Econometrics

TA Session 4

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Overview

- Global Hypothesis Testing
- Multiple Hypothesis Testing
- Monte Carlo Simulations

Let's start by running the following regression:

$$\texttt{colGPA}_i = \beta_1 + \beta_2 \texttt{hsGPA}_i + \beta_3\ \texttt{job19}_i + \beta_4\ \texttt{job20}_i + \beta_5\ \texttt{skipped}_i + \beta_6\ \texttt{bgfriend}_i + \beta_7\ \texttt{alcohol}_i + \varepsilon_i$$

where:

- `colGPA`: college GPA
- `hsGPA`: high school GPA
- `job19`: worked in 2019 (1=yes, 0=no)
- `job20`: worked in 2020 (1=yes, 0=no)
- `skipped`: skipped classes (1=yes, 0=no)
- `bgfriend`: has a boyfriend/girlfriend (1=yes, 0=no)
- `alcohol`: alcohol consumption (1=yes, 0=no)

```
bcuse gpa1, clear
regress colgpa hsGPA job19 job20 skipped bgfriend alcohol
```

Global Hypothesis Testing

What is testing the F value present in the regression output?

Global Hypothesis Testing

We want to test whether our regression model adds explanatory power beyond the mean.

Exercise

1. Indicate **null and alternative hypotheses**.
2. Write the expression of the **F-test statistic** used for this test, and its assumed distribution.
3. Run the restricted model, compute the RSSE (restricted model), SSE (unrestricted model) and F-statistic.
4. Find the critical value of the F-distribution and compute the p-value.

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1. Indicate null and alternative hypotheses.

$$H_0 : \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$$
$$H_a : \text{at least one } \beta_j \neq 0$$

Also can be written as:

$$H_0 : R\beta = r \quad \text{versus} \quad H_a : R\beta \neq r$$

where:

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad q = 6$$

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2. F-test statistic

The F-test statistic is given by:

$$F = \frac{(RSSE - SSE)/q}{SSE/(n-k)} \overset{Under H_0}{\sim} F_{6,141-7}$$

where:

- $RSSE$ is the sum of squared errors for the restricted model
- SSE is the sum of squared errors for the unrestricted model

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3. RSSE and SSE

1. The restricted model includes only an intercept (no explanatory variables):

$$\texttt{colGPA}_i \; = \beta_1 + \epsilon_i \quad \widehat{\texttt{colGPA}}_i = \overline{\texttt{colGPA}}_i$$

2. Compute the RSSE, SSE and F-statistic:

```
// RSSE
summarize colGPA
scalar colgpa_mean = r(mean)
generate double resid_R = colGPA - colgpa_mean
generate double resid_R_sq = resid_R^2
summarize resid_R_sq
scalar RSSE = r(sum)
// SSE
scalar SSE = e(rss)
scalar F = ((RSSE - SSE) / 6) / (SSE / (e(N) - 7))
```

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4. Critical value and p-value

Decision rule:

- If $F > F_{critical}$, reject H_0, otherwise do not reject H_0.

```
// Critical value
scalar F_critical = invF(0.95, 6, e(N) - 7)
display F_critical F
```

- If p-value $< \alpha$, reject H_0, otherwise do not reject H_0.

$$p\text{-value} = Prob\{F_{statistic} > F|H_0\}$$

```
// P-value
scalar p_value = 1 - F(F, 6, e(N) - 7)
display p_value
```

Multiple Hypothesis Testing

Consider testing whether job19 and job20 are jointly significant at $\alpha = 0.05$:

> Exercise
>
> 1. Indicate **null and alternative hypotheses**.
> 2. Write the expression of the **F-test statistic** used for this test, and its assumed distribution.
> 3. Run the restricted model, compute the RSSE (restricted model), SSE (unrestricted model) and F-statistic.
> 4. Find the critical value of the F-distribution and compute the p-value.
> 5. Draw the p-value and the critical value.
> 6. Compare the results with the ones obtained in Stata.

Monte Carlo Simulations

Monte Carlo Casino

Monte Carlo: a lab for estimators

Workflow

1. **Specify a known DGP** (the "true" model).
2. **Generate** many random samples from it.
3. **Estimate** the coefficients repeatedly.
4. **Observe** the estimator's behavior across replications:
   - mean (*bias*)
   - spread (*variance*)
   - shape (*sampling distribution*)

Data-Generating Process (DGP)

We will generate $m = 10000$ samples of size $n = 100$ from the following DGP:

DGP

$$y_i = 4 + 2x_{i2} + 2x_{i3} + \varepsilon_i$$

$$\varepsilon_i \,|\, X_i \sim \text{i.i.d. } N(0, 32)$$

$$x_{i2} \sim U[0, 40], \quad x_{i3} = x_{i2} + v_i, \; v_i \sim N(0, 16)$$

Function in Python

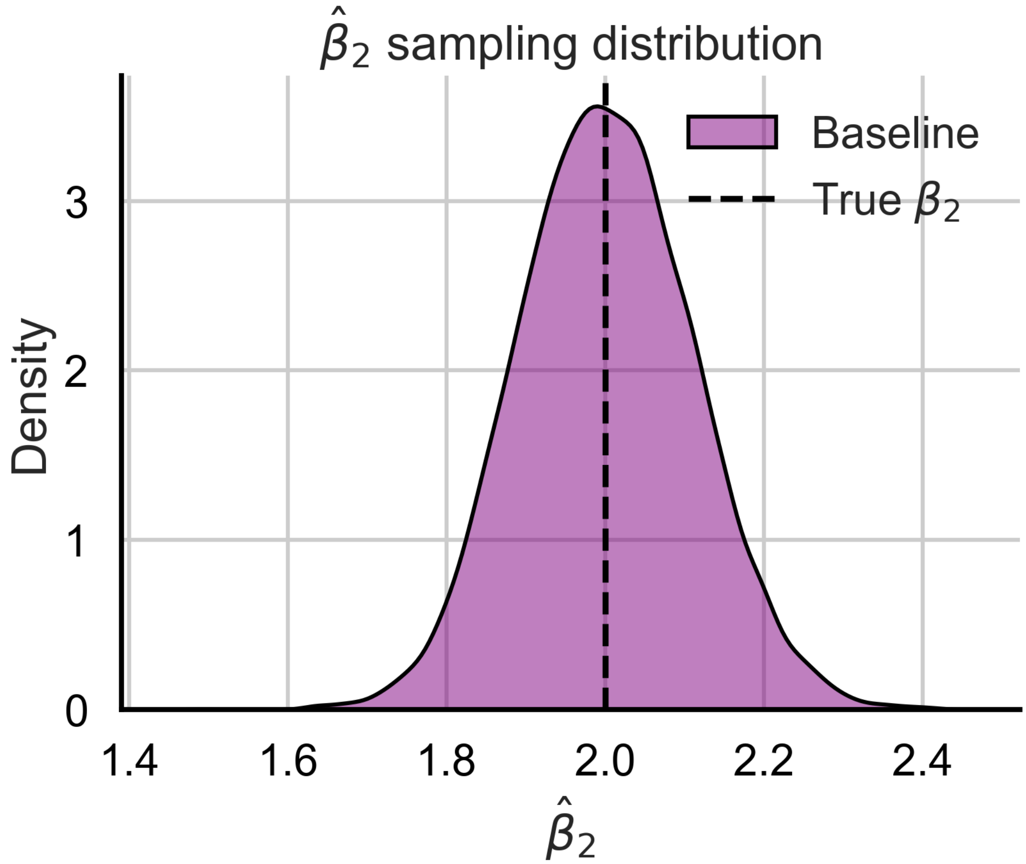
Let's create a function to analyze the behavior of $\hat{\beta}_2$:

```
def simulate_betas(n=100, sigma_eps=32, sigma_v=16, reps=10000, conditional=False):
    """
    Monte Carlo simulation of   from y = 4 + 2x  + 2x  +  .
    """
    betas = []

    # For conditional distribution: fix X once
    if conditional:
        x2_fixed = np.random.uniform(0, 40, n)
        v_fixed  = np.random.normal(0, sigma_v, n)
        x3_fixed = x2_fixed + v_fixed

    for _ in range(reps):
        if conditional:
            x2, x3 = x2_fixed, x3_fixed
        else:
            x2 = np.random.uniform(0, 40, n)
            v  = np.random.normal(0, sigma_v, n)
            x3 = x2 + v

        eps = np.random.normal(0, sigma_eps, n)
        y = 4 + 2*x2 + 2*x3 + eps

        X = sm.add_constant(np.column_stack([x2, x3]))
        model = sm.OLS(y, X).fit()
        betas.append(model.params[1])

    return np.array(betas)
```

Let's run the simulation for the conditional distribution of $\hat{\beta}_2$:

```
betas_base = simulate_betas(n=1000, sigma_eps=32, sigma_v=16, reps=10000)

plt.figure(figsize=(6,5))
sns.kdeplot(betas_base, fill=True, alpha=0.5, color="purple", label="Baseline", edgecolor="bl
```

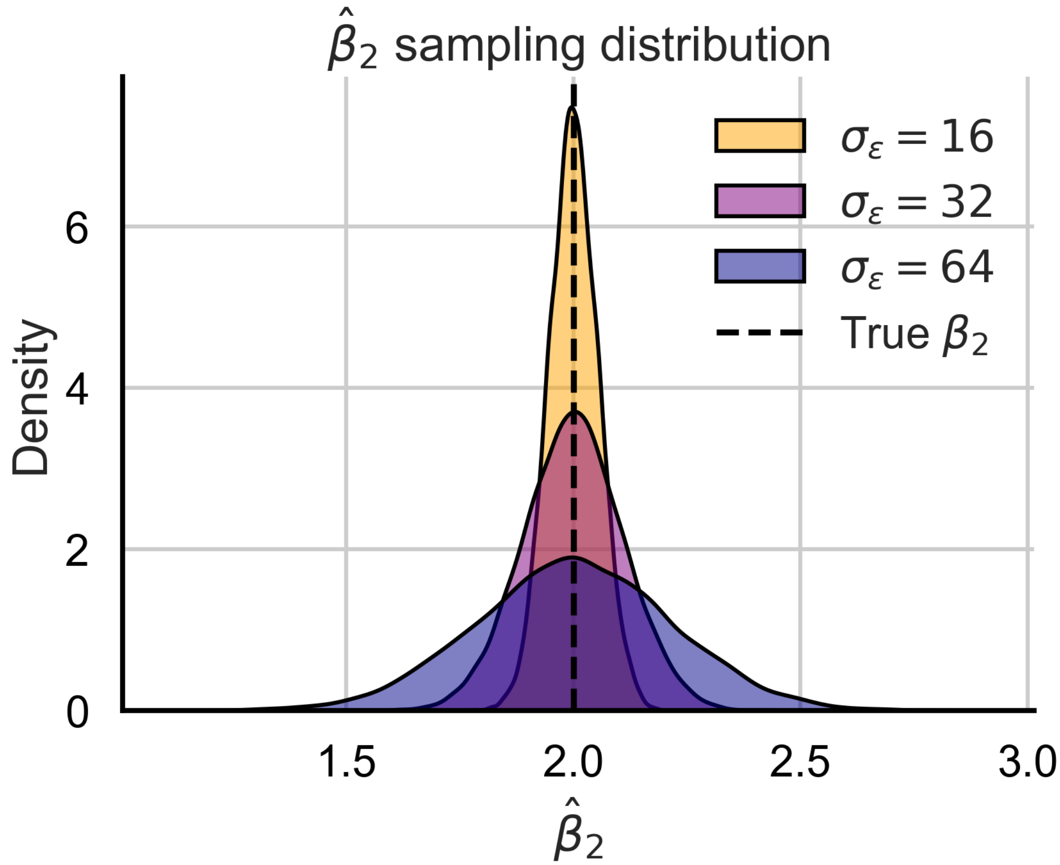
```
plt.axvline(2, color="black", ls="--", label=r"True $\beta_2$")
plt.title(r"$\hat{\beta}_2$ sampling distribution")
plt.xlabel(r"$\hat{\beta}_2$")
plt.show()
```



Now, let's increase σ_ε^2:

```
betas_high_sigma = simulate_betas(n=1000, sigma_eps=64, sigma_v=16, reps=10000)
betas_low_sigma = simulate_betas(n=1000, sigma_eps=16, sigma_v=16, reps=10000)

plt.figure(figsize=(6,5))
for sns, color, label in zip([betas_low_sigma, betas_base, betas_high_sigma], [r"$\sigma_{\va
    sns.kdeplot(sns, fill=True, alpha=0.5, edgecolor="black", color=color, label=label)
```

```
plt.axvline(2, color="black", ls="--", label=r"True $\beta_2$")
plt.title(r"$\hat{\beta}_2$ sampling distribution")
plt.xlabel(r"$\hat{\beta}_2$")
plt.show()
```



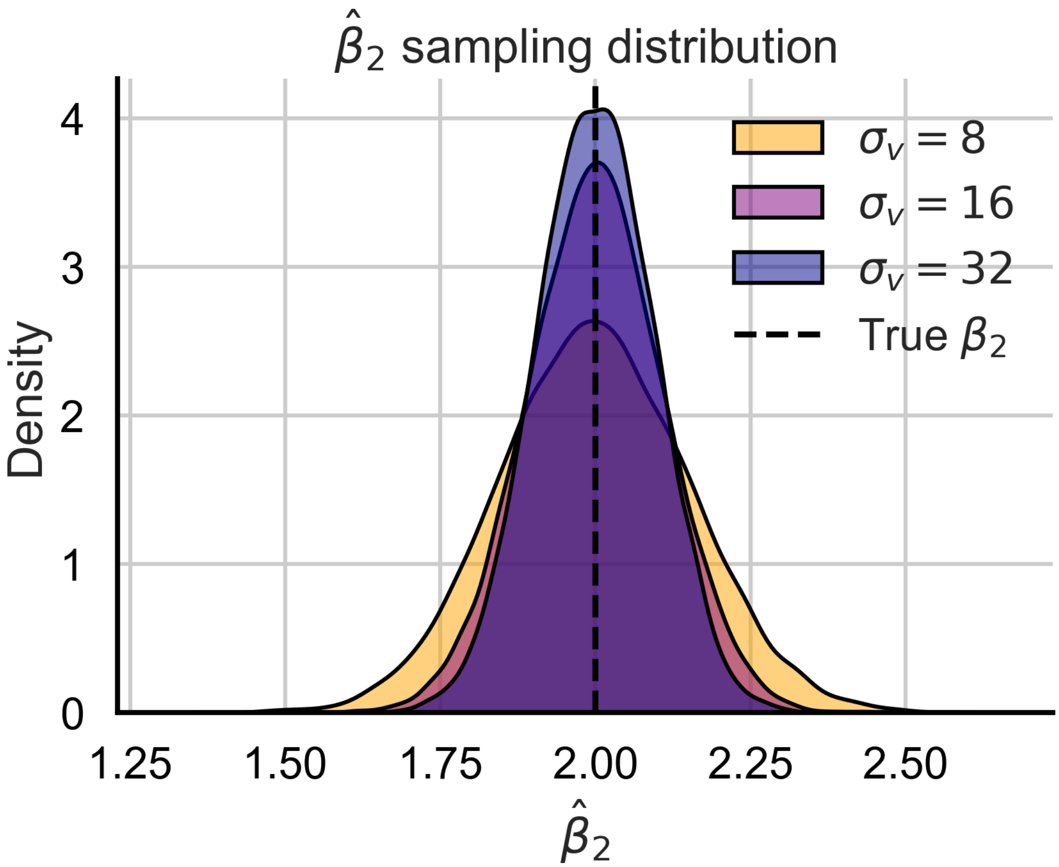
The OLS variance can then be expressed as:

$$Var(\hat{\beta}^2|X) = \sigma_\varepsilon^2 \frac{1}{(1-R_2^2)} \frac{1}{\sum(x_{2i}-\bar{x}_2)^2}$$

Hence, as $\sigma_\varepsilon^2 \uparrow$, the numerator increases and the estimator becomes increasingly unstable.

Now, let's reduce σ_v^2, the collinearity between x_2 and x_3:

```
betas_high_collinear = simulate_betas(n=1000, sigma_eps=32, sigma_v=32, reps=10000)
betas_low_collinear = simulate_betas(n=1000, sigma_eps=32, sigma_v=8, reps=10000)

plt.figure(figsize=(6,5))
for sns, color, label in zip([betas_low_collinear, betas_base, betas_high_collinear], [r"$\s
    sns.kdeplot(sns, fill=True, alpha=0.5, edgecolor="black", color=color, label=label)
plt.axvline(2, color="black", ls="--", label=r"True $\beta_2$")
plt.title(r"$\hat{\beta}_2$ sampling distribution")
plt.xlabel(r"$\hat{\beta}_2$")
plt.show()
```



Formally, as we reduce σ_v^2, x_2 and x_3 become more correlated:

$$\rho(x_2, x_3) = \sqrt{\frac{Var(x_2)}{Var(x_2) + \sigma_v^2}}$$

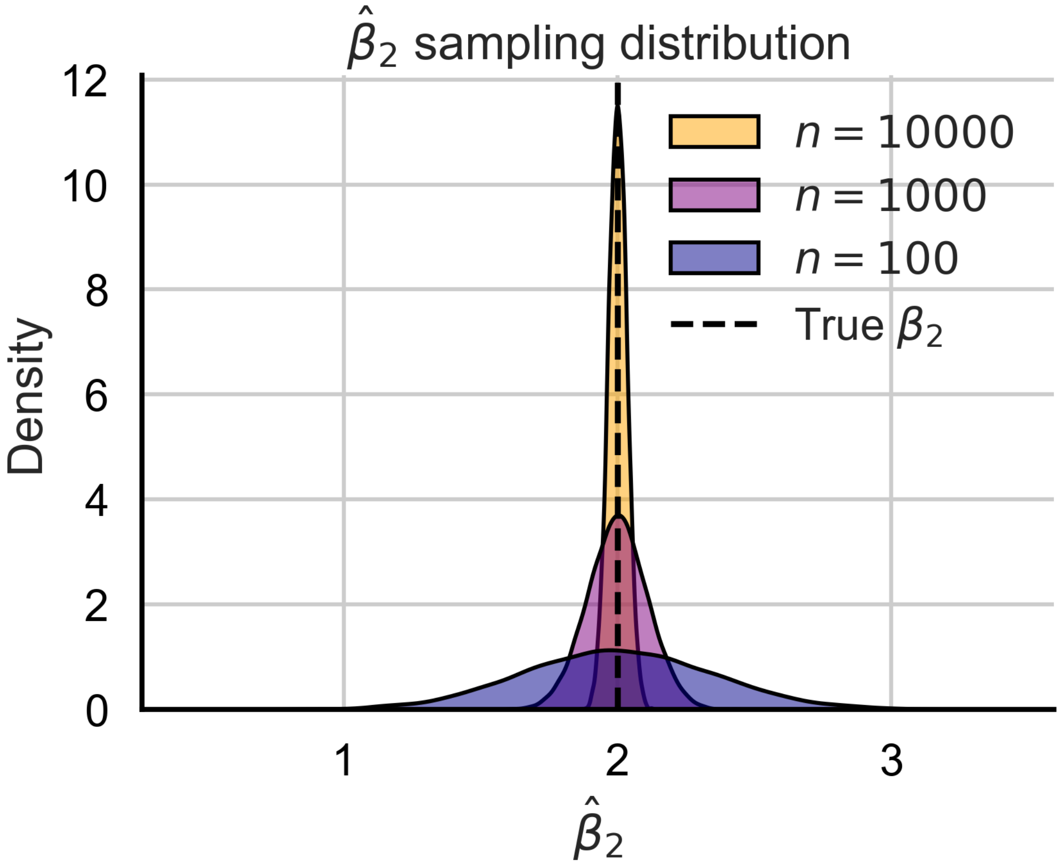
In the auxiliary regression $x_2 = \alpha + \delta x_3 + u_2$,

$$R_2^2 = \rho(x_2, x_3)^2.$$

Hence, as $\rho \uparrow 1$ (or $R_2^2 \uparrow 1$), affecting the VIF and the estimator becomes increasingly unstable — the classic symptom of **multicollinearity**.

Increasing the sample size n:

```
betas_large_sample = simulate_betas(n=10000)
betas_small_sample = simulate_betas(n=100)



plt.figure(figsize=(6,5))
for sns, color, label in zip([betas_small_sample, betas_base, betas_large_sample], [r"$n=100
    sns.kdeplot(sns, fill=True, alpha=0.5, edgecolor="black", color=color, label=label)
plt.axvline(2, color="black", ls="--", label=r"True $\beta_2$")
plt.title(r"$\hat{\beta}_2$ sampling distribution")
plt.xlabel(r"$\hat{\beta}_2$")
plt.show()
```



Consistency:

$$\hat{\beta}_2 \xrightarrow{p} \beta_2 \quad \text{as } n \to \infty$$

In the limit the sampling distribution collapses to a spike at the true value.

Now, running the simulation for the unconditional distribution of $\hat{\beta}_2$:

```
betas_base_cond = simulate_betas()
betas_base_uncond = simulate_betas(conditional=True)


plt.figure(figsize=(6,5))
for sns, color, label in zip([betas_base_uncond, betas_base_cond], [r"$Conditioned on X$", r'
    sns.kdeplot(sns, fill=True, alpha=0.5, edgecolor="black", color=color, label=label)
plt.axvline(2, color="black", ls="--", label=r"True $\beta_2$")
plt.title(r"$\hat{\beta}_2$ sampling distribution")
plt.xlabel(r"$\hat{\beta}_2$")
plt.show()
```