ESCOLA BRASILEIRA DE ECONOMIA E FINANÇAS - EPGE

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Industrial Organization - Problem Set 1

Rio de Janeiro

1st Quarter - 2022

Part 1

Question 1: Let $\delta_{jt} = \beta_{0,j} + \beta_1 x_{jt}^{conv} + \beta_2 x_{jt}^{spec} - \alpha p_{jt} + \xi_{jt}$. Consumer i chooses product j in city t if $u_{ijt} = \delta_{jt} + \epsilon_{ijt} \geq \delta_{j't} + \epsilon_{ij't} = u_{ij't} \ \forall \ j' \in \{0,1,2\}$, where $\delta_{i0t} = 0$. In the Logit model, we assume that $\{\epsilon_{ijt}\}_{ijt}$ are (given ξ) iid over consumers i, firms j, and cities t. Thus, the market-share function of product j in city t is given by $s_{jt}(x,p,\xi) = \int 1\{j \in \underset{k\in\{0,1,2\}}{\text{argmax}}\{\delta_{kt} + \epsilon_{ikt}\}\} dF(\epsilon_{i0t})dF(\epsilon_{i1t})dF(\epsilon_{i2t})$. Now,

$$\begin{aligned}
s_{jt}(x,p,\xi) &= P(\{j \in \underset{k\in\{0,1,2\}}{\text{argmax}}\{\delta_{kt} + \epsilon_{ikt}\}\}) \\
&= P(\delta_{jt} + \epsilon_{ijt} \geq \delta_{j't} + \epsilon_{ij't}, \delta_{jt} + \epsilon_{ijt} \geq \delta_{j''t} + \epsilon_{ij''t}) \\
&= \int_{-\infty}^{+\infty} \int_{\infty}^{\epsilon_{ijt}+\delta_{jt}-\delta_{j't}} \int_{\infty}^{\epsilon_{ijt}+\delta_{jt}-\delta_{j''t}} f(\epsilon_{ijt}) f(\epsilon_{ij't}) f(\epsilon_{ij''t}) d\epsilon_{ijt} d\epsilon_{ij't} d\epsilon_{ij''t} \\
&= \int_{-\infty}^{+\infty} F(\epsilon_{ijt} + \delta_{jt} - \delta_{j't}) F(\epsilon_{ijt} + \delta_{jt} - \delta_{j''t}) f(\epsilon_{ijt}) d\epsilon_{ijt} \\
&= \int_{-\infty}^{+\infty} e^{-e^{-(\epsilon_{ijt}+\delta_{jt}-\delta_{j't})}} e^{-e^{-(\epsilon_{ijt}+\delta_{jt}-\delta_{j''t})}} e^{-\epsilon_{ijt}-e^{-\epsilon_{ijt}}} d\epsilon_{ijt} \\
&\overset{v=e^{-\epsilon_{ijt}}}{=} \int_{0}^{+\infty} e^{-ve^{\delta_{ij't}-\delta_{ijt}}} e^{-ve^{\delta_{ij''t}-\delta_{ijt}}} e^{-v} dv \\
&= \frac{1}{1 + e^{\delta_{ij't}-\delta_{ijt}} + e^{\delta_{ij''t}-\delta_{ijt}}} \\
&= \frac{e^{\delta_{jt}}}{\sum_{j'=0}^{2} e^{\delta_{j't}}}
\end{aligned}$$

which implies that $log(s_{jt}) - log(s_{0t}) = \delta_{jt} = \beta_{0,j} + \beta_1 x_{jt}^{conv} + \beta_2 x_{jt}^{spec} - \alpha p_{jt} + \xi_{jt}$ for $j \in \{1,2\}$.

With this model, I first run a linear regression of the dependent variable log_diff in the dataset on the variables $price$, $channels$, $channels\_spec$, and dummy variables that indicate the firm, in order to control for the systematic difference in firm quality. With this specification, I do not control for endogeneity of the variable 'price'. The results of the regression is shown in the table in Figure 1.

The estimated parameters for $price$, $channels$, and $channels\_spec$ have the expected signs.

Question 2: Now, I use an Instrumental Variable approach to control for the endogeneity of the variable $price$. The instruments used here are the so-called Hausman instruments, which include prices for the same firm in different markets. With no further information about the data, Logit model without intruments

Dep. Variable:	log_diff	R-squared:	0.395
Model:	OLS	Adj. R-squared:	0.389
Method:	Least Squares	F-statistic:	nan
Date:	Sat, 12 Feb 2022	Prob (F-statistic):	nan
Time:	10:30:25	Log-Likelihood:	-341.22
No. Observations:	400	AIC:	692.4
Df Residuals:	395	BIC:	712.4
Df Model:	4		
Covariance Type:	cluster		

	coef	std err	t	P>|t|	[0.025	0.975]
price	-0.0388	0.003	-11.991	0.053	-0.080	0.002
channels	0.0180	0.001	21.624	0.029	0.007	0.029
channels_spec	0.1356	0.022	6.099	0.103	-0.147	0.418
j_1	-0.4848	0.224	-2.165	0.276	-3.331	2.361
j_2	-0.9486	0.211	-4.493	0.139	-3.631	1.734

Omnibus:	4.280	Durbin-Watson:	2.235
Prob(Omnibus):	0.118	Jarque-Bera (JB):	4.335
Skew:	-0.251	Prob(JB):	0.114
Kurtosis:	2.915	Cond. No.	474.

Notes:
[1] Standard Errors are robust to cluster correlation (cluster)

Figure 1: Regression without controlling for the endogeneity of 'price'

there are arguments for two kinds of Hausman instruments: (i - IV_1) using prices of the same firm for different cities within the same region; and (ii - IV_2) using prices of same firm for cities in different regions. For (i), the relevance condition is satisfied if each firm has the same cost structure for each city within a region, and the exclusion condition is satisfied if city-specific demand shocks are uncorrelated among cities within the same region. The arguments made for (ii) are analogous. I, therefore, use different specifications of the model (with different instruments of each type) to assess the robustness of the results. Figure 2 and Figure 3 show, respectively, the estimation results using one instrument of each kind and the 2SLS method. Figure 4 and 5 shows comparisons between using different instruments of the same kind.

The estimated parameters have the expected signs and with the exception of the variables j_1

```
                        IV-2SLS Estimation Summary
==============================================================================
Dep. Variable:               log_diff   R-squared:                      0.3550
Estimator:                    IV-2SLS   Adj. R-squared:                 0.3485
No. Observations:                 400   F-statistic:                  1.51e+18
Date:                Sat, Feb 12 2022   P-value (F-stat)                0.0000
Time:                        12:05:36   Distribution:                  chi2(5)
Cov. Estimator:             clustered

                             Parameter Estimates
==================================================================================
               Parameter  Std. Err.     T-stat    P-value    Lower CI    Upper CI
----------------------------------------------------------------------------------
channels          0.0187     0.0006     33.475     0.0000      0.0176      0.0198
channels_spec     0.1440     0.0151     9.5635     0.0000      0.1145      0.1735
j_1               0.2727     0.1949     1.3996     0.1616     -0.1092      0.6546
j_2              -0.2548     0.1827    -1.3949     0.1630     -0.6129      0.1032
price            -0.0603     0.0033    -18.058     0.0000     -0.0668     -0.0538
==================================================================================

Endogenous: price
Instruments: iv_1
Clustered Covariance (One-Way)
Debiased: False
Num Clusters: 2
```

Figure 2: 2SLS regression using IV_1

```
                        IV-2SLS Estimation Summary
==============================================================================
Dep. Variable:               log_diff   R-squared:                      0.3651
Estimator:                    IV-2SLS   Adj. R-squared:                 0.3587
No. Observations:                 400   F-statistic:                -1.958e+15
Date:                Sat, Feb 12 2022   P-value (F-stat)                1.0000
Time:                        12:26:07   Distribution:                  chi2(5)
Cov. Estimator:             clustered

                             Parameter Estimates
==================================================================================
               Parameter  Std. Err.     T-stat    P-value    Lower CI    Upper CI
----------------------------------------------------------------------------------
channels          0.0186     0.0002     99.372     0.0000      0.0182      0.0190
channels_spec     0.1428     0.0108     13.250     0.0000      0.1217      0.1639
j_1               0.1697     0.5831     0.2910     0.7711     -0.9732      1.3125
j_2              -0.3492     0.5383    -0.6488     0.5165     -1.4042      0.7058
price            -0.0574     0.0143    -4.0015     0.0001     -0.0855     -0.0293
==================================================================================

Endogenous: price
Instruments: iv_2
Clustered Covariance (One-Way)
Debiased: False
Num Clusters: 2
```

Figure 3: 2SLS regression using IV_2

Model Comparison

	iv_11	iv_12	iv_13
Dep. Variable	log_diff	log_diff	log_diff
Estimator	IV-2SLS	IV-2SLS	IV-2SLS
No. Observations	400	400	400
Cov. Est.	clustered	clustered	clustered
R-squared	0.3550	0.3444	0.3523
Adj. R-squared	0.3485	0.3378	0.3457
F-statistic	1.51e+18	1.115e+19	-1.281e+19
P-value (F-stat)	0.0000	0.0000	1.0000
channels	0.0187	0.0188	0.0187
	(33.475)	(32.690)	(33.141)
channels_spec	0.1440	0.1450	0.1442
	(9.5635)	(9.5362)	(9.5412)
j_1	0.2727	0.3672	0.2980
	(1.3996)	(2.0501)	(1.5811)
j_2	-0.2548	-0.1683	-0.2316
	(-1.3949)	(-1.0003)	(-1.3097)
price	-0.0603	-0.0630	-0.0610
	(-18.058)	(-21.764)	(-19.313)
Instruments	iv_1	price	price

T-stats reported in parentheses

Figure 4: Comparison among different IV_1 instruments: different cities within the region

Model Comparison

	iv_21	iv_22	iv_23
Dep. Variable	log_diff	log_diff	log_diff
Estimator	IV-2SLS	IV-2SLS	IV-2SLS
No. Observations	400	400	400
Cov. Est.	clustered	clustered	clustered
R-squared	0.3651	-25.779	-1.3121
Adj. R-squared	0.3587	-26.051	-1.3356
F-statistic	-1.958e+15	1.496e+15	9.644e+16
P-value (F-stat)	1.0000	0.0000	0.0000
channels	0.0186	-0.0005	0.0227
	(99.372)	(-0.0033)	(8.0238)
channels_spec	0.1428	-0.0790	0.1904
	(13.250)	(-0.0444)	(4.7763)
j_1	0.1697	-19.866	4.4650
	(0.2910)	(-0.1251)	(2.0939)
j_2	-0.3492	-18.701	3.5849
	(-0.6488)	(-0.1286)	(1.8395)
price	-0.0574	0.5104	-0.1791
	(-4.0015)	(0.1134)	(-2.8617)
Instruments	iv_2	instruments	instruments

T-stats reported in parentheses

Figure 5: Comparison among different IV_2 instruments: different cities in different regions

and j_2 (the ”firm quality”), all of them are significant ate the usual levels. Using these first two instruments (iv_1 variable is created rolling the $price$ variable within each region with shift equal to 2, iv_2 variable is created rolling the $price$ variable with shift equal to 10 - using the whole dataset), the estimated parameters are very close in the two specifications, although using IV_1 the estimates are more precise. Comparing to the model in question 1, we can see that the parameters of $price$, $channels$, and $channels\_spec$ increased in absolute value. For $price$, this is the expected change, since economic theory suggests that endogeneity softens the effect of price on demand.

With the comparisons, we can see that the estimates vary little with different instruments of the first kind (within region). Here, each instrument (iv_11, iv_12, and iv_13) are prices of different cities. However, the estimates using the second kind of instrument (different regions) present large variation. This might indicate that the at least one of the assumptions about the validity of this kind of instrument does not hold. The model with iv_11 is, therefore, my preferred specification.

Part 2

Let $\delta_{jt} = \beta_{0,j} + \beta_1 x_{jt}^{conv} + \beta_2 x_{jt}^{spec} - \alpha p_{jt} + \xi_{jt}$ as before. Now, the indirect utility of consumer i if she chooses good j in city t is given by $u_{ijt} = \delta_{jt} + \sigma\eta_{it}p_{jt} + \epsilon_{ijt}$. Similarly to what I derived in Question 1, Part 1, the market-share function are now given by

$$s_{jt}(\delta_t; x, p, \xi, \theta) = \int \frac{e^{\delta_{jt}+\sigma\eta_{it}p_{jt}}}{\sum_{j'=0}^{2} e^{\delta_{j't}+\sigma\eta_{it}p_{jt}}} f(\eta_{it}) d\eta_{it}$$

where f is the standard normal distribution pdf and θ include the parameters of interest.

Let $s_{jt}^{-1}(s_t; x, p, \xi, \theta)$ be the inverse market-share function. We have that $s_{jt}^{-1}(s_t; x, p, \xi, \theta) = \beta_{0,j} + \beta_1 x_{jt}^{conv} + \beta_2 x_{jt}^{spec} - \alpha p_{jt} + \xi_{jt}$ for each firm j and market t. Stack the observations to get

$$s^{-1}(s; x, p, \xi, \theta) = X\beta + \xi$$

where $X = [w_{11}, \ldots, w_{2T}]'$, $w_{jt} = (d'_{jt}, x_{jt}^{conv}, x_{jt}^{spec}, p_{jt})'$, d_{jt} is a vector that indicates firm j, $\beta = (\beta_{0,1}, \beta_{0,2}, \beta_1, \beta_2, \alpha)$, and T is the number of markets. Given a vector of instruments z_{jt}, we have the population moment $E[z_{jt}\xi_{jt}] = 0$. The instruments I use are the Hausman instrument of Part 1, the proposed BLP approximated optimal instruments, which, in this setting, are the characteristics of the product of the same firm and the characteristics of the products of the other firm, and the dummy variables which are exogenous. Thus, z_{jt} is a 7×1 vector. Stack them together in the matrix Z. The GMM estimator of (β, σ) is the one that minimizes the function

$$J(\beta, \sigma) = (s^{-1}(s; x, p, \xi, \theta) - X\beta)' Z W Z' (s^{-1}(s; x, p, \xi, \theta) - X\beta)$$

where W is a weight matrix.

Note that $min_{\beta,\sigma} J(\beta, \sigma) = min_{\sigma} min_{\beta} J(\beta, \sigma)$. Now, the first-order condition of $min_{\beta} J(\beta, \sigma)$ gives us the optimal β:

$$\beta(\sigma) = (X'ZWZ'X)^{-1}(X'ZWZ's^{-1}(\sigma))$$

Plugging $\beta(\sigma)$ back into the function J, and letting $\tilde{W} = ZWZ'$, we have

$$\begin{aligned}
J(\sigma) &= (s^{-1}(s; x, p, \xi, \theta) - X\beta(\sigma))'\tilde{W}(s^{-1}(s; x, p, \xi, \theta) - X\beta(\sigma)) \\
&= (s^{-1}(\sigma) - X(X'\tilde{W}X)^{-1}X'\tilde{W}s^{-1}(\sigma))'\tilde{W}(s^{-1}(\sigma) - X(X'\tilde{W}X)^{-1}X'\tilde{W}s^{-1}(\sigma)) \\
&= (s^{-1}(\sigma))'P'\tilde{W}P(s^{-1}(\sigma))
\end{aligned}$$

where $P = I_T - X(X'\tilde{W}X)^{-1}X'\tilde{W}$

The algorithm will be as follows:

1 For a given value of $\overline{\sigma}$, I find the inverse market-share function $s_{jt}^{-1}(s_t; x, p, \xi, \overline{\theta})$ using the Berry contraction and defining the recursion $\delta_{jt}^{i+1} = \delta_{jt}^{i} + log(s_{jt}) - log(s_{jt}(\delta_t^i; x, p, \xi, \overline{\theta}))$, where s_{jt} is the observed market-share. The tolerance used is 10^{-10}. At every step, the market-share function $s_{jt}(\delta; x, p, \xi, \overline{\theta})$ must be evaluated at δ^i. Thus, I estimate the market-share functions by Monte-Carlo simulation, i.e., I let $s_{jt}(\delta_t; x, p, \xi, \overline{\theta}) \approx \frac{1}{H_s}\sum_{s=1}^{H_s} \frac{e^{\delta_{jt}+\overline{\sigma}\eta_{it,s}p_{jt}}}{\sum_{j'=0}^{2} e^{\delta_{j't}+\overline{\sigma}\eta_{it,s}p_{j't}}}$, where $\eta_{it,s}$, $s = 1, \ldots, H_s$ are iid standard normal distributed. In the code, I use $H_s = 500$. The draws are taken at the beginning of the algorithm and remain fixed.

2 With the inverse market-shares in hand, I estimate the parameters of interest using GMM. The minimization can be broke into two steps, since the function of interest is linear on β. For the non-linear parameter σ, I use a grid-search in the grid $[0, 0.2]$ with 40 points, as recommended in the question. I set $W = I_T$.

The estimated parameters are (using tolerance equal to 10^{-10} in the contraction, 500 draws in the integration, and 40 points in the grid) $\sigma = 0.020512$, and $\beta_{0,1} = -0.078285$, $\beta_{0,2} = -0.57513$, $channels = 0.02099$, $channels\_spec = 0.167768$, and $price = -0.056866$. The optimal sigma is around 0.051 if the weighting matrix used is $W = (Z'Z)^{-1}$ as in the 2SLS, in which case the parameters are $\beta_{0,1} = 1.31065$, $\beta_{0,2} = 0.759391$, $channels = 0.022625$, $channels\_spec = 0.171174$, and $price = -0.099329$.

Part 3

The first-order conditions of each firm's maximization problem gives us the system of non-linear equations

$$p = c - \left(\frac{\partial s(p)}{\partial p'} \odot I_2\right)^{-1} s(p)$$

since M, the ownership matrix, is a 2×2 identity matrix in our case.

Given

$$s_j(\delta; x, p, \xi, \theta) = \int \frac{e^{\delta_j + \sigma\eta_i p_j}}{\sum_{j'=0}^{2} e^{\delta_{j'} + \sigma\eta_i p_j}} f(\eta_i) d\eta_i$$

we have that

$$\frac{\partial s_j(\delta; x, p, \xi, \theta)}{\partial p_j} = \int \frac{(\sigma\eta_i - \alpha)e^{\delta_j + \sigma\eta_i p_j}(1 - e^{\delta_{j'} + \sigma\eta_i p_{j'}})}{(\sum_{j'=0}^{2} e^{\delta_{j'} + \sigma\eta_i p_j})^2} f(\eta_i) d\eta_i$$

for $j = 1, 2$. Similarly to what I did in Part 2, I approximate the derivatives using Monte-Carlo integration.

With $\left(\frac{\partial s(p)}{\partial p'} \odot I_2\right)^{-1}$ and $s(p)$ and the representative market given in the question, I find the equilibrium price vector using the recursion

$$p^{i+1} = c - \left(\frac{\partial s(p^i)}{\partial p'} \odot I_2\right)^{-1} s(p^i)$$

Using the parameters estimated using $W = I_7$, and with $x_1^{conv} = x_2^{conv} = 40$, $x_1^{spec} = x_2^{spec} = 3$, $\xi_1 = \xi_2 = 0$, and $mc_1 = mc_2 = 24$, the equilibrium prices are $(p_1, p_2) = (53.000257, 50.327068)$. Increasing the number of special channels of firm 1 from 3 to 8, the equilibrium prices become $(p_1, p_2) = (56.602234, 48.156599)$. As we can see, more channels allowed firm 1 to increase its price due to demand substitution.