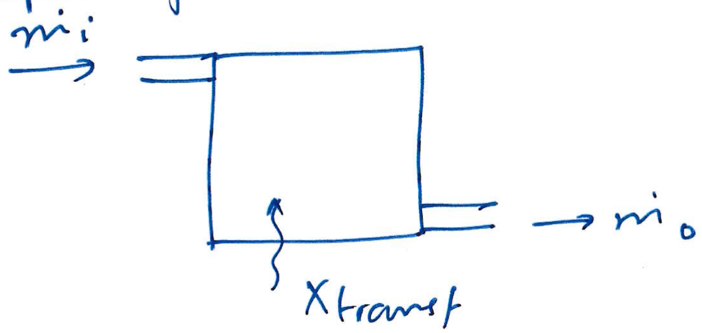
## Open Systems




$$\dot{X}_{in} - \dot{X}_{out} + \dot{X}_{transf} + \dot{X}_{gen} = \frac{\partial}{\partial t} X_{cv}$$

**Mass Balance:** $\dot{m}_i - \dot{m}_o + \dot{m}_{transf} + \dot{m}_{gen} = \frac{\partial m_{cv}}{\partial t}$

If no mass transfer at boundaries & no mass generation.

$$\dot{m}_i - \dot{m}_o = \frac{\partial m_{cv}}{\partial t}$$

Steady State: $\dot{m}_i = \dot{m}_o$

**Energy:** $\dot{m}_i\left(h_i + \frac{V_i^2}{2} + g z_i\right) - \dot{m}_o\left(h_o + \frac{V_o^2}{2} + g z_o\right) + \dot{Q} - \dot{W} = \frac{\partial E_{cv}}{\partial t}$

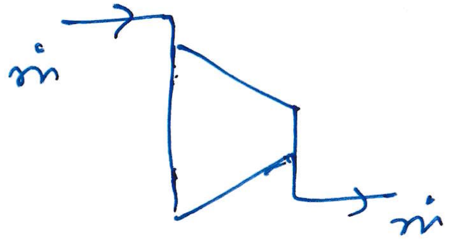
(no energy generation) $\qquad E_{cv} = \left(h + \frac{V^2}{2} + g z\right) m_{cv}$

**Entropy.** $\dot{m}_i s_i - \dot{m}_o s_o + \int \frac{\delta Q_{rev}}{T} + \underbrace{\dot{\Sigma}}_{\text{entropy generated}} = \frac{\partial S_{cv}}{\partial t}$

**Linear momentum:** $\dot{m}_i v_i - \dot{m}_o v_o + \Sigma F = \frac{\partial (m_{cv} v)}{\partial t}$

**Angular momentum:** $(r_i \times \dot{m}_i v_i) - (r_o \times \dot{m}_o v_o) + \Sigma T = \frac{\partial}{\partial t}(r \times m_{cv} v)$.

## Compressor:



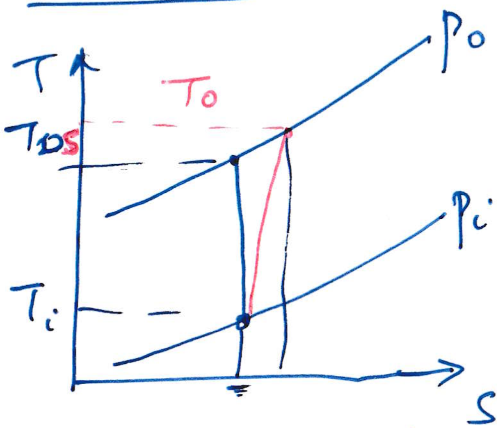

$\dot{m}(h_i) - \dot{m} h_o + \dot{Q} - \dot{W} = 0$

Ideal compressor is reversible adiabatic. $\dot{Q} = 0$

$\dot{W} = \dot{m}(h_i - h_o)$

$w = \dfrac{\dot{W}}{\dot{m}} = h_i - h_o$

Ideal gas: $w = C_p(T_i - T_o)$




$$\left(\frac{T_{os}}{T_i}\right) = \left(\frac{p_o}{p_i}\right)^{\frac{r-1}{r}}$$

Define isentropic efficiency

$$\eta_{iso} = \frac{T_{os} - T_i}{T_o - T_i}$$

$$\Delta s = C_p \ln \frac{T_o}{T_i} - R \ln \frac{p_o}{p_i}$$

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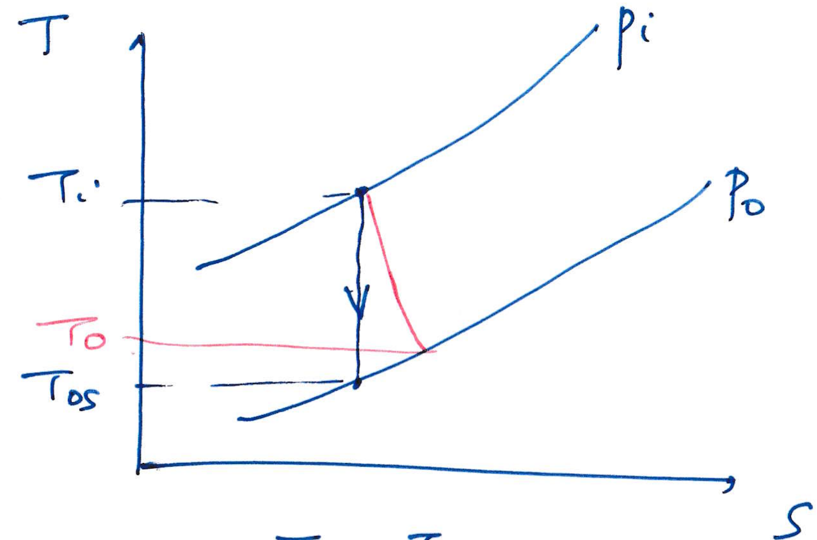
## Turbine:

$\dot{m} h_i - \dot{m} h_o + \dot{Q} - \dot{W} = 0$

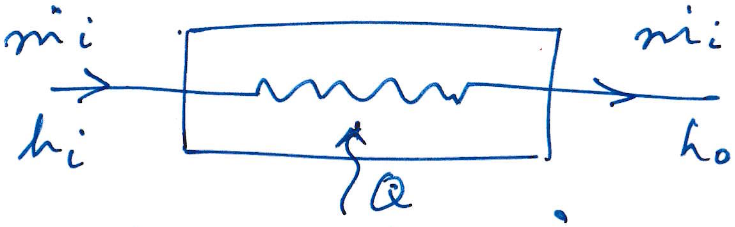
If $\dot{Q} = 0 \Rightarrow \dot{W} = \dot{m}(h_i - h_o)$

$w = h_i - h_o.$




$$\eta_{iso} = \frac{T_i - T_o}{T_i - T_{os}}$$

# Heat Exchanger




$$\dot{m}_i h_i - \dot{m}_i h_o + \dot{Q} = 0$$

$$\dot{Q} = \dot{m}_i (h_o - h_i)$$

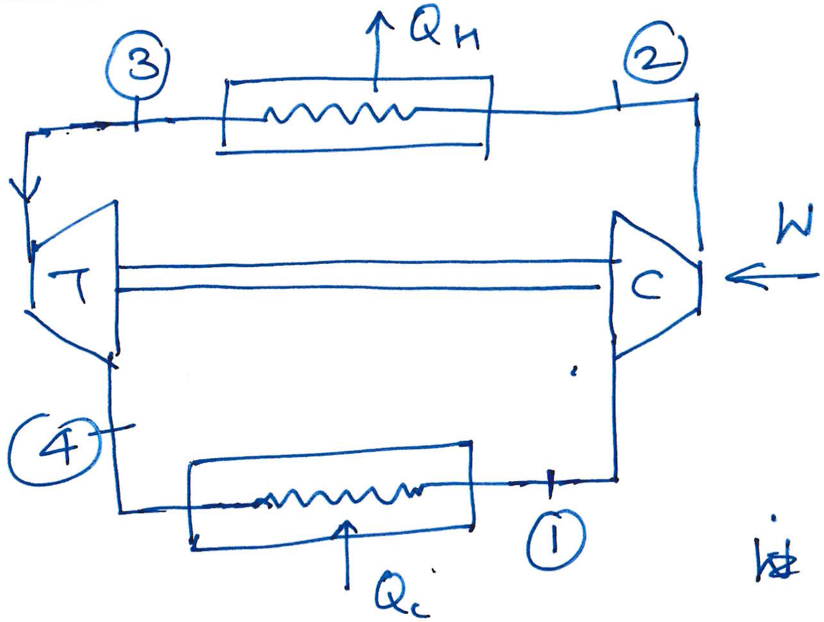
$$q = \frac{\dot{Q}}{\dot{m}_i} = h_o - h_i$$

Ideal gas: $$q = C_p \cancel{(h_o - h_i)}$$
$$= C_p (T_o - T_i)$$

$$\Delta s = C_p \ln \frac{T_o}{T_i} - \cancelto{0}{R \ln \frac{P_o}{P_i}} \quad (P_i = P_o)$$

$$= C_p \ln \frac{T_o}{T_i}$$

# Reversed Brayton Refrigeration Cycle




1–2 : Isentropic compression

2–3 Constant pressure heat rejection

3 → 4 : Isentropic expansion

4 → 1 : Constant pressure heat addition.

$$w_{comp} = h_2 - h_1$$

$$w_{turbine} = h_3 - h_4$$

$$w_{net} = w_{comp} - w_{turbine}$$
$$= (h_2 - h_1) - (h_3 - h_4)$$

$$q_c = h_1 - h_4$$

Coefficient of performance $COP = \dfrac{q_c}{w_{net}} = \dfrac{h_1 - h_4}{(h_2 - h_1) - (h_3 - h_4)}$

$$\boxed{\gamma = 1.4}$$

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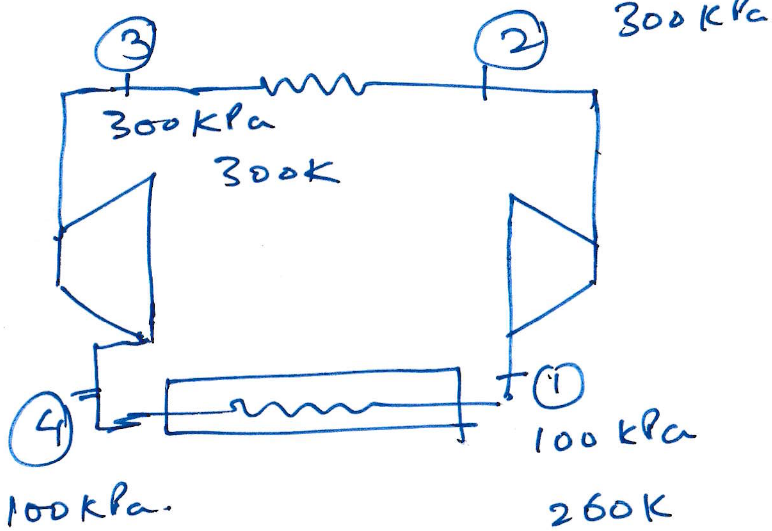
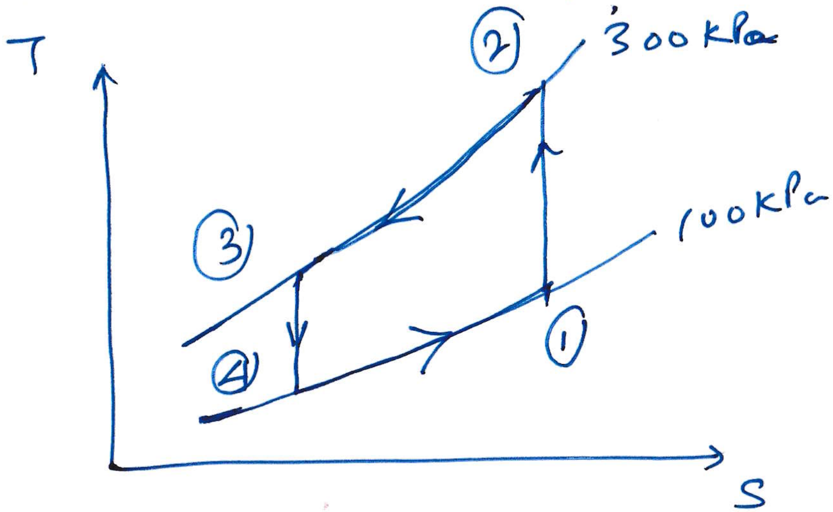
Air enters the compressor of a Brayton refrigeration cycle at 100 kPA, 260K and is compressed adiabatically to 300 kPa. Air enters the turbine at 300 kPa, 300 K and expands adiabatically to 100 kPa. For the cycle, determine the net work per unit mass of air flow, assuming compressor and turbine isentropic efficiencies are both 100%. Also determine the coefficient of performance. If the cooling load is 200 kW, determine the air flow required and the power required to be supplied to the compressor.




$$w_{comp} = h_2 - h_1 = C_p(T_2 - T_1)$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_2}{260} = \left(\frac{300}{100}\right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 355.9\,K$$

$$w_{comp} = (1.005)(355.9 - 260) = 96.4\ kJ/kg$$

$$w_{turbine} = h_3 - h_4 = C_p(T_3 - T_4)$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \cancel{T_3}\ \frac{300}{T_4} = \left(\frac{300}{100}\right)^{\frac{0.4}{1.4}}$$

$$\Rightarrow T_4 = 219.2\,K$$

$$w_{turbine} = (1.005)(300 - 219.2) = 81.2\ kJ/kg$$

$$w_{net} = w_{comp} - w_{turb} = 96.4 - 81.2 = 15.2\ \frac{kJ}{kg}$$

$$COP = \frac{q_c}{w_{net}} = \frac{41}{15.2} = 2.7$$

$$q_c = h_1 - h_4 = C_p(T_1 - T_4) = (1.005)(260 - 219.2) = 41\ kJ/kg.$$

$$\dot{Q} = 200 = \dot{m}(h_1 - h_4) = \dot{m}(41) \Rightarrow \dot{m} = 4.87\ \cancel{\frac{kg}{kg}}\ \frac{kg}{s}$$

$$P = \dot{m}w = (4.87)(15.2) = 74.1\ kW$$