Psychrometrics

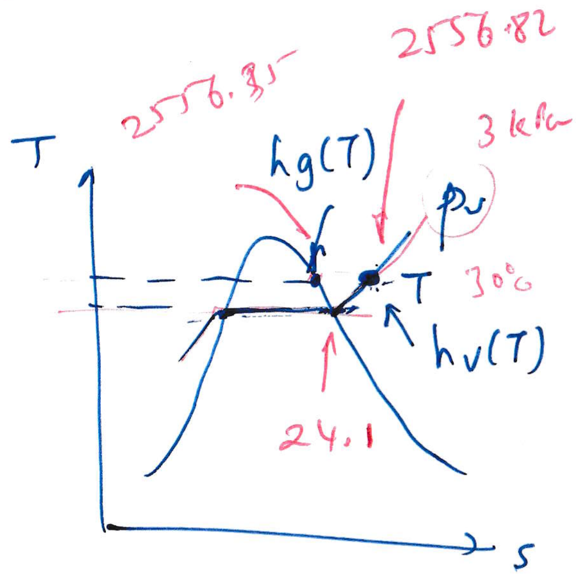
Humidity ratio $\omega \triangleq \dfrac{m_v}{m_a} = 0.622 \dfrac{p_v}{p - p_v}$

p = total pressure

p_v = ~~vap~~ partial pressure of vapor

Relative humidity $\phi \triangleq \dfrac{m_v}{m_{v,sat}} = \dfrac{p_v}{p_{v,sat}}$ ← from tables.

mass of vapor at saturation at the same temperature



Enthalpy

m_a

m_v

$$
\begin{aligned}
H &= H_a + H_v \\
&= m_a h_a + m_v h_v \\
&= m_a \left[h_a + \frac{m_v}{m_a} h_v \right] \\
&= m_a \left[h_a + \omega h_v \right]
\end{aligned}
$$

Specific enthalpy $h \triangleq \dfrac{H}{m_a} = h_a + \omega h_v$ $\quad [h] = kJ/kg\ d.a.$

$$
\begin{aligned}
h(T) &= h_a(T) + \omega h_v(T) \\
&= C_{pa} T + \omega h_v(T)
\end{aligned}
$$

ie $h_v(T) \approx h_g(T)$

$$\boxed{h(T) \approx C_{pa} T + \omega h_g(T)}$$

$$h(T) = C_{pa} T + \omega \left[h_f(T_{dp}) + h_{fg}(T_{dp}) + C_{ps}(T - T_{dp}) \right] \quad 2$$

enthalpy at dew point

latent heat at dew point

specific heat of steam. $\approx 1.9\ kJ/kg\,K$

$$= C_{pa} T + \omega \Big[\underbrace{\big(\overbrace{h_f(T_{dp}) + h_{fg}(T_{dp})}^{h_g(T_{dp})} - C_{ps} T_{dp} \big)}_{\approx 2500\ kJ/kg} + C_{ps} T \Big]$$

$$= C_{pa} T + \omega [2500 + 1.9\, T]$$

$$\boxed{h(T) = 1.005\, T + \omega [2500 + 1.9\, T]}$$

Allows to determine T given $h(T)$

Specific volume

$$v = \frac{\forall}{m_a}$$

Total mass

$$\boxed{\begin{aligned} m &= m_a + m_v \\ &= m_a(1+\omega) \end{aligned}} \longrightarrow m = \frac{p_a \forall}{R_a T} + \frac{p_v}{R_v}\frac{\forall}{T}$$

$$m = m_a + m_v$$

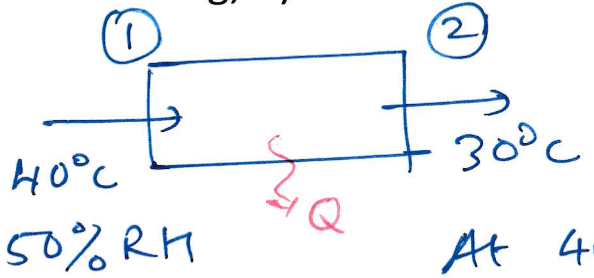
$$= m_a\left[1 + \frac{m_v}{m_a}\right]$$

$$= m_a(1+\omega)$$

$$m_a(1+\omega) = \left(\frac{p_a}{R_a} + \frac{p_v}{R_v}\right)\frac{\forall}{T}$$

$$\boxed{v = \frac{\forall}{m_a} = \frac{T(1+\omega)}{\frac{p_a}{R_a} + \frac{p_v}{R_v}}}$$

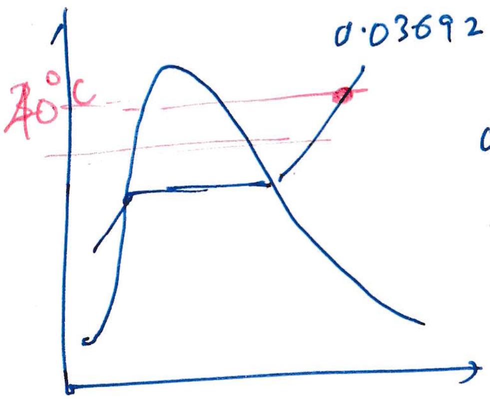
Air at 40C and 50% RH is cooled to 30C. Determine a) humidity ratio before and after cooling, b) relative humidity after cooling, c) Heat removed in the cooling process



$$\omega_1 = 0.622 \frac{P_{v_1}}{P - P_{v_1}} \quad ; \quad \phi_1 = 0.5 = \frac{P_{v_1}}{P_{v_1,sat}}$$

At $40°C$, $P_{v_1,sat} = 0.07384$ bar

$$\phi_1 = 0.5 = \frac{P_{v_1}}{0.07384} \Rightarrow P_{v_1} = 0.03692 \text{ bar.}$$



$$\omega_1 = (0.622) \frac{0.03692}{1.013025 - 0.03692} = 0.02353 \text{ kg / kg d.a}$$

(23.5 g / kg.da)

Since $T_{dewpoint}$ is between $27°C$ & $28°C$, no condensation occurs.

$$\Rightarrow \omega_2 = \omega_1$$

$$P_{v,sat} @ 30°C = 0.04246 \Rightarrow \phi_2 = \frac{0.03692}{0.04246} = 86.95\%$$

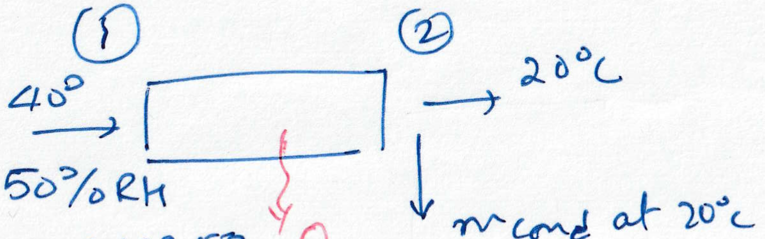
Heat removed $Q = h_1 - h_2$

$$h_1 = C_{pa} T_1 + \omega_1 h_v(T_1) = (1.005)(40) + (0.02353)(2574.3) = 101.78 \text{ kJ/kg da}$$

$$h_2 = C_{pa} T_2 + \omega_2 h_v(T_2) = (1.005)(30) + (0.02353)(2556.3) = 90.3 \text{ kJ/kg d.a.}$$

$$Q = 101.78 - 90.3 = 11.48 \text{ kJ / kg.d.a.}$$

Air at 40C and 50% RH is cooled to 20C. Determine a) humidity ratio before and after cooling, b) relative humidity after cooling c) Heat removed during cooling.

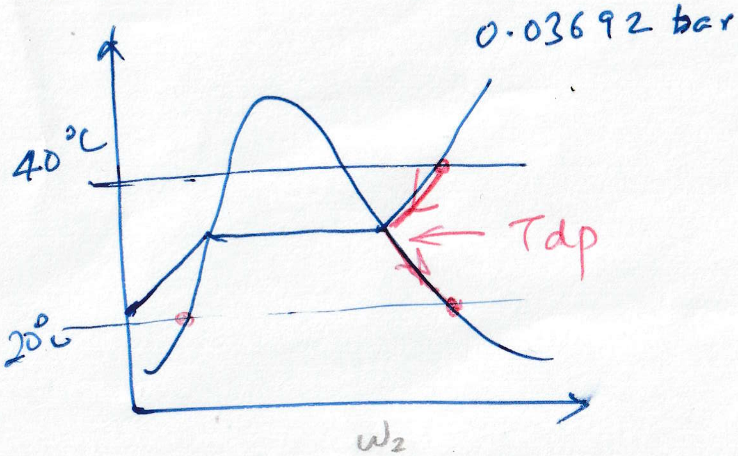


$\omega_1 = 0.02353$

$h_1 = 101.78$

At 20°C $\left.\begin{array}{l} P_{v,sat} = 0.02339 \text{ bar} \\ \phi = 100\% \end{array}\right\} \Rightarrow P_v = 0.02339$ bar

$$\omega_2 = (0.622)\frac{0.02339}{1.013025 - 0.02339} = 0.0147 \frac{\text{kg}}{\text{kg d.a.}}$$



Condensate $= \omega_1 - \omega_2$

$= 0.02353 - 0.0147$

$= 8.83 \times 10^{-3}$ kg/kg d.a.

$$Q = \dot{m}_a h_1 - \dot{m}_a h_2 - \dot{m}_{cond} C_p T$$

$\dot{m}_{v_1} = \dot{m}_{cond} + \dot{m}_{v_2}$

$$= \dot{m}_a\left[h_1 - h_2 - \frac{\dot{m}_{cond}}{\dot{m}_a} C_p T\right]$$

$$= \dot{m}_a\left[h_1 - h_2 - \frac{\dot{m}_{v_1} - \dot{m}_{v_2}}{\dot{m}_a} C_p T\right]$$

$$= \dot{m}_a\left[h_1 - h_2 - \underbrace{(\omega_1 - \omega_2) C_p T}\right]$$

$= (8.83 \times 10^{-3})(4.186)(20)$

$= 0.739$ kJ/kg d.a.

$$h_2 = C_{p_a} T_2 + \omega_2 h_v(T_2)$$

$\Rightarrow = (1.005)(20) + (0.0147)(2538.1)$

$= 57.41$ kJ/kg d.a.

$\Rightarrow Q = 101.78 - 57.41 - 0.739 = \boxed{43.632 \text{ kJ/kg d.a.}}$