# Econometrics

**TA Session 1**

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## TA Materials

All the material for this course — slides, code, and some LaTeX utilities for referencing assignments — will be hosted in the GitHub repository.

```
git clone HHTPS
```

Install uv

```
uv sync
```

Pull the repository and get the updated materials for each session.

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## Overview

- Matrix Algebra with Python and R
- Derivatives with vectors
- Histograms and KDE
- Quick LaTex guide

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## Why These Topics?

Matrix Algebra

- Compact notation for regression & multivariate models
- Solve systems efficiently with Python

Derivatives with Vectors

- Gradients for optimization & ML
- Key for regression, loss functions

Histograms & KDE

- Explore data shape: skewness, modality, outliers
- Nonparametric distribution estimation & model checks

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## System of equations in Matrix Form

Consider the following system of equations:

$$\begin{cases} 2x_1 + 3x_2 + 10x_3 = 5 \\ 4x_1 + x_2 + 12x_3 = 6 \\ 7x_1 + 2x_2 + x_3 = 10 \end{cases}$$

Exercises

- How can we write this system in matrix form?
- Solve it using Python.

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## Matrix Operations using Python

```python
import numpy as np

#Define matrix A and vector b
A = np.array([[2, 3, 10],
                  [4 , 1, 12],
                  [7 , 2, 1]])
b = np.array([5 , 6, 10])

#Transpose A
A_transpose = np.transpose(A)

#Inverse A
A_inverse = np.linalg.inv(A)

x = np.dot(A_inverse, b)
print('Solution: ', x)
```

```
Solution:  [1.21078431 0.74509804 0.03431373]
```

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### Matrix Operations using R

```r
# Define matrix A and vector b
A <- matrix(c(2, 3, 10,
              4, 1, 12,
              7, 2, 1),
            nrow = 3, byrow = TRUE)

b <- c(5, 6, 10)

# Transpose A
A_transpose <- t(A)

# Inverse A
A_inverse <- solve(A)

# Solve Ax = b
x <- solve(A, b)
print(x)
```

## Derivatives of a vector

Gradient with respect to a vector

- In single-variable calculus:
  $\frac{d}{dx} f(x)$ is the slope of a scalar function of one variable.
- In multivariable calculus:
  If $f(z_1, z_2)$, we can group the variables into a vector:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

then the derivative with respect to a vector is the **gradient**:

$$\nabla_z f(z) = \begin{bmatrix} \frac{\partial f}{\partial z_1} \\ \frac{\partial f}{\partial z_2} \end{bmatrix}$$

Take the partial derivative with respect to each coordinate, then stack them in a column vector.

Consider:

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad B = \begin{bmatrix} b_{12} & b_{12} \\ b_{22} & b_{22} \end{bmatrix}$$

Exercises

Verify, by showing all the relevant steps, that:

$$(i) \quad \frac{\partial z'a}{\partial a} = z$$

$$(ii) \quad \frac{\partial a'z}{\partial z} = a$$

$$(iii) \quad \frac{\partial z'Bz}{\partial z} = 2Bz \quad \text{since B is symmetric}$$

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## Density Estimation - Introduction

Given some observations from some variable $X$, we would like to obtain an estimate of its density.

*Parametric vs Nonparametric*

- **Parametric approach**: we would choose a parametric density function (normal, exponential, uniform, etc) and use data to estimate the parameters of this density.

*Problem: what if $X$ doesn't follow a normal distribution?*

- **Nonparametric approach** like histograms and KD estimate the distribution of a variable $X$ without strong parametric assumptions.

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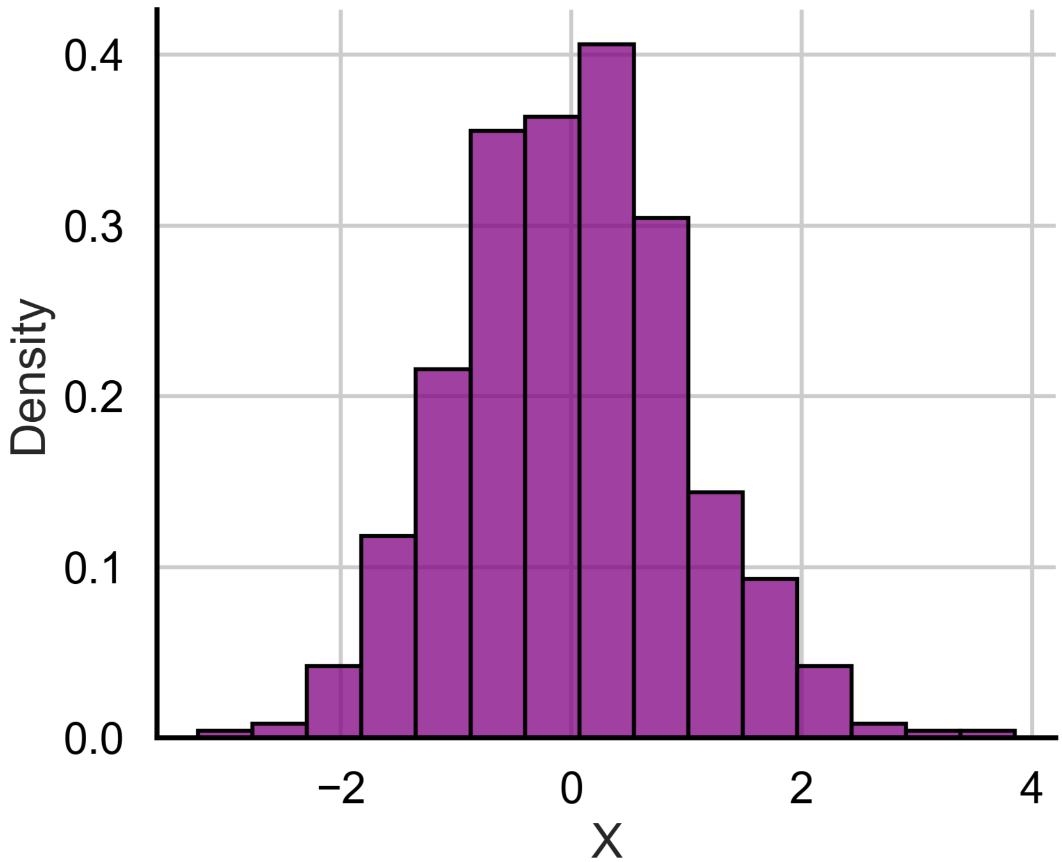
## Histogram

**Key idea:** A histogram is a graphical tool to visualize the distribution of a dataset.

- A **nonparametric method** to estimate the probability density function (PDF)
- Groups data into **intervals (bins)**
- Plots the **frequency** of data points in each bin

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**Concept of Density**

Let's first differentiate between frequency histograms and density histograms.

**Step 1: Bins and Frequency**

| Bins | Frequency |
|---|---|
| 0-5 | 2 |
| 5-10 | 4 |
| 10-15 | 5 |
| 15-20 | 1 |

## Concept of Density

Let's first differentiate between frequency histograms and density histograms.

### Step 2: Add Width

| Bins | Frequency | Width |
|---|---|---|
| 0-5 | 2 | 5 |
| 5-10 | 4 | 5 |
| 10-15 | 5 | 5 |
| 15-20 | 1 | 5 |

## Concept of Density

Let's first differentiate between frequency histograms and density histograms.

### Step 3: Add Density

| Bins | Frequency | Width | Density |
|---|---|---|---|
| 0-5 | 2 | 5 | $2/(12 * 5) = 0.033$ |
| 5-10 | 4 | 5 | $4/(12 * 5) = 0.067$ |
| 10-15 | 5 | 5 | $5/(12 * 5) = 0.083$ |
| 15-20 | 1 | 5 | $1/(12 * 5) = 0.017$ |

## Concept of Density

Let's first differentiate between frequency histograms and density histograms.

| Bins | Frequency | Width | Density |
|---|---|---|---|
| 0-5 | 2 | 5 | $2/(12 * 5) = 0.033$ |
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| 10-15 | 5 | 5 | $5/(12 * 5) = 0.083$ |
| 15-20 | 1 | 5 | $1/(12 * 5) = 0.017$ |

Mathematically, for bin width $h$ and number of observations $n$:

$$\hat{f(x)} = \frac{1}{nh} \sum_{i=1}^{n} \mathbf{1}\{x_i \in \text{bin}(x)\}$$

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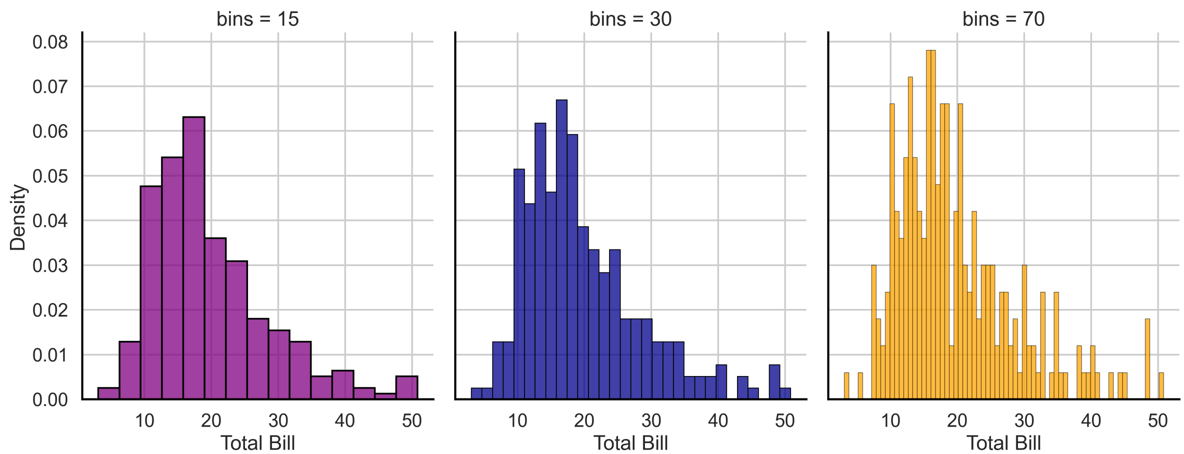
**Coding Example in Python**

Important parameters bins, stat, binwidth, binrange

```
import seaborn as sns
x = sns.load_dataset("tips")["total_bill"]
bins = [15, 30, 70]
for bin in bins:
  sns.histplot(x,bins=bin, stat='density', binwidth=None, binrange=None)
```

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### Coding Example in R

Important parameters: bins, aes(y = ..stat..), binwidth, xlim()

```
library(ggplot2)
library(readr)

tips <- read_csv("https://raw.githubusercontent.com/mwaskom/seaborn-data/master/tips.csv")
x <- tips$total_bill

bins <- c(15, 30, 70)
for (bin in bins) {
  p <- ggplot(data = data.frame(x = x), aes(x = x)) +
    geom_histogram(aes(y = ..density..), bins = bin, color = "black", fill = "skyblue")
  print(p)
}
```

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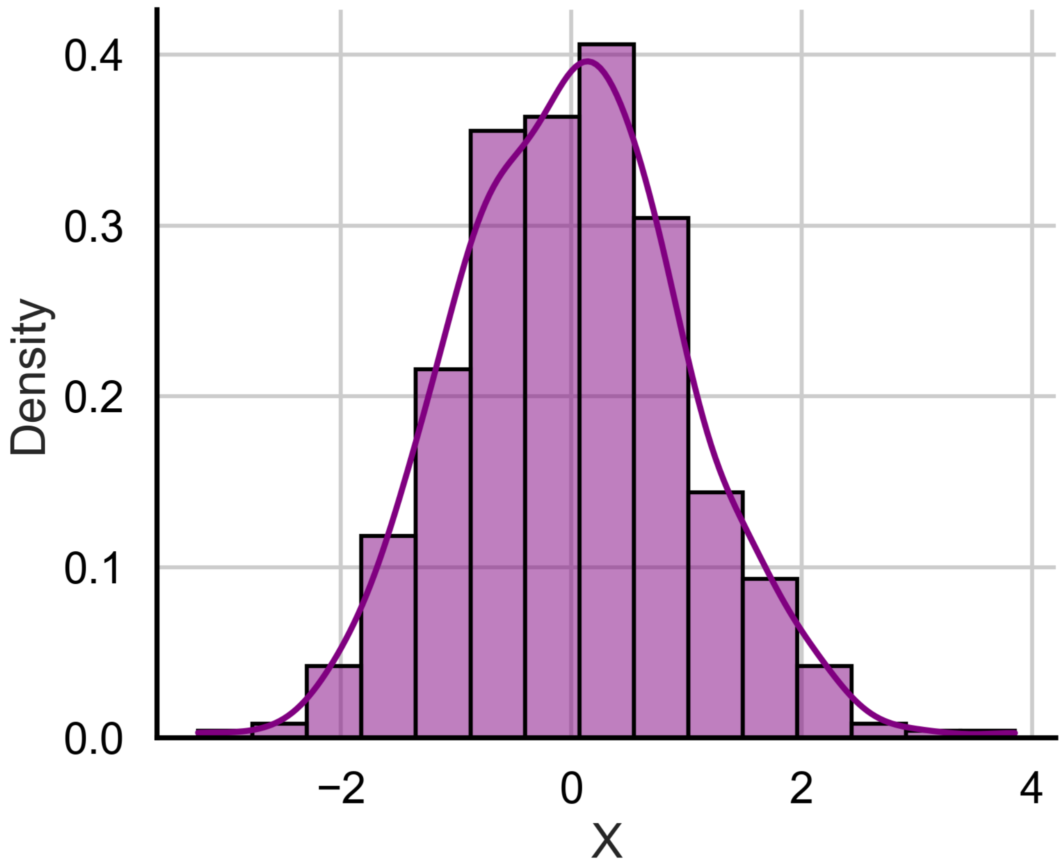
### Kernel Density Estimation

> **Key idea:** Smooth, nonparametric estimate of the probability density function

- Instead of counting frequencies in bins, KDE places a smooth kernel function (usually Gaussian) centered at each data point, and then averages them.

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**Mathematical formulation:**

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

- $N$ is the number of observations
- $w$ is the bandwidth (controls how much the influence of each observation expands)
- and, $K$ is the Kernel, a function that defines the shape and distribution of influence associated with each observation.

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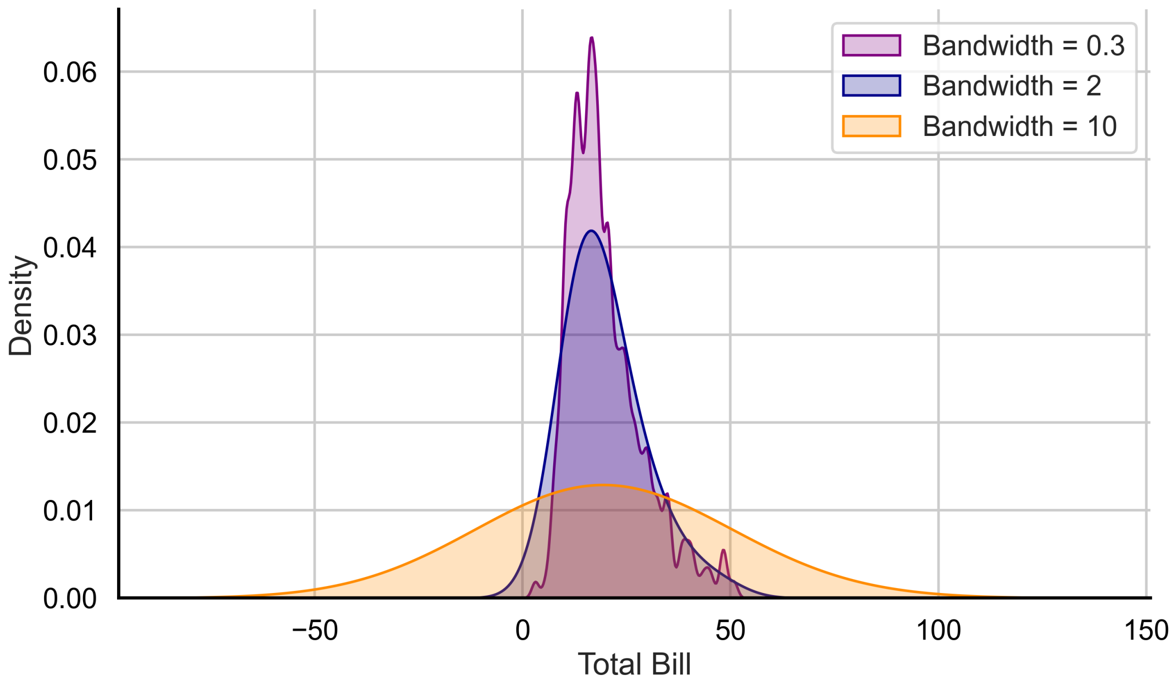
**Coding Example in Python**

```python
import seaborn as sns
x = sns.load_dataset('tips')['total_bill']
for bw in [0.3, 1, 2]:
    sns.kdeplot(x, bw_adjust=bw, label=f"Bandwidth = {bw}")
```




- Small $h$: very wiggly estimate (low bias, high variance).
- Large $h$: oversmoothed estimate (high bias, low variance).

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**Coding example in R**

```r
library(ggplot2)
library(readr)

tips <- read_csv("https://raw.githubusercontent.com/mwaskom/seaborn-data/master/tips.csv")
x <- tips$total_bill
```

```r
for (bw in c(0.3, 1, 2)) {
  print(ggplot(data.frame(x = x), aes(x = x)) +
          geom_density(adjust = bw, color = "blue") +
          ggtitle(paste("Bandwidth =", bw)))
}
```

## Thanks!