Properties: p, v, T, h, s, u

$v = \frac{V}{m}$ m^3/kg ; $h \triangleq u + pv$; $ds = \frac{\delta q}{T}$

Specific heat at constant pressure. $C_p \triangleq \left(\frac{\partial h}{\partial T}\right)_p$

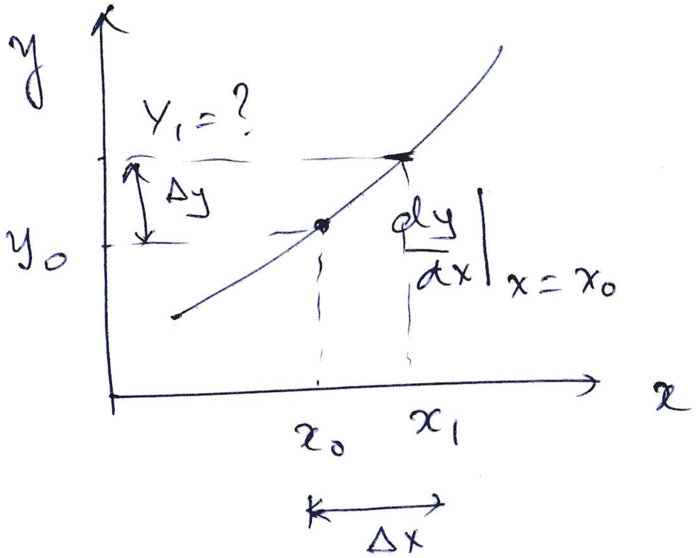
Specific heat at constant volume $C_v \triangleq \left(\frac{\partial u}{\partial T}\right)_v$

Gibb's Phase Rule : $\boxed{F = C - P + 2}$

d.o.f (# variables) # components # phases

1) $C = 1$, $P = 1$ $F = 1 - 1 + 2 = 2$

Air given p, T, find v $pv = RT \Rightarrow v = \frac{RT}{p}$

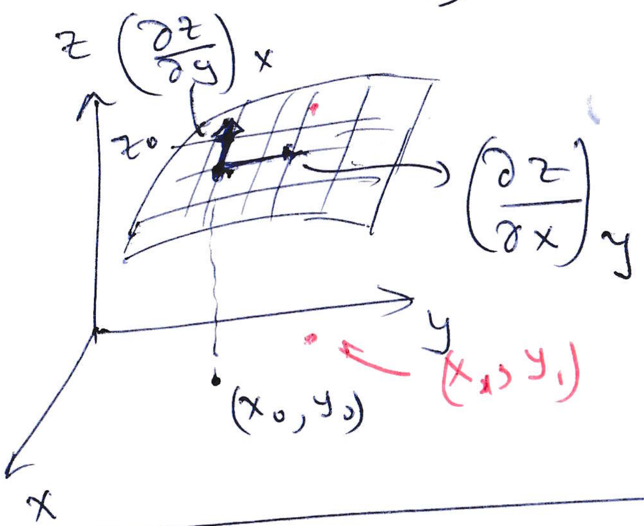


$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{y_1-y_0}{x_1-x_0} \Rightarrow$$

$$y_1 = y_0 + \left(\left.\frac{dy}{dx}\right|_{x=x_0}\right)(x_1-x_0)$$

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{\Delta y}{\Delta x} \Rightarrow \Delta y = \left(\frac{dy}{dx}\right)_{x=x_0} \Delta x$$

$z = z(x, y)$.



$$\Delta z = \left(\frac{\partial z}{\partial x}\right)_y \Delta x + \left(\frac{\partial z}{\partial y}\right)_x \Delta y$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$h = h(p, T)$$

$$dh = \left(\frac{\partial h}{\partial p}\right)_T dp + \underbrace{\left(\frac{\partial h}{\partial T}\right)_p}_{c_p} dT \qquad \boxed{c_p \triangleq \left(\frac{\partial h}{\partial T}\right)_p}$$

For ideal gas $\left(\frac{\partial h}{\partial p}\right)_T = 0$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT \Rightarrow dh = \left(\frac{dh}{dT}\right) dT$$

$$\boxed{dh = c_p\, dT}$$

$$u = u(v, T)$$

$$du = \left(\frac{\partial u}{\partial v}\right)_T dv + \underbrace{\left(\frac{\partial u}{\partial T}\right)_v}_{C_v} dT$$

$$\boxed{C_v \triangleq \left(\frac{\partial u}{\partial T}\right)_v}$$

<u>Ideal gas:</u> $\left(\frac{\partial u}{\partial v}\right)_T = 0$ $\quad C_v = \frac{du}{dT} \Rightarrow \boxed{du = C_v dT}$

<u>Ideal gas</u>: $\quad dh = C_p dT \quad ; \quad du = C_v dT$

$$h = u + pv$$

$$dh = du + \cancel{p\,dv + v}\, d(pv)$$

$$= du + d(RT)$$

$$= du + R\,dT$$

$$C_p dT = C_v dT + R\,dT \Rightarrow \boxed{C_p - C_v = R} \longrightarrow (*)$$

Define adiabatic index $\gamma \triangleq \frac{C_p}{C_v}$

$\gamma = \frac{C_p}{C_v} \Rightarrow C_p = \gamma C_v$. Subs in $(*)$ $\Rightarrow \gamma C_v - C_v = R$

$$\Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}}$$

Since $C_p = \gamma C_v \Rightarrow \boxed{C_p = \frac{\gamma}{\gamma - 1} R}$

First law of thermodynamics

$\Delta u = Q - W$ $\qquad du = \delta q - \delta w \qquad dU = \delta Q - \delta W$

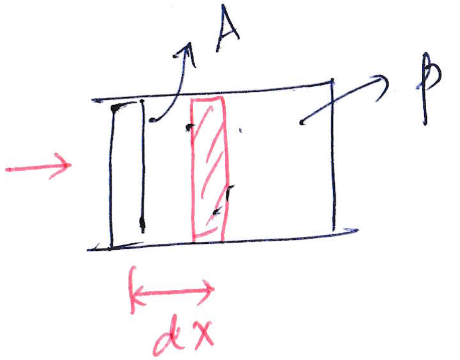
$\Delta u \uparrow$ when ~~Q in~~ $Q > 0$ or $W < 0$

heat supplied to system — work done on the system

$\Delta u \downarrow$ when $Q < 0$ or $W > 0$

heat is rejected from system — work done by system

$\delta w = F\,dx = p\,\underbrace{A\,dx} = p\,dV \implies \delta w = p\,dV$



Entropy

$$ds = \frac{\delta q}{T}$$

Since $\delta q - \delta w = du \Rightarrow \delta q = du + \delta w$

$$= du + p\,dv$$

$$ds = \frac{du + p\,dv}{T} = \frac{du}{T} + \frac{p\,dv}{T}$$

Ideal gas $pv = RT \Rightarrow \frac{p}{T} = \frac{R}{v}$

$$ds = \frac{du}{T} + \frac{R}{v}\,dv$$

$du = C_v\,dT$
(ideal gas)

$$\int ds = \int \frac{C_v\,dT}{T} + \int \frac{R}{v}\,dv$$

$$\Delta S = C_v \int \frac{dT}{T} + R \int \frac{dv}{v}$$

(assuming C_v is constant)

$$\boxed{\Delta s = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}}$$

$h = u + \cancel{pd}\ pv$

$dh = \underbrace{du + pdv}_{\delta q} + vdp \Rightarrow \delta q = dh - vdp$

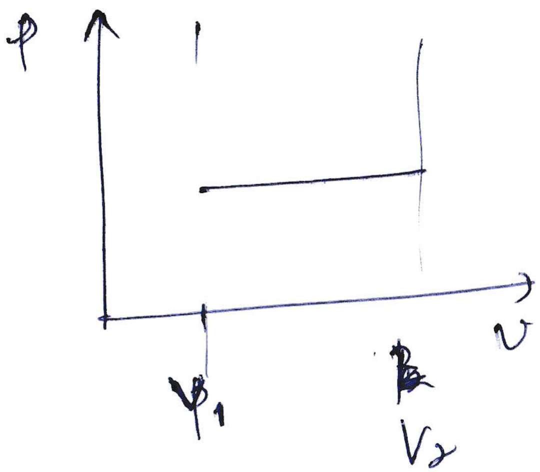
$ds = \frac{\delta q}{T} = \frac{dh - vdp}{T}$

Ideal gas: $dh = C_p dT$; $pv = RT \Rightarrow \frac{v}{T} = \frac{R}{p}$

$\int ds = C_p \int \frac{dT}{T} - \int \frac{R}{p} dp.$

$$\boxed{\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}}$$

Processes: 1) **Constant pressure process** (Ideal gas)



$$p_1 v_1 = R T_1$$
$$p_2 v_2 = R T_2$$

Since $p_1 = p_2 \Rightarrow \dfrac{v_1}{T_1} = \dfrac{v_2}{T_2}$ or $v \sim T$

Work done:

$$\delta w = p\,dv$$
$$W = \int \delta w = \int_{v_1}^{v_2} p\,dv$$
$$= p \int_{v_1}^{v_2} dv$$
$$= p(v_2 - v_1)$$

Change in internal energy $\Delta u = C_v \Delta T$

Heat transfer: $Q - W = \Delta u$ or $Q = \Delta u + W$

$$= C_v \Delta T + p(v_2 - v_1)$$
$$= C_v \Delta T + (p v_2 - p v_1)$$
$$= C_v \Delta T + R(T_2) - R T_1$$
$$= C_v \Delta T + R \Delta T$$
$$= (C_v + R) \Delta T$$
$$= C_p \Delta T$$
$$= \Delta h$$

Change in entropy

$$\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Since $P_1 = P_2$

$$\boxed{\Delta S = C_p \ln \frac{T_2}{T_1}} \leftarrow \text{constant pressure}$$