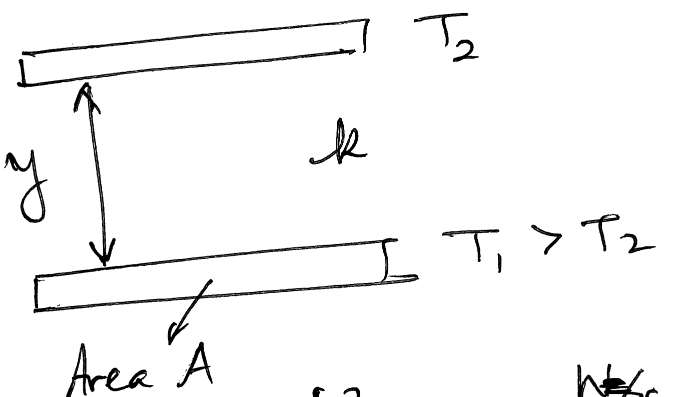
Transport.

1) Heat transport
2) Mass transport
3) Momentum transport.

→ Molecular transport
→ Convective transport.

①

Heat transport by conduction



$$\dot{Q} = kA\frac{T_1 - T_2}{y}$$

$$\dot{q} = \frac{\dot{Q}}{A} = -k\left(\frac{T_2 - T_1}{y}\right)$$

$$\dot{q} = -k\frac{dT}{dy}$$ ← Fourier's law of conduction

$$[k] = \frac{[\dot{q}]}{[dT/dy]} = \frac{W/m^2}{K/m} = \frac{W}{K\,m} = W\,K^{-1}\,m^{-1}$$

$$\dot{q} = \frac{-k}{\rho c_p}\frac{d}{dy}(\rho c_p T)$$

Let $\alpha \stackrel{\Delta}{=} \dfrac{k}{\rho c_p}$ ← Thermal diffusivity.

$$\dot{q} = -\alpha\frac{d}{dy}(\rho c_p T)$$

$\dot{q}$: Rate of energy flow per unit area

$\rho c_p T$: Energy / Enthalpy per unit volume

$$\frac{W}{m^2} = [\alpha]\frac{J/m^3}{m}$$

$$\frac{J}{s\,m^2} = [\alpha]\frac{J}{m^4} \Rightarrow \boxed{[\alpha] = \frac{m^2}{s}}$$

Momentum transfer

$$F = \mu \frac{V}{y} A \Rightarrow \frac{F}{A} = \mu \frac{V}{y}$$

Dynamic viscosity μ

$$\Rightarrow \tau = \mu \frac{V}{y}$$

$$\Rightarrow \boxed{\tau = -\mu \frac{du}{dy}}$$

$$\tau = -\frac{\mu}{\rho} \frac{d(\rho u)}{dy}$$

Momentum per volume

$$[\tau] = [\mu] \left[\frac{du}{dy}\right]$$

$$Pa = [\mu] \frac{m}{s} \cdot \frac{1}{m}$$

$$[\mu] = Pa.s$$

$[\cancel{\mu}] :$ $\mu_{water} \sim 10^{-3}$ Pas

Define $\nu \triangleq \frac{\mu}{\rho}$ Kinematic viscosity.

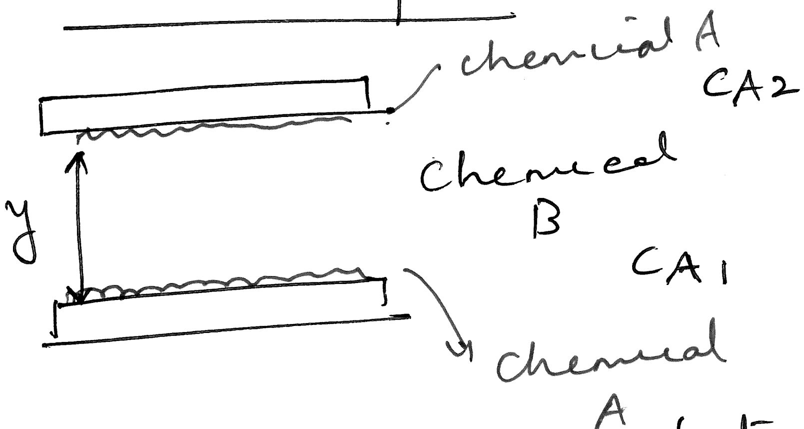
$$\boxed{\tau = -\nu \frac{d}{dy}(\rho u)}$$

$$[\nu] = \frac{[\mu]}{[\rho]} = \frac{Pa \cdot s}{kg/m^3} = \frac{N}{m^2} \frac{s}{kg/m^3}$$

$$= \left(\cancel{kg} \frac{m}{s^{\cancel{2}}}\right) \frac{1}{m^2} \cdot \frac{\cancel{s}}{\frac{\cancel{kg}}{m^3}} = \frac{m^2}{s}$$

$$[\nu] = \frac{m^2}{s}$$

Mass transfer



C_{A1} , C_{A2} = concentrations of chemical A

$$\dot{m}_A = D_{AB}\,\frac{C_{A1} - C_{A2}}{y}\,A$$

$\downarrow$

Diffusion coefficient depends on both A and B

$$\frac{\dot{m}_A}{A} = D_{AB}\,\frac{C_{A1} - C_{A2}}{y}$$

$$j_A = \text{mass flux} \triangleq \frac{\dot{m}_A}{A}$$

$$\boxed{j_A = -D_{AB}\,\frac{dc_A}{dy}}$$

$$[j_A] = [D_{AB}]\,\frac{[dc_A]}{[dy]}$$

$$\frac{\cancel{kg}}{s\,m^2} = [D_{AB}]\,\frac{\cancel{kg}}{m^3}\cdot\frac{1}{m}$$

$$[D_{AB}] = \frac{m^2}{s}$$

Heat transfer	Momentum transfer	Mass transfer
$\dot{q} = -\frac{k}{\rho c_p}\frac{d}{dy}(\rho c_p T)$	$\tau = -\mu \frac{du}{dy}$	$j_A = -D_{AB}\frac{dc_A}{dy}$
$= -\alpha \frac{d}{dy}(\rho c_p T)$	$= -\cancel{\mu} - \nu \frac{d}{dy}(\rho u)$	

$$\text{Flux} = -(\text{transport quantity})(\text{Gradient of a quantity})$$

$\downarrow$
energy
momentum
mass

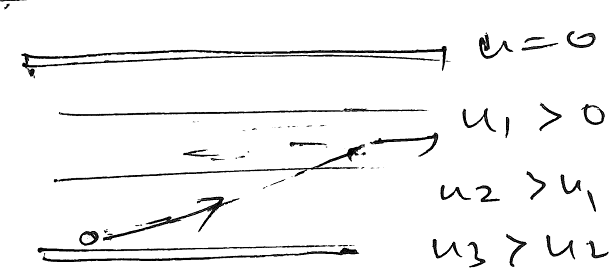
Transport quantity	α	ν	D_{AB}
	m^2/s	m^2/s	m^2/s

$$\boxed{Pr \triangleq \frac{\nu}{\alpha}}$$

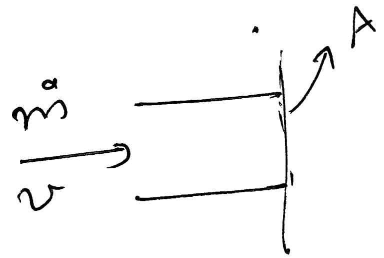
Prandtl number

$$\boxed{Sc \triangleq \frac{\nu}{D_{AB}}}$$

Schmidt Number

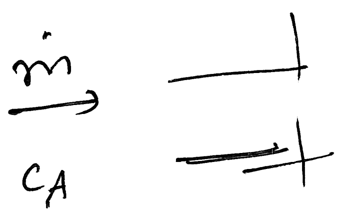


Convective transport



$$\dot{E} = \dot{m} h$$
$$= \dot{m} C_p T$$
$$= \rho A v C_p T$$

$$\boxed{\dot{q}_{conv} = \frac{\dot{E}}{A} = (\rho C_p T)\, v}$$



$$\dot{m}_A = \cancel{\rho} A v C_A$$

$$\boxed{\dot{j}_{conv} = \frac{\dot{m}_A}{A} = (C_A)\, v}$$

$$\dot{M} = \dot{m} v$$
$$= (\rho A v)\, v$$

$$\boxed{\tau = \frac{\dot{M}}{A} = \rho v^2}$$