# Estatística 2 - Lista 7

① (9.7) See that

$$E[y_i \mid x_i = 40] = 40\beta_1 + 40^2\beta_2$$

Thus, the null hypothesis is

$H_0: 40\beta_1 + 40^2\beta_2 = 20 \quad \times \quad H_1: H_0^c$

or $H_0: \beta_1 + 40\beta_2 = 0.5 \quad \times \quad H_1: H_0^c$

We can write this restriction as

$R'\beta = c$, where $R = [1, 40]'$, $\beta = [\beta_1, \beta_2]'$, and $c = 0.5$.

We can then use the t-test $T = \dfrac{\sqrt{n}\,[R'\hat{\beta} - 0.5]}{[R'\hat{V}_\beta R]^{1/2}}$ where $\hat{V}_\beta = \hat{Q}_{xx}^{-1}\hat{\Omega}\hat{Q}_{xx}^{-1}$

and $\hat{\beta}$ is the OLS estimate ~~of~~ $\beta$. Since $T \xrightarrow{d} N(0,1)$, ~~for~~ a two-sided test with asymptotic size $\alpha$, we set $c_\alpha$ such that $\lim_{n\to\infty} P(|T| > c_\alpha) = P(|Z| > c_\alpha) = 2(1 - \Phi(c_\alpha)) = \alpha$.

and reject $H_0$ if $|T| > c_\alpha$.

(9.8) The tests are related. See that the model $y_i = x_{1i}'\beta_1 + x_{2i}'\beta_2 + e_i$ can be written as $y_i = x_{1i}'(\beta_1 + \beta_2) + (x_{2i} - x_{1i})'\beta_2 + e_i$.

Let $\gamma_1 = \beta_1 + \beta_2$ and $\gamma_2 = \beta_2$. Thus, a test for $H_0: \gamma_2 = 0 \times H_1: \gamma_2 \neq 0$ is the same as a test for $H_0: \beta_2 = 0 \times H_1: \beta_2 \neq 0$.

(2) (a) For the first Wald test, we will use the null

$H_0: R_1'\hat{\beta} = 0_{2\times 1}$ where $R_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}_{3\times 2}$. For the second, we will use the null

$r'(\hat{\beta}) = 0$ where $r(\hat{\beta}) = \begin{bmatrix} \exp(\hat{\beta}_1) - \exp(\hat{\beta}_2) \\ \exp(\hat{\beta}_2) - \exp(\hat{\beta}_3) \end{bmatrix}_{2\times 1}$

with $R = \frac{\partial}{\partial \beta} r(\beta)' = \begin{bmatrix} \exp(\beta_1) & 0 \\ -\exp(\beta_2) & \exp(\beta_2) \\ 0 & -\exp(\beta_3) \end{bmatrix}_{3\times 2}$

and $\hat{R}_2 = \frac{\partial}{\partial \beta} r(\beta)' \Big|_{\beta = \hat{\beta}}$.

The first Wald test has test statistic

$W_1 = \hat{\beta}' R_1 (R_1' \hat{V}_{\hat{\beta}} R_1)^{-1} R_1' \hat{\beta}$ where $\hat{V}_{\hat{\beta}} = n(X'X)^{-1}\hat{\Omega}(X'X)^{-1}$

and $\hat{\Omega} = \frac{1}{n}\sum_{i=1}^{n} x_i x_i' \hat{e}_i^2$

The second one has test statistic

$W_2 = (r(\hat{\beta}))' (\hat{R}_2' \hat{V}_{\hat{\beta}} \hat{R}_2)^{-1} (r(\hat{\beta}))$

(b) The r.v. $1\{W_s^j > c\}$, $s = 1, \dots, B$, are iid. We have that $E[p^j] = E[1\{W_s^j > c\}] = P(W_s^j > c) \xrightarrow[n\to\infty]{} 0.1$ since $W_s^j \xrightarrow[n\to\infty]{d} \chi_2^2$ and $c$ is set so that $P(\chi_2^2 > c) = 0.1$.

(c) For a fixed $n$, by the WLLN,

$p^j \xrightarrow[B\to\infty]{p} E[1\{W_s^j > c\}] = P(W_s^j > c)$

which is usually different from the values computed in (b), then we'd want them to be approximately equal.

(d) The estimates are $\hat{p}^1 = 0.129$ and $\hat{p}^2 = 0.104$. Thus, the second test has a better finite sample performance for this sample size, DGP, and model.

(e) We can use the CLT to test the hypothesis $H_0: P(W_s^j > c) = 0.1$ x $H_1: P(W_s^j > c) \neq 0.1$. The t-statistic for this test is

$$T^j = \sqrt{B}\,\frac{\hat{p}^j - 0.1}{\sqrt{0.1(0.9)}} \overset{a}{\sim} N(0,1).$$

We have $T^1 = \sqrt{1000} \cdot \frac{0.029}{\sqrt{0.09}} = 3.056$ and

$$T^2 = \sqrt{1000} \cdot \frac{0.004}{\sqrt{0.09}} = 0.421$$

Hence, we can reject $H_0$ in the first test at level 0.05 and we cannot reject $H_0$ in the second. We then have evidence that for this sample size, DGP, and model, $W^2$ is a good test with size 0.1, but $W^1$ is not.