Properties: P, V, T, h, u, s

Enthalpy : $h = u + pv$

Entropy : $ds = \frac{\delta q_{rev}}{T}$

$C_p \triangleq \left(\frac{\partial h}{\partial T}\right)_p$; $C_v \triangleq \left(\frac{\partial u}{\partial T}\right)_v$ $\boxed{\gamma \triangleq C_p / C_v}$

Adiabatic index

Ideal gas: $C_p = \frac{dh}{dT} \Rightarrow dh = C_p\, dT$

$C_v = \frac{du}{dT} \Rightarrow du = C_v\, dT$

$\boxed{C_p - C_v = R} \Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}}$; $C_p = \frac{\gamma R}{\gamma - 1}$

/

$$\Delta s = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$= C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Isothermal process: $T = c$

1) Relation between p & v.

$$p_1 v_1 = mRT_1$$
$$p_2 v_2 = mRT_2$$

Since $T_1 = T_2 \Rightarrow p_1 v_1 = p_2 v_2$ or $pv = c$

Ideal gas: $\Delta u = 0$; $\Delta h = 0$

Work done: $w = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{c}{v} dv$

$$= c \ln\left(\frac{v_2}{v_1}\right)$$

$$= p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) \left(= p_2 v_2 \ln \frac{v_2}{v_1} = p_1 v_1 \ln \frac{p_1}{p_2} = mRT \ln \frac{v_2}{v_1} \ldots \right)$$

Heat transfer: $q - w = \cancel{\Delta u}^{\,0} \Rightarrow q = w$

$$= p_1 v_1 \ln \frac{v_2}{v_1}$$

Entropy change: $\Delta s = R \ln \frac{v_2}{v_1}$

$$= -R \ln \frac{p_2}{p_1}$$

Reversible adiabatic: $q = 0$

1st law: $q - w = \Delta u \Rightarrow -w = \Delta u$

In differential form: $-\delta w = du$

$$-p\,dv = c_v\,dT$$

Ideal gas: $pv = RT \Rightarrow p = \frac{RT}{v}$

$$-\frac{RT}{v}dv = c_v\,dT$$

$$\frac{c_v}{R}\frac{dT}{T} + \frac{dv}{v} = 0$$

$$\frac{c_v}{R}\int_{T_1}^{T_2}\frac{dT}{T} + \int_{v_1}^{v_2}\frac{dv}{v} = 0$$

$$\frac{c_v}{R}\ln\frac{T_2}{T_1} + \ln\frac{v_2}{v_1} = 0$$

$$\ln\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}} + \ln\frac{v_2}{v_1} = 0$$

$$\ln\left[\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}\left(\frac{v_2}{v_1}\right)\right] = 0$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}\left(\frac{v_2}{v_1}\right) = 1$$

$$\Rightarrow T_2^{1/r-1}\,v_2 = T_1^{\frac{1}{r-1}}\,v_1$$

(Since $c_v = \frac{R}{r-1} \Rightarrow \frac{c_v}{R} = \frac{1}{r-1}$)

$$T_2 v_2^{r-1} = T_1 v_1^{r-1}$$

$$\boxed{T v^{r-1} = c}$$

Since $pv \sim T \Rightarrow pv \cdot v^{r-1} = c$

$$\Rightarrow \boxed{pv^r = c}$$

Eliminating v using $pv \sim T$ (ie $v \sim \frac{T}{p}$)

$$p\left(\frac{T}{p}\right)^r = c \Rightarrow p\frac{T^r}{p^r} = c$$

$$\Rightarrow \cancel{p}\frac{T^r}{p^{r-1}} = c$$

ie $\boxed{p \sim T^{\frac{r}{r-1}} \quad \text{or} \quad T \sim p^{\frac{r-1}{r}}}$

$$\delta w = +p\,dv$$

Since $pv^r = c \Rightarrow p = \frac{c}{v^r}$

$$\delta w = \frac{c\,dv}{v^r}$$

$$= c\,v^{-r}\,dv$$

$$w = c\int_{v_1}^{v_2} v^{-r}\,dv = c\,\frac{v^{-r+1}}{-r+1}\Bigg|_{v_1}^{v_2}$$

$$= \frac{c\,v_2^{-r+1} - c\,v_1^{-r+1}}{-r+1}$$

$$= \frac{c\,v_1^{-r}\cdot v_1 - c\,v_2^{-r}\cdot v_2}{r-1}$$

$$= \frac{p_1 v_1 - p_2 v_2}{r-1}$$

$$= \frac{mRT_1 - mRT_2}{r-1} = mR\,\frac{(T_1 - T_2)}{r-1}$$

$$\Delta s = \frac{\cancelto{0}{\delta q}}{T}$$

$$\Delta s = 0$$

(isentropic)

$$\Delta u = \cancelto{0}{q} - w = -\frac{p_1 v_1 - p_2 v_2}{r-1}$$

~~$\Delta h =$~~ $dh = d(u + pv)$

$$= \underbrace{du + p\,dv}_{\cancelto{0}{\delta q}} + v\,dp$$

$$\Rightarrow dh = v\,dp$$

$$\Delta h = \int v\,dp$$

$$pv^r = c$$

$$v = \frac{c}{p^{1/r}}$$

Polytropic process

$$pv^n = c$$

$$TV^{n-1} = c \quad ; \quad P \sim T^{\frac{n}{n-1}} \quad ; \quad T \sim P^{\frac{n-1}{n}}$$

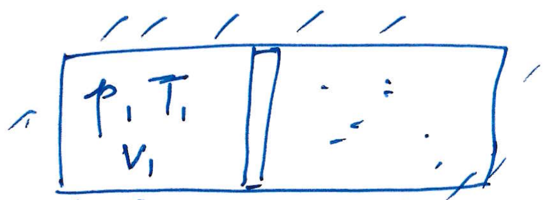
$$\cancel{q} \quad w = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$\Delta u = C_v \Delta T \qquad \Delta h = C_p \Delta T$$

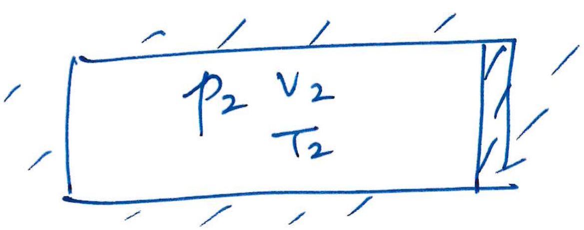
$$q = \Delta u + w$$

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = \cancel{C_p}\ C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Free expansion



State ①



$$q = 0$$

$$w = 0$$

$$\Delta u = 0 \Rightarrow \Delta T = 0 \Rightarrow \Delta h = 0$$

$$\left.\begin{aligned} P_1 V_1 &= mRT_1 \\ P_2 V_2 &= mRT_2 \end{aligned}\right\} \Rightarrow P_1 V_1 = P_2 V_2$$

$$\Delta s = -R \ln \frac{P_2}{P_1} = R \ln \frac{V_2}{V_1}$$

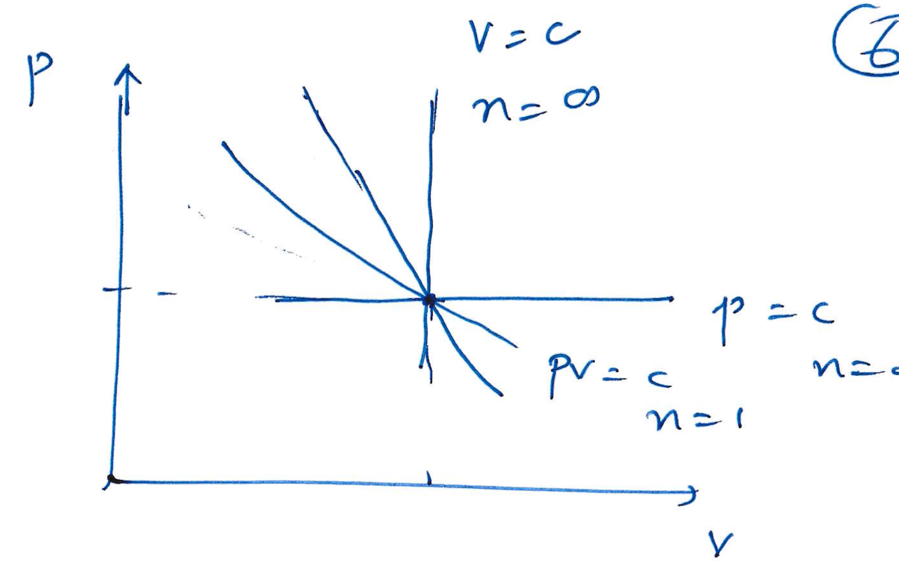
Constant pressure $p = c$

Constant volume $V = c$

Isothermal $pV = c$

Rev. adiabatic $pV^{\gamma} = c$

Polytropic $pV^{n} = c$



$p = c \Rightarrow pV^{0} = c$

$p^{0} V = c$ $\qquad pV^{n} = c \Rightarrow p^{1/n} V = c$

when $n \to \infty$ $\quad 1/n \to 0$

$\Rightarrow V = c$