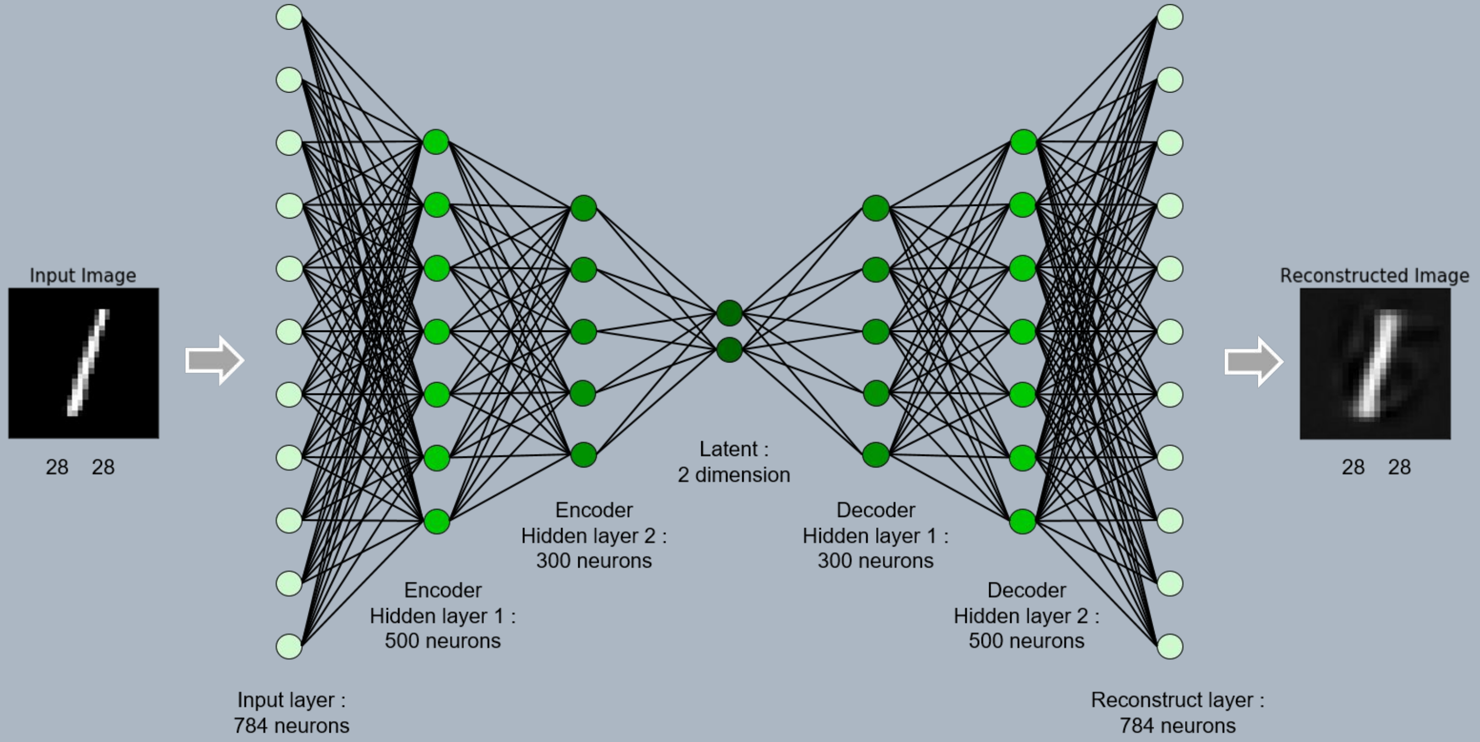
Variational AutoEncoders

Ágora @ TSC

10/06/2025

by Lucía Schmidt Santiago

AutoEncoders



Kaggle. Intro to Autoencoders

Variational AutoEncoders

Auto-Encoding Variational Bayes

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Abstract

How can we perform efficient inference and learning in directed probabilistic models, in the presence of continuous latent variables with intractable posterior distributions, and large datasets? We introduce a stochastic variational inference and learning algorithm that scales to large datasets and, under some mild differentiability conditions, even works in the intractable case. Our contributions is two-fold. First, we show that a reparameterization of the variational lower bound yields a lower bound estimator that can be straightforwardly optimized using standard stochastic gradient methods. Second, we show that for i.i.d. datasets with continuous latent variables per datapoint, posterior inference can be made especially efficient by fitting an approximate inference model (also called a recognition model) to the intractable posterior using the proposed lower bound estimator. Theoretical advantages are reflected in experimental results.

Kingma, D. P., & Welling, M. (2013, December). Auto-encoding variational bayes.

Generative Process

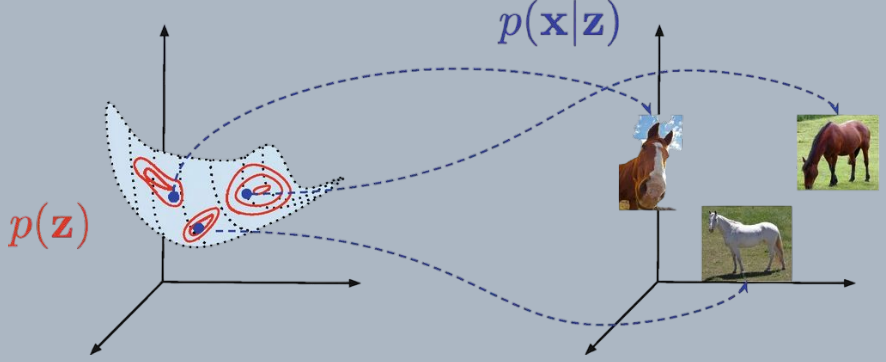
 $\mathbf{x} \in \mathcal{X}^D \quad$ (e.g., for images, $\mathcal{X} \in \{0, 1, \ldots, 255\}$)

 $\mathbf{z} \in \mathcal{Z}^M \quad$ (e.g., $\mathcal{Z} = \mathbb{R}$)

low-dimensional manifold

$$\mathbf{z} \sim p(\mathbf{z})$$

$$\mathbf{x} \sim p_\theta(\mathbf{x} \mid \mathbf{z})$$



Tomczak, J.M. (2022). Latent Variable Models. In: Deep Generative Modeling. Springer, Cham.

Generative Process

marginal likelihood / evidence

$$p(\mathbf{x}) = \int p(\mathbf{x} \mid \mathbf{z})\, p(\mathbf{z})\, dz$$

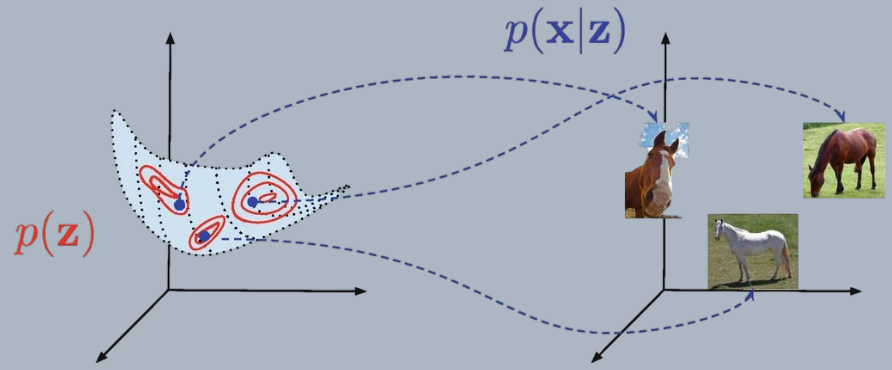
 $\mathbf{x} \in \mathcal{X}^D$

 $\mathbf{z} \in \mathcal{Z}^M$

low-dimensional manifold

$$\mathbf{z} \sim p(\mathbf{z})$$

$$\mathbf{x} \sim p_\theta(\mathbf{x} \mid \mathbf{z})$$



Tomczak, J.M. (2022). Latent Variable Models. In: Deep Generative Modeling. Springer, Cham.

Variational Inference

$$p(\mathbf{z} \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid \mathbf{z})\, p(\mathbf{z})}{p(\mathbf{x})}$$

$$\int p(\mathbf{x} \mid \mathbf{z})\, p(\mathbf{z})\, d\mathbf{z}$$

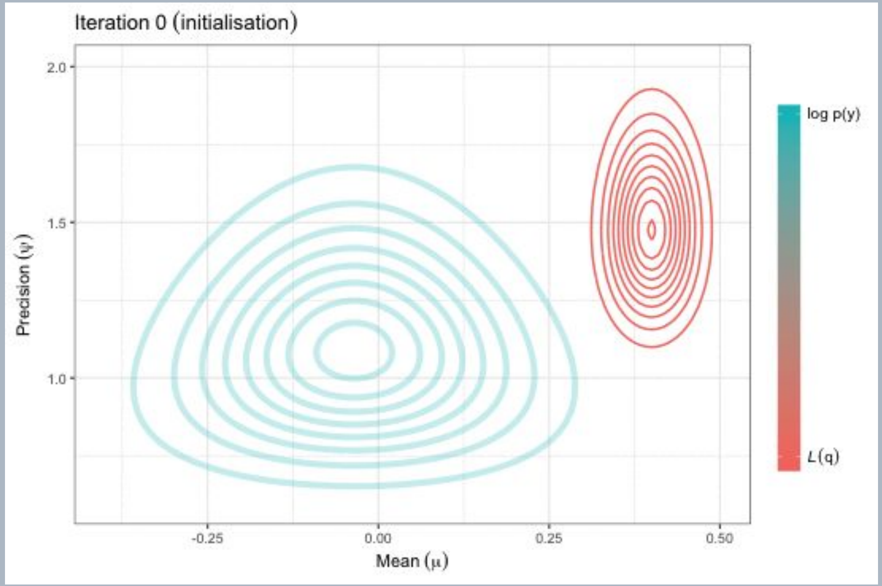
Variational Inference

Approximate the posterior distribution:

$$p(\mathbf{z} \mid \mathbf{x})$$

Use a **surrogate** function:

$$q(\mathbf{z} \mid \mathbf{x})$$



A Beginner's Guide to Variational Inference. Haziq Jamil

ELBO

Evidence Lower Bound

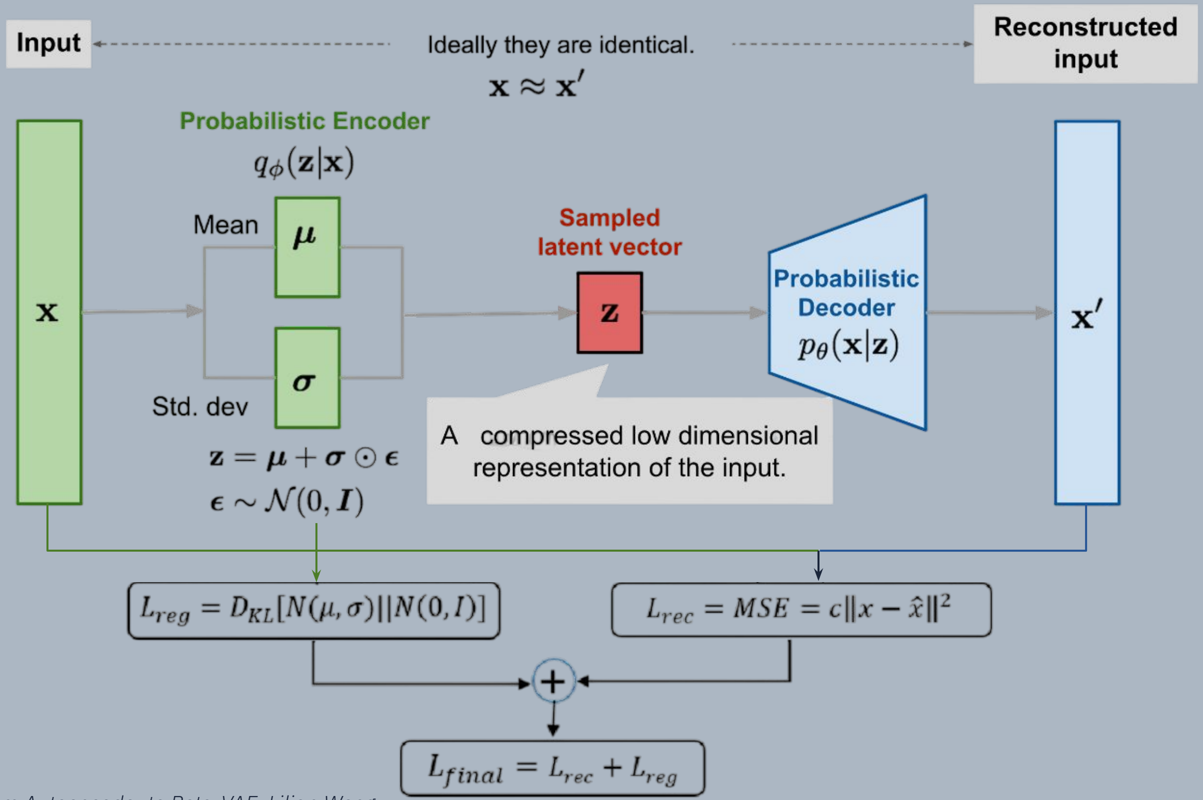
reconstruction error

true log-likelihood

Kullback-Leibler Divergence

$$\log p_\theta(\mathbf{x}) \geq \mathcal{L}(\theta, \phi; \mathbf{x}, \mathbf{z}) = \mathbb{E}q\phi(\mathbf{z} \mid \mathbf{x})[\log p_\theta(\mathbf{x} \mid \mathbf{z})] - D_{KL}(q_\phi(\mathbf{z} \mid \mathbf{x}) \parallel p(\mathbf{z}))$$

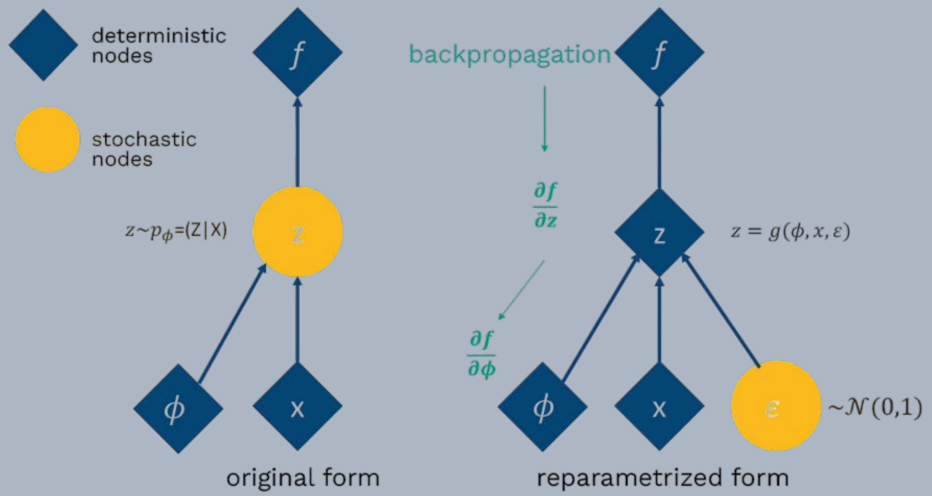
VAE Overview



From Autoencoder to Beta-VAE. Lilian Weng

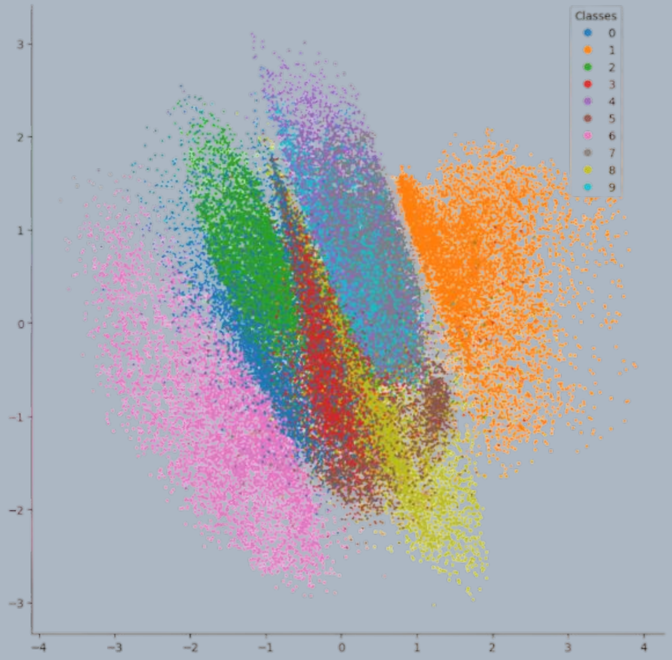
Reparametrization Trick

🌊 Possible errors can propagate through the network → backpropagation

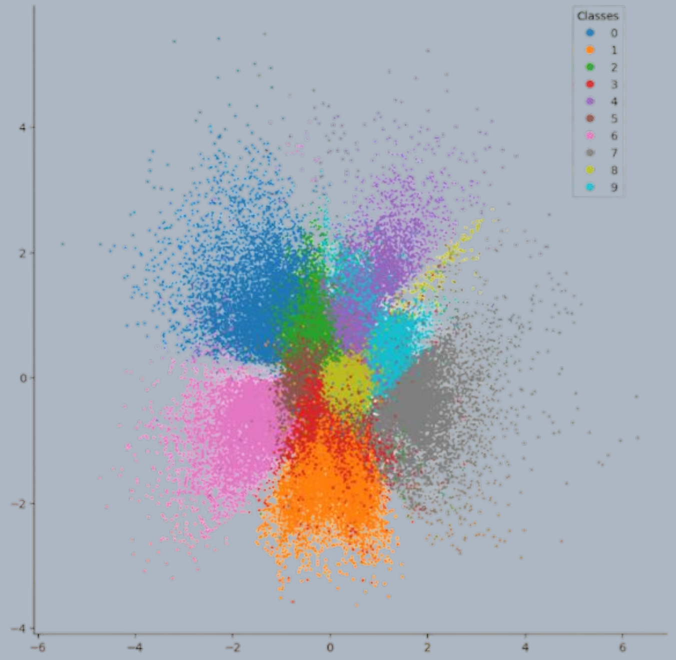


Generative neural models learn latent probability distributions. Jörg Kindermann. Lamarr Institute (2021)

Visualization of the latent space



(**a**) Latent space distribution by label for AE.



(**b**) Latent space distribution by label for VAE.

Ataeiasad, F. at al. (2024). Out-of-Distribution Detection with Memory-Augmented Variational Autoencoder. Mathematics, 12(19), 3153.