Topic: Infinite limits and vertical asymptotes

Question: Evaluate the limit.

$$\lim_{x \to 2} \frac{\sqrt{x^2+1}}{(x-2)^2}$$

Answer choices:

A 0

B 1

C ∞

D $-\infty$

Solution: C

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\frac{\sqrt{2^2+1}}{(2-2)^2}$$

$$\frac{\sqrt{5}}{0^2}$$

$$\frac{\sqrt{5}}{0}$$

So we'll test values close to, and on either side of, $x=2$ to see how the function is behaving close to that point.

$$f(1.9999)=\frac{\sqrt{1.9999^2+1}}{(1.9999-2)^2}\approx\frac{\sqrt{4.9999}}{0.00000001}\approx 225{,}000{,}000$$

$$f(2.0001)=\frac{\sqrt{2.0001^2+1}}{(2.0001-1)^2}\approx\frac{\sqrt{5.0004}}{0.00000001}\approx 225{,}000{,}000$$

The specific values we find aren't important. All that matters is that we realize that these extremely large values tell us that the function is approaching ∞ on both sides of $x=2$, so we can say that the value of the limit is ∞.

$$\lim_{x\to 2}\frac{\sqrt{x^2+1}}{(x-2)^2}=\infty$$

Topic: Infinite limits and vertical asymptotes

Question: Evaluate the limit.

$$\lim_{x \to \frac{\pi}{2}} \tan x$$

Answer choices:

A $\lim_{x \to \frac{\pi}{2}^-} \tan x = -\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan x = \infty$

B $\lim_{x \to \frac{\pi}{2}^-} \tan x = \infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan x = -\infty$

C $\lim_{x \to \frac{\pi}{2}^-} \tan x = -\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan x = -\infty$

D $\lim_{x \to \frac{\pi}{2}^-} \tan x = \infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan x = \infty$

Solution: B

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\tan\frac{\pi}{2}$$

$$\frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$$

$$\frac{1}{0}$$

So we'll test values close to, and on either side of, $x = \pi/2$ to see how the function is behaving close to that point.

$$f\left(\frac{49\pi}{100}\right) = \tan\frac{49\pi}{100} \approx 31.82$$

$$f\left(\frac{51\pi}{100}\right) = \tan\frac{51\pi}{100} \approx -31.82$$

The function is approaching ∞ to the left of $x = \pi/2$ and $-\infty$ to the right of $x = \pi/2$, so the general limit does not exist. But the one-sided limits are

$$\lim_{x\to\frac{\pi}{2}^-} \tan x = \infty$$

$$\lim_{x\to\frac{\pi}{2}^+} \tan x = -\infty$$

Topic: Infinite limits and vertical asymptotes

Question: Evaluate the limit.

$$\lim_{x \to \pi} \cot x$$

Answer choices:

A 0

B 1

C ∞

D Does not exist (DNE)

Solution: D

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\cot \pi$$

$$\frac{\cos \pi}{\sin \pi}$$

$$\frac{-1}{0}$$

So we'll test values close to, and on either side of, $x = \pi$ to see how the function is behaving close to that point.

$$f\left(\frac{99\pi}{100}\right) = \cot \frac{99\pi}{100} \approx -31.82$$

$$f\left(\frac{101\pi}{100}\right) = \cot \frac{101\pi}{100} \approx 31.82$$

The function is approaching $-\infty$ to the left of $x = \pi$ and ∞ to the right of $x = \pi$, so the general limit does not exist.