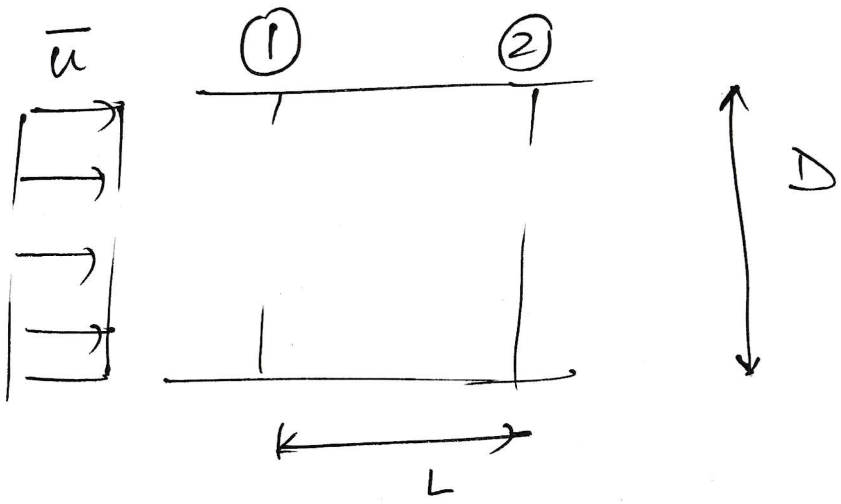
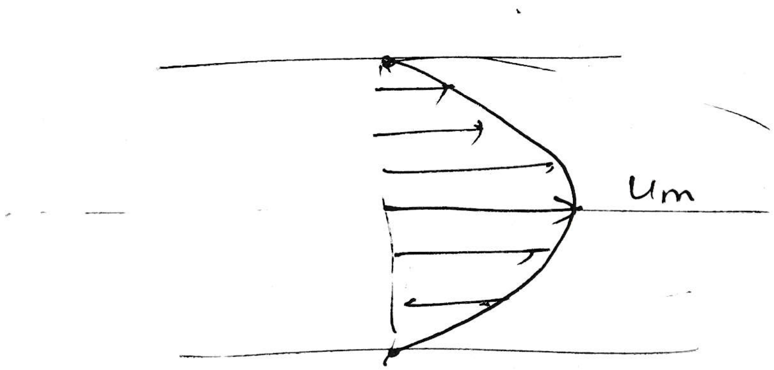
Internal flows



1) $$\Delta p = f_D \rho \frac{\bar{u}^2}{2} \frac{L}{D}$$

$$\frac{\Delta p}{\rho g} = f_D \frac{\bar{u}^2}{2g} \frac{L}{D}$$ ← head version

f_D ↓ Darcy-Weisbach friction factor.



(2) Define friction velocity

$\tau_w = \rho u_*^2$ i.e. $u_* \triangleq \sqrt{\frac{\tau_w}{\rho}}$

(3) $$f_D = 8\left(\frac{u_*}{\bar{u}}\right)^2$$

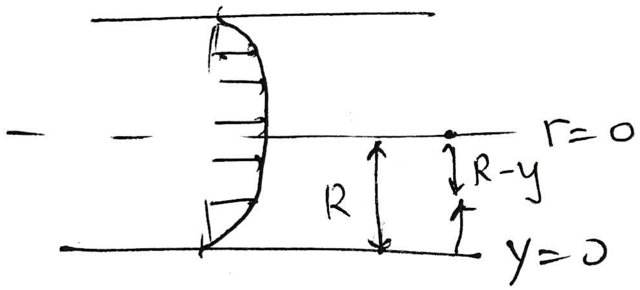
$$C_f = 2\left(\frac{u_*}{\bar{u}}\right)^2 = \frac{f_D}{4}$$

$$\left(\tau_w = \rho u_*^2 = \frac{1}{2} C_f \rho \bar{u}^2\right)$$

Laminar flow

$$u = u_m\left(1 - \frac{r^2}{R^2}\right); \quad \bar{u} = \frac{u_m}{2}; \quad f_D = \frac{64}{Re} \quad \left(Re \triangleq \frac{\bar{u}D}{\nu}\right)$$

Turbulent flow



$$\frac{u}{u_m} = \left(\frac{y}{\delta}\right)^{1/7}$$

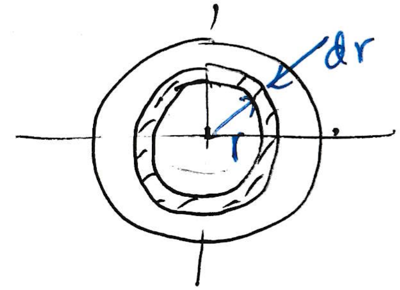
δ = thickness of B.L.

u_m = ~~max vel~~ free stream velocity

Since $r = R - y \Rightarrow y = R - r$

and $\delta = R$

$$u(r) = u_m \left(\frac{R-r}{R}\right)^{1/7}$$



$$\dot{V} = \bar{u}\,\pi R^2 = \int u(r)\, dA$$

$$= \int_0^R u(r)\, 2\pi r\, dr$$

$$= \int_0^R u_m \left(\frac{R-r}{R}\right)^{1/7} 2\pi r\, dr$$

$$= 2\pi u_m \int_0^R \left(\frac{R-r}{R}\right)^{1/7} r\, dr$$

$$= 2\pi u_m \int_1^0 t^{1/7} R(1-t)(-R\,dt)$$

$$= 2\pi R^2 u_m \int_0^1 t^{1/7} (1-t)\, dt.$$

$$= 2\pi R^2 u_m \frac{49}{120} = \frac{98 u_m \pi R^2}{120}$$

Let $\frac{R-r}{R} = t$

$R - r = Rt$

$r = R(1-t)$

$dr = -R\,dt$

when $r = 0$, $t = 1$

$r = R$, $t = 0$

$$\int_0^1 t^{1/7}(1-t)\,dt = \int_0^1 \left(t^{1/7} - t^{8/7}\right)dt$$

$$= \frac{t^{8/7}}{8/7} - \frac{t^{15/7}}{15/7}\Bigg|_0^1$$

$$= \frac{7}{8} - \frac{7}{15}$$

$$= \frac{\cancel{98}}{120} \quad \frac{49}{120}$$

$$\dot{V} = \bar{u}\pi R^2 = \frac{98 u_m \pi R^2}{120} \Rightarrow \boxed{\bar{u} = \frac{98}{120} u_m}$$

Shear stress at wall.

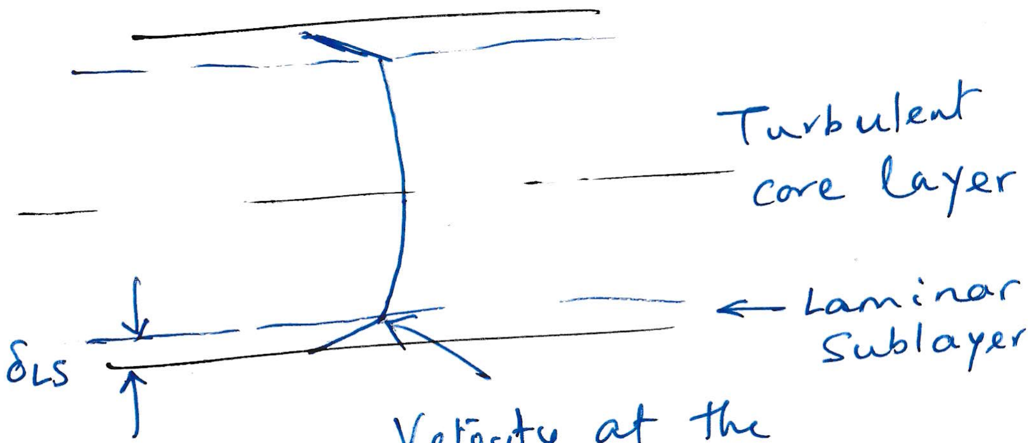
$$\tau_w = -\mu \frac{du}{dy}\Big|_{y=0}$$

Since $u = u_m\left(\frac{y}{\delta}\right)^{1/7} \Rightarrow \frac{du}{dy} = u_m \frac{1}{7}\left(\frac{y}{\delta}\right)^{\frac{1}{7}-1}$

$$= \frac{u_m}{7}\left(\frac{y}{\delta}\right)^{-6/7}$$

$$\frac{du}{dy}\Big|_{y=0} \longrightarrow \infty$$

Two layer model



$$\frac{u}{u_m} = \left(\frac{y}{\delta}\right)^{1/7} = \left(\frac{R-r}{R}\right)^{\frac{1}{7}}$$

linear variation of velocity with distance from the wall.

Velocity at the boundary between laminar sublayer and turbulent core $= \beta u_*$ $\left(u_* = \sqrt{\frac{\tau_w}{\rho}},\ \beta \text{ is some number} \right)$

Laminar Sublayer

$$u(y) = \frac{y}{\delta_{LS}} \beta u_* \Rightarrow \frac{du}{dy} = \frac{\beta u_*}{\delta_{LS}}$$

$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0} = \mu \frac{\beta u_*}{\delta_{LS}}$$

But $\tau_w = \rho u_*^2$

Equating $\tau_w = \rho u_*^2 = \mu \frac{\beta u_*}{\delta_{LS}} \Rightarrow \delta_{LS} = \beta\left(\frac{\mu}{\rho}\right)\frac{u_*}{u_*^2}$

$$\delta_{LS} = \frac{\beta \nu}{u_*}$$

~~At~~ Turbulent core : (Bottom of turbulent core.

$y = \delta_{LS} = \beta \nu / u_*$

$\delta \approx R$

$u(\delta_{LS}) = \beta u_*$)

$$\frac{u}{u_m} = \left(\frac{y}{\delta}\right)^{1/7} \Rightarrow \frac{\beta u_*}{u_m} = \left(\frac{\delta_{LS}}{R}\right)^{\frac{1}{7}}$$

$$\beta \frac{u_*}{u_m} = \cancel{\beta \nu u_*} \left(\frac{\beta \nu}{u_* R}\right)^{1/7}$$

We want to find f_D or C_f, both require $\left(\frac{u_*}{\bar{u}}\right)^2$

$$\left(\frac{u_*}{\bar{u}}\right) = \left(\frac{u_*}{u_m}\frac{u_m}{\bar{u}}\right)$$

~~Th~~ We have found $\bar{u} = u_m \frac{98}{120}$

$$\beta \frac{u_*}{u_m} = \beta^{1/7} \frac{\nu^{1/7}}{u_*^{1/7} R^{1/7}} \Rightarrow \frac{u_*^{8/7}}{u_m} = \frac{1}{\beta^{6/7}} \frac{\nu^{1/7}}{\cancel{R^{1/7}}} \frac{2^{1/7}}{D^{1/7}}$$

$$\Rightarrow \frac{u_*^{8/7}}{u_m u_m^{1/7}} = \frac{2^{1/7}}{\beta^{6/7}} \frac{\nu^{1/7}}{D^{1/7} u_m^{1/7}}$$

$$\Rightarrow \left(\frac{u_*}{u_m}\right)^{8/7} = \frac{2^{1/7}}{\beta^{6/7}} \left(\frac{\nu}{D u_m}\right)^{1/7}$$

$$\left(\frac{u_*}{u_m}\right)^{\frac{8}{7}\cdot\frac{7}{4}} = \frac{2^{\frac{1}{7}\cdot\frac{7}{4}}}{\beta^{\frac{6}{7}\cdot\frac{7}{4}}}\left(\frac{\nu}{D u_m}\right)^{\frac{1}{7}\cdot\frac{7}{4}}$$

$$\left(\frac{u_*}{u_m}\right)^{2} = \frac{2^{1/4}}{\beta^{3/2}}\left(\frac{\nu}{D u_m}\right)^{1/4}$$

$$= \frac{2^{1/4}}{\beta^{3/2}}\left(\frac{\nu}{D\bar{u}}\,\frac{120}{98}\right)^{1/4}$$

Since $\bar{u} = \frac{u_m 98}{120}$

$\Rightarrow u_m = \bar{u}\,\frac{120}{98}$

$$= \frac{2^{1/4}}{\beta^{3/2}}\left(\frac{98}{120}\right)^{1/4}\left(\frac{\nu}{D\bar{u}}\right)^{1/4}.$$

$$f_D = 8\left(\frac{u_*}{\bar{u}}\right)^2 = 8\left(\frac{u_*}{u_m}\,\frac{u_m}{\bar{u}}\right)^2 = 8\left(\frac{u_*}{u_m}\right)^2\left(\frac{u_m}{\bar{u}}\right)^2$$

$$= 8\cdot\frac{2^{1/4}}{\beta^{3/2}}\left(\frac{98}{120}\right)^{1/4}\frac{1}{Re^{1/4}}\left(\frac{120}{98}\right)^2$$

Experimentally $\beta = 12$

$$= 8\cdot\frac{2^{1/4}}{(12)^{3/2}}\left(\frac{120}{98}\right)^{7/4} Re^{-1/4}$$

$$\Rightarrow \boxed{f_D = 0.326\,Re^{-1/4}}$$

$$\boxed{C_f = \frac{f_D}{4} = 0.082\, Re^{-1/4}}$$

$$\Delta p = f_D\, \rho \frac{\bar{u}^2}{2} \frac{L}{D}$$

Heat transfer

Chilton - Colburn analogy

$$\frac{C_f}{2} Re\, \delta = Nu\, \delta_T \Rightarrow$$

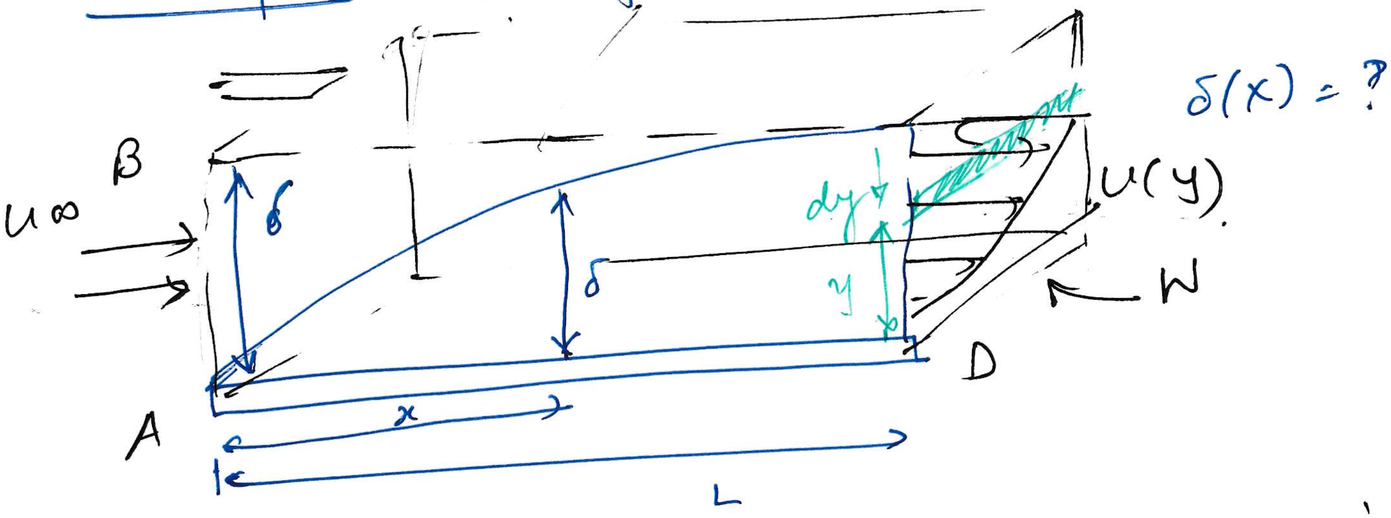
$$Nu = \frac{C_f}{2} Re \left(\frac{\delta}{\delta_T}\right)$$

$$= \frac{0.082\, Re^{-1/4}}{2} Re \left(Pr\right)^{1/3}$$

$$\boxed{Nu = 0.04\, Re^{3/4} (Pr)^{1/3}}$$

$$4000 < Re < 20{,}000$$

Flat plate Integral Momentum Method.



$$\dot{m}_{AB} - \dot{m}_{CD} - \dot{m}_{BC} = 0$$

$$\dot{m}_{AB} = \rho(u_\infty)\delta W$$

$$\dot{m}_{CD} = \int d\dot{m}$$

$$= \int_\delta \rho\, u(y)\, dA$$

$$= \int_0^\delta \rho\, u(y)\, W dy$$