ESCOLA BRASILEIRA DE ECONOMIA E FINANÇAS - EPGE

Luciano Fabio Busatto Venturim

Industrial Organization - Problem Set 2

Rio de Janeiro

1st Quarter - 2022

Part 1: Setup

Question 1a: The Bellman Equation for the firm's maximization problem is

$$V(x_n, a_{nt}, \epsilon_{0nt}, \epsilon_{1nt}) = \max\{\theta x_n(1-\alpha a_{nt}) + \epsilon_{0nt} + \rho \int V(x_n, \min\{a_{nt}+1, 10\}, \epsilon_{0nt+1}, \epsilon_{1nt+1}) dG_\epsilon(\epsilon_{nt}),$$
$$- \phi + \epsilon_{1nt} + \rho \int V(x_n, 0, \epsilon_{0nt+1}, \epsilon_{1nt+1}) dG_\epsilon(\epsilon_{nt+1})\}$$

Question 1b: Integrating $V(x_n, a_{nt}, \epsilon_{0nt}, \epsilon_{1nt})$, we have

$$\begin{aligned} V(x_n, a_{nt}) &:= \int V(x_n, a_{nt}, \epsilon_{0nt}, \epsilon_{1nt}) dG_\epsilon(\epsilon_{nt}) \\ &= \int \max\{\theta x_n(1-\alpha a_{nt}) + \epsilon_{0nt} + \rho V(x_n, \min\{a_{nt}+1, 10\}), -\phi + \epsilon_{1nt} + \rho V(x_n, 0)\} dG_\epsilon(\epsilon_{nt}) \end{aligned}$$

Since ϵ_{0nt} and ϵ_{1nt} are iid extreme value type 1 distributed, we can write

$$V(x_n, a_{nt}) = log\left(exp(\theta x_n(1-\alpha a_{nt}) + \rho V(x_n, \min\{a_{nt}+1, 10\}) + exp(-\phi + \rho V(x_n, 0))\right) + \gamma$$

To simplify the notation, let $c_{nt} \in \{0, 1\}$ denote the decision of replacing the machine in market n and period t, where $c_{nt} = 1$ represents the replacement, and $v(0, x_n, a_{nt}) = \theta x_n(1 - \alpha a_{nt}) + \rho V(x_n, \min\{a_{nt}+1, 10\})$ and $v(1, c_n, a_{nt}) = -\phi + \rho V(x_n, 0)$. Thus,

$$V(x_n.a_{nt}) = log(exp(v(0, x_n, a_{nt})) + exp(v(1, x_n, a_{nt}))) + \gamma$$

Question 1c: The Conditional Choice Probabilities (CCP) are given by

$$\begin{aligned} Pr(c_{nt} = 0 | x_n, a_{nt}) &= \int 1\left\{0 = \underset{c_{nt}\in\{0,1\}}{max}\, v(c_{nt}, x_n, a_{nt}) + \epsilon_{c_{nt}nt}\right\} dG_\epsilon(\epsilon_{nt}) \\ &= \frac{exp(v(0, x_n, a_{nt}))}{exp(v(0, x_n, a_{nt})) + exp(v(1, x_n, a_{nt}))} \end{aligned}$$

and

$$Pr(c_{nt} = 1 | x_n, a_{nt}) = \frac{exp(v(1, x_n, a_{nt}))}{exp(v(0, x_n, a_{nt})) + exp(v(1, x_n, a_{nt}))}$$

Question 3: The results of the linear probability model are presented in Figure 1.

See that the coefficients of the variables $MarketSize$ and age are positive: the older the machine, the smaller the profits and the bigger it is the incentive to replace it, so the probability of

OLS Estimation Summary

Dep. Variable:	replacement	R-squared:	0.1748
Estimator:	OLS	Adj. R-squared:	0.1739
No. Observations:	1900	F-statistic:	353.63
Date:	Mon, Mar 14 2022	P-value (F-stat)	0.0000
Time:	10:28:42	Distribution:	chi2(2)
Cov. Estimator:	robust		

Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-0.0941	0.0225	-4.1785	0.0000	-0.1382	-0.0500
MarketSize	0.1479	0.0228	6.4751	0.0000	0.1032	0.1927
age	0.0860	0.0047	18.332	0.0000	0.0768	0.0951

Figure 1: Linear Probability Model

replacement increases with age, as expected. However, in this linear probability model, the bigger the market size, the larger is the probability of replacement, which is not to be expected, since bigger market sizes increases the incentive to avoid periods of no-production when replacing the machine.

Part 2: Estimation

Question 2: Using the optimization method "BFGS" and initial values $(\theta, \alpha, \phi) = (1, 0.1, 1)$, the optimal parameters (the ones that maximize the loglikelihood criterion function) are $(\theta, \alpha, \phi) = (1.45546067, 0.26154854, 2.58972716)$

Part 3: Counterfactual

Question 1: Figure 2 shows the conditional probabilities of replacing the machine ($P(c_{nt} = 1|x_n = 1, a_{nt})$) when the market size equals 1 for the estimated parameters and for the counterfactual case, in which the cost of replacement of the machine is reduced in half.

As expected, the probability of replacement increases for every possible age, and increases

Conditional probabilities of replacing the machine when market size is 1

Age	Estimated	Counterfactual
0	0.039570	0.106886
1	0.102411	0.214597
2	0.208050	0.351786
3	0.344282	0.493995
4	0.486808	0.621394
5	0.615294	0.725019
6	0.720228	0.804257
7	0.800673	0.862518
8	0.859908	0.904294
9	0.902338	0.933741
10	0.931128	0.953749

Figure 2: Conditional probability of replacing the machine

more for new machines. It is interesting to note that for these parameters, the reduction of the cost in 50% have the same effect on the probability as the increase of 1 year in the age of the machine.