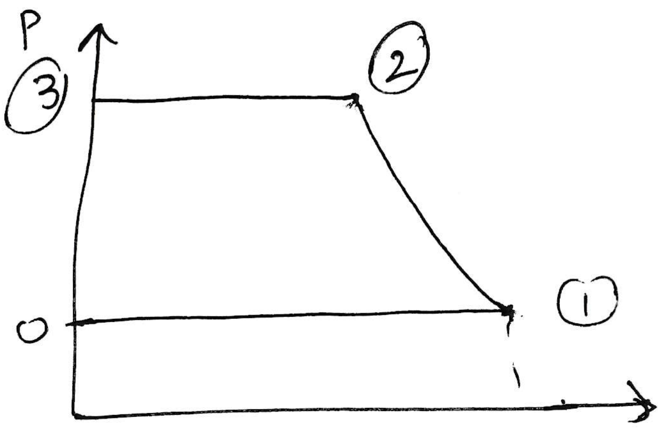
Reciprocating compressors

Without clearance

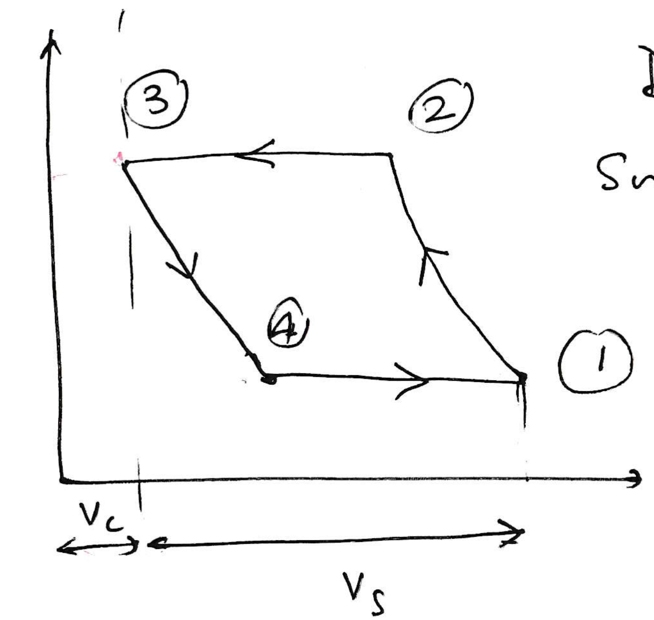


$$W_{net} = \frac{n}{n-1} \underbrace{m}_{=1} R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

polytropic index

$pv^n = c$

With clearance



$$W_{net} = \frac{n}{n-1} (m_1 - m_4) R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

Define volumetric efficiency $\eta_{vol} \triangleq \frac{V_1 - V_4}{V_s}$

Since $V_1 = V_c + V_s \Rightarrow \eta_{vol} = \frac{V_c + V_s - V_4}{V_s}$

$$= 1 + \frac{V_c}{V_s} - \frac{V_4}{V_s}$$

Since $3 \to 4$ is a polytropic process,

$p_3 V_3^n = p_4 V_4^n \Rightarrow V_4 = \left(\frac{p_3}{p_4}\right)^{1/n} V_3$ $= V_c$

pressure ratio r_p

$$\eta_{vol} = 1 + \frac{V_c}{V_s} - r^{1/n} \frac{V_c}{V_s} = 1 - K(r_p^{1/n} - 1)$$

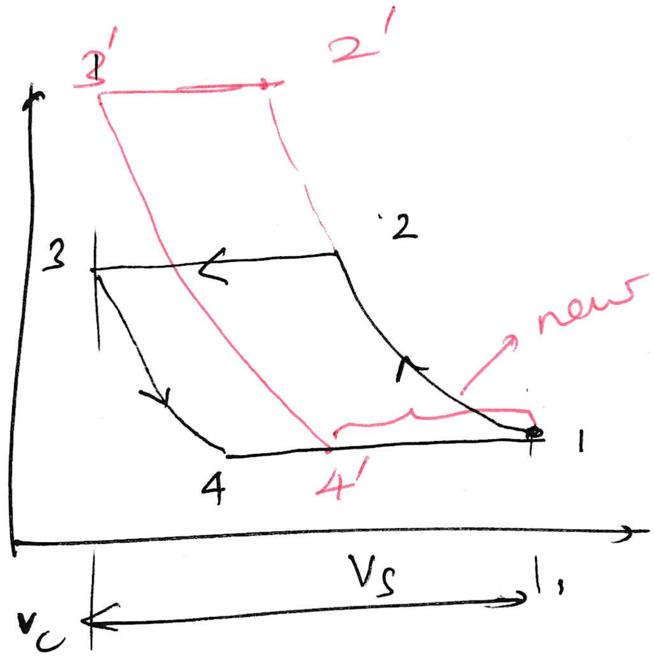
where $K = \frac{V_c}{V_s}$ (clearance ratio)

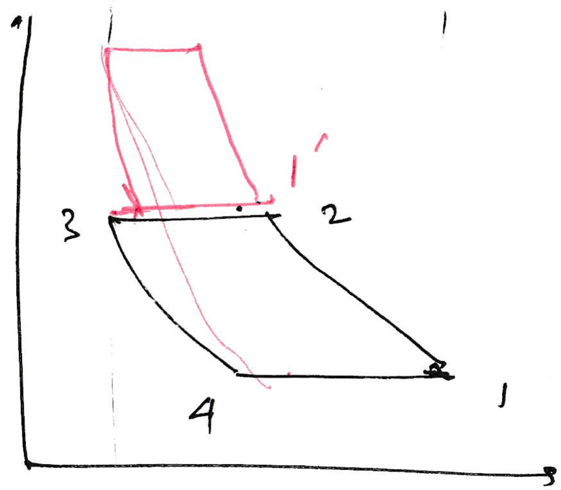
$K = 5\%$ $\quad n = 1.2$

$r_p = 8$

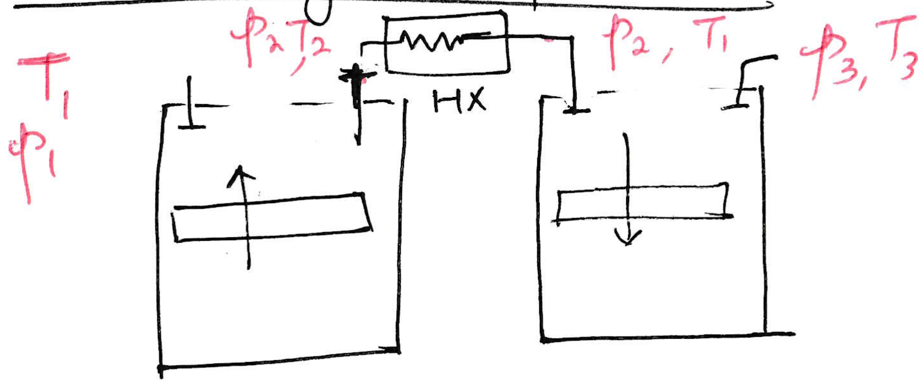
$$\eta_{vol} = 1 - 0.05\left(8^{1/1.2} - 1\right) = 76.7\%$$

$r_p = 16$ $\quad \eta_{vol} = 1 - 0.05\left(16^{1/1.2} - 1\right) = 54.6\%.$





Multi-stage compression



Perfect intercooling :-
Temperature at exit of intercooler is same as temperature at inlet to the 1st stage

$$W = \underbrace{\frac{n}{n-1}(m_1 - m_4) R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}_{1^{st}\ \text{stage}} + \underbrace{\frac{n}{n-1}(m_1 - m_4) R T_1 \left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]}_{2^{nd}\ \text{stage}}$$

$$= \frac{n}{n-1}(m_1 - m_4) R T_1 \underbrace{\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]}_{\phi}$$

$$\left(-1 + \frac{1}{n} - 1\right.$$

$$\frac{dW}{dp_2} = 0 \Rightarrow \frac{d\phi}{dp_2} = 0 \Rightarrow \phi = \frac{p_2^{\left(\frac{n-1}{n}\right)}}{p_1^{(n-1)/n}} + p_3^{\frac{n-1}{n}} \cdot p_2^{-\frac{n-1}{n}} - 2$$

$$\frac{d\phi}{dp_2} = \frac{1}{p_1^{\frac{n-1}{n}}} \cancel{\left(\frac{n-1}{n}\right)} p_2^{-1/n} + p_3^{\frac{n-1}{n}} \left(-\cancel{\left(\frac{n-1}{n}\right)}\right) p_2^{-2+\frac{1}{n}} = 0$$

$$\frac{p_2^{-1/n}}{p_1^{\frac{n-1}{n}}} = p_3^{\frac{n-1}{n}} \cdot p_2^{-2+\frac{1}{n}}$$

$$\frac{p_2^{-\frac{1}{n}}}{p_2^{-2+1/n}} = (p_1 p_3)^{\frac{n-1}{n}}$$

$$p_2^{-\frac{1}{n}+2-\frac{1}{n}} = (p_1 p_3)^{\frac{n-1}{n}} \qquad 2-\frac{2}{n} = 2\left(\frac{n-1}{n}\right)$$

$$p_2^{2\left(\frac{n-1}{n}\right)} = (p_1 p_3)^{\frac{n-1}{n}}$$

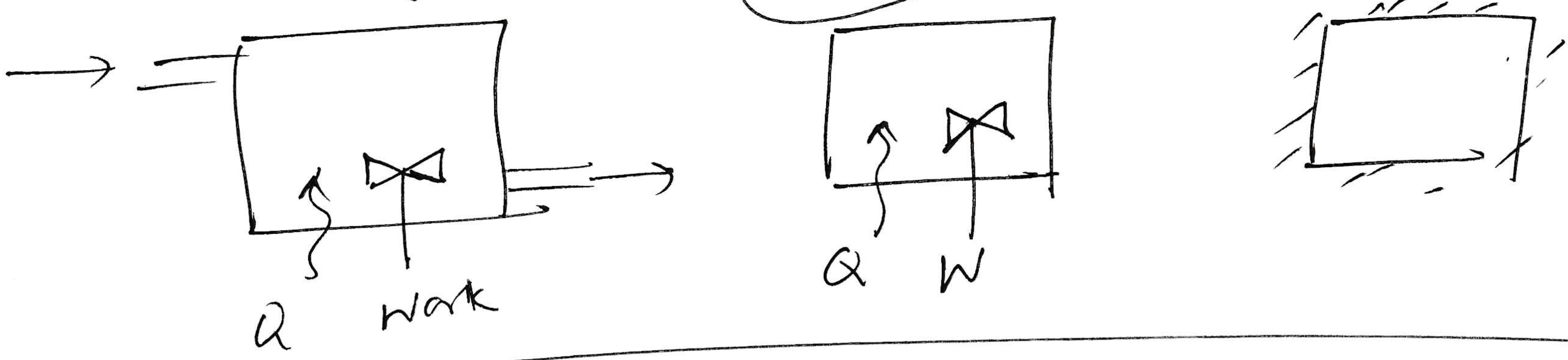
Raise both sides to $\frac{n}{n-1}$

$$p_2^2 = p_1 p_3 \Rightarrow p_2 = \sqrt{p_1 p_3} \leftarrow$$

p_2 is geometric mean of p_1 & p_3

$$\frac{p_2}{p_1} = \frac{p_3}{p_2}$$

Open System   Closed   Isolated



X – Mass, energy, entropy, linear momentum, angular momentum

Open System:

X is some quantity

$$\dot{X}_{in} - \dot{X}_{out} + \dot{X}_{gen} + \dot{X}_{transf} = \frac{dX_{cv}}{dt}$$

Mass: $\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{gen} + \dot{m}_{transf} = \frac{dm_{cv}}{dt}$

Energy: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} + (\dot{Q} - \dot{W}) = \frac{dE_{cv}}{dt}$

$\cancel{\dot{E}_{in}}$ $\dot{E} = \dot{m}\left(h_i + \frac{V_i^2}{2} + gz\right)$