Information and Coding

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Contents

Principles

- Let $x^n = x_1 x_2 \ldots x_n$ be the sequence of values (scalars or vectors) produced by an information source until time n.
- Predictive coding is based on encoding sequence $r^n = r_1 r_2 \ldots r_n$, instead of the original sequence x^n, where

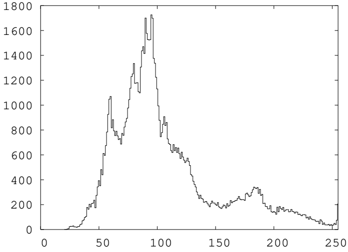
$$r_n = x_n - \hat{x}_n$$

  and

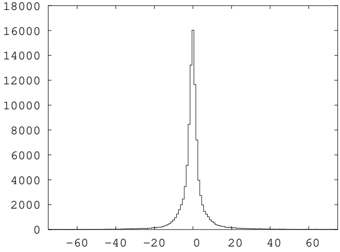
$$\hat{x}_n = p(x^{n-1}) = p(x_1 x_2 \ldots x_{n-1})$$

- The $\hat{x}_n$ are the estimates and the values of the sequence r^n are the residuals.
- Function $p()$ is the estimator or predictor.
- The aim of predictive coding is to have $H(r^n) < H(x^n)$.

Example



Original
$H = 7.26$ bits/symbol



Predictor 1 JPEG
$H = 4.49$ bits/symbol

Simple 1D prediction

- Simple polynomial predictors used in some audio encoders:

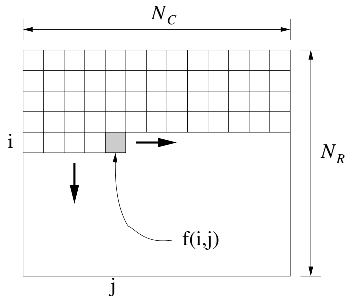
$$\begin{cases} \hat{x}_n^{(0)} = 0 \\ \hat{x}_n^{(1)} = x_{n-1} \\ \hat{x}_n^{(2)} = 2x_{n-1} - x_{n-2} \\ \hat{x}_n^{(3)} = 3x_{n-1} - 3x_{n-2} + x_{n-3} \end{cases}$$

and the corresponding residuals, computed efficiently:

$$\begin{cases} \hat{r}_n^{(0)} = x_n \\ \hat{r}_n^{(1)} = r_n^{(0)} - r_{n-1}^{(0)} \\ \hat{r}_n^{(2)} = r_n^{(1)} - r_{n-1}^{(1)} \\ \hat{r}_n^{(3)} = r_n^{(2)} - r_{n-1}^{(2)} \end{cases}$$

Predictive image coding techniques

- Typically, images are encoded from left to right, top to bottom, i.e., in raster-scan order:



- In this case, the sequence x^n is obtained by concatenating the first $\lfloor n/N_c \rfloor$ image rows, plus the $n \bmod N_c$ pixels from row number $\lfloor n/N_c \rfloor + 1$.

Predictive image coding techniques

- Other approaches use hierarchical decompositions (or multi-resolution).
- This is the case of the HINT method (Hierarchical INTerpolation):

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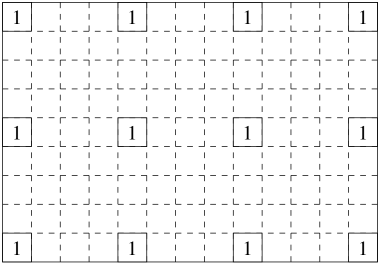

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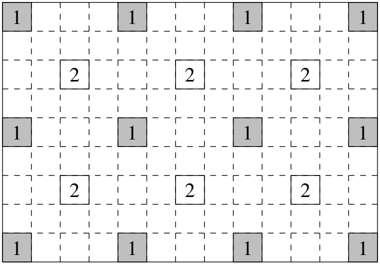

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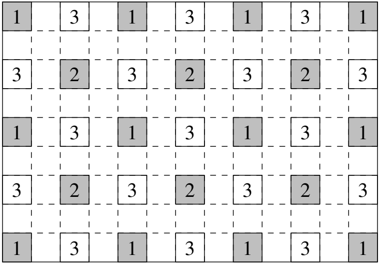

Predictive image coding techniques

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- This is the case of the HINT method (Hierarchical INTerpolation):

1		3		1		3		1		3		1
	4		4		4		4		4		4	
3		2		3		2		3		2		3
	4		4		4		4		4		4	
1		3		1		3		1		3		1
	4		4		4		4		4		4	
3		2		3		2		3		2		3
	4		4		4		4		4		4	
1		3		1		3		1		3		1

Predictive image coding techniques

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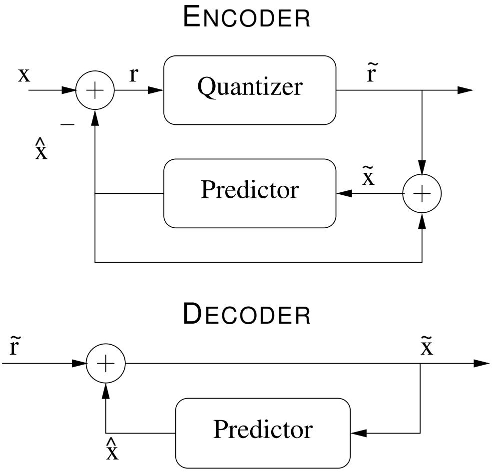
1	5	3	5	1	5	3	5	1	5	3	5	1
5	4	5	4	5	4	5	4	5	4	5	4	5
3	5	2	5	3	5	2	5	3	5	2	5	3
5	4	5	4	5	4	5	4	5	4	5	4	5
1	5	3	5	1	5	3	5	1	5	3	5	1
5	4	5	4	5	4	5	4	5	4	5	4	5
3	5	2	5	3	5	2	5	3	5	2	5	3
5	4	5	4	5	4	5	4	5	4	5	4	5
1	5	3	5	1	5	3	5	1	5	3	5	1

Predictors

- Good predictors are fundamental in predictive coding.
- For efficient encoding, the estimated values should be as close as possible to the real values, i.e., the r_k values should be small.
- The decoder must be able to generate the same sequence, $\hat{x}^n$, of estimated values.
- In other words, the predictor cannot introduce any error during encoding / decoding.
- Therefore, the predictor must be causal, and, in lossy coding, the predictor at the encoder must use the reconstructed values, $\tilde{x}^{n-1}$, instead of the original values, x^{n-1}.

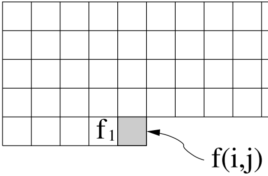
Predictors

Example:



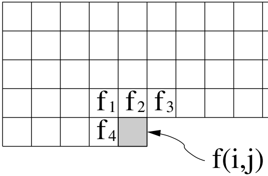
Predictors

- Generally, the complexity of the predictor depends on two aspects:
  - The number of values used for calculating the estimates (the order of the predictor).
  - The spatial (or temporal) configuration of these values.
- Consider the example of a spatial predictor of order 1, where the estimated value is given by the immediately preceding value:



Predictors

- This type of predictor can be easily extended to higher orders, using the last *k* processed pixels of the image.
- However, for orders higher than 3 or 4, the efficiency does not increase significantly.
- This happens because images are 2D signals, not 1D sequences of data.
- Therefore, generally, the spatial configurations used for predictive image coding have a 2D shape:

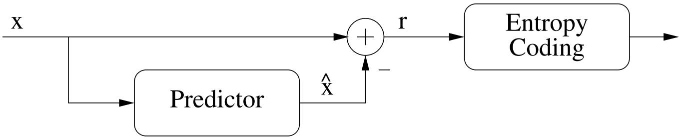


Lossless predictive coding

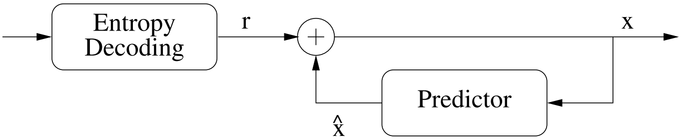
- One of the main advantages of predictive coding is allowing a simple design of lossless encoders.
- In fact, most of the lossless encoders for audio and image rely on predictive coding techniques.
- However, for lossless coding, there is an additional constraint regarding the predictor: the estimates generated must be platform independent.
- Generally, this constraint implies that the predictor can use only integer arithmetic.

Lossless predictive coding

ENCODER

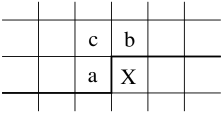


DECODER



Linear prediction: the lossless mode of JPEG

- The lossless mode of JPEG (ISO/IEC 10918-1, ITU-T T.81, 1992) provides seven linear predictors:



Mode	Predictor
1	a
2	b
3	c
4	$a+b-c$
5	$a+(b-c)/2$
6	$b+(a-c)/2$
7	$(a+b)/2$

- Generally, the performance of the several predictors may vary considerably from image to image.
- If encoding time is not a problem, then all of them can be tested and the one with the best compression rate chosen.

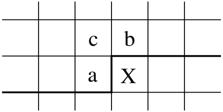
Linear prediction: the lossless mode of JPEG

Example:

Predictor	1	2	3	4	5	6	7
Entropy	4.49	4.21	4.74	4.17	4.16	**4.04**	4.10

The nonlinear predictor of JPEG-LS

- JPEG-LS (ISO/IEC 14495-1, ITU-T T.87, 1999) uses a predictor based on the same spatial configuration as that of JPEG:



- However, instead of a linear predictor, it uses the nonlinear predictor

$$\hat{x} = \begin{cases} \min(a,b) & \text{if} \quad c \geq \max(a,b) \\ \max(a,b) & \text{if} \quad c \leq \min(a,b) \\ a+b-c & \text{otherwise} \end{cases}$$

- Note that the linear part of this predictor ($a+b-c$) is the same as predictor number 4 of JPEG.

The nonlinear predictor of JPEG-LS

Example:

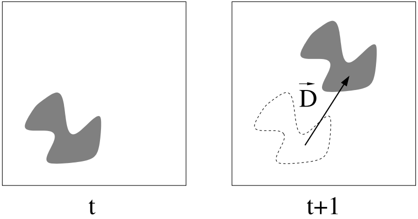
(a)

(b)

Predictor	1	2	3	4	5	6	7	JLS
Entropy (a)	4.49	4.21	4.74	4.18	4.16	4.04	4.10	**3.98**
Entropy (b)	5.60	5.05	5.82	5.19	5.23	4.97	5.15	**4.93**

Motion compensation

- Typically, the differences between one frame and the previous frame of a video sequence are due to the motion of the several elements of the scene.
- Exceptions occur when there are scene changes, zoom-in / zoom-out operations and camera translation.



- To exploit this redundancy, it is frequent to use temporal prediction (interframe compression), which relies on motion compensation.

Motion compensation

- Conditional replenishment video coding:
  - Finds zones in the video frame where there were changes with respect to the previous frame.
  - Only those zones are encoded.
  - This technique does not use motion compensation. It just performs a detection of temporal activity.
- Video coding based on motion compensation involves the following steps:
  - Estimation of the motion vectors.
  - Compensation, i.e., temporal prediction.
  - Encoding of the motion vectors.
  - Encoding of the prediction residuals.

Motion compensation

- There are a large number of techniques for motion detection, but one of them is clearly the most common approach for video coding.
- For each frame block (for example, of $N \times N$ pixels), it seeks the position where it minimizes some measure in relation to the previous frame (the reference frame).
- Note that this approach tries to find the position that minimizes a measure of interest, which might not correspond to the true motion in the scene...

Motion compensation

- Typically, we want to minimize some measure $C(i,j)$, such as

$$C(i,j) = \sum_{r=1}^{N} \sum_{c=1}^{N} d\Big(g(r,c,t) - g(i+r,j+c,t+1)\Big)$$

  where $d(\cdot)$ is, for example, $(\cdot)^2$ or $|\cdot|$.

- Due to complexity constraints, searching is limited to a neighborhood of $(N+2\Delta) \times (N+2\Delta)$ pixels around the block, i.e., $-\Delta \leq i,j \leq \Delta$.
- If exhaustive search is used, it is guaranteed to find the minimum of $C(i,j)$...
- This approach is generally computationally too demanding, hence other sub-optimal techniques have been proposed.

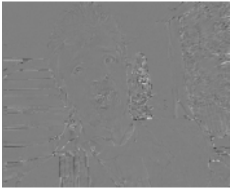
Motion compensation

- Example:

Frame 200

Frame 201

Direct difference
$H = 5.23$ bpp



Motion compensation
$H = 4.38$ bpp

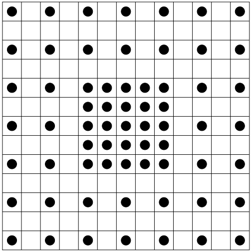
Motion compensation

Δ	Block	Entropy			
		Residual	Nominal MV	MV	Total
3	4	4.29	0.50	0.31	4.60
7	4	4.03	1.00	0.39	4.42
15	4	3.91	2.00	0.47	4.38
3	8	4.63	0.12	0.07	4.70
7	8	4.43	0.25	0.09	4.52
15	8	4.36	0.50	0.10	4.46
3	16	4.84	0.03	0.01	4.85
7	16	4.76	0.06	0.02	4.78
15	16	4.72	0.12	0.02	4.74

Motion compensation

- Several of the sup-optimal approaches for finding the best reference block rely on spatial sub-sampling.
- For example, considering that the most probable zone for finding the reference block is in the near neighborhood of the block, then we may use the following scheme:

Total: 169 blocks

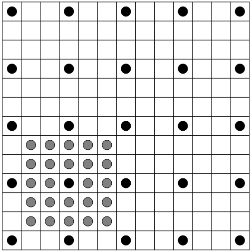


Sub-optimal: 65 blocks

Motion compensation

- If we consider that after finding a reasonably good reference block it is probable that others better than itself can be found in the near neighborhood, then we can use a greedy search:

Total: 169 blocks



Sub-optimal: 49 blocks

- A number of other variants of local search have been proposed…

Motion compensation

- For example, the logarithmic search: