Calculus 1 Workbook Solutions

Derivative rules

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POWER RULE

■ 1. Find the derivative of $f(x) = 7x^3 - 17x^2 + 51x - 25$ using the power rule.

Solution:

Differentiating $f(x) = 7x^3 - 17x^2 + 51x - 25$ term-by-term gives

$$f'(x) = 7(3)x^{3-1} - 17(2)x^{2-1} + 51(1)x^{1-1} - 25(0)x^{0-1}$$

$$f'(x) = 21x^2 - 34x^1 + 51x^0 - 0x^{-1}$$

$$f'(x) = 21x^2 - 34x + 51(1) - 0$$

$$f'(x) = 21x^2 - 34x + 51$$

■ 2. Find the derivative of $g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$ using the power rule.

Solution:

Differentiating $g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$ term-by-term gives

$$g'(x) = 2(4)x^{4-1} + 8(3)x^{3-1} + 6(2)x^{2-1} - 32(1)x^{1-1} + 16(0)x^{0-1}$$

$$g'(x) = 8x^3 + 24x^2 + 12x^1 - 32x^0 + 0x^{-1}$$

$$g'(x) = 8x^3 + 24x^2 + 12x - 32(1) + 0$$

$$g'(x) = 8x^3 + 24x^2 + 12x - 32$$

■ 3. Find the derivative of $h(x) = 22x^3 - 19x^2 + 13x - 17$ using the power rule.

Solution:

Differentiating $h(x) = 22x^3 - 19x^2 + 13x - 17$ term-by-term gives

$$h'(x) = 22(3)x^{3-1} - 19(2)x^{2-1} + 13(1)x^{1-1} - 17(0)x^{0-1}$$

$$h'(x) = 66x^2 - 38x^1 + 13x^0 - 0x^{-1}$$

$$h'(x) = 66x^2 - 38x + 13(1) - 0$$

$$h'(x) = 66x^2 - 38x + 13$$

■ 4. Find the derivative of $h(s) = s^4 - s^3 + 3s - 7$ using the power rule.

Solution:

Differentiating $h(s) = s^4 - s^3 + 3s - 7$ term-by-term gives

$$h'(s) = 4s^{4-1} - 3s^{3-1} + 3(1)s^{1-1} - 7(0)x^{0-1}$$

$$h'(s) = 4s^3 - 3s^2 + 3s^0 - 0$$

$$h'(s) = 4s^3 - 3s^2 + 3$$

5. Find the derivative using the power rule.

$$g(t) = \frac{2}{3}t^3 - \frac{5}{2}t^6$$

Solution:

Differentiating the function term-by-term gives

$$g'(t) = \frac{2}{3}(3)t^{3-1} - \frac{5}{2}(6)t^{6-1}$$

$$g'(t) = 2t^2 - 15t^5$$

6. Find the derivative of $f(x) = 20x^{100} + 5x^{21} - 3x - 1$ using the power rule.

Solution:

Differentiating $f(x) = 20x^{100} + 5x^{21} - 3x - 1$ term-by-term gives

$$f'(x) = 20(100)x^{100-1} + 5(21)x^{21-1} - 3(1)x^{1-1} - 1(0)x^{0-1}$$

$$f'(x) = 2{,}000x^{99} + 105x^{20} - 3 - 0$$

$$f'(x) = 2{,}000x^{99} + 105x^{20} - 3$$

POWER RULE FOR NEGATIVE POWERS

1. Find the derivative of the function using the power rule.

$$f(x)=\frac{7}{x^2}-\frac{5}{x^4}+\frac{2}{x}$$

Solution:

Rearrange $f(x)$ to use the power rule.

$$f(x)=7x^{-2}-5x^{-4}+2x^{-1}$$

Differentiating term-by-term gives

$$f'(x)=7(-2)x^{-2-1}-5(-4)x^{-4-1}+2(-1)x^{-1-1}$$

$$f'(x)=-14x^{-3}+20x^{-5}-2x^{-2}$$

Move the variables back to the denominator to make positive exponents.

$$f'(x)=-\frac{14}{x^3}+\frac{20}{x^5}-\frac{2}{x^2}$$

2. Find the derivative of the function using the power rule.

$$g(x)=\frac{1}{9x^4}+\frac{2}{3x^5}-\frac{1}{x}$$

Solution:

Rearrange $g(x)$ to use the power rule.

$$g(x) = \frac{1}{9}x^{-4} + \frac{2}{3}x^{-5} - x^{-1}$$

Differentiating term-by-term gives

$$g'(x) = \frac{1}{9}(-4)x^{-4-1} + \frac{2}{3}(-5)x^{-5-1} - (-1)x^{-1-1}$$

$$g'(x) = -\frac{4}{9}x^{-5} - \frac{10}{3}x^{-6} + x^{-2}$$

Move the variables back to the denominator to make positive exponents.

$$g'(x) = -\frac{4}{9x^5} - \frac{10}{3x^6} + \frac{1}{x^2}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = -\frac{7}{6x^6} - \frac{1}{4x^4} + \frac{9}{2x^2}$$

Solution:

Rearrange $h(x)$ to use the power rule.

$$h(x) = -\frac{7}{6}x^{-6} - \frac{1}{4}x^{-4} + \frac{9}{2}x^{-2}$$

Differentiating term-by-term gives

$$h'(x) = -\frac{7}{6}(-6)x^{-6-1} - \frac{1}{4}(-4)x^{-4-1} + \frac{9}{2}(-2)x^{-2-1}$$

$$h'(x) = 7x^{-7} + x^{-5} - 9x^{-3}$$

Move the variables back to the denominator to make positive exponents.

$$h'(x) = \frac{7}{x^7} + \frac{1}{x^5} - \frac{9}{x^3}$$

■ 4. Find the derivative of the function using the power rule.

$$g(x) = \frac{3}{x^2} + \frac{3}{2x^4} + \frac{1}{2}$$

Solution:

Rearrange $g(x)$ to use the power rule.

$$g(x) = 3x^{-2} + \frac{3}{2}x^{-4} + \frac{1}{2}$$

Differentiating term-by-term gives

$$g'(x) = 3(-2)x^{-2-1} + \frac{3}{2}(-4)x^{-4-1} + 0$$

$$g'(x) = -6x^{-3} - 6x^{-5}$$

Move the variables back to the denominator to make positive exponents.

$$g'(x) = -\frac{6}{x^3} - \frac{6}{x^5}$$

■ 5. Find the derivative of the function using the power rule.

$$f(x) = -2x^{-4} + \frac{1}{x^2} + 7x$$

Solution:

Rearrange $f(x)$ to use the power rule.

$$f(x) = -2x^{-4} + x^{-2} + 7x$$

Differentiating term-by-term gives

$$f'(x) = -2(-4)x^{-4-1} + (-2)x^{-2-1} + 7x^{1-1}$$

$$f'(x) = 8x^{-5} - 2x^{-3} + 7$$

Move the variables back to the denominator to make positive exponents.

$$f'(x) = \frac{8}{x^5} - \frac{2}{x^3} + 7$$

6. Find the derivative of the function using the power rule, if a, b, and c are constants.

$$f(x) = 2ax^{-3a} + \frac{b}{cx^{2c}} - 2a$$

Solution:

Rearrange $f(x)$ to use the power rule.

$$f(x) = 2ax^{-3a} + \frac{b}{c}x^{-2c} - 2a$$

Differentiating term-by-term gives

$$f'(x) = 2a(-3a)x^{-3a-1} + \frac{b}{c}(-2c)x^{-2c-1} - 2a(0)x^{0-1}$$

$$f'(x) = -6a^2x^{-3a-1} - 2bx^{-2c-1}$$

Move the variables back to the denominator to make positive exponents.

$$f'(x) = -\frac{6a^2}{x^{3a+1}} - \frac{2b}{x^{2c+1}}$$

POWER RULE FOR FRACTIONAL POWERS

■ 1. Find the derivative of the function using the power rule.

$$f(x) = 4x^{\frac{3}{2}} - 6x^{\frac{5}{3}}$$

Solution:

Differentiating the function term-by-term gives

$$f'(x) = 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} - 6\left(\frac{5}{3}\right)x^{\frac{5}{3}-1}$$

$$f'(x) = 6x^{\frac{3}{2}-\frac{2}{2}} - 10x^{\frac{5}{3}-\frac{3}{3}}$$

$$f'(x) = 6x^{\frac{1}{2}} - 10x^{\frac{2}{3}}$$

■ 2. Find the derivative of the function using the power rule.

$$g(x) = 6x^{\sqrt{3}} - 4x^{\sqrt{5}}$$

Solution:

Differentiating the function term-by-term gives

$$g'(x) = 6\sqrt{3}x^{\sqrt{3}-1} - 4\sqrt{5}x^{\sqrt{5}-1}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = \frac{1}{3}x^{\frac{6}{5}} + \frac{1}{4}x^{\frac{8}{3}} - \frac{1}{5}x^{\frac{5}{2}}$$

Solution:

Differentiating the function term-by-term gives

$$h'(x) = \frac{1}{3}\left(\frac{6}{5}\right)x^{\frac{6}{5}-1} + \frac{1}{4}\left(\frac{8}{3}\right)x^{\frac{8}{3}-1} - \frac{1}{5}\left(\frac{5}{2}\right)x^{\frac{5}{2}-1}$$

$$h'(x) = \frac{2}{5}x^{\frac{6}{5}-\frac{5}{5}} + \frac{2}{3}x^{\frac{8}{3}-\frac{3}{3}} - \frac{1}{2}x^{\frac{5}{2}-\frac{2}{2}}$$

$$h'(x) = \frac{2}{5}x^{\frac{1}{5}} + \frac{2}{3}x^{\frac{5}{3}} - \frac{1}{2}x^{\frac{3}{2}}$$

■ 4. Find the derivative of the function using the power rule.

$$h(x) = \sqrt{x} + 2\sqrt[3]{x} - 3\sqrt[5]{x^2}$$

Solution:

Rewrite the function with fractional powers.

$$h(x) = x^{\frac{1}{2}} + 2x^{\frac{1}{3}} - 3x^{\frac{2}{5}}$$

Then differentiating term-by-term gives

$$h'(x)=\frac{1}{2}x^{\frac{1}{2}-1}+2\left(\frac{1}{3}\right)x^{\frac{1}{3}-1}-3\left(\frac{2}{5}\right)x^{\frac{2}{5}-1}$$

$$h'(x)=\frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}}+\frac{2}{3}x^{\frac{1}{3}-\frac{3}{3}}-\frac{6}{5}x^{\frac{2}{5}-\frac{5}{5}}$$

$$h'(x)=\frac{1}{2}x^{-\frac{1}{2}}+\frac{2}{3}x^{-\frac{2}{3}}-\frac{6}{5}x^{-\frac{3}{5}}$$

$$h'(x)=\frac{1}{2x^{\frac{1}{2}}}+\frac{2}{3x^{\frac{2}{3}}}-\frac{6}{5x^{\frac{3}{5}}}$$

Change back from fractional powers to roots.

$$h'(x)=\frac{1}{2\sqrt{x}}+\frac{2}{3\sqrt[3]{x^2}}-\frac{6}{5\sqrt[5]{x^3}}$$

■ 5. Find the derivative of the function using the power rule.

$$f(z)=\frac{3}{\sqrt{z^5}}+\frac{5}{4z^4}-2z^{-2}$$

Solution:

Rewrite the function.

$$f(z)=3z^{-\frac{5}{2}}+\frac{5}{4}z^{-4}-2z^{-2}$$

Then differentiating term-by-term gives

$$f'(z) = 3\left(-\frac{5}{2}\right)z^{-\frac{5}{2}-1} + \frac{5}{4}(-4)z^{-4-1} - 2(-2)z^{-2-1}$$

$$f'(z) = -\frac{15}{2}z^{-\frac{5}{2}-\frac{2}{2}} - 5z^{-5} + 4z^{-3}$$

$$f'(z) = -\frac{15}{2}z^{-\frac{7}{2}} - 5z^{-5} + 4z^{-3}$$

$$f'(z) = -\frac{15}{2z^{\frac{7}{2}}} - \frac{5}{z^5} + \frac{4}{z^3}$$

Change back from fractional powers to roots.

$$f'(z) = -\frac{15}{2\sqrt{z^7}} - \frac{5}{z^5} + \frac{4}{z^3}$$

■ 6. Find the derivative of the function using the power rule.

$$h(t) = \frac{2}{3t^6} + \frac{t^4}{4} - 9t^3 + \sqrt{t^3} + \frac{1}{2\sqrt[3]{t^2}}$$

Solution:

Rewrite the function.

$$h(t) = \frac{2}{3}t^{-6} + \frac{1}{4}t^4 - 9t^3 + t^{\frac{3}{2}} + \frac{1}{2}t^{-\frac{2}{3}}$$

Then differentiating term-by-term gives

$$h'(t) = \frac{2}{3}(-6)t^{-6-1} + \frac{1}{4}(4)t^{4-1} - 9(3)t^{3-1} + \frac{3}{2}t^{\frac{3}{2}-1} + \frac{1}{2}\left(-\frac{2}{3}\right)t^{-\frac{2}{3}-1}$$

$$h'(t) = -4t^{-7} + t^3 - 27t^2 + \frac{3}{2}t^{\frac{3}{2}-\frac{2}{2}} - \frac{1}{3}t^{-\frac{2}{3}-\frac{3}{3}}$$

$$h'(t) = -4t^{-7} + t^3 - 27t^2 + \frac{3}{2}t^{\frac{1}{2}} - \frac{1}{3}t^{-\frac{5}{3}}$$

Change back from fractional powers to roots.

$$h'(t) = -\frac{4}{t^7} + t^3 - 27t^2 + \frac{3}{2}\sqrt{t} - \frac{1}{3\sqrt[3]{t^5}}$$

PRODUCT RULE WITH TWO FUNCTIONS

1. Use the product rule to find the derivative of the function.

$$h(x) = (3x + 5)(2x^2 - 3x + 1)$$

Solution:

Let

$$f(x) = 3x + 5$$

$$f'(x) = 3$$

$$g(x) = 2x^2 - 3x + 1$$

$$g'(x) = 4x - 3$$

By product rule, the derivative is

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(x) = (3x + 5)(4x - 3) + 3(2x^2 - 3x + 1)$$

Expand the derivative, then collect like terms.

$$h'(x) = 12x^2 - 9x + 20x - 15 + 6x^2 - 9x + 3$$

$$h'(x) = 18x^2 + 2x - 12$$

■ 2. Use the product rule to find the derivative of the function.

$$h(x) = 8x^3\sqrt[3]{x^2}$$

Solution:

Let

$$f(x) = 8x^3$$

$$f'(x) = 24x^2$$

$$g(x) = \sqrt[3]{x^2}$$

$$g'(x) = \frac{2}{3}x^{-\frac{1}{3}}$$

Then by product rule, the derivative is

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(x) = 8x^3 \cdot \frac{2}{3}x^{-\frac{1}{3}} + 24x^2 \cdot \sqrt[3]{x^2}$$

$$h'(x) = \frac{16}{3}x^{\frac{8}{3}} + 24x^{\frac{8}{3}}$$

$$h'(x) = \frac{88}{3}x^{\frac{8}{3}}$$

■ 3. Use the product rule to find the derivative of the function.

$$h(x) = (5x^2 - x)\left(\frac{1}{x^4} - 6\right)$$

Solution:

Let

$$f(x) = 5x^2 - x$$

$$f'(x) = 10x - 1$$

$$g(x) = \frac{1}{x^4} - 6$$

$$g'(x) = -4x^{-5}$$

Then by product rule, the derivative is

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(x) = (5x^2 - x)(-4x^{-5}) + (10x - 1)\left(\frac{1}{x^4} - 6\right)$$

$$h'(x) = -20x^{-3} + 4x^{-4} + \frac{10}{x^3} - \frac{1}{x^4} - 60x + 6$$

$$h'(x) = -\frac{20}{x^3} + \frac{4}{x^4} + \frac{10}{x^3} - \frac{1}{x^4} - 60x + 6$$

$$h'(x) = -\frac{10}{x^3} + \frac{3}{x^4} - 60x + 6$$

■ 4. Use the product rule to find the derivative of the function.

$$h(x) = (1 + \sqrt{x^3})(x^{-2} - 3\sqrt[3]{x})$$

Solution:

Let

$$f(x) = 1 + \sqrt{x^3}$$

$$f'(x) = \frac{3}{2}\sqrt{x}$$

$$g(x) = x^{-2} - 3\sqrt[3]{x}$$

$$g'(x) = -2x^{-3} - x^{-\frac{2}{3}}$$

Then by product rule, the derivative is

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(x) = (1 + \sqrt{x^3})(-2x^{-3} - x^{-\frac{2}{3}}) + \left(\frac{3}{2}\sqrt{x}\right)(x^{-2} - 3\sqrt[3]{x})$$

$$h'(x) = -2x^{-3} - x^{-\frac{2}{3}} - 2x^{-\frac{3}{2}} - x^{\frac{5}{6}} + \frac{3}{2}x^{-\frac{3}{2}} - \frac{9}{2}x^{\frac{5}{6}}$$

$$h'(x) = -2x^{-3} - x^{-\frac{2}{3}} - \frac{1}{2}x^{-\frac{3}{2}} - \frac{11}{2}x^{\frac{5}{6}}$$

$$h'(x) = -\frac{2}{x^3} - \frac{1}{\sqrt[3]{x^2}} - \frac{1}{2\sqrt{x^3}} - \frac{11}{2}\sqrt[6]{x^5}$$

■ 5. If $f(3) = -4$, $f'(3) = 2$, $g(3) = -1$, and $g'(3) = 3$, determine the value of $(fg)'(3)$.

Solution:

Evaluating the product rule formula at $x = 3$ gives

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(3) = f(3)g'(3) + f'(3)g(3)$$

$$h'(3) = (-4)(3) + 2(-1)$$

$$h'(3) = -12 - 2$$

$$h'(3) = -14$$

■ 6. If $h(x) = 2x^3g(x)$, $g(-4) = -5$, and $g'(-4) = 1$, determine the value of $h'(-4)$.

Solution:

We're given $g(-4) = -5$ and $g'(-4) = 1$, but let's also set $f(x) = 2x^3$ and $f'(x) = 6x^2$. Then

$$f(-4) = 2(-4)^3 = -128$$

$$f'(-4) = 6(-4)^2 = 96$$

So evaluating the product rule formula at $x = -4$ gives

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(-4) = f(-4)g'(-4) + f'(-4)g(-4)$$

$$h'(-4) = (-128)(1) + 96(-5)$$

$$h'(-4) = -128 - 480$$

$$h'(-4) = -608$$

PRODUCT RULE WITH THREE OR MORE FUNCTIONS

1. Use the product rule to find the derivative of the function.

$$y = 5x^4(2x - x^2)\left(\frac{1}{x^2} - 5\right)$$

Solution:

Let

$$f(x) = 5x^4 \qquad g(x) = 2x - x^2 \qquad h(x) = \frac{1}{x^2} - 5$$

$$f'(x) = 20x^3 \qquad g'(x) = 2 - 2x \qquad h'(x) = -\frac{2}{x^3}$$

Then by product rule, the derivative is

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$y' = (20x^3)(2x - x^2)\left(\frac{1}{x^2} - 5\right) + (5x^4)(2 - 2x)\left(\frac{1}{x^2} - 5\right) + (5x^4)(2x - x^2)\left(-\frac{2}{x^3}\right)$$

$$y' = (40x^4 - 20x^5)\left(\frac{1}{x^2} - 5\right) + (10x^4 - 10x^5)\left(\frac{1}{x^2} - 5\right) + (10x^5 - 5x^6)\left(-\frac{2}{x^3}\right)$$

$$y' = 40x^2 - 200x^4 - 20x^3 + 100x^5 + 10x^2 - 50x^4 - 10x^3 + 50x^5 - 20x^2 + 10x^3$$

$$y' = 150x^5 - 250x^4 - 20x^3 + 30x^2$$

■ 2. Use the product rule to find the derivative of the function.

$$y = 30\left(\frac{1}{x^3} + x^2\right)(2x^4 - x^2 - x)$$

Solution:

Let

$$f(x) = 30 \qquad g(x) = \frac{1}{x^3} + x^2 \qquad h(x) = 2x^4 - x^2 - x$$

$$f'(x) = 0 \qquad g'(x) = -\frac{3}{x^4} + 2x \qquad h'(x) = 8x^3 - 2x - 1$$

Then by product rule, the derivative is

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$y' = (0)\left(\frac{1}{x^3} + x^2\right)(2x^4 - x^2 - x) + (30)\left(-\frac{3}{x^4} + 2x\right)(2x^4 - x^2 - x)$$

$$+(30)\left(\frac{1}{x^3} + x^2\right)(8x^3 - 2x - 1)$$

$$y' = 0 + (30)\left(-6 + \frac{3}{x^2} + \frac{3}{x^3} + 4x^5 - 2x^3 - 2x^2\right)$$

$$+(30)\left(8 - \frac{2}{x^2} - \frac{1}{x^3} + 8x^5 - 2x^3 - x^2\right)$$

$$y' = -180 + \frac{90}{x^2} + \frac{90}{x^3} + 120x^5 - 60x^3 - 60x^2$$

$$+240 - \frac{60}{x^2} - \frac{30}{x^3} + 240x^5 - 60x^3 - 30x^2$$

$$y' = 360x^5 - 120x^3 - 90x^2 + \frac{30}{x^2} + \frac{60}{x^3} + 60$$

■ 3. Use the product rule to find the derivative of the function.

$$y = (x^2 - 3x + 5)(7 + 2x - 5x^2)(2 - 2\sqrt{x})$$

Solution:

Let

$$f(x) = x^2 - 3x + 5 \qquad g(x) = 7 + 2x - 5x^2 \qquad h(x) = 2 - 2\sqrt{x}$$

$$f'(x) = 2x - 3 \qquad g'(x) = 2 - 10x \qquad h'(x) = -\frac{1}{\sqrt{x}}$$

Then by product rule, the derivative is

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$y' = (2x - 3)(7 + 2x - 5x^2)(2 - 2\sqrt{x}) + (x^2 - 3x + 5)(2 - 10x)(2 - 2\sqrt{x})$$

$$+(x^2 - 3x + 5)(7 + 2x - 5x^2)\left(-\frac{1}{\sqrt{x}}\right)$$

$$y' = (14x + 4x^2 - 10x^3 - 21 - 6x + 15x^2)(2 - 2\sqrt{x})$$

$$+(2x^2 - 10x^3 - 6x + 30x^2 + 10 - 50x)(2 - 2\sqrt{x})$$

$$+(7x^2 + 2x^3 - 5x^4 - 21x - 6x^2 + 15x^3 + 35 + 10x - 25x^2)\left(-\frac{1}{\sqrt{x}}\right)$$

$$y' = (-10x^3 + 19x^2 + 8x - 21)(2 - 2\sqrt{x}) + (-10x^3 + 32x^2 - 56x + 10)(2 - 2\sqrt{x})$$

$$+(-5x^4 + 17x^3 - 24x^2 - 11x + 35)(-\frac{1}{\sqrt{x}})$$

$$y' = -40x^3 + 102x^2 - 96x + 45x^{\frac{7}{2}} - 119x^{\frac{5}{2}} + 120x^{\frac{3}{2}} + 33\sqrt{x} - \frac{35}{\sqrt{x}} - 22$$

■ 4. Use the product rule to find the derivative of the function.

$$y = \left(x - \frac{3}{x}\right)(x^2 + 4x)(7x^4)\left(-5x^2 - \frac{1}{2}\right)$$

Solution:

Let

$$f(x) = x - \frac{3}{x} \qquad g(x) = x^2 + 4x \qquad h(x) = 7x^4 \qquad k(x) = -5x^2 - \frac{1}{2}$$

$$f'(x) = 1 + \frac{3}{x^2} \qquad g'(x) = 2x + 4 \qquad h'(x) = 28x^3 \qquad k'(x) = -10x$$

Then by product rule, the derivative is

$$y' = f'(x)g(x)h(x)k(x) + f(x)g'(x)h(x)k(x) + f(x)g(x)h'(x)k(x) + f(x)g(x)h(x)k'(x)$$

$$y' = \left(1 + \frac{3}{x^2}\right)(x^2 + 4x)(7x^4)\left(-5x^2 - \frac{1}{2}\right) + \left(x - \frac{3}{x}\right)(2x + 4)(7x^4)\left(-5x^2 - \frac{1}{2}\right)$$

$$+\left(x - \frac{3}{x}\right)(x^2 + 4x)(28x^3)\left(-5x^2 - \frac{1}{2}\right) + \left(x - \frac{3}{x}\right)(x^2 + 4x)(7x^4)(-10x)$$

$$y' = \left(1 + \frac{3}{x^2}\right)(7x^6 + 28x^5)\left(-5x^2 - \frac{1}{2}\right) + \left(x - \frac{3}{x}\right)(14x^5 + 28x^4)\left(-5x^2 - \frac{1}{2}\right)$$

$$+\left(x - \frac{3}{x}\right)(28x^5 + 112x^4)\left(-5x^2 - \frac{1}{2}\right) + \left(x - \frac{3}{x}\right)(7x^6 + 28x^5)(-10x)$$

$$y' = (7x^6 + 28x^5 + 21x^4 + 84x^3)\left(-5x^2 - \frac{1}{2}\right)$$

$$+(14x^6 + 28x^5 - 42x^4 - 84x^3)\left(-5x^2 - \frac{1}{2}\right)$$

$$+(28x^6 + 112x^5 - 84x^4 - 336x^3)\left(-5x^2 - \frac{1}{2}\right)$$

$$+(7x^7 + 28x^6 - 21x^5 - 84x^4)(-10x)$$

$$y' = -35x^8 - \frac{7}{2}x^6 - 140x^7 - 14x^5 - 105x^6 - \frac{21}{2}x^4 - 420x^5 - 42x^3$$

$$-70x^8 - 7x^6 - 140x^7 - 14x^5 + 210x^6 + 21x^4 + 420x^5 + 42x^3$$

$$-140x^8 - 14x^6 - 560x^7 - 56x^5 + 420x^6 + 42x^4 + 1{,}680x^5 + 168x^3$$

$$-70x^8 - 280x^7 + 210x^6 + 840x^5$$

$$y' = -35x^8 - 70x^8 - 140x^8 - 70x^8$$

$$-140x^7 - 140x^7 - 560x^7 - 280x^7$$

$$-\frac{7}{2}x^6 - 105x^6 - 7x^6 + 210x^6 - 14x^6 + 420x^6 + 210x^6$$

$$-14x^5 - 420x^5 - 14x^5 + 420x^5 - 56x^5 + 1{,}680x^5 + 840x^5$$

$$-\frac{21}{2}x^4 + 21x^4 + 42x^4$$

$$-42x^3 + 42x^3 + 168x^3$$

$$y' = -315x^8 - 1{,}120x^7 + \frac{1{,}421}{2}x^6 + 2{,}436x^5 + \frac{105}{2}x^4 + 168x^3$$

■ 5. Use $f(-2) = 5$, $f'(-2) = -7$, $g(-2) = -8$, $g'(-2) = -3$, $h(-2) = 1$ and $h'(-2) = 0$ to determine the value of $(fgh)'(-2)$.

Solution:

Evaluate the product rule formula for three functions at $x = -2$.

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$(fgh)'(2) = f'(-2)g(-2)h(-2) + f(-2)g'(-2)h(-2) + f(-2)g(-2)h'(-2)$$

$$(fgh)'(2) = (-7)(-8)(1) + (5)(-3)(1) + (5)(-8)(0)$$

$$(fgh)'(2) = 56 - 15 + 0$$

$$(fgh)'(2) = 41$$

■ 6. Use $f(5) = 4$, $f'(5) = 2$, $g(5) = -2$, $g'(5) = 3$, $h(5) = -3$, and $h'(5) = -8$ if $y = [x^2 - f(x)]g(x)h(x)$, to determine the value of $y'(5)$.

Solution:

Evaluate the product rule formula at $x = 5$.

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$y' = 2xg(x)h(x) + x^2g'(x)h(x) + x^2g(x)h'(x) - f'(x)g(x)h(x)$$

$$-f(x)g'(x)h(x) - f(x)g(x)h'(x)$$

$$y'(5) = 2(5)g(5)h(5) + 5^2g'(5)h(5) + 5^2g(5)h'(5) - f'(5)g(5)h(5)$$

$$-f(5)g'(5)h(5) - f(5)g(5)h'(5)$$

$$y'(5) = 10g(5)h(5) + 25g'(5)h(5) + 25g(5)h'(5) - f'(5)g(5)h(5)$$

$$-f(5)g'(5)h(5) - f(5)g(5)h'(5)$$

Substitute $f(5) = 4$, $f'(5) = 2$, $g(5) = -2$, $g'(5) = 3$, $h(5) = -3$, and $h'(5) = -8$.

$$y'(5) = 10(-2)(-3) + 25(3)(-3) + 25(-2)(-8) - 2(-2)(-3)$$

$$-4(3)(-3) - 4(-2)(-8)$$

$$y'(5) = 10(6) + 25(-9) + 25(16) - 2(6) - 4(-9) - 4(16)$$

$$y'(5) = 60 - 225 + 400 - 12 + 36 - 64$$

$$y'(5) = 195$$

QUOTIENT RULE

■ 1. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2x + 6}{7x + 5}$$

Solution:

Let

$$f(x) = 2x + 6$$

$$f'(x) = 2$$

$$g(x) = 7x + 5$$

$$g'(x) = 7$$

Then the derivative is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{2 \cdot (7x + 5) - (2x + 6) \cdot 7}{(7x + 5)^2}$$

$$h'(x) = \frac{14x + 10 - 14x - 42}{(7x + 5)^2}$$

$$h'(x) = -\frac{32}{(7x+5)^2}$$

■ 2. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{\sqrt[3]{x}}{1+2x^2}$$

Solution:

Let

$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

$$g(x) = 1 + 2x^2$$

$$g'(x) = 4x$$

Then the derivative is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{\frac{1}{3\sqrt[3]{x^2}} \cdot (1+2x^2) - (\sqrt[3]{x}) \cdot 4x}{(1+2x^2)^2}$$

$$h'(x) = \frac{\frac{1}{3\sqrt[3]{x^2}} + \frac{2}{3}\sqrt[3]{x^4} - 4\sqrt[3]{x^4}}{(1+2x^2)^2}$$

$$h'(x) = \frac{\frac{1}{3\sqrt[3]{x^2}} - \frac{10}{3}\sqrt[3]{x^4}}{(1+2x^2)^2}$$

■ 3. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{-8x}{5x+2}$$

Solution:

Let

$$f(x) = -8x$$

$$f'(x) = -8$$

$$g(x) = 5x + 2$$

$$g'(x) = 5$$

Then the derivative is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{-8 \cdot (5x + 2) - (-8x) \cdot 5}{(5x + 2)^2}$$

$$h'(x) = \frac{-40x - 16 + 40x}{(5x + 2)^2}$$

$$h'(x) = -\frac{16}{(5x + 2)^2}$$

■ 4. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2 - 4x + 5x^2}{5x + x^3}$$

Solution:

Let

$$f(x) = 2 - 4x + 5x^2$$

$$f'(x) = -4 + 10x$$

$$g(x) = 5x + x^3$$

$$g'(x) = 5 + 3x^2$$

Then the derivative is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{(-4 + 10x)(5x + x^3) - (2 - 4x + 5x^2)(5 + 3x^2)}{(5x + x^3)^2}$$

$$h'(x) = \frac{(-20x - 4x^3 + 50x^2 + 10x^4) - (10 + 6x^2 - 20x - 12x^3 + 25x^2 + 15x^4)}{(5x + x^3)^2}$$

$$h'(x) = \frac{-20x - 4x^3 + 50x^2 + 10x^4 - 10 - 6x^2 + 20x + 12x^3 - 25x^2 - 15x^4}{(5x + x^3)^2}$$

$$h'(x) = \frac{-5x^4 + 8x^3 + 19x^2 - 10}{(5x + x^3)^2}$$

5. Use the quotient rule to find the derivative of the function.

$$k(x) = \frac{(2 - 3x)(1 + x)}{2 + 3x^2}$$

Solution:

This function is given in the form

$$k(x) = \frac{f(x)h(x)}{g(x)}$$

which means the combination of quotient rule and product rule will be

$$k'(x) = \frac{[f(x)h(x)]'g(x) - f(x)h(x)g'(x)}{[g(x)]^2}$$

$$k'(x) = \frac{[f'(x)h(x) + f(x)h'(x)]g(x) - f(x)h(x)g'(x)}{[g(x)]^2}$$

Let

$$f(x) = 2 - 3x$$

$$f'(x) = -3$$

$$h(x) = 1 + x$$

$$h'(x) = 1$$

$$g(x) = 2 + 3x^2$$

$$g'(x) = 6x$$

Then the derivative is

$$k'(x) = \frac{[(-3)(1+x) + (2-3x)(1)](2+3x^2) - (2-3x)(1+x)(6x)}{(2+3x^2)^2}$$

$$k'(x) = \frac{(-3 - 3x + 2 - 3x)(2+3x^2) - (2-3x)(6x+6x^2)}{(2+3x^2)^2}$$

$$k'(x) = \frac{(-1 - 6x)(2+3x^2) - (2-3x)(6x+6x^2)}{(2+3x^2)^2}$$

$$k'(x) = \frac{(-2 - 3x^2 - 12x - 18x^3) - (12x + 12x^2 - 18x^2 - 18x^3)}{(2+3x^2)^2}$$

$$k'(x) = \frac{-2 - 3x^2 - 12x - 18x^3 - 12x - 12x^2 + 18x^2 + 18x^3}{(2+3x^2)^2}$$

$$k'(x) = \frac{3x^2 - 24x - 2}{(2 + 3x^2)^2}$$

■ 6. Use $f(5) = 4$, $f'(5) = 2$, $g(5) = -2$, $g'(5) = 3$, $h(5) = -3$, and $h'(5) = -8$ to determine the value of $k'(5)$.

$$k'(5) = \left(\frac{fg}{h}\right)'(5)$$

Solution:

This function is given in the form

$$k(x) = \frac{f(x)g(x)}{h(x)}$$

which means the combination of quotient rule and product rule will be

$$k'(x) = \frac{[f(x)g(x)]'h(x) - f(x)g(x)h'(x)}{[h(x)]^2}$$

$$k'(x) = \frac{[f'(x)g(x) + f(x)g'(x)]h(x) - f(x)g(x)h'(x)}{[h(x)]^2}$$

$$k'(5) = \frac{[f'(5)g(5) + f(5)g'(5)]h(5) - f(5)g(5)h'(5)}{[h(5)]^2}$$

Substitute the values we were given.

$$k'(5) = \frac{[2(-2) + 4(3)](-3) - 4(-2)(-8)}{(-3)^2}$$

$$k'(5) = \frac{(-4 + 12)(-3) - 64}{9}$$

$$k'(5) = \frac{8(-3) - 64}{9}$$

$$k'(5) = -\frac{88}{9}$$

TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = 3x^{-4} + x^2 \cot x$.

Solution:

Let's look at one term at a time. The derivative of $3x^{-4}$ is

$$-12x^{-5}$$

To find the derivative of $x^2 \cot x$, we'll need to use product rule. If we set $f(x) = x^2$, $f'(x) = 2x$, $g(x) = \cot x$, and $g'(x) = -\csc^2 x$, then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$(x^2)(-\csc^2 x) + (2x)(\cot x)$$

$$-x^2 \csc^2 x + 2x \cot x$$

Putting these derivatives together, we get

$$f'(x) = -12x^{-5} - x^2 \csc^2 x + 2x \cot x$$

■ 2. Find $h'(x)$.

$$h(x) = \frac{\sin x}{5 - 2\cos x}$$

Solution:

We'll need to use quotient rule. If we set $f(x) = \sin x$, $f'(x) = \cos x$, $g(x) = 5 - 2\cos x$, and $g'(x) = 2\sin x$, then we can plug directly into the quotient rule formula.

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{\cos x(5 - 2\cos x) - \sin x(2\sin x)}{(5 - 2\cos x)^2}$$

$$h'(x) = \frac{5\cos x - 2\cos^2 x - 2\sin^2 x}{(5 - 2\cos x)^2}$$

$$h'(x) = \frac{5\cos x - 2}{(5 - 2\cos x)^2}$$

■ 3. Find $h'(x)$ if $h(x) = 3\sin x\cos x + 5\sec x$.

Solution:

To find the derivative of $3\sin x\cos x$, we'll need to use product rule. If we set $f(x) = 3\sin x$, $f'(x) = 3\cos x$, $g(x) = \cos x$, and $g'(x) = -\sin x$, then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$(3\sin x)(-\sin x) + (3\cos x)(\cos x)$$

$$-3\sin^2 x + 3\cos^2 x$$

The derivative of $5\sec x$ is

$$5\sec x\tan x$$

Putting these derivatives together, we get

$$h'(x) = -3\sin^2 x + 3\cos^2 x + 5\sec x\tan x$$

■ 4. Find the derivative of the trigonometric function.

$$y = 3 - 2\sqrt{x}\csc x$$

Solution:

The derivative of 3 is 0. To find the derivative of $-2\sqrt{x}\csc x$, we'll need to use product rule. If we set

$$f(x) = -2\sqrt{x}$$

$$f'(x) = -\frac{1}{\sqrt{x}}$$

$$g(x) = \csc x$$

$$g'(x) = -\csc x\cot x$$

then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$(-2\sqrt{x})(-\csc x \cot x) + \left(-\frac{1}{\sqrt{x}}\right)(\csc x)$$

$$2\sqrt{x}\csc x \cot x - \frac{\csc x}{\sqrt{x}}$$

Putting these derivatives together, we get

$$y' = 2\sqrt{x}\csc x \cot x - \frac{\csc x}{\sqrt{x}}$$

■ 5. Find the derivative of the trigonometric function.

$$y = \frac{2}{4\cos x - 5\sin x}$$

Solution:

We can use reciprocal rule,

$$y' = \frac{-ag'(x)}{[g(x)]^2}$$

to find the derivative. If $a = 2$, $-a = -2$, $g(x) = 4\cos x - 5\sin x$, and $g'(x) = -4\sin x - 5\cos x$, then we can plug directly into the reciprocal rule formula.

$$y' = \frac{-2(-4\sin x - 5\cos x)}{(4\cos x - 5\sin x)^2}$$

$$y' = \frac{8\sin x + 10\cos x}{(4\cos x - 5\sin x)^2}$$

■ 6. Find the derivative of y.

$$y = 2x^4 + \frac{x\tan x}{x^2+1}$$

Solution:

Let's look at one term at a time. The derivative of $2x^4$ is

$$8x^3$$

To find the derivative of the fraction, we'll need to use the quotient and product rules.

$$\frac{(x\sec^2 x + \tan x)(x^2+1) - x\tan x(2x)}{(x^2+1)^2}$$

$$\frac{x^3\sec^2 x + x^2\tan x + x\sec^2 x + \tan x - 2x^2\tan x}{(x^2+1)^2}$$

$$\frac{x^3\sec^2 x - x^2\tan x + x\sec^2 x + \tan x}{(x^2+1)^2}$$

Putting these derivatives together, we get

$$y' = 8x^3 + \frac{x^3\sec^2 x - x^2\tan x + x\sec^2 x + \tan x}{(x^2+1)^2}$$

EXPONENTIAL DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = (x^3 - x)e^x$.

Solution:

Use product rule to take the derivative.

$$f'(x) = \frac{d}{dx}(x^3 - x) \cdot e^x + (x^3 - x) \cdot \frac{d}{dx}e^x$$

$$f'(x) = (3x^2 - 1) \cdot e^x + (x^3 - x) \cdot e^x$$

Factor to simplify.

$$f'(x) = e^x(3x^2 - 1 + x^3 - x)$$

$$f'(x) = e^x(x^3 + 3x^2 - x - 1)$$

■ 2. Find $g'(x)$ if $g(x) = 5^x(x^2 - 7x + 1)$.

Solution:

Use product rule to take the derivative.

$$g'(x) = \frac{d}{dx}(5^x) \cdot (x^2 - 7x + 1) + 5^x \cdot \frac{d}{dx}(x^2 - 7x + 1)$$

$$g'(x) = 5^x \ln 5(x^2 - 7x + 1) + 5^x(2x - 7)$$

3. Find $h'(x)$ if $h(x) = \sin xe^x - x^2 \cos x$.

Solution:

Use product rule to take the derivative of the first and second term.

$$h'(x) = \frac{d}{dx} \sin x \cdot e^x + \sin x \cdot \frac{d}{dx} e^x - \left(\frac{d}{dx} x^2 \cdot \cos x + x^2 \cdot \frac{d}{dx} \cos x \right)$$

$$h'(x) = \cos x \cdot e^x + \sin x \cdot e^x - (2x \cdot \cos x + x^2 \cdot (-\sin x))$$

$$h'(x) = e^x \cos x + e^x \sin x - 2x \cos x + x^2 \sin x$$

Factor to simplify.

$$h'(x) = \cos x(e^x - 2x) + \sin x(e^x + x^2)$$

4. Find $f'(x)$.

$$f(x) = \frac{4e^x}{3e^x - 1}$$

Solution:

Use quotient rule to take the derivative.

$$f'(x) = \frac{\frac{d}{dx}(4e^x) \cdot (3e^x - 1) - 4e^x \cdot \frac{d}{dx}(3e^x - 1)}{(3e^x - 1)^2}$$

$$f'(x) = \frac{4e^x \cdot (3e^x - 1) - 4e^x \cdot (3e^x)}{(3e^x - 1)^2}$$

$$f'(x) = \frac{12e^{2x} - 4e^x - 12e^{2x}}{(3e^x - 1)^2}$$

$$f'(x) = -\frac{4e^x}{(3e^x - 1)^2}$$

■ 5. Find $g'(x)$ if $g(x) = 8^x + 3e^x \cot x$.

Solution:

Use product rule to take the derivative of the second term.

$$g'(x) = 8^x \ln 8 + \frac{d}{dx}(3e^x) \cdot \cot x + 3e^x \cdot \frac{d}{dx}(\cot x)$$

$$g'(x) = 8^x \ln 8 + 3e^x \cdot \cot x + 3e^x \cdot (-\csc^2 x)$$

$$g'(x) = 8^x \ln 8 + 3e^x \cot x - 3e^x \csc^2 x$$

■ 6. Find $h'(x)$ if $h(x) = \dfrac{x^3 e^x}{x + 3^x}$.

Solution:

Use quotient rule to take the derivative.

$$h'(x) = \frac{\frac{d}{dx}(x^3e^x) \cdot (x + 3^x) - x^3e^x \cdot \frac{d}{dx}(x + 3^x)}{(x + 3^x)^2}$$

Use product rule to take the derivative x^3e^x.

$$\frac{d}{dx}(x^3) \cdot (e^x) + x^3 \cdot \frac{d}{dx}(e^x)$$

$$3x^2e^x + x^3e^x$$

Then the derivative of the function is

$$h'(x) = \frac{(3x^2e^x + x^3e^x)(x + 3^x) - x^3e^x(1 + 3^x \ln 3)}{(x + 3^x)^2}$$

LOGARITHMIC DERIVATIVES

■ 1. Find $f'(x)$.

$$f(x) = 2\log_5 x - 11\log_{13} x$$

Solution:

Differentiate by applying the derivative formula for $f(x) = \log_a x$,

$$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$$

So the derivative is

$$f'(x) = 2\left(\frac{1}{x\ln 5}\right) - 11\left(\frac{1}{x\ln 13}\right)$$

$$f'(x) = \frac{2}{x\ln 5} - \frac{11}{x\ln 13}$$

■ 2. Find $g'(x)$.

$$g(x) = \log_4 x - x^6\ln x$$

Solution:

We need to take the derivative one term at a time, applying the derivative formulas for the log and natural log. We'll also need to apply product rule to the second term.

$$g'(x) = \frac{1}{x \ln 4} - \left[(6x^5)(\ln x) + x^6\left(\frac{1}{x}\right)\right]$$

$$g'(x) = \frac{1}{x \ln 4} - 6x^5 \ln x - x^5$$

■ 3. Find $h'(x)$.

$$h(x) = \log_7 x \ln x$$

Solution:

We need to take the derivative by applying the derivative formulas for the log and natural log. We'll also need to apply product rule.

$$h'(x) = \frac{1}{x \ln 7}(\ln x) + \log_7 x\left(\frac{1}{x}\right)$$

$$h'(x) = \frac{\ln x}{x \ln 7} + \frac{\log_7 x}{x}$$

Using properties of logs, we get

$$h'(x) = \frac{\log_7 x}{x} + \frac{\log_7 x}{x}$$

$$h'(x) = \frac{2 \log_7 x}{x}$$

■ 4. Find $y'(x)$.

$$y = \frac{1 + 7 \ln x}{6x^4}$$

Solution:

We need to take the derivative by applying the derivative formulas for the natural log. We'll also need to apply quotient rule.

$$y' = \frac{\frac{7}{x}(6x^4) - (1 + 7 \ln x)(24x^3)}{(6x^4)^2}$$

$$y' = \frac{42x^3 - 24x^3 - 168x^3 \ln x}{36x^8}$$

$$y' = \frac{18x^3 - 168x^3 \ln x}{36x^8}$$

$$y' = \frac{3 - 28 \ln x}{6x^5}$$

■ 5. Find $y'(x)$.

$$y = \frac{x^3 + \log_5 x}{5^x}$$

Solution:

We need to take the derivative by applying the derivative formulas for the log. We'll also need to apply quotient rule.

$$y' = \frac{\left(3x^2 + \frac{1}{x \ln 5}\right)(5^x) - (x^3 + \log_5 x)(5^x \ln 5)}{(5^x)^2}$$

$$y' = \frac{3x^2 + \frac{1}{x \ln 5} - (x^3 + \log_5 x)(\ln 5)}{5^x}$$

$$y' = \frac{3x^2 + \frac{1}{x \ln 5} - x^3 \ln 5 - \log_5 x \ln 5}{5^x}$$

■ 6. Find $y'(x)$.

$$y = \frac{x^7 e^x}{\ln x}$$

Solution:

We need to take the derivative by applying the derivative formulas for the natural log. We'll also need to apply product and quotient rule.

$$y' = \frac{(7x^6 e^x + x^7 e^x)(\ln x) - (x^7 e^x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$y' = \frac{7x^6e^x \ln x + x^7e^x \ln x - x^6e^x}{(\ln x)^2}$$

$$y' = \frac{x^6e^x(7 \ln x + x \ln x - 1)}{(\ln x)^2}$$

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