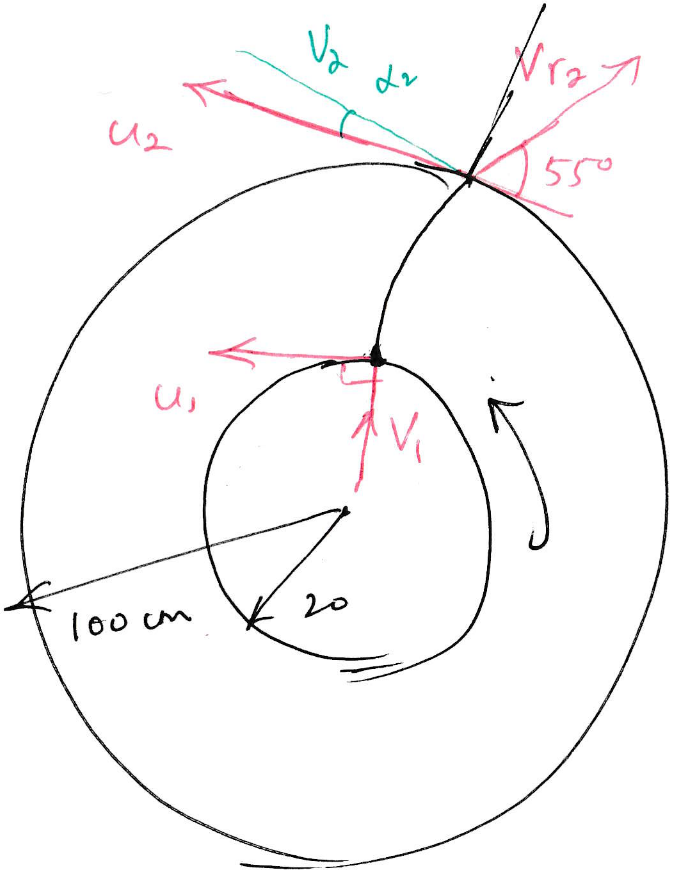
A centrifugal compressor compresses 6.28 m^3/s air with inlet temperature 25C and pressure 1 bar. The inner radius of the impeller is 20 cm, the outer radius is 100 cm. The impeller width is uniform 5 cm. The outlet angles of the blade is 55 degrees. The compressor operates at 2700 rpm. Its isentropic efficiency is 90%. Inlet is radial.

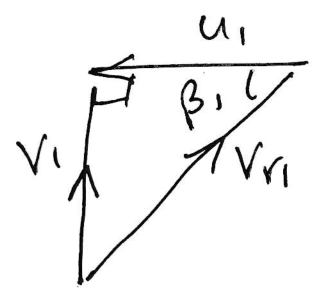


$N = 2700$ rpm $\Rightarrow \omega = \frac{2\pi N}{60} = 282.7$ rad/s

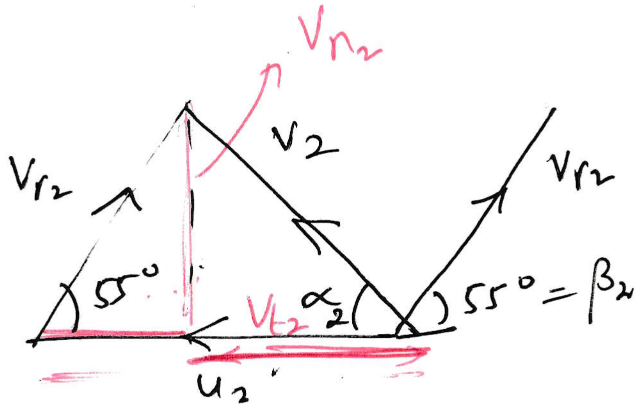
$u_1 = R_1 \omega_1 = (0.2)(282.7) = 56.5$ m/s

$u_2 = R_2 \omega = (1)(282.7) = 282.7$ m/s.

Inlet $\alpha_1 = 90^\circ$



Outlet.



$V_{t_1} = 0$

$V_{n_1} = V_1$

$\dot{V} = (\cancel{2\pi R_1 b_1}) V_{n_1} = (\cancel{2\pi R_2 b_2}) V_{n_2}$

$(b_1 = b_2 = 5)$

$\dot{V} = 2\pi R_1 b V_{n_1} = 2\pi R_2 b V_{n_2}$

$6.28 = (2\pi)(0.2)(0.05) V_{n_1}$

$\Rightarrow V_{n_1} =$

$V_2 = u_2 + V_{r_2}$

$V_{t_2} = u_2 - V_{r_2} \cos\beta_2$

$V_{r_2} \sin\beta_2 = V_2 \sin\alpha_2 = V_{n_2}$

$V_{t_2} = u_2 - \frac{V_{n_2}}{\sin\beta_2} \cos\beta_2$

$V_{t_2} = u_2 - V_{n_2} \cot\beta_2$

$$\dot{V} = (2\pi R_1 b_1) V_{n_1} \Rightarrow V_{n_1} = \frac{\dot{V}}{(2\pi R_1 b_1)}$$

$$= \frac{6.28}{(2\pi)(0.2)(0.05)}$$

$$V_{n_1} = 100 \text{ m/s}$$

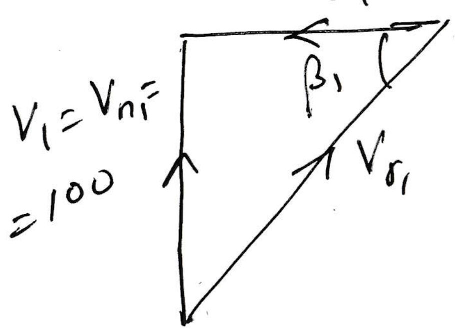
$$\cancel{2\pi} R_1 \cancel{b_1} V_{n_1} = \cancel{2\pi} R_2 \cancel{b_2} V_{n_2} \quad (b_1 = b_2 = 5)$$

$$V_{n_2} = \frac{R_1 V_{n_1}}{R_2} = \left(\frac{20}{100}\right)(100)$$

$$V_{n_2} = 20 \text{ m/s}.$$

Inlet $\quad \underline{V_1} = \underline{V_{r_1}} + \underline{u_1}$

$u_1 = 56.5$

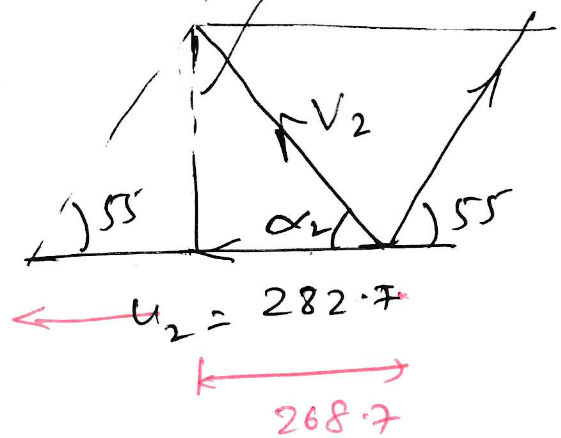


$\cancel{\tan\beta_1} = \tan\beta_1 = \frac{100}{56.5}$

$$\Rightarrow \boxed{\beta_1 = 60.5^\circ}$$

Outlet

$V_{n_2} = 20$ m/s.



$$V_{t_2} = u_2 - V_{n_2} \cot\beta_2$$

$$= 282.7 - 20 \cot 55$$

$$= 268.7 \text{ m/s}$$

$$\alpha_2 = \tan^{-1} \frac{V_{n_2}}{V_{t_2}}$$

$$= \tan^{-1} \frac{20}{268.7}$$

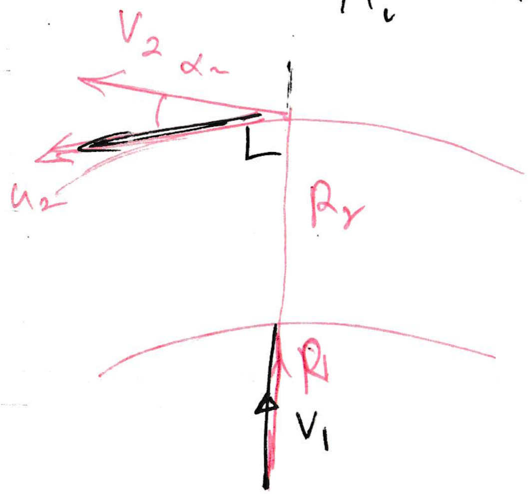
$$\boxed{\alpha_2 = 4.3^\circ}$$

Angular momentum Conservation

③

$$\dot{H}_1 - \dot{H}_0 + T = 0 \Rightarrow T = \dot{H}_0 - \dot{H}_1$$

$$= \dot{m} V_{t_2} R_2 - \cancel{\dot{m} V_{t_1} R_1}^{\,0}$$



Power $P = T\omega$

$$= \dot{m} V_{t_2} R_2 \omega$$

$$= \dot{m} V_{t_2} u_2$$

work done per unit mass $w = \dfrac{P}{\dot{m}} = V_{t_2} u_2$

$$= \underbrace{(u_2 - V_{n_2} \cot\beta_2)}_{268.7} \underbrace{u_2}_{287.7}$$

$$= 77305 \text{ J/kg}.$$

$$P = \dot{m} w$$

$$\dot{m} = \cancel{\rho Q}\ \rho \dot{V}$$

$$P \cancel{v} = mRT$$

$$P = \rho R T$$

$$\Rightarrow \rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287)(298)} = 1.17 \text{ kg/m}^3 \Rightarrow \dot{m} = \rho \dot{V}$$

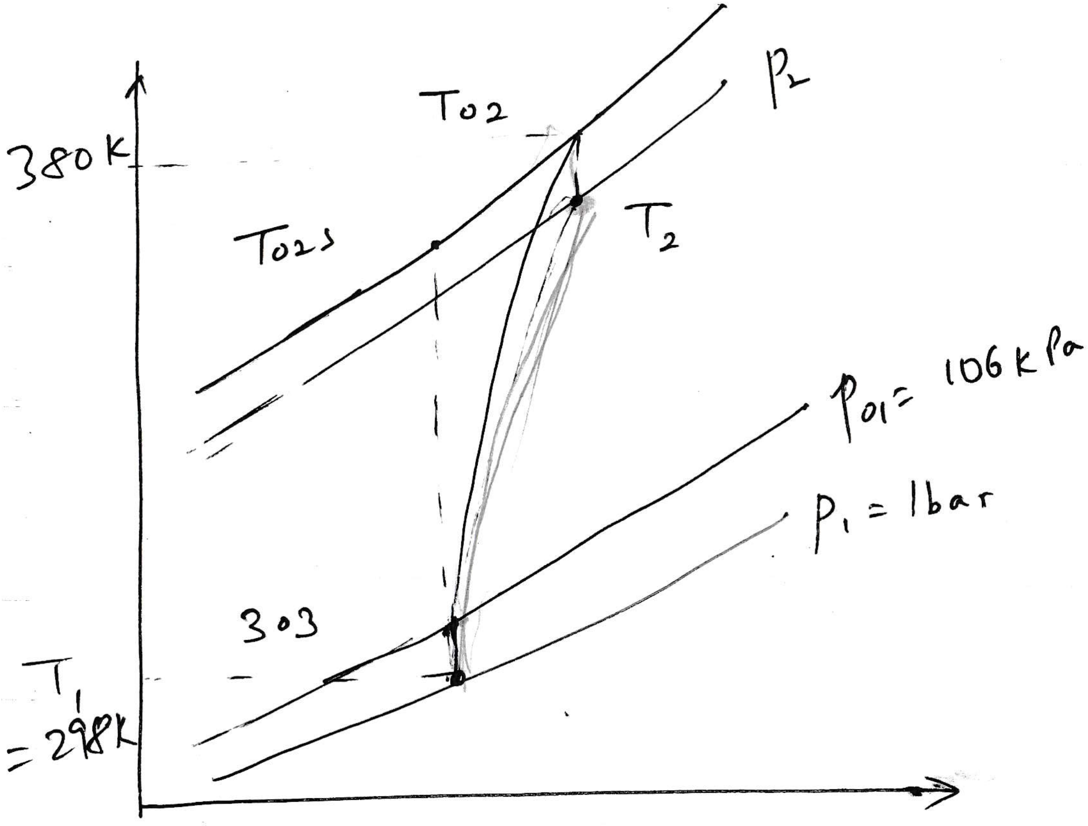
$$= (1.17)(6.28)$$

$$= 7.34 \text{ kg/s}$$

$$P = \dot{m} w$$

$$= (7.34)(77.31)$$

$$\boxed{P = 567.7 \text{ kW}}$$



(4)

$$\cancel{h_{01}} = h_1 + \frac{V_1^2}{2}$$

$$C_p T_{01} = C_p T_1 + \frac{V_1^2}{2}$$

$$T_{01} = T_1 + \frac{V_1^2}{2C_p}$$

$$= 298 + \frac{(100)^2}{(2)(1005)}$$

$$= 303\text{ K}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow$$

$$\frac{P_{01}}{100} = \left(\frac{303}{298}\right)^{\frac{1.4}{0.4}} = 106\text{ kPa.}$$

$$\underbrace{h_1 + \frac{V_1^2}{2}}_{h_{01}} + w = \underbrace{h_2 + \frac{V_2^2}{2}}_{h_{02}}$$

Stagnation enthalpy

$$\longrightarrow h_{01} + w = h_{02}$$

$$w = h_{02} - h_{01}$$

$$= C_p(T_{02} - T_{01})$$

$$T_{02} = T_{01} + \frac{w}{C_p}$$

$$= 303 + \frac{77305}{1005}$$

$$= 380\text{ K}$$

$$\eta = 0.9 = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} \Rightarrow 0.9 = \frac{T_{02s} - 303}{380 - 303} \Rightarrow T_{02s} = 372.3\,K$$

$$\frac{P_{02}}{P_{01}} = \left(\frac{T_{02s}}{T_{01}}\right)^{\gamma/\gamma-1} \Rightarrow \frac{P_{02}}{106} = \left(\frac{372.3}{303}\right)^{\frac{1.4}{0.4}} \Rightarrow P_{02} = 218\,kPa$$

$$h_{02} = h_2 + \frac{V_2^2}{2} \Rightarrow C_p T_{02} = C_p T_2 + \frac{V_2^2}{2}$$

$$\Rightarrow T_2 = T_{02} - \frac{V_2^2}{2C_p}$$

$$= 380 - \frac{(269.4)^2}{(2)(1005)}$$

$$V_2^2 = V_{t_2}^2 + V_{n_2}^2$$

$$= 268.7^2 + 20^2$$

$$= (269.4)^2$$

$$\boxed{T_2 = 343.8\,K}$$

$$\frac{P_2}{P_{02}} = \left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \frac{P_2}{218} = \left(\frac{343.8}{380}\right)^{\frac{1.4}{0.4}} \Rightarrow \boxed{P_2 = 154\,kPa}$$

$$\Delta s = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.005 \ln\frac{343.8}{298} - 0.287 \ln\frac{154}{100}$$

$$= 0.0198\ kJ/kg\,K$$

$$\Delta S = \dot{m}\,\Delta s = (7.34)(0.0198) = 0.145\ kJ/K$$