(1)

$\rightsquigarrow T_\infty$ $\rightsquigarrow C_{A\infty}$ $\rightsquigarrow u_\infty$

T_s C_{As}

$\dot{q} = h(T_s - T_\infty)$

$j_A = h_M (C_{As} - C_{A\infty})$

$\tau_w = \frac{C_f}{2} \rho u_\infty^2$

shear stress

rate of momentum

Transport Mechanism

T_2

y y

T_1

B A

C_{As}

Molecular $\dot{q} = -k \frac{dT}{dy} = -\frac{k}{\rho c_p} \frac{d(\rho c_p T)}{dy}$

$j_A = -D_{AB} \frac{dc_A}{dy}$

$\tau_w = -\nu \frac{d(\rho u)}{dy}$

Convective $\dot{q} = (\rho C_p T) u$

$j_A = u c_A$

$\tau = (\rho u) u$

At Boundary $y=0,\ u=0$

$\dot{q} = -k \left.\frac{dT}{dy}\right|_{y=0}$

$j_A = -D_{AB} \left.\frac{dc_A}{dy}\right|_{y=0}$

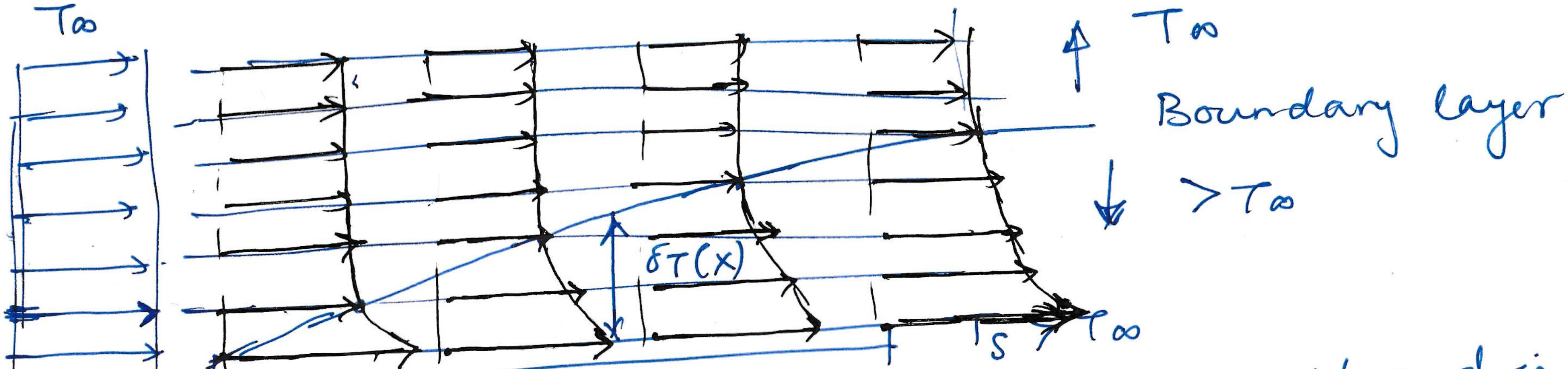
$\tau_w = -\nu \left.\frac{d(\rho u)}{dy}\right|_{y=0}$

$\alpha \triangleq k/\rho c_p$ $[\alpha] = m^2/s$

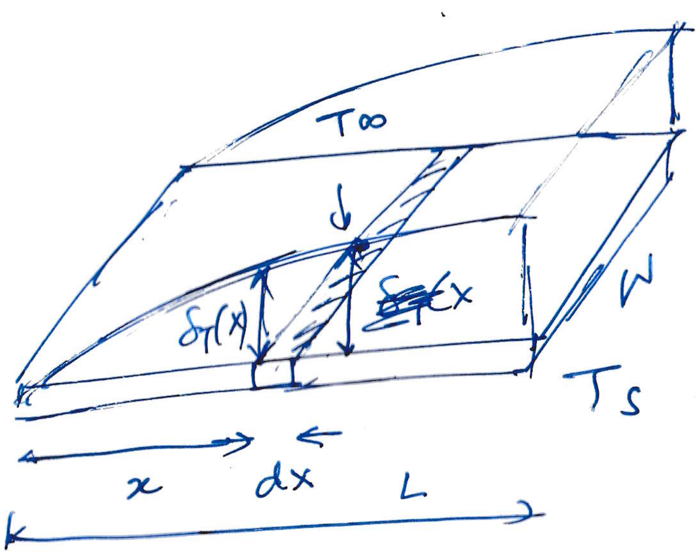
$[D_{AB}] = m^2/s$

$[\nu] = m^2/s$

$Pr \triangleq \frac{\nu}{\alpha}$; $Sc \triangleq \frac{\nu}{D_{AB}}$



$\delta_T(x)$ = Boundary layer height and is a function of distance along the plate.



$$d\dot{q} = h\,\cancel{W}\,dx\,(T_s - T_\infty) = k\,\cancel{W}\,dx\,\frac{(T_s - T_\infty)}{\delta_T}$$

$$\Rightarrow h(x) = \frac{k}{\delta_T(x)}$$ ← local heat transfer coefficient

$$\dot{Q} = \int_0^L d\dot{q} = \int_0^L h(x)\,W\,dx\,(T_s - T_\infty) = \bar{h}(T_s - T_\infty)WL$$

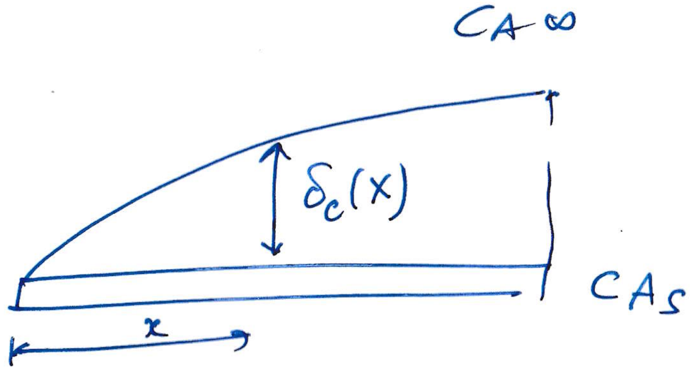
$$\Rightarrow \boxed{\bar{h} = \frac{1}{L}\int_0^L h(x)\,dx}$$ overall heat transfer coefficient

$$h(x) = \frac{k}{\delta_T(x)} \Rightarrow \frac{h(x)}{k} = \frac{1}{\delta_T(x)} \Rightarrow \underbrace{\frac{h(x)\,x}{k}}_{} = \frac{x}{\delta_T(x)}$$

Nusselt number $Nu(x) \triangleq \frac{h(x)\,x}{k}$

Overall Nusselt number $\triangleq \frac{\bar{h}L}{k} = \frac{L}{\delta_T}$

Mass Transfer

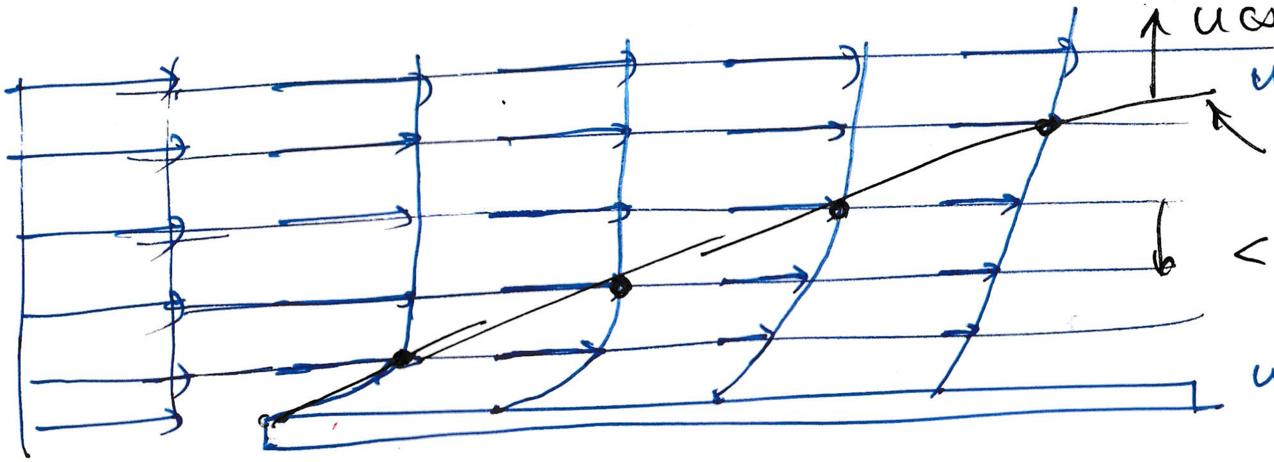


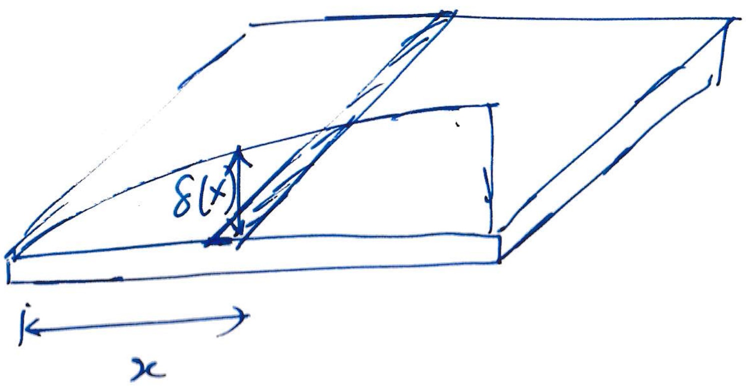
$$h_M(x) = \frac{D_{AB}}{\delta_c(x)}$$

$$\frac{h_M(x)\,x}{D_{AB}} = \frac{x}{\delta_c(x)}$$

$$Nu_M(x) \triangleq \frac{h_M(x)\,x}{D_{AB}}$$ ← local Nusselt number for mass transfer.

$$Nu_M \triangleq \frac{\overline{h}_M L}{D_{AB}} = \frac{L}{\delta_c}$$





$$dF = \frac{C_f(x)}{2} \rho u_\infty^2 W dx = \mu \frac{u_\infty}{\delta} (W dx)$$

$$\frac{C_f}{2} \frac{\rho}{\mu} u_\infty = \frac{1}{\delta}$$

$$\frac{C_f}{2} \frac{u_\infty x}{(\mu/\rho)} = \frac{x}{\delta}$$

$$\frac{C_f}{2} \frac{u_\infty x}{\nu} = \frac{x}{\delta}$$

Let $Re \triangleq \frac{u_\infty x}{\nu}$ ← local Reynold's number

$$\frac{C_f}{2} Re(x) = \frac{x}{\delta}$$

$$\frac{\bar{C}_f}{2} Re_L = \frac{L}{\delta}$$

$$Re_L = \frac{u_\infty L}{\delta}$$

Heat transfer | Mass transfer | Momentum transfer (5)

$$Nu = \frac{\bar{h} L}{K} = \frac{L}{\delta_T} \qquad Nu_M = \frac{\bar{h}_M L}{D_{AB}} = \frac{L}{\delta_C} \qquad \frac{C_f}{2} Re = \frac{L}{\delta}$$

$$\Rightarrow \quad L = Nu\, \delta_T \qquad L = Nu_M\, \delta_C \qquad L = \frac{C_f}{2} Re\, \delta$$

$$L = Nu\, \delta_T = Nu_M\, \delta_C = \frac{C_f}{2} Re\, \delta$$

<u>Reynold's analogy</u>

$$\delta_T = \delta_C = \delta$$

$$Nu = Nu_M = \frac{C_f}{2} Re$$

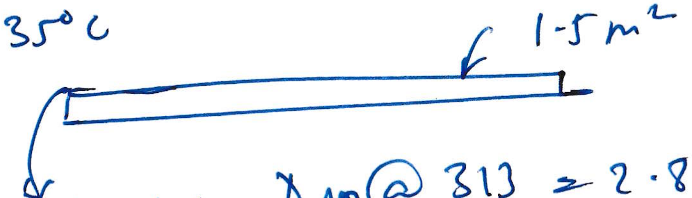
<u>Chilton- Colburn Analogy</u>

$$\frac{\delta}{\delta_T} = (Pr)^{1/3} \quad ; \quad \frac{\delta}{\delta_C} = (Sc)^{1/3}$$

$$Nu = \frac{C_f}{2} Re \frac{\delta}{\delta_T} = \frac{C_f}{2} Re (Pr)^{1/3} \quad ; \quad Nu_M = \frac{C_f}{2} Re \frac{\delta}{\delta_C} = \frac{C_f}{2} Re (Sc)^{1/3}$$

Water evaporates from a wetted surface of rectangular shape when air at 1 atm and 35C blows over the surface at 15 m/s. Heat transfer measurements indicate that for air at 1 atm and 35C, the average heat transfer coefficient is given by the empirical relation $h = 21\, v_\infty^{0.6}$ where h is in W/m^2K and v_∞ is air speed in m/s. Estimate the mass transfer coefficient and the rate of evaporation of the water from the surface if the area is $1.5m^2$. For air, $\nu = 16.47 \times 10^{-6} m^2/s$ and the diffusion coefficient for water to air at 313 K is $2.88 \times 10^{-5} m^2/s$. $Pr = 0.7$

15 m/s Air
1 atm
35°C



Water $D_{AB} @ 313 = 2.88 \times 10^{-5}\ m^2/s$

$h_M = ?$

& $\dot{m}_A = ?$

given $D_{AB} @ 313$, we need $D_{AB} @ 308$

$$D_{AB} \sim T^{3/2} \Rightarrow \frac{D_{AB}(308)}{D_{AB}(313)} = \left(\frac{308}{313}\right)^{3/2}$$

$$Sc = \frac{\nu}{D_{AB}}$$

$$Pr = 0.7 = \frac{\nu}{\alpha}$$

$$= 0.7 = \frac{16.47 \times 10^{-6}}{\alpha}$$

$$\Rightarrow \alpha = \frac{16.47 \times 10^{-6}}{0.7} = \frac{k}{\rho C_p} \Rightarrow k = \rho C_p\, \alpha$$

$$\dot{m}_A = h_M A (C_{A_s} - \cancel{C_{A_\infty}})$$

$$C_{A_s} = \frac{P_{sat}}{R_v T}$$

$$Nu_M\, \delta_c = Nu\, \delta_T$$

$$Nu_M \frac{\delta_c}{\delta} = Nu \frac{\delta_T}{\delta}$$

$$Nu_M (Sc)^{-1/3} = Nu (Pr)^{-1/3}$$

$$\frac{h_M L}{D_{AB}} (Sc)^{-1/3} = \frac{h L}{k} (Pr)^{-1/3}$$