Ball thrown up from the ground

In this derivative application, we're dealing with the vertical motion pattern of an object that's launched straight up from the ground (or some other height), travels up until it reaches a maximum, and then falls back down to earth.

In these problems, we're always interested in three positions along the flight path of the object:

1. the initial position where the object begins when it's first thrown up,
2. the maximum position where the object stops traveling up and starts traveling back down again, and
3. the position at which the object hits the ground, finishing its flight.

At each of these three positions, we're always interested in three values:

1. the time at which the object reaches the position,
2. the height of the object at that position, and
3. the velocity of the object at that position.

If the ball is thrown from the ground, instead of from some higher spot, then its initial height is $y=0$. And when it hits the ground again after falling back down, its final height will again be $y=0$.

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We also can say that the ball flight begins at time $t = 0$. And when the ball reaches its maximum point, the velocity is $v = 0$, since this is the point where the object changes the direction, from increasing on its way up to decreasing on its way down.

Therefore, before we even begin the problem, we can partially fill in the table of values that describes the flight path.

Initial	**Maximum**	**End**
$t = 0$		
	$v = 0$	
$y = 0$		$y = 0$

So when we solve these kinds of problems, we're usually working on filling in the missing values in this table. We'll usually start with a position function $y(t)$ that models the flight path of the object, and an initial velocity. If the position function includes g, it's the gravitational constant $g = 32$ ft/s^2 or $g = 9.8$ m/s^2, and we can substitute for it.

If we differentiate the position function, we get the velocity function, and we can set the velocity function equal to 0, we'll be able to solve for the time t at which the object reaches its maximum height.

Then we can plug that value of t into the position function in order to find the object's maximum height. At that point, we'll have the first two columns from the table completely filled in.

To find the time when the object hits the ground at the end of its path, set the position function equal to 0, and solve for t. We should find $t = 0$ and one other value for t. This other value for t is the time at which the object hits the ground.

Then if we evaluate the velocity function (the derivative) at this second value of t, we'll get the velocity of the object when it hits the ground.

Let's work through a full example.

Example

A ball is thrown straight up from the ground with an initial velocity of $v_0 = 32$ ft/s. Find its height and velocity when it leaves the ground, when it reaches its maximum height, and when it hits the ground again.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

First, substitute $g = 32$, since the gravitational constant is included in the function.

$$y(t) = -\frac{1}{2}(32)t^2 + v_0t + y_0$$

$$y(t) = -16t^2 + v_0t + y_0$$

The initial velocity is given as $v_0 = 32$, so substitute this value as well.

$$y(t) = -16t^2 + 32t + y_0$$

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Substitute the initial height of the ball, $y_0 = 0$.

$$y(t) = -16t^2 + 32t + 0$$

$$y(t) = -16t^2 + 32t$$

Fill out the vertical motion table with what's given so far in the question.

Initial	**Maximum**	**End**
$t = 0$		
$v = 32$	$v = 0$	
$y = 0$		$y = 0$

To fill out the rest of the table, start by setting the original function equal to 0, which will give us the time(s) at which the ball is on the ground.

$$-16t^2 + 32t = 0$$

$$-16t(t-2) = 0$$

$$t = 0, 2$$

This tells us that the ball is on the ground when $t = 0$ and $t = 2$, so we can add $t = 2$ to the vertical motion chart.

Initial	**Maximum**	**End**
$t = 0$		$t = 2$
$v = 32$	$v = 0$	

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$y = 0$	$y = 0$

Take the derivative of the original position function to find the velocity function.

$$v(t) = y'(t) = -32t + 32$$

Find the velocity at $t = 2$.

$$v(2) = -32(2) + 32$$

$$v(2) = -64 + 32$$

$$v(2) = -32$$

Add this to the vertical motion chart.

Initial	**Maximum**	**End**
$t = 0$		$t = 2$
$v = 32$	$v = 0$	$v = -32$
$y = 0$		$y = 0$

Set the velocity function equal to 0 to find the time at which the ball reaches its maximum height.

$$-32t + 32 = 0$$

$$32t = 32$$

$$t = 1$$

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Add this to the vertical motion chart.

Initial	Maximum	End
$t = 0$	$t = 1$	$t = 2$
$v = 32$	$v = 0$	$v = -32$
$y = 0$		$y = 0$

Substitute $t = 1$ into the original position function to find the maximum height of the ball.

$$y(1) = -16(1)^2 + 32(1)$$

$$y(1) = -16(1) + 32(1)$$

$$y(1) = -16 + 32$$

$$y(1) = 16$$

Add this to the vertical motion chart.

Initial	Maximum	End
$t = 0$	$t = 1$	$t = 2$
$v = 32$	$v = 0$	$v = -32$
$y = 0$	$y = 16$	$y = 0$

If we want to summarize this information, we can say that the ball leaves the ground with an initial velocity of 32 ft/s, reaches its maximum height of 16 feet after $t = 1$ seconds, at which point it stops increasing in height,

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starts falling back toward the ground, and eventually hits the ground at a velocity of -32 ft/s after $t = 2$ second.

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