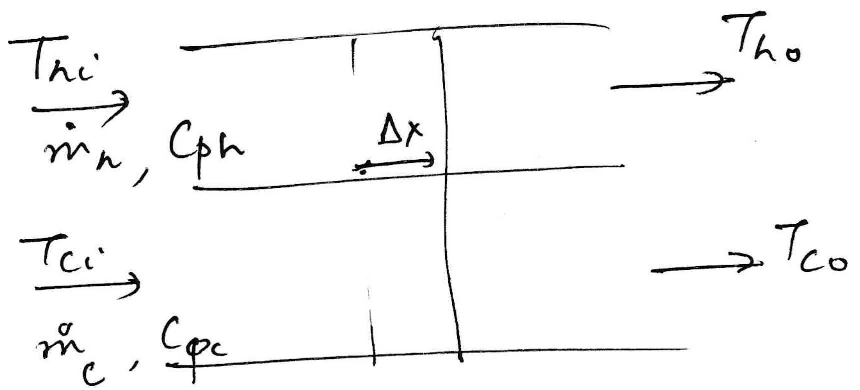
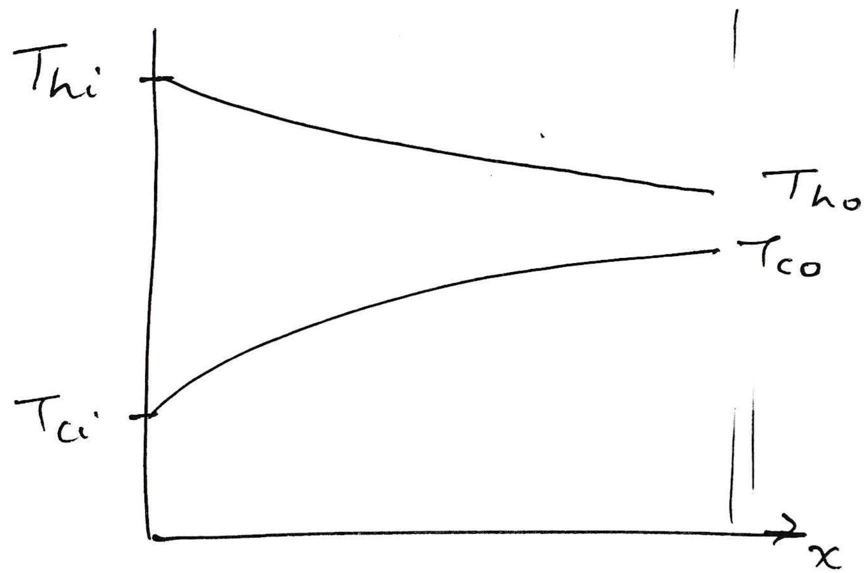
Parallel flow heat exchangers





$$\dot{Q} = \underbrace{\dot{m}_h C_{ph}}_{\parallel \Delta \; C_h}(T_{hi} - T_{ho}) = \underbrace{\dot{m}_c C_{pc}}_{\parallel \Delta \; C_c}(T_{co} - T_{ci})$$

$$\dot{Q} = C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci}).$$

$$\Rightarrow \quad T_{hi} - T_{ho} = \frac{\dot{Q}}{C_h} \quad ; \quad T_{co} - T_{ci} = \frac{\dot{Q}}{C_c}$$

$$\Rightarrow \quad (T_{hi} - T_{ho}) + (T_{co} - T_{ci}) = \dot{Q}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\Rightarrow \quad (T_{hi} - T_{ci}) - (T_{ho} - T_{co}) = \dot{Q}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\Delta T_i - \Delta T_o = \dot{Q}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\Rightarrow \quad \left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \frac{\Delta T_i - \Delta T_o}{\dot{Q}}$$

From Page 9 for previous lecture.

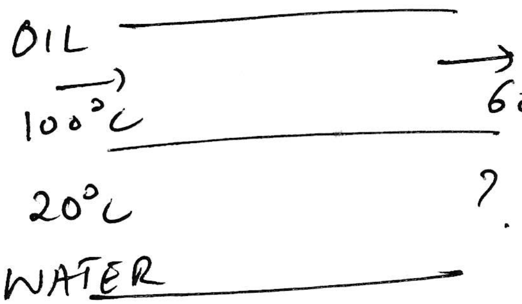
$$\boxed{\ln \frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) UA}$$

$$= - \frac{\Delta T_i - \Delta T_o}{\dot{Q}} UA$$

$$\dot{Q} = UA \frac{\Delta T_i - \Delta T_o}{-\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$= UA \underbrace{\left(\frac{\Delta T_i - \Delta T_o}{\ln \frac{\Delta T_i}{\Delta T_o}} \right)}_{\text{log mean temperature difference (LMTD)}}$$

Oil at 100C is to be cooled to 60C in a parallel-flow heat exchanger by means of water that is available at 20C. The flow rates of the oil and the water are respectively 0.2 kg/s and 0.1 kg/s. The overall heat transfer coefficient is 40 Wm^2K^{-1}. The specific heat of oil is 1.8kJ/kg K. What is the surface area of the heat exchanger?



$\dot{m}_o = 0.2$

$C_{po} = 1.8\ kJ/kgK$

$\dot{m}_w = 0.1$

$C_{pw} = 4.2\ kJ/kgK.$

$$\dot{Q} = \dot{m}_h C_{ph}(T_{hi} - T_{ho}) = \dot{m}_c C_{pc}(T_{co} - T_{ci})$$

$$\dot{Q} = (0.2)(1.8)(100-60) = 14.4\ kW$$

$$= (0.1)(4.2)(T_{co} - 20)$$

$$\Rightarrow T_{co} = 54.3°C.$$

$$\dot{Q} = UA\,(LMTD)$$

$$\Delta T_i = T_{hi} - T_{ci} = 100 - 20 = 80°C$$

$$\Delta T_o = T_{ho} - T_{co} = 60 - 54.3 = 5.7°C$$

$$LMTD = \frac{\Delta T_i - \Delta T_o}{\ln \frac{\Delta T_i}{\Delta T_o}}$$

$$= \frac{80 - 5.7}{\ln\left(\frac{80}{5.7}\right)} = 28.13°C$$

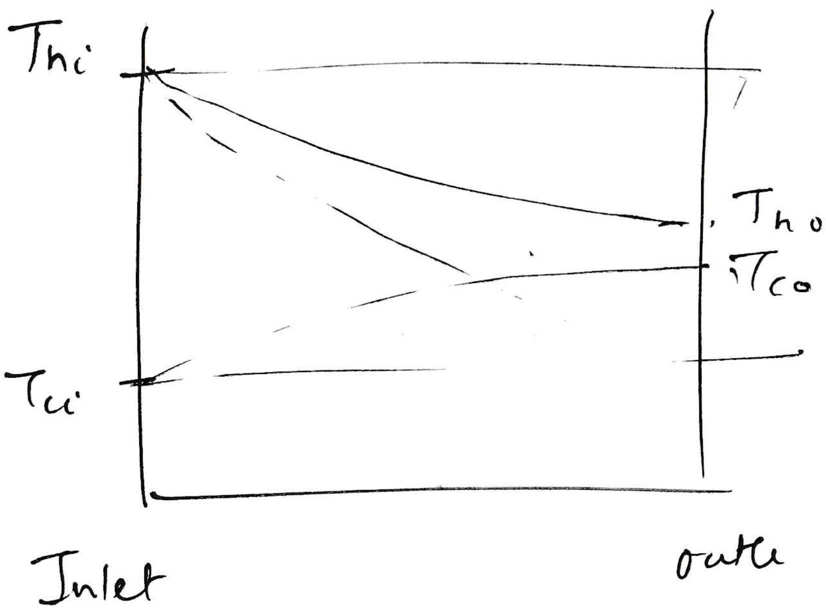
$$(14.4)(10^3) = (40)\,A\,(28.13) \Rightarrow \boxed{A = 12.8\,m^2}$$

Effectiveness - NTU Method

NTU = net transfer units.

$$\text{Effectiveness} = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{\dot{q}}{\dot{q}_{max}}$$

$$\dot{q}_{max} = C_h (T_{hi} - T_{ci})$$



In general

$$\dot{q}_{max} = C_{min} (T_{hi} - T_{ci})$$

where $C_{min} = \min(C_h, C_c)$

$C_{max} = \max(C_h, C_c)$

Define $C_r \triangleq \frac{C_{min}}{C_{max}}$

Effectiveness $\epsilon = \dfrac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \dfrac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})}$

Let $C_{min} = C_c$; $C_{max} = C_h$

$$\epsilon = \frac{C_{max}(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})} = \frac{\cancel{C_{min}}(T_{co} - T_{ci})}{\cancel{C_{min}}(T_{hi} - T_{ci})} \Rightarrow$$

$$\epsilon = \frac{1}{C_r} \cdot \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}$$

From Page 2, $\ln \dfrac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = -\left(\dfrac{1}{C_h} + \dfrac{1}{C_c}\right) UA$

$$= -\left(\frac{1}{C_{max}} + \frac{1}{C_{min}}\right) UA$$

$$= -\frac{1}{C_{min}}\left(\frac{C_{min}}{C_{max}} + 1\right) UA$$

$$= -\left(\frac{UA}{C_{min}}\right)(1 + C_r)$$

Let $NTU \triangleq \dfrac{UA}{C_{min}}$

net transfer unit

$$= -NTU(1 + C_r)$$

$$\Rightarrow \frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = e^{-NTU(1+C_r)}$$

$$LHS = \frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}$$

$$= \frac{T_{ho} - T_{hi} + T_{hi} - T_{ci} + T_{ci} - T_{co}}{T_{hi} - T_{ci}}$$

$$= -\left(\frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}}\right) + 1 - \left(\frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}\right)$$

From page 5, $\frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} = \cancel{\epsilon C_{min}} = \epsilon C_r$; $\frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = \epsilon$

$$LHS = 1 - \epsilon C_r - \epsilon$$

$$= 1 - \epsilon(C_r + 1)$$

$$1 - \epsilon(C_r + 1) = e^{-NTU(1+C_r)} \Rightarrow \boxed{\epsilon = \frac{1 - e^{-NTU(1+C_r)}}{1 + C_r}}$$

$$\boxed{NTU = -\frac{\ln(1 - \epsilon(C_r + 1))}{1 + C_r}}$$

Oil at 100C is to be cooled in a parallel heat exchanger with area 9 m^2 by means of water that is available at 20C. The flow rates of the oil and the water are respectively 0.2 kg/s and 0.1 kg/s. The overall heat transfer coefficient is 40 $Wm^{-2}K^{-1}$. The specific heat of oil is 1.8kJ/kg K. What are the temperatures of the oil and water at their exits?

OIL 100°C $\longrightarrow$ $\dot{m}_o = 0.2$; $C_{ph} = 1.8$

$A = 9\,m^2$

20°C WATER $\longrightarrow$ $\dot{m}_w = 0.1$ $C_{pw} = 4.2$

$C_h = \dot{m}_h C_{ph} = (0.2)(1.8) = 0.36$

$C_c = \dot{m}_c C_{pc} = (0.1)(4.2) = 0.42$

$C_{min} = 0.36$, $C_{max} = 0.42$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{0.36}{0.42} = 0.857$$

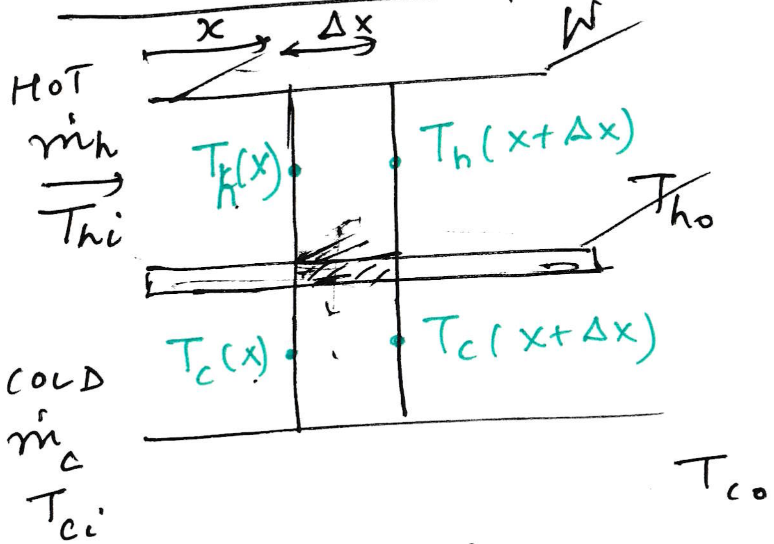
$$NTU = \frac{UA}{C_{min}} = \frac{(40)(9)}{0.36} = 1000$$

$$\epsilon = \frac{1 - e^{-NTU(1+C_r)}}{1+C_r} = \frac{1 - e^{-1000(1+0.857)}}{1+0.857} = 0.539$$

$$\epsilon = \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} \Rightarrow 0.539 = \frac{100 - T_{ho}}{100 - 20} \Rightarrow T_{ho} = 56.9\,°C$$

$$= \frac{(T_{co} - T_{ci})}{C_r(T_{hi} - T_{ci})} \Rightarrow 0.539 = \frac{T_{co} - 20}{0.857(100-20)} \Rightarrow T_{co} = 56.9\,°C$$

Parallel flow heat exchanger



i = inlet , o = outlet

$$\dot Q = \underbrace{\dot m_h C_{ph}}_{C_h}(T_{hi} - T_{ho}) = \underbrace{\dot m_c C_{pc}}_{C_c}(T_{co} - T_{ci})$$

Define $\quad C_h \triangleq \dot m_h C_{ph}\ ; \quad C_c \triangleq \dot m_c C_{pc}$

Balance for hot fluid.

$$\cancel{\dot m_h c}.\ C_h T_h(x) - C_h T_h(x+\Delta x) - \delta \dot q = 0$$

$$-\delta \dot q = C_h\left(T_h(x+\Delta x) - T_h(x)\right)$$

$$= C_h\left[T_h(x) + \frac{dT_h}{dx}\Delta x - T_h(x)\right]$$

$$= C_h \frac{dT_h}{dx}\Delta x \rightarrow (1)$$

Cold fluid: $\quad C_c T_c(x) - C_c T_c(x+\Delta x) + \delta \dot q = 0$

$$\delta \dot q = C_c\left(T_c(x+\Delta x) - T_c(x)\right)$$

$$= C_c \frac{dT_c}{dx}\Delta x \rightarrow (2)$$

$$\left.\begin{aligned} (1) &\Rightarrow \quad \frac{dT_h}{dx}\Delta x = \frac{-\delta \dot q}{C_h} \\ (2) &\Rightarrow \quad \frac{dT_c}{dx}\Delta x = \frac{\delta \dot q}{C_c} \end{aligned}\right\} \xrightarrow[\text{(2) from (1)}]{\text{Subtract}} \left(\frac{dT_h}{dx} - \frac{dT_c}{dx}\right)\Delta x = -\delta \dot q\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

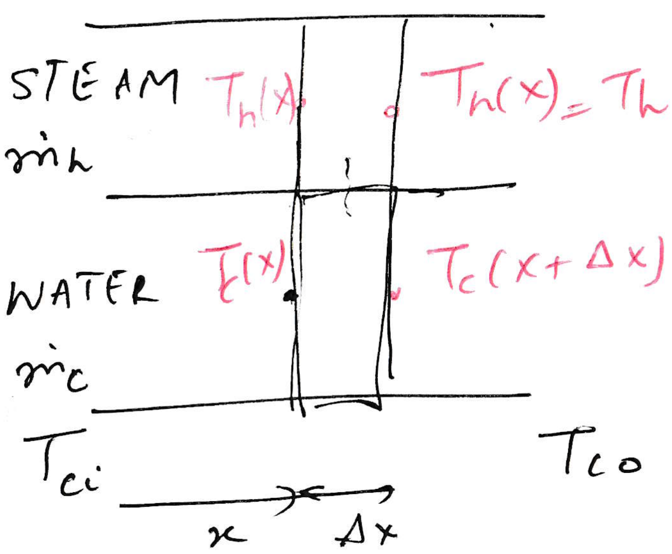
$$\delta \dot{q} = U (W \Delta x)\left(T_h(x) - T_c(x)\right)$$

$$\frac{d}{dx}(T_h - T_c)\,\cancel{\Delta} x = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) U W \cancel{\Delta x} \left(T_h(x) - T_c(x)\right)$$

$$\int \frac{d(T_h - T_c)}{(T_h - T_c)} = -\int_0^L \left(\frac{1}{C_h} + \frac{1}{C_c}\right) U W \, dx$$

$$\ln(T_h - T_c)\Big|_{\text{inlet}}^{\text{outlet}} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) U \underline{WL}$$

$$\ln \frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) U A$$



$$\dot{Q} = \dot{m}_h h_{fg} = \dot{m}_c C_{pc}(T_{co} - T_{ci})$$

$$\dot{Q} = \dot{m}_h h_{fg} = C_c(T_{co} - T_{ci})$$

<u>Balance on cold fluid</u>

$$C_c T_c(x) - C_c(T_c(x+\Delta x)) + \delta\dot{q} = 0$$

$$\delta\dot{q} = C_c\left(T_c(x+\Delta x) - T_c(x)\right)$$

$$= C_c \frac{dT_c}{dx}\Delta x$$

But $\delta\dot{q} = U(\Delta x W)(T_h - T_c(x))$

$$U \cancel{\Delta x} W(T_h - T_c(x)) = C_c \frac{dT_c}{dx}\cancel{\Delta x}$$

$$\int_{x=0}^{x=L} \frac{UW}{C_c}\,dx = \int_{\text{inlet}}^{\text{outlet}} \frac{dT_c}{T_h - T_c(x)} \Rightarrow \frac{UWL}{C_c} = -\ln\left(T_h - T_c(x)\right)\Big|_{\text{in}}^{\text{out}}$$

$$\Rightarrow \ln\left(\frac{T_h - T_{co}}{T_h - T_{ci}}\right) = -\frac{UA}{C_c} = -\frac{UA}{C_{min}} = -NTU.$$

$$\Rightarrow \frac{T_h - T_{co}}{T_h - T_{ci}} = e^{NTU}.$$

$$\frac{T_h - T_{ci} + T_{ci} - T_{co}}{T_h - T_{ci}} = e^{-NTU}.$$

$$1 - \frac{T_{co} - T_{ci}}{T_h - T_{ci}} = e^{-NTU}$$

$$1 - \epsilon = e^{-NTU} \rightarrow \boxed{\epsilon = 1 - e^{-NTU}}$$

TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation		
Parallel flo	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflo	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\mathrm{NTU})$		(11.35a)

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation		
Parallel flo	$\mathrm{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$		(11.28b)
Counterflo	$\mathrm{NTU} = \dfrac{1}{C_r - 1} \ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$	$(C_r < 1)$	
	$\mathrm{NTU} = \dfrac{\varepsilon}{1 - \varepsilon}$	$(C_r = 1)$	(11.29b)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1 + C_r^2)^{-1/2} \ln\left(\dfrac{E - 1}{E + 1}\right)$		(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$		(11.30c)
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with		
	$\varepsilon_1 = \dfrac{F - 1}{F - C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$		(11.31b, c, d)
Cross-flow (single pass			
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$		(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$		(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1 - \varepsilon)$		(11.35b)