ESCOLA BRASILEIRA DE ECONOMIA E FINANÇAS - EPGE

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Industrial Organization - Problem Set 3

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Question 1: Let $b(v)$ be the symmetric equilibrium strategy and $G_i(b) = Prob(\max_{j \neq i} b_j \leq b)$ be the probability that player i's bid b is bigger than the others. In a symmetric equilibrium, $G_i = G \ \forall \ i$. If the reservation price has distribution H, then the probability of bid b winning the auction is $G(b)H(b)$. We must have

$$b(v) = \text{argmax}_b (v - b)G(b)H(b)$$

The First Order Condition of this problem gives us

$$\begin{aligned} G(b(v))H(b(v)) &= (v - b(v))[G'(b(v))H(b(v)) + G(b(v))H'(b(v))] \\ v &= b(v) + \frac{G(b(v))H(b(v))}{G'(b(v))H(b(v)) + G(b(v))H'(b(v))} \end{aligned}$$

Question 2: Now, $G(b) = Prob(\max_{j \neq i} b(V_j) \leq b) = \prod_{j \neq i} Prob(b(V_j) \leq b)$. Let B be the distribution of bids. Thus, $G(b) = B(b)^2$, so $G'(b) = 2B(b)B'(b)$. I estimate B and B' using a Normal kernel function and Silverman's rule of thumb for the bandwidth. Thus, I let $\hat{G}(b) = \hat{B}(b)^2$ and $\hat{G}'(b) = 2\hat{B}(b)\hat{B}'(b)$.

Assuming that H is a log-normal distribution, I estimate the parameters μ_r and σ_r through Maximum Likelihood. Let $X = 1\{\max_i b_i \leq R\}$. i.e., $x = 1$ means that the winning bid is rejected. The PDF of X is

$$\begin{aligned} P(X = x) &= P(\max_i b_i \leq R)^x (1 - P(\max_i b_i \leq R))^{1-x} \\ &= (1 - P(\max_i b_i \geq R))^x P(\max_i b_i \geq R)^{1-x} \\ &= (1 - P(\mu_r + \sigma_r Z \leq log(\max_i b_i)))^x P(\mu_r + \sigma_r Z \leq log(\max_i b_i))^{1-x} \\ &= \left[1 - \Phi\left(\frac{log(\max_i b_i) - \mu_r}{\sigma_r}\right)\right]^x \left[\Phi\left(\frac{log(\max_i b_i) - \mu_r}{\sigma_r}\right)\right]^{1-x} \end{aligned}$$

where Φ is the standard Normal distribution CDF. For T auctions, the loglikelihood function is given by

$$l(\mu_r, \sigma_r) = \sum_{i=1}^{T} x_t \left[1 - \Phi\left(\frac{log(\max_i b_{it}) - \mu_r}{\sigma_r}\right)\right] + (1 - x_t)\left[\Phi\left(\frac{log(\max_i b_{it}) - \mu_r}{\sigma_r}\right)\right]$$

We then estimate $\hat{v}_i = b_i + \dfrac{\hat{G}(b_i)\hat{H}(b_i)}{\hat{G}'(b_i)\hat{H}(b_i) + \hat{G}(b_i)\hat{H}'(b_i)}$, and use these estimates to estimate the density f of private values using a normal kernel function.

Figure 1 shows the estimated pdf of the estimated private values.

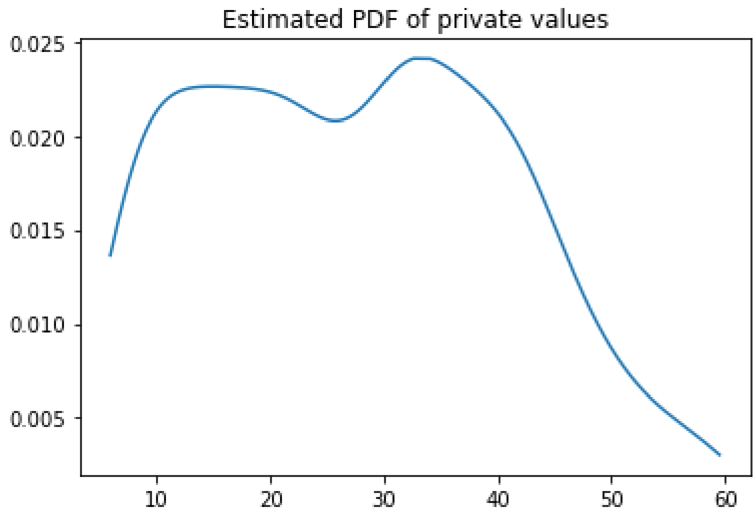

Figure 1: Estimated PDF of private values

Question 3: $M(r)$ is the CDF of the second larger private value, i.e.,

$$\begin{aligned}
M(r) &= P(Y_{2:3} \le r) \\
&= P(Y_{2:3} \le r, Y_{3:3} \ge r) + P(Y_{2:3} \le r, Y_{3:3} \le r) \\
&= \binom{3}{1} F(r)^2(1 - F(r)) + F(r)^3 \\
&= 3F(r)^2(1 - F(r)) + F(r)^3
\end{aligned}$$

So $m(r) = 6F(r)f(r)(1 - F(r)) - 3F(r)^2 f(r) + 3F(r)^2 f(r) = 6F(r)(1 - F(r))f(r)$. Thus,

$$\begin{aligned}
E[Revenue(r)] &= N \times \left[r(1 - F(r))(3F(r)^2(1 - F(r)) + F(r)^3) + \int_r^{\overline{v}} 6y(1 - F(y))^2 F(y) f(y) dy \right] \\
&= N \times \left[r(1 - F(r))(F(r)^2)(3 - 2F(r)) + \int_r^{\overline{v}} 6y(1 - F(y))^2 F(y) f(y) dy \right]
\end{aligned}$$

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For each $r \in [4, \overline{v}]$, I use the estimated $\hat{F}(r)$ and $\hat{f}(r)$, and approximate the integral as

$$\sum_{r \leq r_i \leq \overline{v}} 6r_i(1 - F(r_i))^2 F(r_i) f(r_i) dr$$

in which the r_i's are the points in the grid for r and $dr = \dfrac{32 - 4}{n\_points}$, where n_points is the number of points in the grid.

Figure 2 shows the estimated expected revenue as a function of the reserve prices. As we can see, the expected revenue is maximized when the reserve price is around 26.

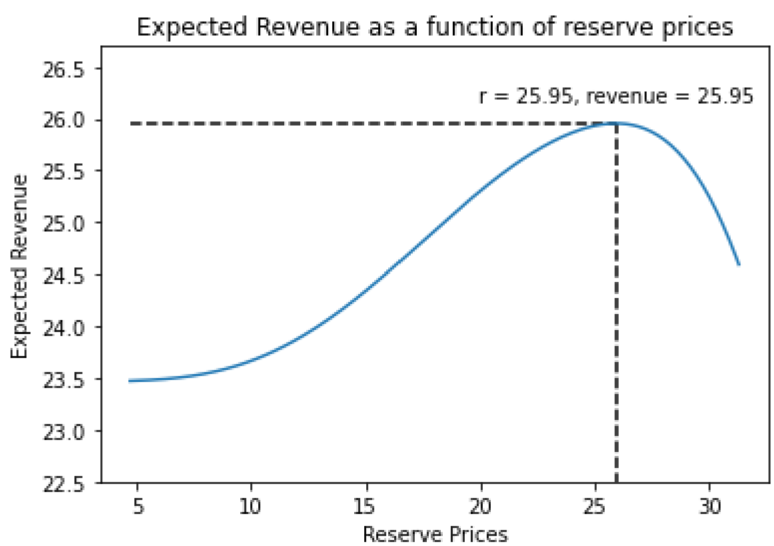

Figure 2: Expected revenue as a function of reserve prices