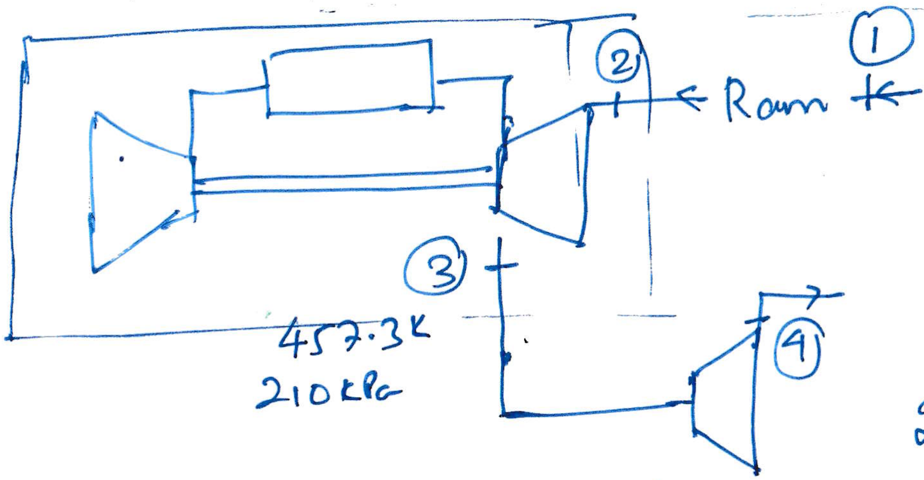
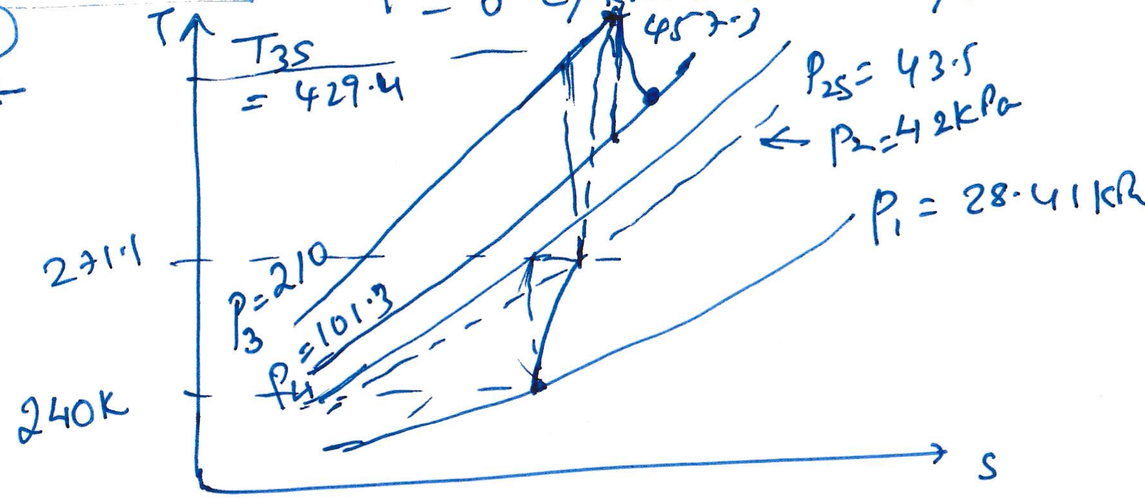
A plane uses a simple air-refrigeration for cabin air-conditioning. It cruises at 900 kmph at an altitude of 10,000 m. The compression ratio in the main compressor is 5. The cabin pressure is to be 101.3 kPa. Determine the temperature of air entering the cabin from the refrigeration system. Assume lapse rate is $6^o\ C/km$ and ground temperature is 27C. Isentropic efficiency of the compressor and turbine are 85%. Assume ram efficiency of 90%.

$\Gamma = 6°C/km = 0.006°C/m$





$V_1 = 900 \text{ kmph} = 250 \text{ m/s}$

$T_1 = T_0 - \Gamma z = 27 - (6)(10) = -33 = 240\text{K}$

$\frac{P_1}{P_0} = \left(1 - \frac{\Gamma z}{T_0}\right)^{g/R\Gamma}$ $\frac{r}{r-1}$ $= \left(1 - \cancel{6\times 10}\ \frac{6°C/km \times 10km}{300}\right)^{\frac{9.81}{(287)(0.006)}} = 0.2805$

$P_1 = (0.2805)\, P_0 = (0.2805)(101.3) = 28.41 \text{ kPa}$

$\cancel{s_1 - s_2 = C_p \log\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (1005)\ln\left(\frac{240}{}\right.}$

$\boxed{1-2}$ $T_2 = T_1 + \frac{V^2}{2c_p}$

$= 240 + \frac{250^2}{(2)(1005)} = 271.1\text{K}$

$\left(\frac{P_{2s}}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{r}{r-1}} = \frac{P_{2s}}{28.41} = \left(\frac{271.1}{240}\right)^{\frac{1.4}{0.4}} = 43.5 \text{ kPa}$

$\frac{P_2 - P_1}{P_{2s} - P_1} = 0.9 \Rightarrow \frac{P_2 - 28.4}{43.5 - 28.4} = 0.9 \Rightarrow P_2 = 42 \text{ kPa}$

$$\Delta s_{1-2s} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_{2s}}{p_1} = (1.005) \ln \frac{271.1}{240} - (0.287) \ln \frac{43.5}{28.41} = 1.9 \times 10^{-4} \approx 0$$

$$\Delta s_{1-2} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = (1.005) \ln \frac{271.1}{240} - (0.287) \ln \frac{42}{28.41} = 0.0103 \text{ kJ/K}.$$

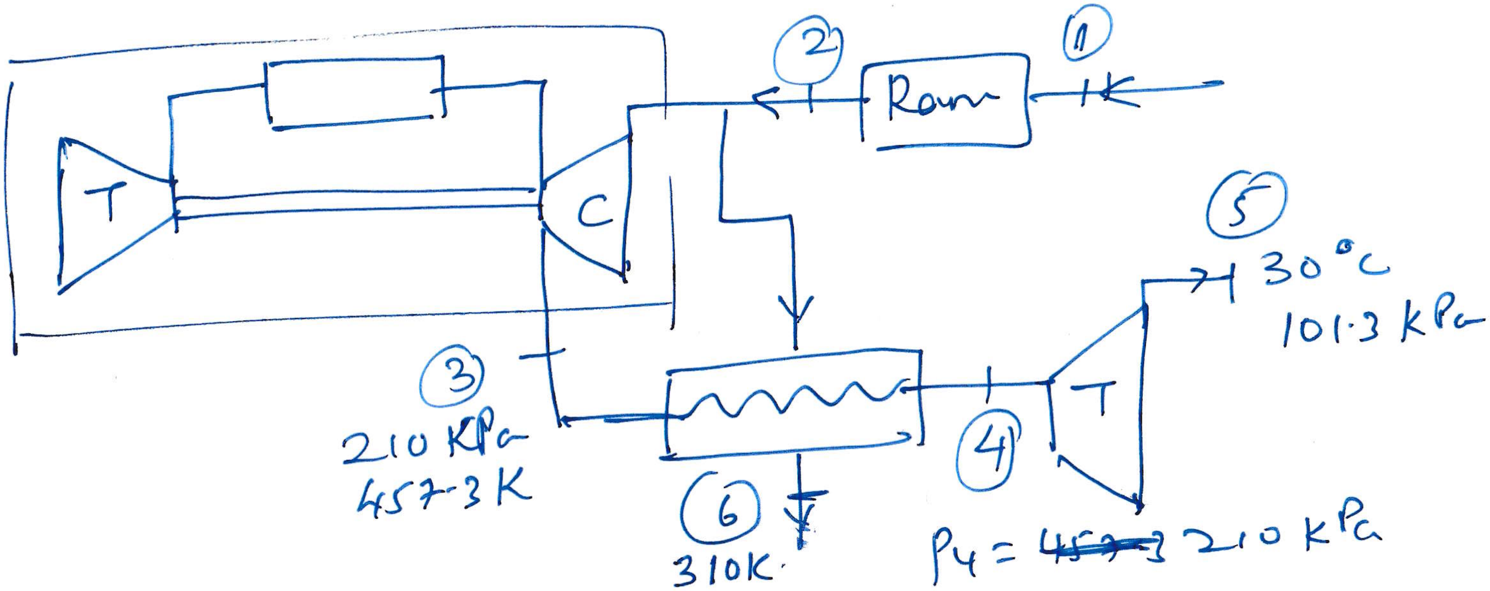
$\underline{2 \to 3}$ Compression $\frac{p_3}{p_2} = 5 \Rightarrow p_3 = (42)(5) = 210 \text{ kPa}.$

$$\frac{T_{3s}}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{3s}}{271.1} = (5)^{\frac{0.4}{1.4}} = 429.4\text{K}.$$

$$\eta = \frac{T_{3s} - T_2}{T_3 - T_2} \Rightarrow 0.85 = \frac{429.4 - 271.1}{T_3 - 271.1} \Rightarrow T_3 = 457.3\text{K}.$$

$\underline{3 \to 4}$ Turbine $\frac{T_{4s}}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{4s}}{457.3} = \left(\frac{101.3}{210}\right)^{\frac{0.4}{1.4}} \Rightarrow T_{4s} = 371.3\text{K}$

$$\eta = \frac{T_4 - T_3}{T_{4s} - T_3} \Rightarrow 0.85 = \frac{T_4 - 457.3}{371.3 - 457.3} \Rightarrow \boxed{T_4 = 384.2\text{K}}$$



$$\frac{T_4}{T_5} = \left(\frac{P_4}{P_5}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_4}{303} = \left(\frac{210}{101.3}\right)^{\frac{0.4}{1.4}} = 373\ K$$

To find the mass of air to cool 1 kg of air in the intercooler

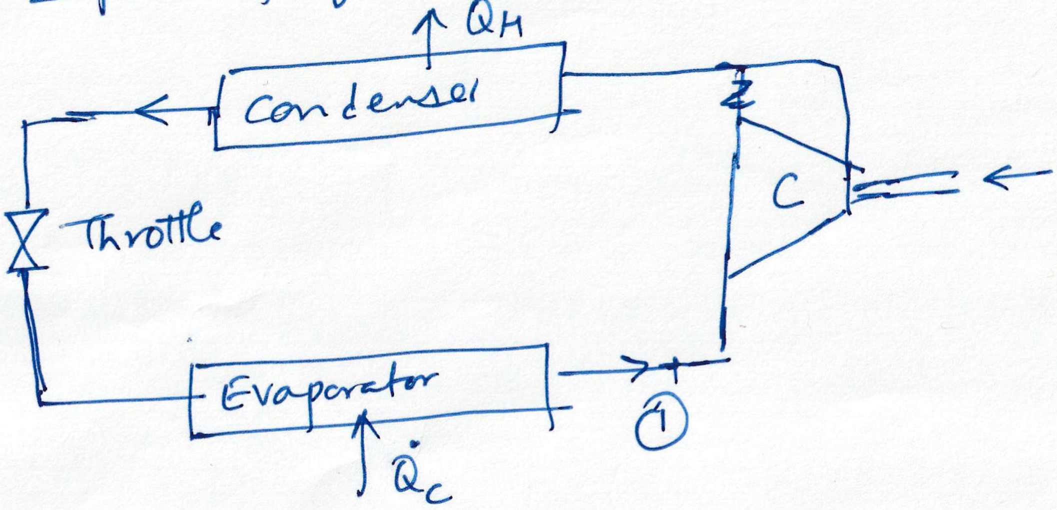
$$(1)(h_3 - h_4) = (h_6 - h_2)\dot{m}$$

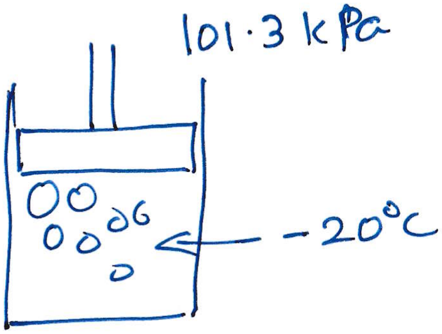
$$\cancel{C_p}(T_3 - T_4) = \cancel{C_p}(T_6 - T_2)\dot{m} \Rightarrow \dot{m} = \frac{T_3 - T_4}{T_6 - T_2}$$

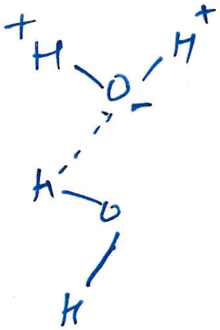
$$= \frac{457.3 - 373}{310 - 271.1}$$

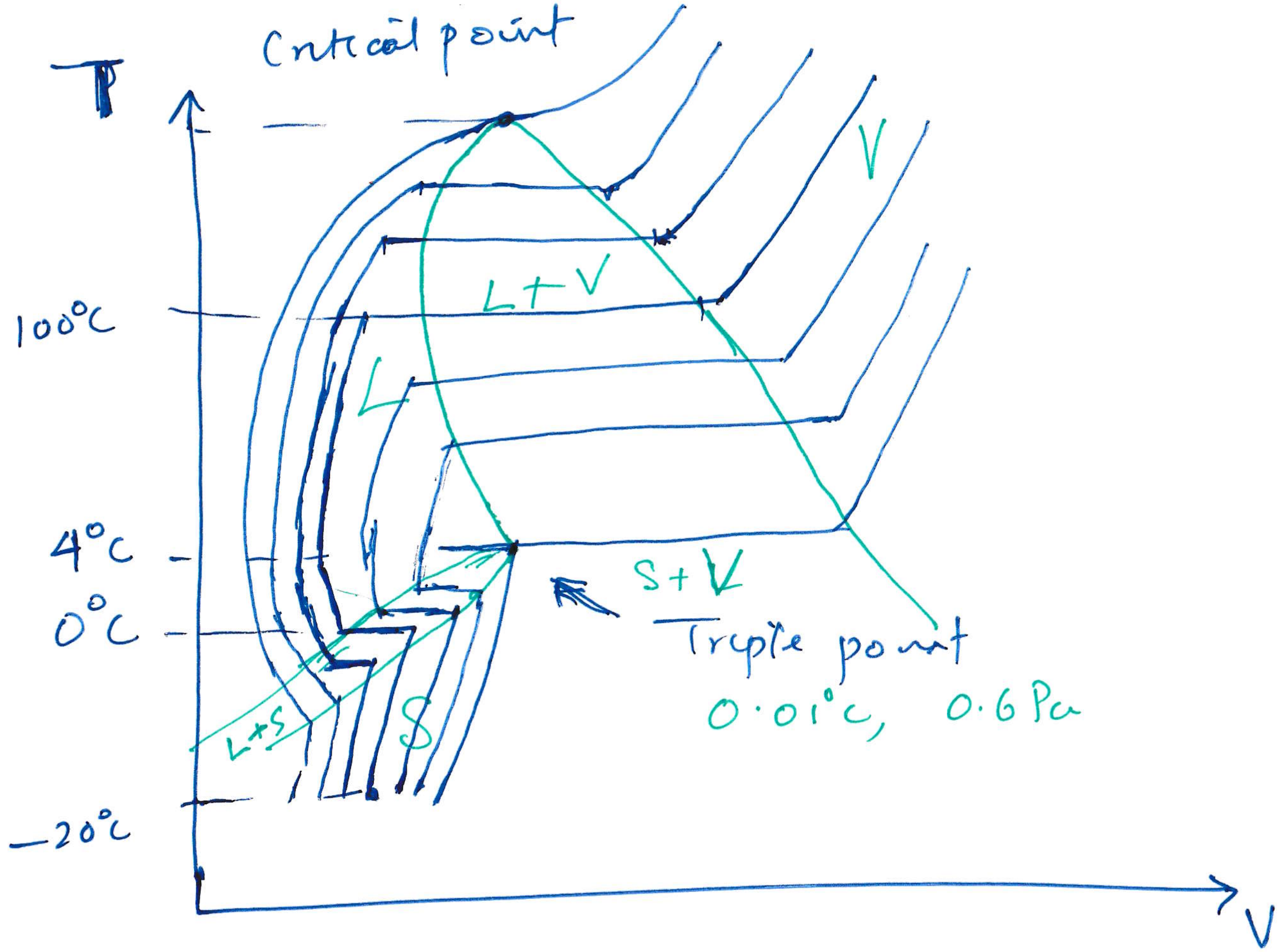
$$= 2.16\ kg/kg \text{ of air}$$

Vapor Refrigeration Cycle




101·3 kPa
-20°C


H
O
H
H
O
H


Critical point
T
V
L+V
100°C
L
4°C
0°C
S+V
Triple point
0·01°C, 0.6 Pa
L+S
S
-20°C
V