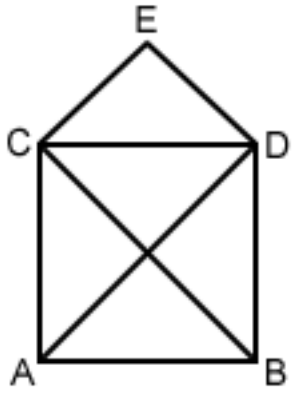
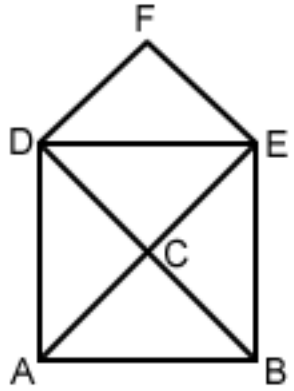
House of Lines

You are supposed to draw a house on paper without lifting your pencil or drawing any of the lines twice. If you label the corners A through E as in figure 1, one possible solution would be: A -> B -> D -> C -> E -> D -> A -> C -> B. How many possible solutions are there?



You can make it more challenging by letting the player change the pencil's direction at the point where the diagonals intersect. In that case, we would label the corners as in figure 2 and a possible solution would be: A -> C -> B -> A -> D -> C -> E -> D -> F -> E -> B. Now how many solutions are there?



Task:
Write a PHP program that finds and lists all possible ways of drawing the figure using the above notation. Once done, enhance the program to ignore solutions that are reflections of solutions already in the result set.