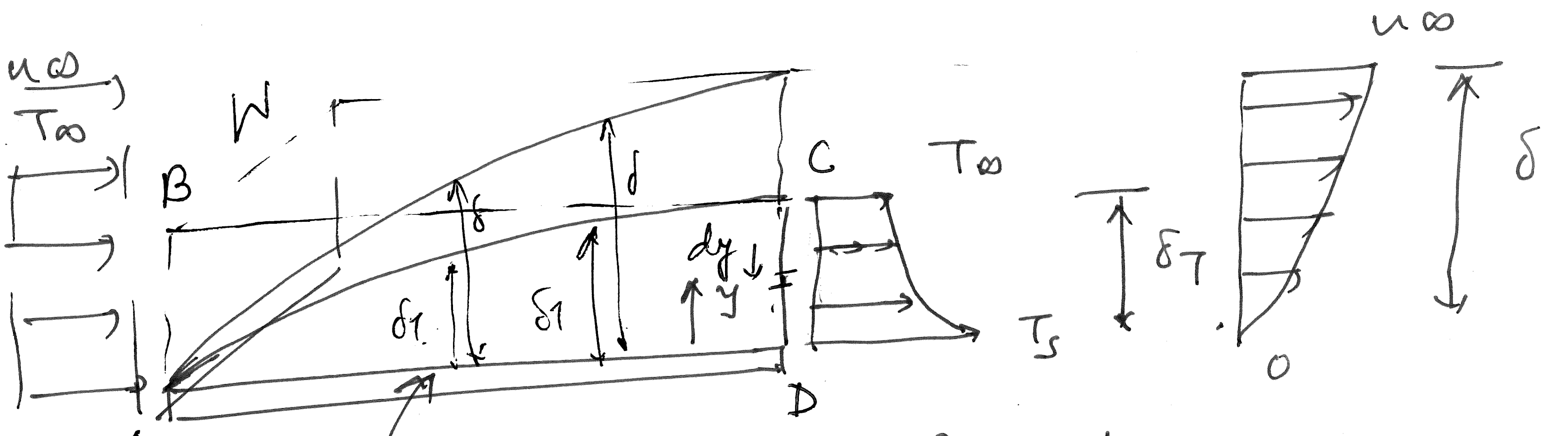
Heat transfer   Integral Momentum method to find Nu.



T_s

$$\dot{E}_{AB} - \dot{E}_{CD} - \dot{E}_{BC} + \dot{Q} = 0$$

$$\dot{E}_{AB} = \dot{m}_{AB} h_{AB} = \underbrace{\rho u_\infty (\delta_T W)}_{\dot{m}} \underbrace{c_p T_\infty}_{h}$$

$$\dot{E}_{CD} = \int h(y)\, d\dot{m} = \int_0^{\delta_T} \underbrace{c_p T(y)}_{h(y)} \underbrace{\rho u(y)(dy\, W)}_{d\dot{m}}$$

$$\dot{E}_{BC} = \dot{m}_{BC} h_{BC} = \dot{m}_{BC} c_p T_\infty$$

$$= \left(\rho u_\infty W \delta_T - \int_0^{\delta_T} \rho u(y)\, dy\, W\right) c_p T_\infty$$

$$= \left(\rho u_\infty W \delta_T c_p T_\infty\right) - c_p T_\infty \rho W \int_0^{\delta_T} u(y)\, dy.$$

Mass Balance

$$\dot{m}_{AB} - \dot{m}_{CD} - \dot{m}_{BC} = 0$$

$$\dot{m}_{AB} = \rho u_\infty W L$$

$$\dot{m}_{CD} = \int \rho u(y)\, dy\, W$$

$$\dot{m}_{BC} = \dot{m}_{AB} - \dot{m}_{CD}$$

$$= \rho u_\infty W \delta_T - \int_0^{\delta_T} \rho u(y)\, dy\, W$$

Subs in $\dot{E}_{AB} - \dot{E}_{CD} - \dot{E}_{BC} + \dot{Q} = 0$

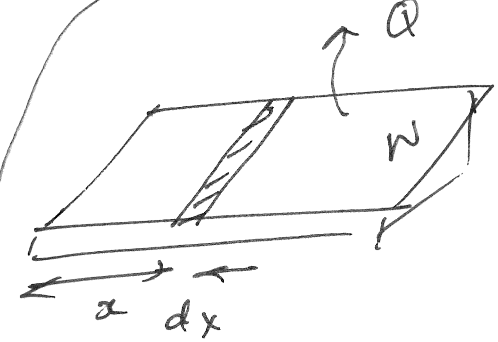
$$\Rightarrow \quad \dot{Q} = \dot{E}_{CD} + \dot{E}_{BC} - \dot{E}_{AB}$$

$$\dot{Q} = \rho C_p W \int_0^{\delta_T} u(y)\,T(y)\,dy - C_p \rho W \int_0^{\delta_T} T_\infty\, u(y)\,dy$$

$$\dot{Q} = \rho C_p W \int_0^{\delta_T(x)} \left(T(y) - T_\infty\right) u(y)\,dy$$ ← total heat transfer rate for $x=0$ to x

<u>Procedure</u>: 1) Assume a profile for $u(y)$ and $T(y)$ and solve for $\dot{Q}$

Compute $\dot{q} = \frac{1}{W}\frac{d\dot{Q}}{dx}$ ← heat flux



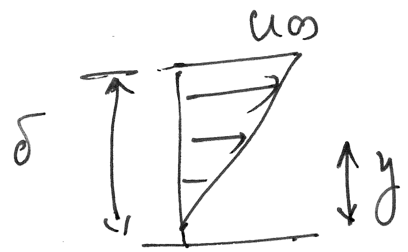
$$\dot{Q} = \int q(x)\, W\, dx$$

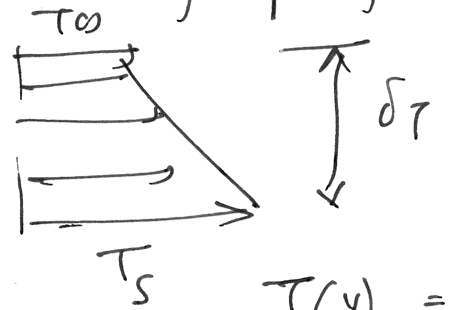
$$\Rightarrow \quad q(x) = \frac{1}{W}\frac{d\dot{Q}}{dx}$$

2) $\dot{q} = -k \left.\frac{dT}{dy}\right|_{y=0}$

(3) Equate the two $\dot{q}$

Example: Linear velocity profile / Linear temp profile





$$u(y) = u_\infty \frac{y}{\delta}$$

$$T(y) = T_s - \frac{y}{\delta_T}(T_s - T_\infty) \quad \text{or} \quad \frac{T(y) - T_\infty}{T_s - T_\infty} = \left(1 - \frac{y}{\delta_T}\right)$$

$$\dot{Q} = \rho c_p W \int_0^{\delta_T(x)} (T(y) - T_\infty)\, u(y)\, dy$$

$$= \rho c_p W \int_0^{\delta_T(x)} (T_s - T_\infty)\left(1 - \frac{y}{\delta_T}\right) u_\infty \frac{y}{\delta}\, dy$$

$$= \rho c_p W (T_s - T_\infty) u_\infty \int_0^{\delta_T} \frac{y}{\delta}\left(1 - \frac{y}{\delta_T}\right) dy$$

Write $\frac{y}{\delta} = \frac{y}{\delta_T}\frac{\delta_T}{\delta}$ $\quad \boxed{\text{Assume: } \frac{\delta_T}{\delta} \text{ is a constant}}$ $(*)$

$$= \rho c_p W (T_s - T_\infty) u_\infty \int_0^{\delta_T} \frac{y}{\delta_T}\left(\frac{\delta_T}{\delta}\right)\left(1 - \frac{y}{\delta_T}\right) dy$$

$$= \rho c_p W (T_s - T_\infty) u_\infty \beta \int_0^{\delta_T} \frac{y}{\delta_T}\left(1 - \frac{y}{\delta_T}\right) dy \quad \text{where } \boxed{\beta \stackrel{\Delta}{=} \frac{\delta_T}{\delta}}$$

$$\int_0^{\delta_T} \frac{y}{\delta_T}\left(1-\frac{y}{\delta_T}\right) dy = \int_0^{\delta_T}\left(\frac{y}{\delta_T}-\frac{y^2}{\delta_T^2}\right) dy$$

$$= \frac{1}{\delta_T}\frac{y^2}{2} - \frac{y^3}{3\delta_T^2}\Big|_0^{\delta_T}$$

$$= \frac{1}{6}\delta_T$$

$$\dot{Q} = \rho C_p W (T_s - T_\infty) u_\infty \beta \frac{1}{6}\delta_T$$

$\nearrow$ Since β = constant $\frac{d\beta}{dx} = 0$

$$\Rightarrow \quad \dot{q} = \frac{1}{W}\frac{d\dot{Q}}{dx} = \rho C_p (T_s - T_\infty) u_\infty \frac{\beta}{6} \cancel{\frac{\delta_T}{6}} \frac{d\delta_T}{dx}$$

<u>Step2</u>: $\dot{q} = -k\frac{dT}{dy}\Big|_{y=0}$

$$T(y) - T_\infty = (T_s - T_\infty)\left(1-\frac{y}{\delta_T}\right) \Rightarrow \frac{dT}{dy} = (T_s - T_\infty)\left(-\frac{1}{\delta_T}\right)$$

$$\dot{q} = +k(T_s - T_\infty)\frac{1}{\delta_T}$$

<u>Equating</u>: $\dot{q} = \rho C_p \cancel{(T_s - T_\infty)} u_\infty \beta \frac{1}{6}\frac{d\delta_T}{dx} = k\cancel{(T_s - T_\infty)}\frac{1}{\delta_T}$

$$\delta_T \, d\delta_T = \left(\frac{k}{\rho C_p}\right)\frac{1}{u_\infty}\frac{1}{\beta} \, 6\,dx$$

$$= \frac{6\alpha}{u_\infty \beta} dx$$

$$\frac{\delta_T^2}{2} = \frac{6\,\alpha}{\cancel{u\beta}\; u_\infty \beta}\, x \qquad (5)$$

$$\Rightarrow \quad \cancel{\frac{\delta_T}{2}}\quad \delta_T^2 = 12\,\frac{\alpha}{u_\infty \beta}\, x$$

From integral momentum equations from last lecture

$$\delta^2 = 12\,\frac{\nu x}{u_\infty}$$

Dividing

$$\frac{\delta_T^2}{\delta^2} = \frac{\dfrac{\cancel{12}\,\alpha \cancel{x}}{u_\infty \beta}}{\dfrac{\cancel{12}\,\nu \cancel{x}}{u_\infty}} \Rightarrow \left(\frac{\delta_T}{\delta}\right)^2 = \left(\frac{\alpha}{\nu}\right)\frac{1}{\beta}$$

$$\beta^2 = \left(\frac{\alpha}{\nu}\right)\frac{1}{\beta}$$

$$\Rightarrow \quad \beta^3 = \frac{\alpha}{\nu}$$

$$\Rightarrow \quad \beta^3 = Pr^{-1}$$

$$\Rightarrow \quad \beta = Pr^{-1/3}$$

$$\boxed{\frac{\delta_T}{\delta} = Pr^{-1/3} \quad \text{or} \quad \frac{\delta}{\delta_T} = Pr^{1/3}} \quad (*)$$

$$\delta_T^2 = 12 \frac{\alpha}{u_\infty} x \frac{1}{\beta}$$

$$\frac{\delta_T^2}{x^2} = 12 \frac{\alpha}{u_\infty x} \frac{1}{\beta}$$

$$= 12 \frac{\alpha}{\nu} \left(\frac{\nu}{u_\infty x}\right) \frac{1}{\beta}$$

$$= 12\, Pr^{-1}\, Re^{-1}\, Pr^{1/3}$$

$$\left(\frac{\delta_T}{x}\right)^2 = 12\, Pr^{-2/3}\, Re^{-1} \Rightarrow \boxed{\frac{\delta_T}{x} = \sqrt{12}\, Pr^{-1/3}\, Re^{-1/2}}$$

$$Nu = \frac{hx}{k} \quad ; \quad h = \frac{k}{\delta_T} \Rightarrow hx = \frac{kx}{\delta_T}$$

$$\frac{hx}{k} = \frac{x}{\delta_T}$$

$$\boxed{Nu_x = \frac{hx}{k} = \frac{1}{\sqrt{12}} Pr^{1/3} Re^{1/2}}$$

$$\dot{Q} = \rho C_p W (T_s - T_\infty)\, u_\infty\, \beta\, \frac{1}{6}\, \delta_T$$

$$= \rho C_p W (T_s - T_\infty)\, u_\infty\, \beta\, \frac{1}{6} \sqrt{12}\, Pr^{-1/3}\, Re^{-1/2}\, L$$

$$\dot{Q} = \rho C_p (WL)\, u_\infty\, \beta\, \frac{1}{\cancel{36}}\frac{1}{6}\sqrt{12}\,(Pr^{-1/3})(Re^{-1/2})(T_s - T_\infty)$$ 7

$$= \left(\frac{\rho C_p}{k}\right) k \left(\frac{u_\infty L}{\nu}\right)\frac{\nu}{L} \quad '' \quad '' \quad ''$$

$$= \frac{\nu}{\alpha}\, Re\, \frac{k}{L}\, \beta\, \sqrt{12}\,(Pr)^{-1/3}(Re^{-1/2})(T_s - T_\infty)(WL)$$

$$= (Pr)(Pr)^{-1/3}\, Re\, \frac{k}{L}\, \sqrt{12}\,(Pr)^{-1/3}\, Re^{-1/2}\,(T_s - T_\infty)\,\underset{A}{\underbrace{WL}}$$

$$(T_s - T_\infty)\, A$$

$$= \frac{\sqrt{12}}{6}\, \underbrace{Pr^{1/3}\, Re^{1/2}\, \frac{k}{L}}_{h}$$

$$h = \frac{\sqrt{12}}{6} Pr^{1/3} Re^{1/2} \frac{k}{L}$$

$$\dot{Q} = h A (T_s - T_\infty)$$

$$Nu = \frac{hL}{k} = \frac{\sqrt{12}}{\cancel{6}} Pr^{1/3} Re^{1/2}$$