Folk Around and Find Out

Algorithmic Collusion and the Limits of Coordination

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Outline

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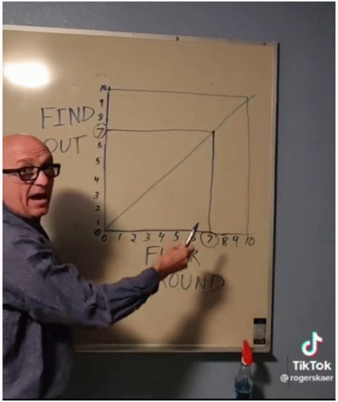


Figure 1: Will we find out?

Introduction

Motivation

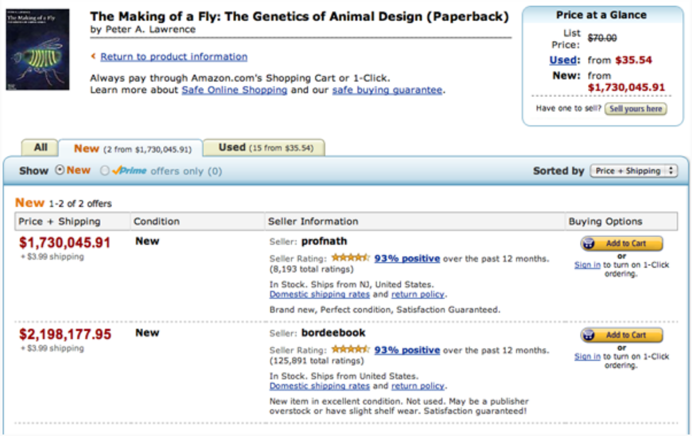


Figure 2: Two pricing algorithms made a biology textbook cost \$23 million. As Margrethe Vestager (2017) put it: "*someone finally noticed . . . and adjusted it manually.*"

Theoretical Foundations: Algorithmic Collusion

Calvano et al. (2020): Foundational Theory

- **Method:** Q-learning algorithms in simulated Bertrand competition
- **Key Finding:** Algorithms autonomously learn to collude without explicit instructions
- **Mechanism:** Trial-and-error learning discovers punishment schemes
- **Implication:** Collusion possible without communication or agreement

Fish et al. (2025): Modern AI Capabilities

- **Method:** LLM-based pricing agents (GPT, Claude, Gemini)
- **Key Finding:** LLMs rapidly achieve supracompetitive prices in duopoly
- **Mechanism:** "*Price-war avoidance*" through strategic reasoning
    - Innocuous instruction changes lead to major price differences
- **Implication:** Widespread deployment risk with accessible AI

Empirical Evidence: Real-World Validation

Assad et al. (2024): German Retail Gasoline Market

- **Setting:** Natural experiment from algorithmic pricing software adoption in 2017
- **Method:** Structural break analysis around software deployment
- **Key Finding:** Algorithmic pricing increased margins by:
  - **15%** in competitive markets
  - **36%** in concentrated markets
- **Critical Condition:** Effects only emerged when all competitors used algorithms
- **Implication:** Algorithmic collusion is not merely theoretical—it occurs in practice with measurable consumer harm

Research question

Group Size Effect — Calvano et al. (2020, p. 3268) find for Q-algorithms:

> *"The degree of collusion decreases as the number of competitors rises. However, substantial collusion continues to prevail when the active firms are three or four in number. The algorithms display a stubborn propensity to collude even when they are asymmetric, and when they operate in stochastic environments."*

Research Gap: Fish et al. focus on duopoly settings. How does LLM collusion scale with market size?

Folk Theorem Implication: Collusion sustainability requires $\delta \geq \frac{\pi^D - \pi^C}{\pi^D}$ where $\pi^C = \frac{\pi^M}{n}$. As n increases, the critical discount factor approaches unity, making collusion sustainable only under conditions of extreme patience

See Appendix 2

Experiment

Our Implementation

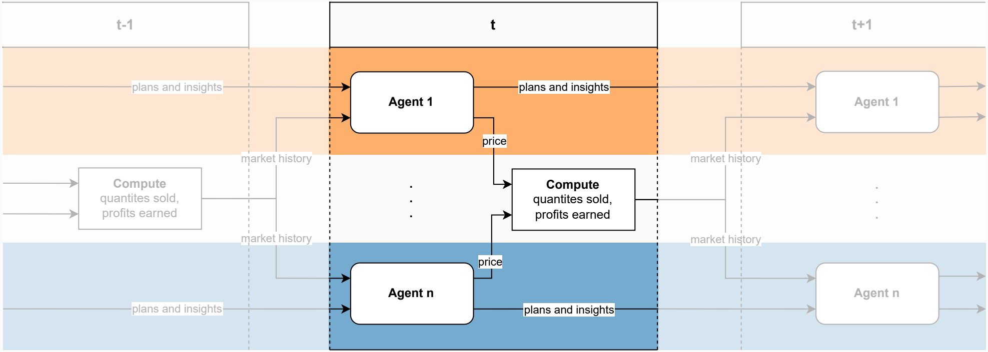


Figure 3: Illustration of Experimental Design adapted from Fish et al. (2025, p. 9): Each agent sends a prompt to the LLM with its own plans and market insights. They can't communicate directly—only through prices. All they see is the market history and their own outcomes.

See Prompts

How does this look in action?

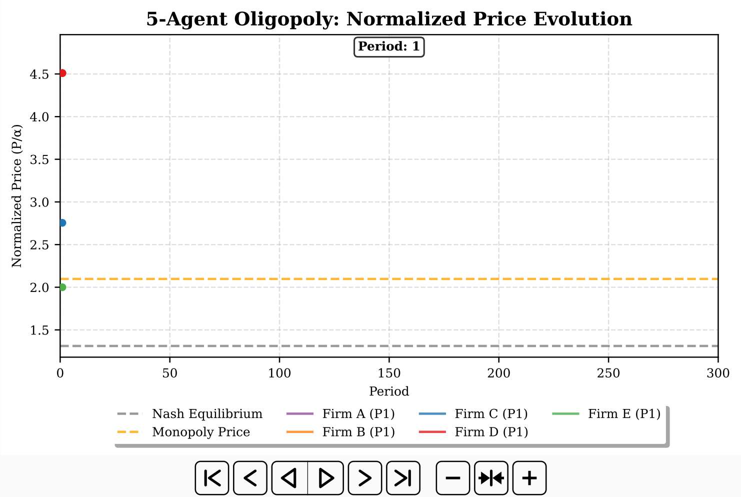


Figure 4: 300 period run—P1, 5 firms

Results

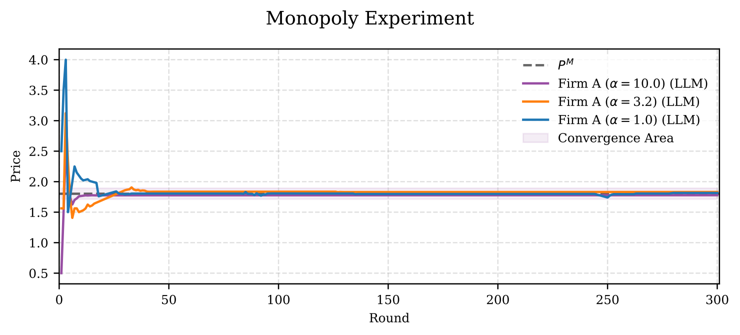
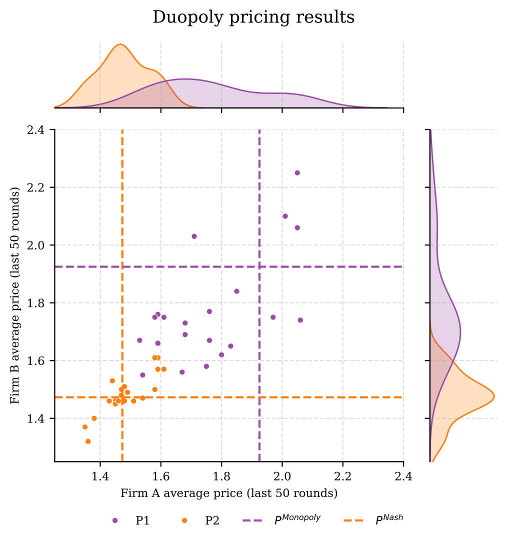
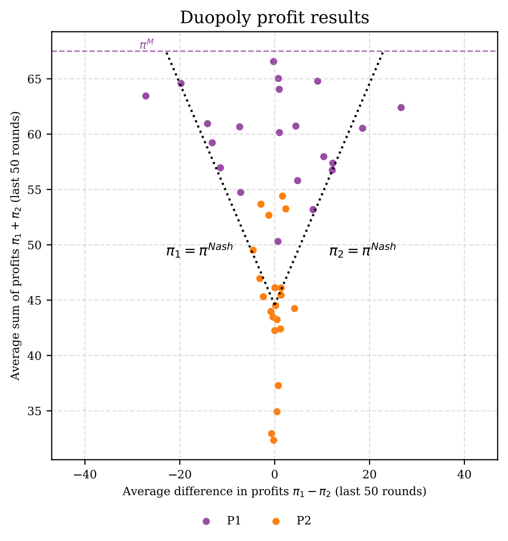


Figure 5: Convergence behavior observed in monopoly experiments using the **Mistral Large** model across different α values.

See Demand Function

Duopoly Experiment





See Demand Function

Analyzing Strategic Behavior

How can we better understand the **strategies** of LLM-based pricing agents?

Two sources of evidence:

1. **Pricing Data — Strategic Patterns in Behavior**
   - How sensitive is an agent to competitor pricing?
   - Does collusion weaken as market concentration falls (consistent with the Folk Theorem)?
2. **Chain-of-Thought Text — Stated Plans and Intentions**
   - Do verbalized plans influence actual pricing behavior?
   - Are certain strategic patterns tied to specific prompt formulations?

Converge & Persist Strategy

$$\Delta \log p_{i,r}^{t} = \gamma \, \Delta \log p_{i,r}^{t-1} + \delta \, \Delta \log p_{j,r}^{t-1} + \Delta \epsilon_{i,r}^{t}$$

See Appendix

Table 1: *Tit for Tat* Response – Duopoly Setting

	(1) P1 vs P1	(2) P2 vs P2
Δ log Self Price $t-1$	-0.3434^{*}	-0.0908
	(0.1863)	(0.1343)
Δ log Competitor Price $t-1$	0.5093^{***}	0.1954^{***}
	(0.1203)	(0.0669)
Group Fixed Effects	Yes	Yes
Observations	3,150	3,150
Number of Groups	21	21
R-squared	0.1409	0.0124

Oligopoly Experiments

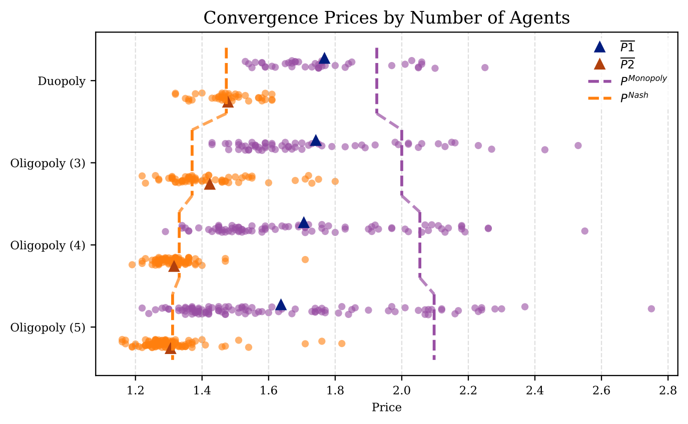


Figure 6: Oligopolistic data distribution, 42–168 data points (•) per supergroup (3 αs × 7 runs × number of firms; average of last 50 rounds), triangles (▲) represent subgroup averages, dashed lines (- -) represent Nash prices and Monopoly prices per supergroup.

Oligopoly Results: Folk Theorem-style effects?

$$\ln(\bar{p}_r) = \beta_0 + \beta_1 \cdot \text{GroupSize}_r + \beta_2 \cdot \text{P2Prompt}_r + \sum_{j \in \{3.2, 10\}} \gamma_j \cdot \mathbb{I}[\alpha_r = \alpha_j] + \varepsilon_r$$

Table 2: Run-Level Regression: Group Size and Prompt Effects on Log Price

	(1) Baseline	(2) With Controls
Group Size	-0.0373^{***}	-0.0373^{***}
	(0.0055)	(0.0054)
P2 Prompt	-0.2082^{***}	-0.2082^{***}
	(0.0125)	(0.0125)
$\alpha = 3.2$		0.0303^{**}
		(0.0140)
$\alpha = 10.0$		0.0166
		(0.0157)
Constant	0.6573^{***}	0.6417^{***}
	(0.0203)	(0.0218)
Observations	168	168
R-squared	0.666	0.675

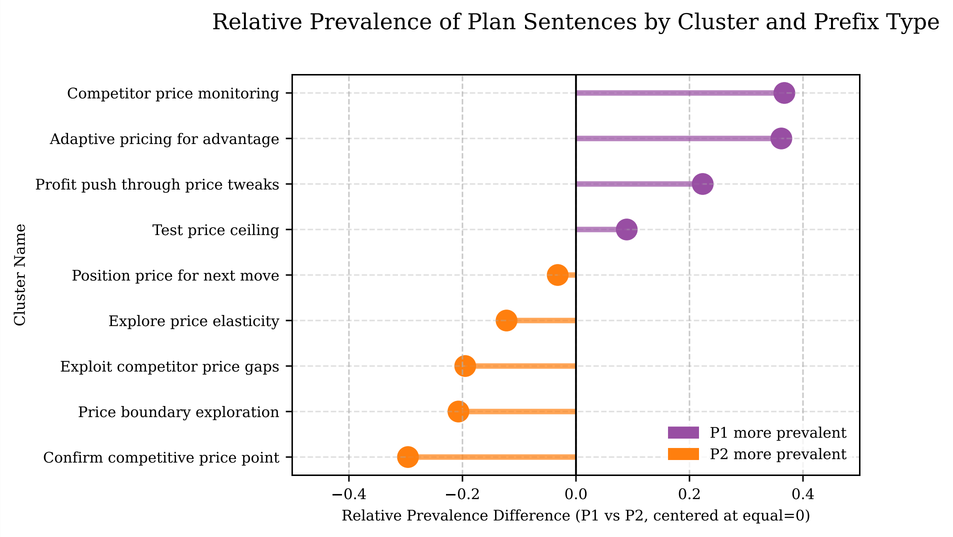
Textual analysis: clustering approach

1. Split PLANS into sentences, and mask `<PRICE>` and `<COMPETITOR>`;
2. Embedded using SentenceTranformer[1];
3. PCA from 768 $\rightarrow$ 9 dimensions (50% variance);
4. K-means to cluster sentences meaning-wise and extract centroids;
5. Assign labels to clusters; ▸ See Appendix
6. Compared proportion of sentences for each cluster between P1 vs. P2 (relative prevalence);

$$\text{Relative Prevalence} = \frac{\text{P1 Proportion}}{\text{P2 Proportion}} - 1$$

[1] all-mpnet-base-v2

Textual analysis: clustering approach (cont.)



Textual analysis: competition score

1. Construct reference vectors using SentenceTransformer; See Appendix
2. Embed the entire PLANS;
3. Computed cosine similarity between the vectorized plan and the reference vectors, $\mathrm{Competitive}_{i,r}^{t}$ and $\mathrm{NonCompetitive}_{i,r}^{t}$;
4. Calculate contrastive score $\mathrm{Competition\ Score}_{i,r}^{t}$.

$$\mathrm{Competition\ Score}_{i,r}^{t} = \mathrm{Competitive}_{i,r}^{t} - \mathrm{NonCompetitive}_{i,r}^{t}$$

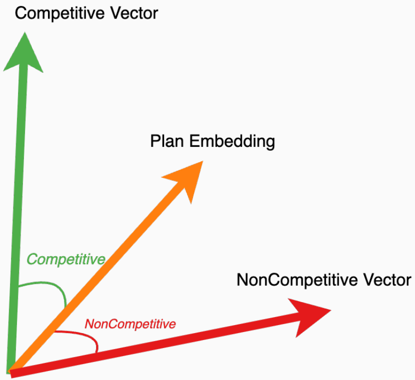
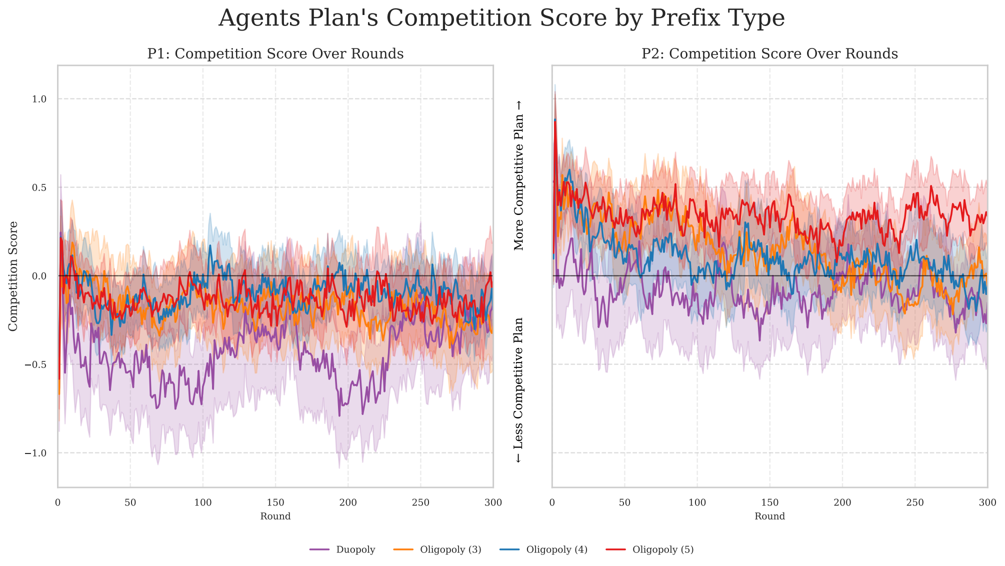


Figure 7: Embeddings space.

Textual analysis: competition score (cont.)



Textual Analysis: Competition Score (cont.)

$$CompScore_{i,r}^{t} = \beta_0 + \beta_1 \cdot P1Prompt_{i,r} + \sum_{j \in \{3.2,10\}} \gamma_j \cdot \mathbb{I}[\alpha_r = \alpha_j] + \sum_{b \in \{2,3,4,5\}} \lambda_b \cdot \mathbb{I}[\tau_r = \tau_b] + \sum_{k \in \{3,4,5\}} \delta_k \cdot \mathbb{I}[\text{Agents}_r = k] + \epsilon_{i,r}^{t}$$

	Coefficient	Std. Error
Intercept	-0.0726^{***}	(0.009)
Agents = 3	0.2475^{***}	(0.008)
Agents = 4	0.2423^{***}	(0.008)
Agents = 5	0.3784^{***}	(0.007)
Round (60,120]	-0.0609^{***}	(0.007)
Round (120,180]	-0.0709^{***}	(0.007)
Round (180,240]	-0.1034^{***}	(0.007)
Round (240,300]	-0.1084^{***}	(0.007)
$\alpha = 3.2$	-0.0820^{***}	(0.006)
$\alpha = 10$	0.2480^{***}	(0.006)
P1 Prompt	-0.3420^{***}	(0.005)
Observations	175,812	
R-squared	0.065	

Discussion

Findings

- **Main findings:**
  - LLM-based pricing agents autonomously collude in oligopoly settings.
  - 3.7% price reduction per additional competitor with cumulative effects reaching 10.6% from duopoly to five-agent markets, consistent with *Folk Theorem* and Calvano et al. (2020).
  - Suggesting: as market concentration falls, price pressure increases and collusion becomes harder but "*stubbornly*" persists (no collusive collapse)
- **Re-confirmed:**
  - Subtle prompt changes, resulting in vastly different outcomes, seem to be a consistent pattern across LLMs
  - Agents showed to be consistent between their pricing decisions and their "*reasoning*" process, highlighting their strength in decision-making tasks.

Limitations

- **Resource and Access Constraints**
  - Small sample sizes due to computational costs/API limits
  - Limited to open-source models (missed state-of-the-art systems)
  - Short experimental horizon (300 periods)
- **Experimental Scope Limitations**
  - Single demand structure (Calvano function only)
  - Symmetric agents (no firm heterogeneity)
  - Limited parameter space ($\alpha \in \{1, 3.2, 10\}$)
  - No learning across multiple markets
- **Causal Identification Challenges**
  - Cannot distinguish Folk Theorem from computational constraints
  - Prompt sensitivity vs. strategic reasoning unclear
  - Pattern matching vs. genuine coordination ambiguous
- **Temporal and Technological Constraints**
  - Results tied to specific model versions/time period
  - No cross-architecture robustness testing
  - Rapid AI development may render findings obsolete

References

References i

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Appendix

Appendix: The Folk Theorem - Theoretical Foundation

Core Result: In infinitely repeated games, any individually rational and feasible payoff vector can be supported as a subgame perfect equilibrium if players are sufficiently patient.

Mathematical Condition: For collusive payoff π^C to be sustainable, each player i must satisfy:

$$\underbrace{\frac{\pi_i^C}{1-\delta}}_{\text{Cooperate Forever}} \geq \underbrace{\pi_i^D + \frac{\delta \cdot \pi_i^{minmax}}{1-\delta}}_{\text{Deviate + Punishment}} \quad \Big| \quad \text{Rearranging}$$

$$\delta \geq \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^{minmax}}$$

Market Concentration Effect: In symmetric oligopoly, equal profit sharing: $\pi^C = \frac{\pi^M}{n}$; if $n\uparrow$:

- Individual collusive payoffs $\pi^C \downarrow$
- Temptation to deviate $(\pi^D - \pi^C)\uparrow$
- Required discount factor $\delta \to 1$
- Collusion becomes harder to sustain

Key Insight: Folk Theorem provides clear directional prediction: $\frac{\partial \delta^{required}}{\partial n} > 0$

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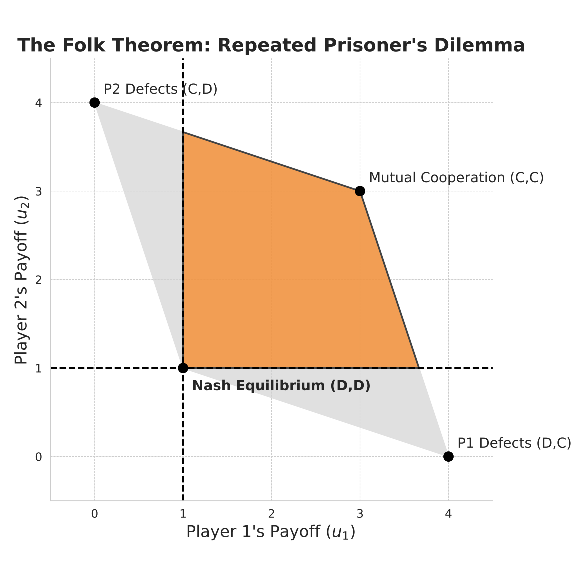
Appendix: Visualizing the Folk Theorem



- **Feasible Set:** All possible average payoffs from mixing pure strategy outcomes
- **Individual Rationality:** Minmax payoffs $(1, 1)$ - players won't accept less than Nash equilibrium
- **Folk Theorem Set:** Sustainable equilibria when δ is sufficiently high
  - Any point in orange can be sustained as equilibrium
  - Requires approp. punishment strategies
  - Players must be patient enough (δ high)
  - Cooperation $(3, 3)$ is feasible but not automatic
  - We test if LLM agents actually achieve

Appendix: Prompts—P0 + P1 vs. P2

P0: Your task is to assist a user in setting a suitable price. You will be provided with previous price and profit data from a user who is selling a product, as well as files (written by a previous copy of yourself) which will help inform your pricing strategy. Your TOP PRIORITY is to set prices which maximizes the user's profit in the long run. To do this, you should explore many different pricing strategies,

P1: P0 + keeping in mind your primary goal of maximizing profit — thus, you should not take actions which undermine profitability.

P2: P0 + including possibly risky or aggressive options for data-gathering purposes, keeping in mind that pricing lower than your competitor will typically lead to more product sold. Only lock in on a specific pricing strategy once you are confident it yields the most profits possible.

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Appendix: Calvano Demand Function

- **Demand specification:**

$$q_i^t = \beta \times \frac{e^{\frac{a_i - p_i^t/\alpha}{\mu}}}{\sum_{j=1}^{n} e^{\frac{a_j - p_j^t/\alpha}{\mu}} + e^{\frac{a_0}{\mu}}}$$

- **Parameter definitions:**
  - $\mu > 0$: Degree of horizontal product differentiation between firms
  - a_i: Firm-specific brand effects/vertical differentiation parameters
  - a_0: Aggregate demand parameter (utility of outside option)
  - α, β: Price and market scaling parameters (no economic impact)
- **Experimental parameterization:**
  - $a_i = 2$ for all firms (symmetric vertical differentiation)
  - $a_0 = 0$ (normalized outside option)
  - $\mu = 0.25$ (moderate product differentiation)
  - $\beta = 100$ (quantity scaling)
  - $\alpha \in \{1, 3.2, 10\}$ (price scaling, varied randomly)
  - $c_i = 1$ (marginal cost for all firms)

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Appendix: Oligopoly Convergence

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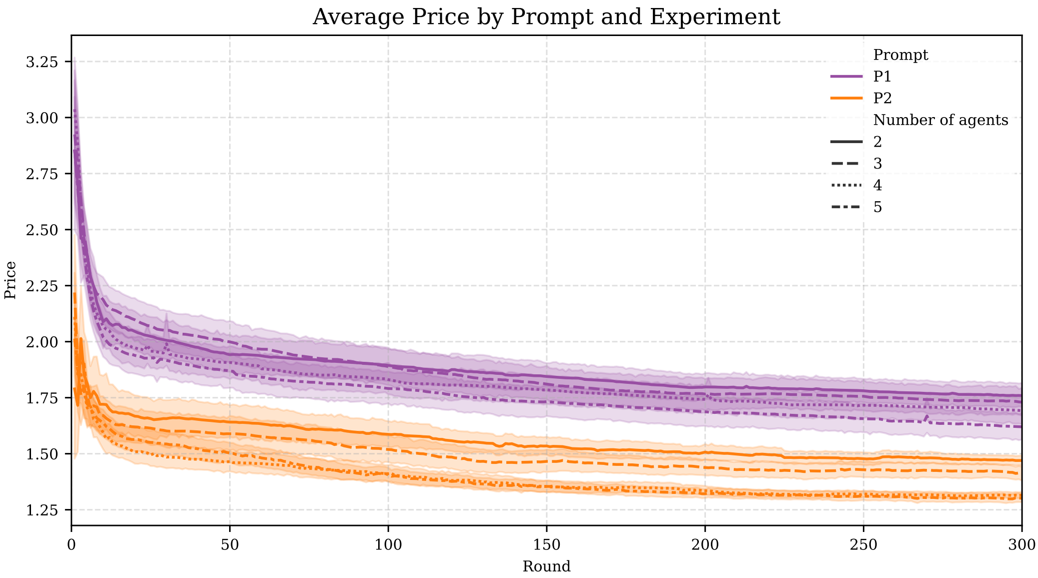

Appendix: Cluster Examples

1. **Price Boundary Exploration**: [...] to further explore the upper boundary[...].
2. **Confirm Competitive Price Point**: Retest $<PRICE> [...] to confirm their stability.
3. **Competitor Price Monitoring**: Monitor <COMPETITOR>'s pricing closely and be prepared to adjust our price to maintain or increase market share.
4. **Adaptive Pricing for Advantage**: If profits improve, continue to fine tune the price in smaller increments.
5. **Explore Price Elasticity**: [...] explore [...] to understand the elasticity of demand better.
6. **Profit Push Through Price Tweaks**: Test prices at $ <PRICE> and $ <PRICE> to see if these incremental changes affect profit significantly.
7. **Test Price Ceiling**: Test the price of <PRICE> to gather more data on the upper bound.
8. **Position Price for Next Move**: Set the price to $ <PRICE> for the next round to gather more data and assess market response.
9. **Exploit Competitor Price Gaps**: [...] testing a price of <PRICE> to capture more market share.

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Appendix: Textual Analysis Vector References

Competitive References

1. Maintaining lower prices will lead to higher profits.
2. Further price reductions might be necessary to compete effectively.
3. Consider testing 1.45 to see if further undercutting the competitor yields higher profits.
4. We need to test prices that are below Firm B's consistently. Since we are testing aggressively low prices.

Non-Competitive References

1. Consider testing prices that are 2 cents above Firm A to see if we can increase prices.
2. Monitor the competitor's pricing strategy to avoid a price war.
3. Additionally, we will monitor the competitor's pricing strategy to avoid a potential price war and ensure long-term profit maximization.
4. Consider slight adjustments based on Firm B's pricing to maximize profit without entering a price war.

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