$\ddot{x} + 2\dot{x} - 3x = 0 \qquad x(0) = 1 \;;\; \dot{x}(0) = 1$

Let $x(t) = e^{\lambda t} \Rightarrow \dot{x}(t) = \lambda e^{\lambda t} \Rightarrow \ddot{x}(t) = \lambda^2 e^{\lambda t}$

$\lambda^2 e^{\lambda t} + 2\lambda e^{\lambda t} - 3e^{\lambda t} = 0 \Rightarrow \lambda^2 + 2\lambda - 3 = 0$

$\Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda+3) - 1(\lambda+3) = 0$

$\Rightarrow \lambda = -3, \; \lambda = 1$

General solution: $x(t) = c_1 e^{-3t} + c_2 e^{t} \Rightarrow \dot{x}(t) = -3c_1 e^{-3t} + c_2 e^{t}$

Initial Condition: $x(0) = 1 = c_1 + c_2 \;;\; \dot{x}(0) = 1 = -3c_1 + c_2$

$$\begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow c_1 = \frac{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}} = 0 \qquad c_2 = \frac{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}} = 1$$

$x(t) = 0e^{-3t} + 1e^{t} = e^{t}$

0

$$\ddot{x} + 2\dot{x} - 3x = 0$$

$$\ddot{x} \rightarrow \dot{x}_2, \quad \dot{x} \rightarrow x_2, \quad x \rightarrow x_1$$

$$x = x_1 \Rightarrow \dot{x} = \boxed{\dot{x}_1 = x_2}$$

Since $\dot{x} = x_2$, $\Rightarrow \ddot{x} = \dot{x}_2$

$$\dot{x}_2 + 2x_2 - 3x_1 = 0 \Rightarrow \boxed{\dot{x}_2 = 3x_1 - 2x_2}$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 3x_1 - 2x_2 \end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Let $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_{10} e^{\lambda t} \\ x_{20} e^{\lambda t} \end{bmatrix} = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} e^{\lambda t} \Rightarrow \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_{10} \lambda e^{\lambda t} \\ x_{20} \lambda e^{\lambda t} \end{bmatrix} = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \lambda e^{\lambda t}$

$$\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \lambda \cancel{e^{\lambda t}} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \cancel{e^{\lambda t}}$$

$$\lambda \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \right) \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda & -1 \\ -3 & \lambda + 2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For nontrivial $\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$, $\begin{vmatrix} \lambda & -1 \\ -3 & \lambda+2 \end{vmatrix} = 0 \Rightarrow \lambda(\lambda+2) - 3 = 0$ (3)

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$

$\Rightarrow \lambda = -3, \lambda = 1$

$\lambda = -3$ $\begin{bmatrix} -3 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -3x_{10} - x_{20} = 0$

Let $x_{10} = 1 \Rightarrow x_{20} = -3$

Solution $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-3t}$ ✓

$\lambda = 1$ $\begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_{10} - x_{20} = 0$

Let $x_{10} = 1 \Rightarrow x_{20} = 1$

Solution: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}$ ✓

general Solution: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}$

Initial condition $x(0) = 1$; $\dot{x}(0) = 1 \Rightarrow x_1(0) = 1$; $x_2(0) = 1$

$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

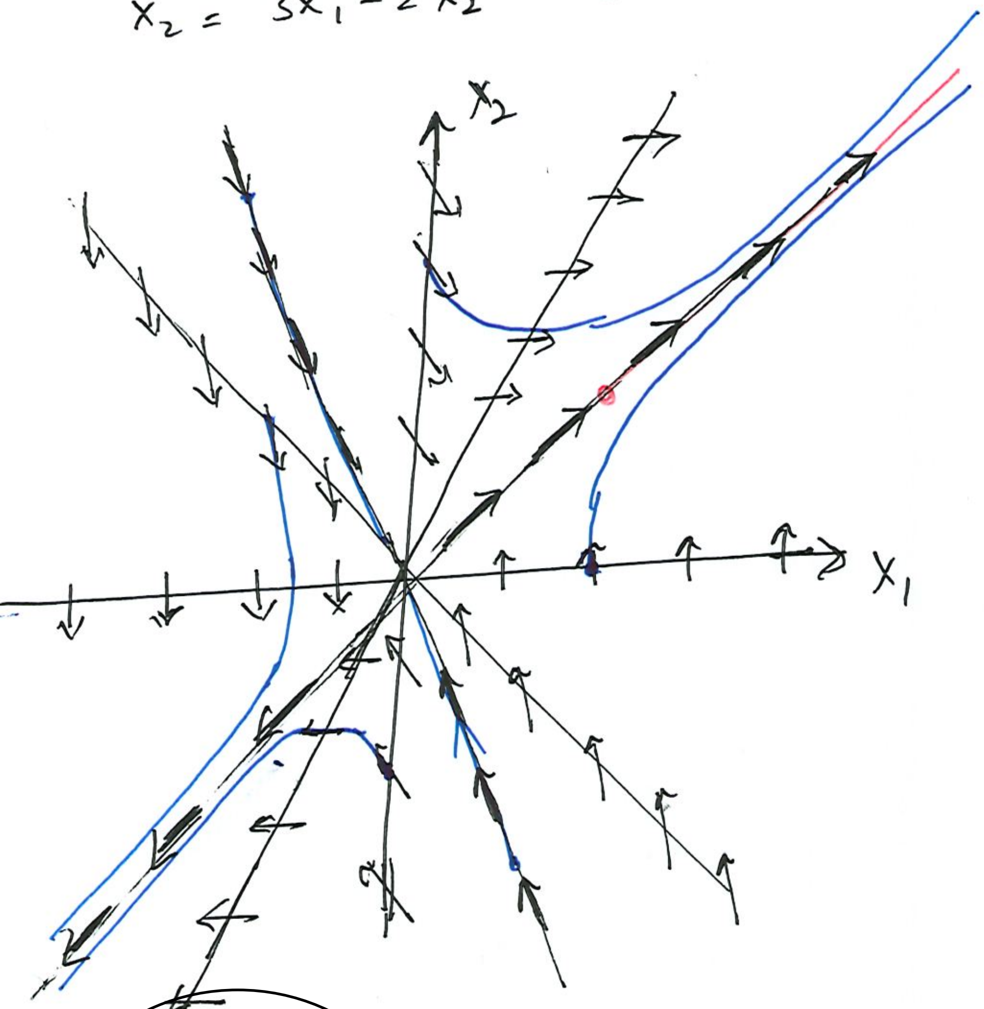
2

Subs IC in general Solution:

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow c_2 = 1 \Rightarrow \boxed{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}}$

Method of isoclines

$$\left.\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= 3x_1 - 2x_2\end{aligned}\right\} \quad \frac{dx_2}{dx_1} = \frac{dx_2}{dt}\frac{dt}{dx_1} = \frac{\dot{x}_2}{\dot{x}_1} = \frac{3x_1 - 2x_2}{x_2}$$



1) Along x_2-axis ; $x_1 = 0$

$$\frac{dx_2}{dx_1} = -2$$

2) Along x_1-axis : $x_2 = 0$

$$\frac{dx_2}{dx_1} \to \infty$$

3) Where is $\frac{dx_2}{dx_1} = 0$

$$\frac{dx_2}{dx_1} = 0 \Rightarrow 3x_1 - 2x_2 = 0 \Rightarrow x_2 = \frac{3}{2}x_1$$

(4) Along $x_1 = x_2$

$$\frac{dx_2}{dx_1} = \frac{3x_1 - 2x_1}{x_1} = 1$$

(5) Along $x_1 = -x_2$

$$\frac{dx_2}{dx_1} = \frac{-3x_2 - 2x_2}{x_2} = -5$$

$\cancel{x = -3x}$ $x_2 = -3x_1$

$$\frac{dx_2}{dx_1} = \frac{3x_1 + 6x_1}{-3x_1} = -3$$

(6) Is there a line with slope m such that $\frac{dx_2}{dx_1} = m$?

$$\frac{dx_2}{dx_1} = m = \frac{3x_1 - 2mx_1}{mx_1} \Rightarrow m^2 = 3 - 2m$$

$$\Rightarrow m^2 + 2m - 3 = 0$$

$$\Rightarrow m = -3,\ m = 1 \checkmark$$

$\ddot{x} + 2\dot{x} + 5x = 0 \qquad x(0) = 1 \qquad \dot{x}(0) = 1$

Let $x = e^{\lambda t} \Rightarrow \dot{x} = \lambda e^{\lambda t} \Rightarrow \ddot{x} = \lambda^2 e^{\lambda t}$

$$\lambda^2 + 2\lambda + 5 = 0 \Rightarrow \lambda = \frac{-2 \pm \sqrt{4 - (4)(5)}}{2} = \frac{-2 \pm \sqrt{(-16)}}{2}$$

$$= \frac{-2 \pm \sqrt{(-1)(16)}}{2}$$

$$= \frac{-2 \pm 4j}{2} = -1 \pm 2j$$

$$x(t) = c_1 e^{(-1+2j)t} + c_2 e^{(-1-2j)t}$$

$$= e^{-t}\left(c_1 e^{2jt} + c_2 e^{-2jt}\right)$$

$$= e^{-t}\left[\left(c_1 \cos 2t + j c_1 \sin 2t\right) + \left(c_2 \cos 2t - j c_2 \sin 2t\right)\right]$$

$$= e^{-t}\left[\underbrace{(c_1 + c_2)}_{A} \cos 2t + \underbrace{j(c_1 - c_2)}_{B} \sin 2t\right]$$

$$x(t) = e^{-t}\left[A \cos 2t + B \sin 2t\right] \Rightarrow x(0) = 1 = A \Rightarrow \boxed{A = 1}$$

$$\dot{x}(t) = e^{-t}\left[-2A \sin 2t + 2B \cos 2t\right] - e^{-t}\left[A \cos 2t + \cancel{2B \cos 2t}\ B \sin 2t\right]$$

$$\dot{x}(0) = 1 = 2B - A \Rightarrow 1 = 2B - 1 \Rightarrow \boxed{B = 1}$$

$$x(t) = e^{-t}\left(1 \cos 2t + 1 \sin 2t\right)$$

$$= e^{-t}\left(\sqrt{2} \cos \pi/4 \cos 2t + \sqrt{2} \sin \pi/4 \sin 2t\right)$$

$$= \sqrt{2}\, e^{-t} \cos\left(2t - \pi/4\right)$$

4

Let $1 = C\cos\phi$

$1 = C\sin\phi$

$1^2 + 1^2 = C^2(\cos^2\phi + \sin^2\phi)$

$\Rightarrow C = \sqrt{2}$

$\tan\phi = 1$

$\Rightarrow \phi = \pi/4.$

$$\boxed{x(t) = \sqrt{2}\, e^{-t} \cos(2t - \pi/4)}$$

5

1

$\ddot{x} + 2\dot{x} - 3x = 0 \qquad x(0) = 1 \;;\; \dot{x}(0) = 1$

Let $x(t) = e^{\lambda t} \Rightarrow \dot{x}(t) = \lambda e^{\lambda t} \Rightarrow \ddot{x}(t) = \lambda^2 e^{\lambda t}$

$\lambda^2 e^{\lambda t} + 2\lambda e^{\lambda t} - 3 e^{\lambda t} = 0 \Rightarrow \lambda^2 + 2\lambda - 3 = 0$

$\Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda+3) - 1(\lambda+3) = 0$

$\Rightarrow \lambda = -3,\ \lambda = 1$

General solution: $x(t) = c_1 e^{-3t} + c_2 e^{t} \Rightarrow \dot{x}(t) = -3c_1 e^{-3t} + c_2 e^{t}$

Initial Conditions $x(0) = 1 = c_1 + c_2 \;;\; \dot{x}(0) = 1 = -3c_1 + c_2$

$$\begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow c_1 = \frac{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}} = 0 \qquad c_2 = \frac{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}} = 1$$

$x(t) = 0e^{-3t} + 1e^{t} = e^{t}$

6

$$\ddot{x} + 2\dot{x} - 3x = 0 \qquad x = x_1 \Rightarrow \dot{x} = \boxed{\dot{x}_1 = x_2}$$

$$\ddot{x} \to \dot{x}_2, \quad \dot{x} \to x_2, \quad x \to x_1$$

Since $\dot{x} = x_2$, $\Rightarrow \ddot{x} = \dot{x}_2$

$$\dot{x}_2 + 2x_2 - 3x_1 = 0 \Rightarrow \boxed{\dot{x}_2 = 3x_1 - 2x_2}$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 3x_1 - 2x_2 \end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Let $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_{10} e^{\lambda t} \\ x_{20} e^{\lambda t} \end{bmatrix} = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} e^{\lambda t} \Rightarrow \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_{10} \lambda e^{\lambda t} \\ x_{20} \lambda e^{\lambda t} \end{bmatrix} = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \lambda e^{\lambda t}$

$$\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \lambda \cancel{e^{\lambda t}} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \cancel{e^{\lambda t}}$$

$$\lambda \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \right) \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda & -1 \\ -3 & \lambda + 2 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

7

3

For nontrivial $\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$, $\begin{vmatrix} \lambda & -1 \\ -3 & \lambda+2 \end{vmatrix} = 0 \Rightarrow \lambda(\lambda+2) - 3 = 0$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$

$\Rightarrow \lambda = -3, \ \lambda = 1$

$\underline{\lambda = -3}$ $\quad \begin{bmatrix} -3 & -1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -3x_{10} - x_{20} = 0$

Let $x_{10} = 1 \Rightarrow x_{20} = -3$

Solution $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-3t}$ ✓

$\underline{\lambda = 1}$ $\quad \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_{10} - x_{20} = 0$

Let $x_{10} = 1 \Rightarrow x_{20} = 1$

<u>Solution:</u> $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}$ ✓

<u>general Solution</u>: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}$

<u>Initial condition</u> $x(0) = 1$; $\dot{x}(0) = 1 \Rightarrow x_1(0) = 1$; $x_2(0) = 1$

$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

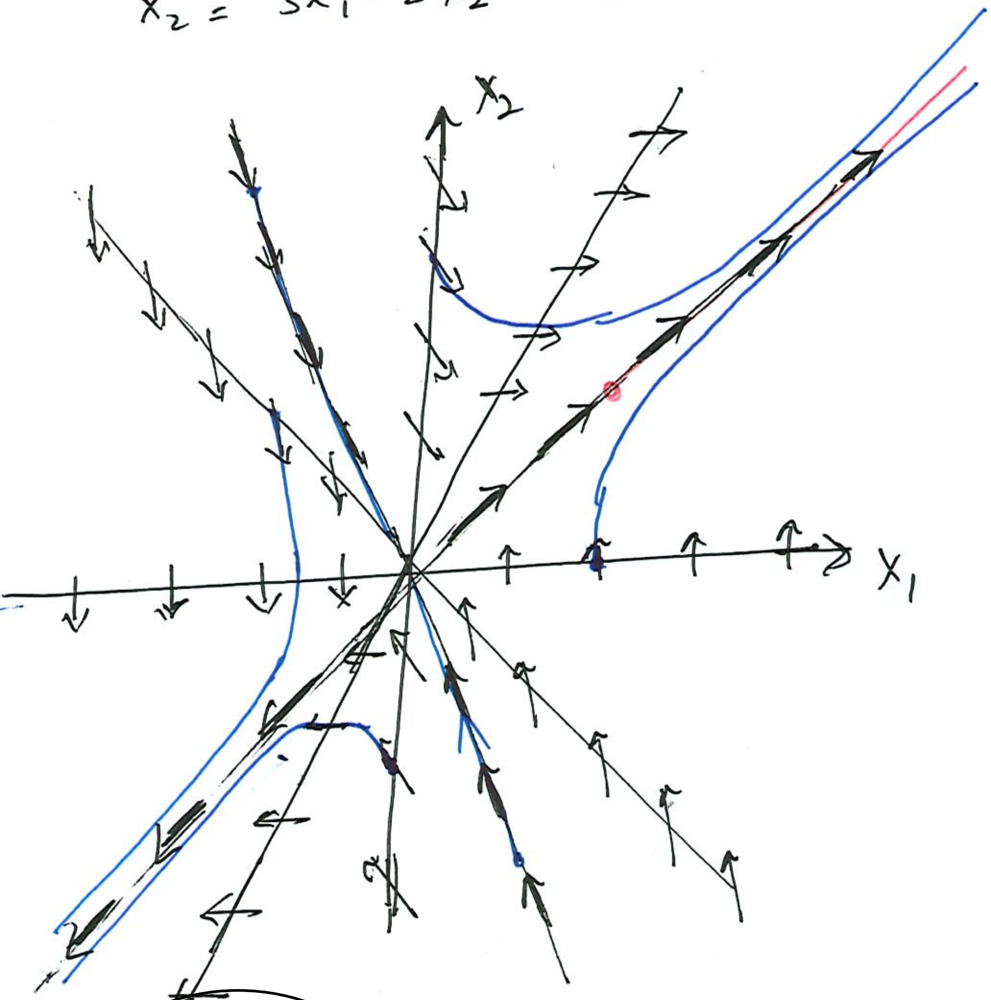
8

Subs IC in general Solution:

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow c_2 = 1 \Rightarrow \boxed{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}}$

Method of isoclines.

$$\left.\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= 3x_1 - 2x_2\end{aligned}\right\} \quad \frac{dx_2}{dx_1} = \frac{dx_2}{dt}\frac{dt}{dx_1} = \frac{\dot{x}_2}{\dot{x}_1} = \frac{3x_1 - 2x_2}{x_2}$$



1) Along x_2-axis ; $x_1 = 0$

$$\frac{dx_2}{dx_1} = -2$$

2) Along x_1-axis : $x_2 = 0$

$$\frac{dx_2}{dx_1} \to \infty$$

3) Where is $\frac{dx_2}{dx_1} = 0$

$$\frac{dx_2}{dx_1} = 0 \Rightarrow 3x_1 - 2x_2 = 0 \Rightarrow x_2 = \frac{3}{2}x_1$$

(4) Along $x_1 = x_2$

$$\frac{dx_2}{dx_1} = \frac{3x_1 - 2x_1}{x_1} = \cancel{5}\,1$$

(5) Along $x_1 = -x_2$

$$\frac{dx_2}{dx_1} = \frac{-3x_2 - 2x_2}{x_2} = -5$$

9

$\cancel{x = -3x}$ $x_2 = -3x_1$

$$\frac{dx_2}{dx_1} = \frac{3x_1 + 6x_1}{-3x_1} = -3$$

(6) Is there a line with slope m such that $\frac{dx_2}{dx_1} = m$?

$$\frac{dx_2}{dx_1} = m = \frac{3x_1 - 2mx_1}{mx_1} \Rightarrow m^2 = 3 - 2m \Rightarrow m^2 + 2m - 3 = 0 \Rightarrow m = -3,\ m = 1$$

$\ddot{x} + 2\dot{x} + 5x = 0 \qquad x(0) = 1 \qquad \dot{x}(0) = 1$

Let $x = e^{\lambda t} \Rightarrow \dot{x} = \lambda e^{\lambda t} \Rightarrow \ddot{x} = \lambda^2 e^{\lambda t}$

$\lambda^2 + 2\lambda + 5 = 0 \Rightarrow \lambda = \frac{-2 \pm \sqrt{4 - (4)(5)}}{2} = \frac{-2 \pm \sqrt{(-16)}}{2}$

$= \frac{-2 \pm \sqrt{(-1)(16)}}{2}$

$= \frac{-2 \pm 4j}{2} = -1 \pm 2j$

$x(t) = c_1 e^{(-1+2j)t} + c_2 e^{(-1-2j)t}$

$= e^{-t}\left(c_1 e^{2jt} + c_2 e^{-2jt}\right)$

$= e^{-t}\left[(c_1 \cos 2t + j c_1 \sin 2t) + (c_2 \cos 2t - j c_2 \sin 2t)\right]$

$= e^{-t}\left[\underbrace{(c_1 + c_2)}_{A} \cos 2t + \underbrace{j(c_1 - c_2)}_{B} \sin 2t\right]$

$x(t) = e^{-t}\left[A \cos 2t + B \sin 2t\right] \Rightarrow x(0) = 1 = A \Rightarrow \boxed{A = 1}$

$\dot{x}(t) = e^{-t}\left[-2A \sin 2t + 2B \cos 2t\right] - e^{-t}\left[A \cos 2t + \cancel{2}B \,\cancel{\cos 2t}\, \sin 2t\right]$

$\dot{x}(0) = 1 = 2B - A \Rightarrow 1 = 2B - 1 \Rightarrow \boxed{B = 1}$

$x(t) = e^{-t}(1 \cos 2t + 1 \sin 2t)$

$= e^{-t}\left(\sqrt{2} \cos \pi/4 \cos 2t + \sqrt{2} \sin \pi/4 \sin 2t\right)$

$= \sqrt{2}\, e^{-t} \cos\left(2t - \pi/4\right)$

10

Let $\quad 1 = C\cos\phi \qquad\qquad \tan\phi = 1$

$\quad\quad 1 = C\sin\phi \qquad\qquad \Rightarrow \phi = \pi/4.$

$1^2 + 1^2 = C^2(\cos^2\phi + \sin^2\phi)$

$\Rightarrow C = \sqrt{2}$

$$\boxed{x(t) = \sqrt{2}\, e^{-t} \cos(2t - \pi/4)}$$

5

$\dot{x}_1 = x_2$

When $x_2 > 0 \Rightarrow \dot{x}_1 > 0 \Rightarrow x_1 \uparrow$

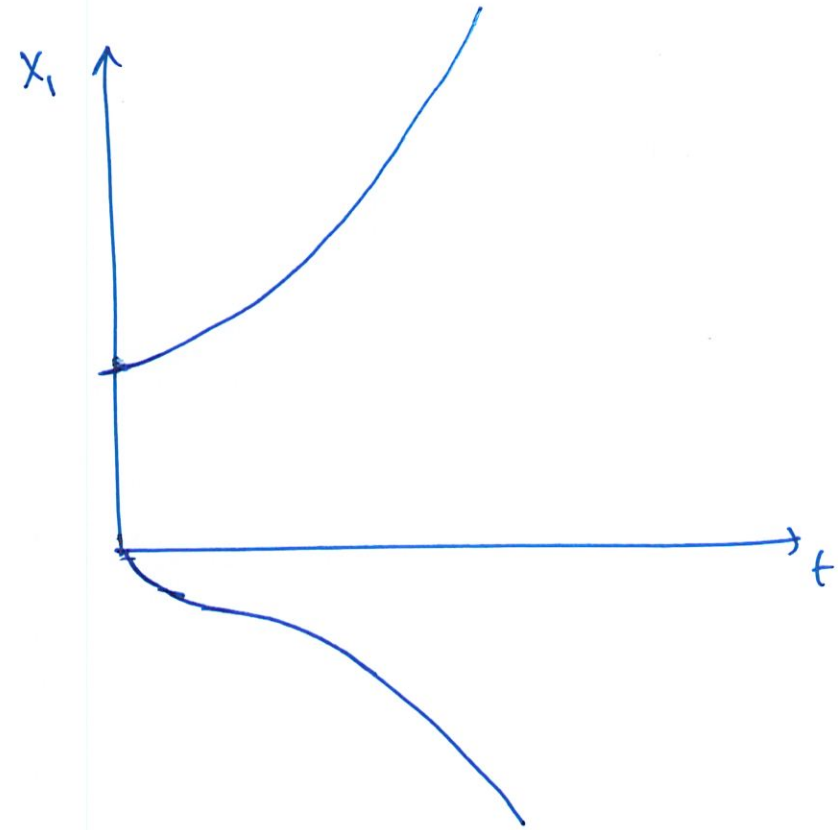
When $x_2 < 0 \Rightarrow \dot{x}_1 < 0 \Rightarrow x_1 \downarrow$

For the tick marks along x_1-axis (ie $x_2 = 0$)

$\dot{x}_2 = 3x_1$

If $x_1 > 0$, $\dot{x}_2 > 0 \Rightarrow x_2 \uparrow$

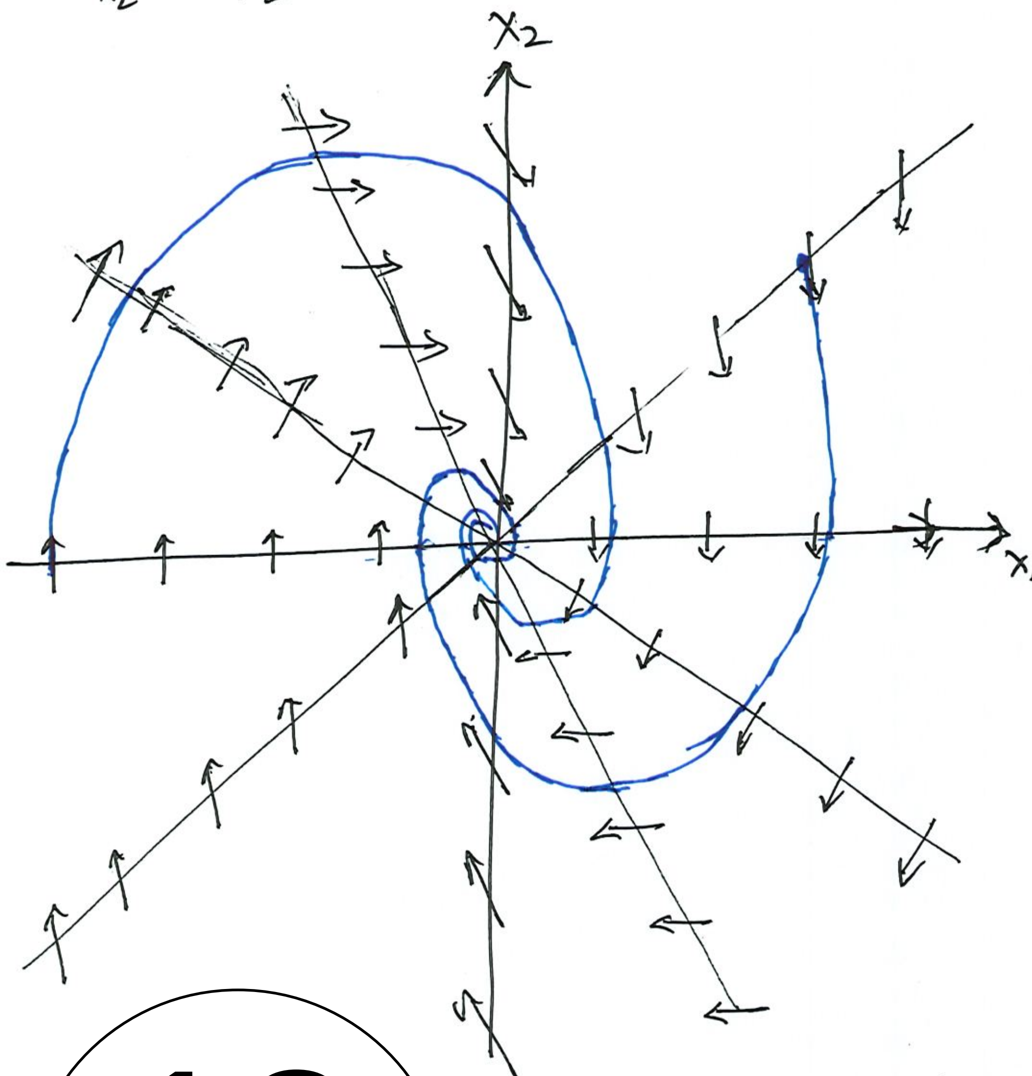
$x_1 < 0$, $\dot{x}_2 < 0 \Rightarrow x_2 \downarrow$



12

$$\underset{\downarrow\, \dot{x}_2}{\ddot{x}} + 2\underset{\downarrow\, x_2}{\dot{x}} + 5\underset{\downarrow\, x_1}{x} = 0$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -5x_1 - 2x_2$$
$$\Rightarrow \frac{dx_2}{dx_1} = \frac{-5x_1 - 2x_2}{x_2}$$



13

$\dot{x}_1 = x_2$ For $x_2 > 0$, $\dot{x}_1 > 0 \Rightarrow x_1 \uparrow$

$x_2 < 0$, $\dot{x}_1 < 0 \Rightarrow x_1 \downarrow$

Along x_1-axis $\dot{x}_2 = -5x_1 - 2\cancel{x_2}^{\,0}$

$\Rightarrow \dot{x}_2 = -5x_1$ For $x_1 > 0$ $\dot{x}_2 < 0 \Rightarrow x_2 \downarrow$

1) Along x_2-axis, $x_1 = 0$

$$\frac{dx_2}{dx_1} = -2$$

2) Along x-axis, $x_2 = 0$

$$\frac{dx_2}{dx_1} \to \infty$$

(3) $\frac{dx_2}{dx_1} = 0 \Rightarrow -5x_1 - 2x_2 = 0$

$\Rightarrow x_2 = -5/2\, x_1$

(4) Along $x_1 = x_2$

$$\frac{dx_2}{dx_1} = -7$$

(5) Along $x_1 = -x_2$

$$\frac{dx_2}{dx_1} = \frac{+5x_2 - 2x_2}{x_2} = 3$$

(6) Is there any $x_2 = mx_1$ s.t $\frac{dx_2}{dx_1} = m$?

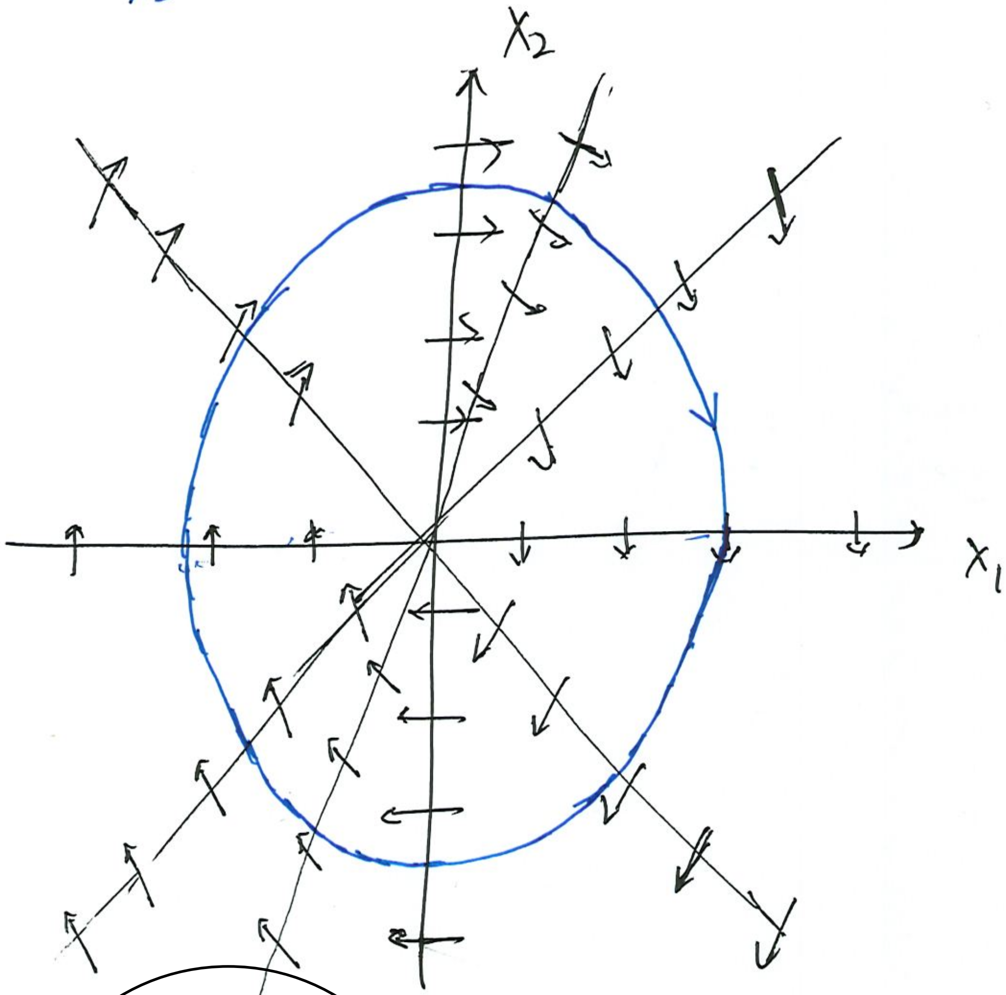
$$m = \frac{-5x_1 - 2mx_1}{mx_1}$$

$$m^2 + 2m + 5 = 0$$

$\to$ no real roots

$\ddot{x} + 4x = 0$ $\quad$ Let $x_1 = x$, $\quad x_2 = \dot{x}$ $\Rightarrow$ $\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -4x_1 \end{aligned}}$

$\ddot{x} \downarrow \dot{x}_2$, $\quad x \downarrow x_1$



$$\frac{dx_2}{dx_1} = \frac{-4x_1}{x_2}$$

1) Along $x_1 = 0$ (x_2-axis)

$$\frac{dx_2}{dx_1} = 0$$

(2) Along $x_2 = 0$, $\frac{dx_2}{dx_1} \to \infty$

(3) Along $x_1 = x_2$

$$\frac{dx_2}{dx_1} = -4$$

(4) Along $x_1 = -x_2$

$$\frac{dx_2}{dx_1} = 4$$

(5) Along $x_1 = \frac{1}{4} x_2$

ie $x_2 = 4x_1$

$$\frac{dx_2}{dx_1} = -1$$