1

1) variation of density / pressure with altitude.

a) Derived hydrostatic equation $\frac{dp}{dz} = -\rho g$

b) Assume ideal gas $p = \rho R T$ and $T(z) = T_0 - \Gamma z$

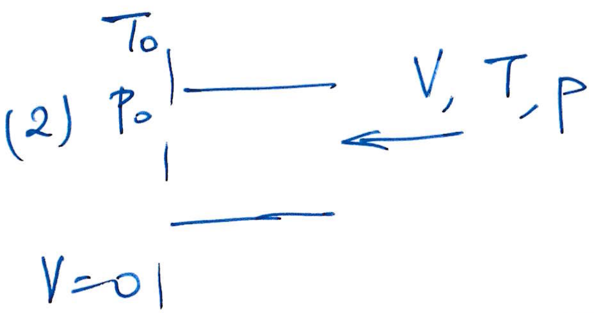
and derived $\frac{p}{p_0} = \left(1 - \frac{\Gamma z}{T_0}\right)^{\frac{g}{R\Gamma}}$ $T_0 = T(z=0)$

Γ = lapse rate

c) Starting with $\frac{dp}{dz} = -\rho g$ and $pv^{\gamma} = c$,

derive. $\Gamma = \left(\frac{\gamma - 1}{\gamma}\right)\frac{g}{R} \Rightarrow \frac{g}{R\Gamma} = 1$

(2)



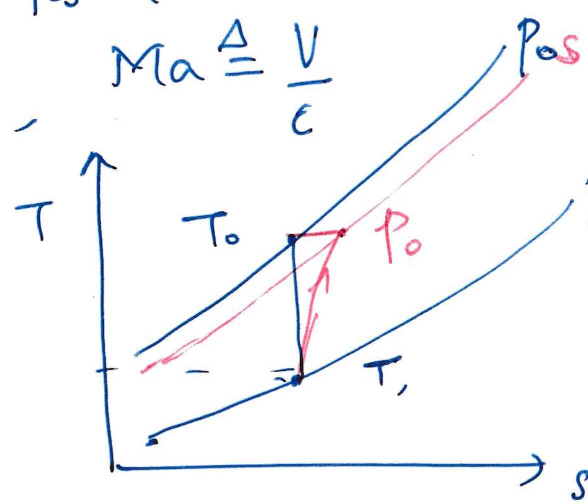
$h_0 = h + \frac{V^2}{2} \Rightarrow T_0 = T + \frac{V^2}{2C_p}$,

$\frac{p}{p_{0s}} = \left(\frac{T}{T_0}\right)^{\cancel{\frac{\gamma-1}{\gamma}}\ \gamma/\gamma-1.}$

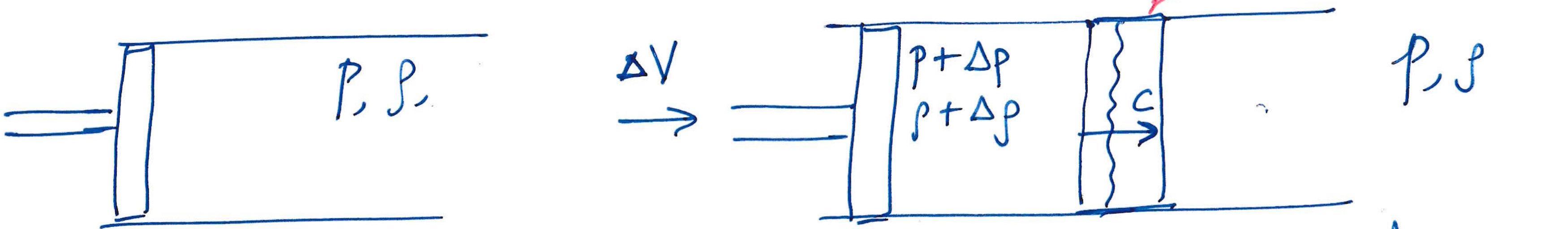
With $\boxed{c^2 = \gamma R T}$ (speed of sound), $Ma \triangleq \frac{V}{c}$

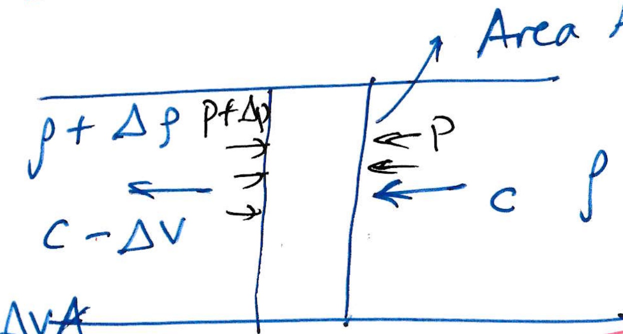
$T_0 = T\left[1 + \frac{\gamma - 1}{2} Ma^2\right]$

$\eta_{isentropic} = \frac{p_0 - p}{p_{0s} - p}$



Speed of sound





Mass Balance:

$$\dot m_{in} = \dot m_{out}$$

$$\rho \cancel{A} c = (\rho+\Delta\rho) A (c-\Delta V)$$

$$= \rho \cancel{A} c - \rho \Delta V \cancel{A} + \Delta\rho \cancel{A} c - \Delta\rho \Delta V \cancel{A}$$

If $\Delta\rho \Delta V \approx 0 \Rightarrow -\rho \Delta V + \Delta\rho\, c = 0 \Rightarrow \boxed{\rho \Delta V = \Delta\rho\, c}$

Momentum Balance: $\dot M_{in} - \dot M_{out} + \sum F = 0$

$\dot M_{in} = -\dot m c$, $\dot M_{out} = -\dot m (c-\Delta V)$; $\sum F = (p+\Delta p)A - pA$

$$-\dot m c - (-\dot m (c-\Delta V)) + (p+\Delta p)A - pA = 0$$

$$-\dot m \Delta V + \Delta p \cancel{A} = 0$$

Since $\dot m = \rho A c \Rightarrow -\rho A c \Delta V + \Delta p A = 0 \Rightarrow -\rho c \Delta V + \Delta p = 0$

$$\Rightarrow -c^2 \Delta\rho + \Delta p = 0$$

$$\Rightarrow c^2 = \frac{\Delta p}{\Delta \rho}$$

Taking limits as $\Delta\rho \to 0$

$$c^2 = dp/d\rho \quad \text{or} \quad c = \sqrt{dp/d\rho}$$

Assuming adiabatic process, $pv^{\gamma} = c$

$$p = \frac{c}{(v)^{\gamma}} \quad \left(\frac{1}{v}\right)^{\gamma} c$$

$$p = c\rho^{\gamma}$$

$$\frac{dp}{d\rho} = c\gamma\rho^{\gamma-1}$$

$$= \gamma \frac{c\rho^{\gamma}}{\rho}$$

$$= \gamma \frac{p}{\rho}$$

$$= \gamma RT$$

$$c = \sqrt{\frac{dp}{d\rho}}$$

$$\boxed{c = \sqrt{\gamma RT}}$$

$$pv = RT$$

$$p = \left(\frac{1}{v}\right) RT$$

$$= \rho RT$$

$$\frac{p}{\rho} = RT$$

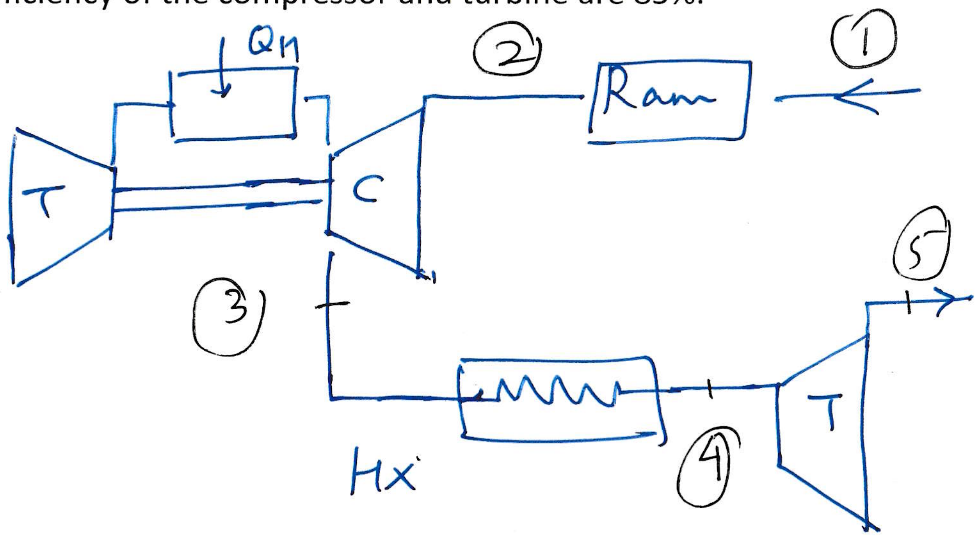
Ex: $T = 27°C = 300\,K$

$$c = \sqrt{(1.4)(287)(300)}$$

$$c = 347\ m/s$$

A plane uses a simple air-refrigeration for cabin air-conditioning. It cruises at 900 kmph at an altitude of 10,000 m. The compression ratio in the main compressor is 5. The cabin ~~temperature~~ is 101.3 kPa. Determine the temperature of air entering the cabin from the refrigeration system. Assume lapse rate is $6^o\ C/km$ and ground temperature is 27C. Isentropic efficiency of the compressor and turbine are 85%.

$C_{p\,air} = 1.005\ kJ/kg\,K$



1 : $T(z) = T_0 - \Gamma z = 27 - (6)(10) = -33^\circ C = 240\,K$

$\frac{P_1}{P_0}\ \cancel{\frac{P_\phi}{P_\phi}} = \left(1 - \frac{\Gamma z}{T_0}\right)^{\frac{\gamma}{\gamma - 1}} \Rightarrow \frac{P_{\cancel{\phi}}}{101.3} = \left(1 - \frac{(6)(10)}{300}\right)^{\frac{1.4}{0.4}} \Rightarrow P_{\cancel{\phi}} = 46.4\ kPa.$

$\left(\frac{g}{R\Gamma}\right)$

2 $T_2 = T_1 + \frac{V^2}{2C_p} \Rightarrow T_2 = 240 + \frac{(250)^2}{(2)(1005)} = 271.1\,K$

$V = 900\ kmph = 250\ m/s$

$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma - 1} \Rightarrow \frac{P_2}{46.4} = \left(\frac{271.1}{240}\right)^{\frac{1.4}{0.4}} \Rightarrow P_2 = 71.1\,kPa$

(3)

$$\frac{p_3}{p_2} = 5 \text{ (given)} \Rightarrow \frac{p_3}{71.1} = 5 \Rightarrow \boxed{p_3 = 355.4 \text{ kPa}}$$

$$\frac{T_{3s}}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{r-1}{r}} \Rightarrow \frac{T_{3s}}{271.1} = (5)^{\frac{0.4}{1.4}} \Rightarrow T_{3s} = 429.4 \text{ K}$$

$$\eta_{isentropic} = 0.85 = \frac{T_{3s} - T_2}{T_3 - T_2} \Rightarrow 0.85 = \frac{429.4 - 271.1}{T_3 - 271.1}$$

$$\Rightarrow \boxed{T_3 = 457.3 \text{ K}}$$