 4: 541.00
(23)	Player 2: -1442.50 Player 3: -206.09
(24)	Player 2: -1373.18 Player 4: 610.31
(34)	Player 3: -151.31 Player 4: 595.77
(123)	Player 1: 959.82 Player 2: -823.07 Player 3: -198.96
(124)	Player 1: 881.38 Player 2: -826.80 Player 4: 539.00
(134)	Player 1: 336.79 Player 3: -149.51 Player 4: 592.18
(234)	Player 2: -1373.18 Player 3: -151.31 Player 4: 665.09
(1234)	Player 1: 889.00 Player 2: -820.98 Player 3: -143.69 Player 4: 594.27

Table 7. Output of the random simulation.

Smart Grid is higher than in every other coalition. Otherwise, for the opposite reason, the grand coalition is preferable for the buyers.

We can investigate if the grand coalition is stable using the concept of the core. In this case, the set of allocations $\mathbf{x} = (-889.00, -820.98, 143.69, 594.27)$ given by the Shapley value does not form the core of the game. For example, considering $\mathcal{S} = (1, 2)$ and using equation 6, $98.02 \not\geq 127.88$. Therefore the core is empty and the grand coalition is not stable. This is because some player has the incentive to change coalition to gain more or spend less.

7. Conclusion

In this paper we investigated the problem of Smart Grid management using a game theoretical approach to explain the behavior of different players. We successfully implement a numerical simulation of some real world scenario, obtaining consistent results in terms of coalitions between the players of the grid and in terms of the exchange of energy.

Despite these aspects, an increment in the number of players would requires a huge effort in terms of computation resources. For this reason, even if we wanted to implement a real city scenario, it will be necessary to use some approximations. A possible solution could be the implementation of asymptotic Shapley value. Moreover, even if our model is able to produce consistent results, it is still needed a penalty term proportional to the energy lost, in order to discourage the trading between far away houses. This could be implemented considering a trading price that depends also on distances between houses.

In conclusion, even if some other aspects should be investigated more deeply, the two-steps Shapley value model enables us to understand which are the optimal coalitions in terms of the total reward or cost a player spend or gain inside a Smart Grid.

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