LISTA # 3 - LEILÕES

In this problem set you will assist the government to improve the design of its auctions. You are allowed to discuss the Problem Set in groups, but each student must write his/her own answers. You must submit a 'pdf' with your answers and code file(s).

Market description

The Widget Authority is responsible for regularly selling widgets. Widgets are sold through first-price sealed bid auctions. The Authority can reject bids if they think the bid is too low. However, they don't have any clear set of rules to determine what too low is, their behavior is now seemingly random. The new head of the Authority wants to make the process more transparent and is considering a fixed reserve price (minimum bid) policy.

In this problem set you will help him to understand the consequences of this new policy. You are provided with auction outcomes from this market. They are in the file 'auctions.txt.' **Each row represents one auction.** There are always three firms bidding in this market. Their bids are in columns 1 to 3. The forth column is a dummy variable which is equal to 1 if the highest bid is rejected by the Authority.

Questions

Assume firms have private, independent and equally distributed values for widgets $V \sim F(\cdot)$ and that they perceive bid rejection behavior by the Authority as a random reserve price, *i.e.*, $R \sim H(\cdot)$.

1. Write down the bidder's problem and corresponding FOC when he/she takes as given the strategy of opponent bidders in a symmetric equilibrium. You should have an expression for a bidder's private value as function of his own bid, the highest opponent bid distribution and the reserve price distribution.

2. Use the bid data and the FOC to recover, for each bid, the private values. We will need the primitive distribution of private values to assess behavior under a different reserve price rule. Plot the density of private values.[1]

   Tips: For every bid, you will need to compute two objects and its derivatives: $G(\cdot)$ and $H(\cdot)$.

   (a) Estimating $G(\cdot)$ and $G'(\cdot)$ at every bid is standard and can be done non-parametrically by Kernel. You can use a Normal kernel function and Silverman's rule of thumb for the bandwidth.

[1] Kernels perform poorly at the tails of the distribution. So set the grid used for plotting (x-axis) to run from the 0.025 to the 0.975 quantiles of estimated private values distribution.

i. If we want to evaluate a kernel density at a set of points $\{b_i\}_{i=1}^{H}$ using as a sample a set $\{B_j\}_{j=1}^{N}$, we need to compute[2]

$$\hat{f}(b_i) = \frac{1}{N \times h} \sum_{j=1}^{N} K\left(\frac{b_i - B_j}{h}\right), \text{ for } i = 1, \dots, H.$$

ii. Try writing a matrix $\tilde{K}$, whose element (i, j) is $K\left(\frac{b_i - B_j}{h}\right)$. Avoid using "for loops" here, as it will slow down your code.

iii. The (column) vector with the kernel density evaluated at all points you need can be computed by the mean across columns of $\tilde{K}$ and adjusting by the bandwidth. The same may be done for evaluating kernel CDFs with the proper changes.

(b) $H(\cdot)$ and $H'(\cdot)$ are trickier since much less information is known about this random reserve price. You only know if the maximum bid in an auction is above (accepted) or below (rejected) this reserve price. Here we can assume a log-normal distribution for $H(\cdot)$, *i.e.*, $R = \exp(\mu_r + \sigma_r Z)$, where $Z \sim N(0, 1)$.

i. Estimate μ_r and σ_r by ML using the bid rejection information.

ii. Evaluate $H(\cdot)$ and $H'(\cdot)$ at every bid using the estimated distribution.

3. Use the estimated distribution of private values to compute the expected revenue under fixed reserve prices in the range $[4, 32]$. Plot expected revenues against reserve prices.

Tips:

i In the simple IPV case, the expected revenue for the seller given reserve price r is[3]

$$E[Revenue] = N \times \left(r(1 - F(r))M(r) + \int_r^{\bar{v}} y(1 - F(y))m(y)dy\right),$$

where $M(\cdot)$ and $m(\cdot)$ are the CDF and pdf of the highest opposing private value. Use your knowledge of order statistics to write $M(r)$ as a function of $F(r)$.

ii Use the sample of recovered PV from 2 to recover the functions that determine the expected revenue as needed by kernel. Kernels are bad at approximating values at the tails of the distribution, so you can work here with a trimmed sample of the recovered PV. You can trim 5% of the distribution extremes (leaving 2.5% at each tail). You can also use the max PV from the trimmed distribution as $\bar{v}$.

iii Create a loop around the previous points for $r \in [4, 32]$ and plot the graphs.

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[2] In this specific point, to recover private values you will need to evaluate the density at the own sample points. Therefore the sets $\{b_i\}_{i=1}^{H}$ and $\{B_j\}_{j=1}^{N}$ will be the same. When you plot the private values density, $\{b_i\}_{i=1}^{H}$ will be a grid of points.

[3] For a derivation, see page 22 in Krishna's book.