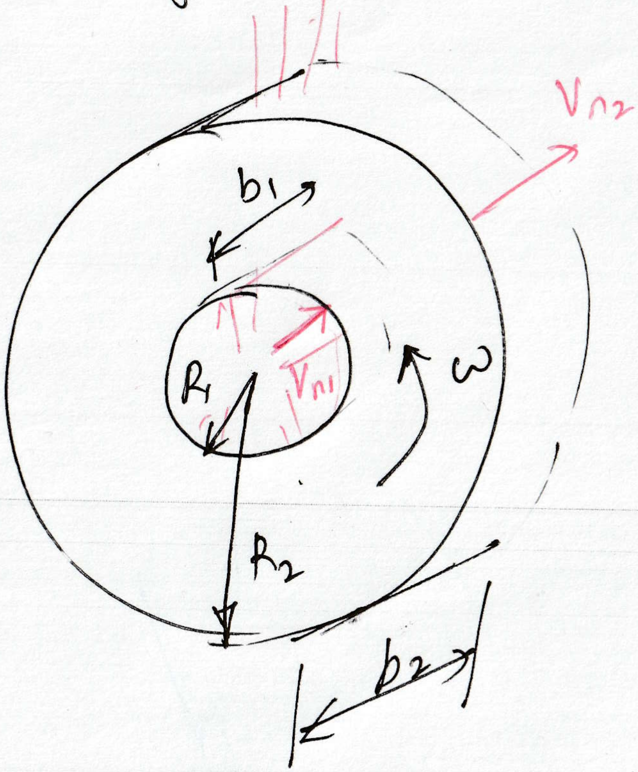
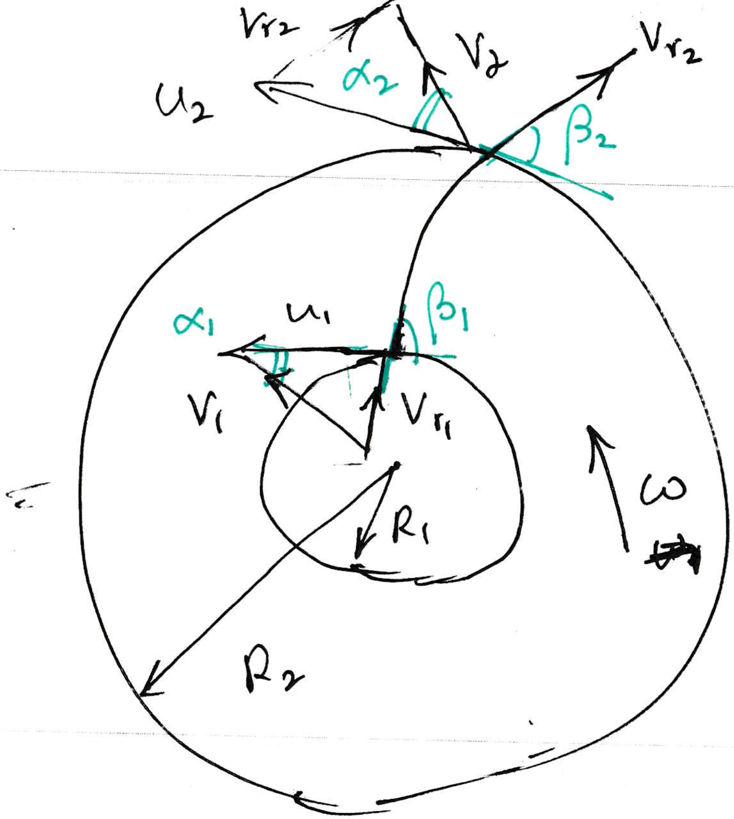
Turbomachines 1) Centrifugal 2) Axial

1) Centrifugal machines



$$\dot{\forall} = A V_{n_1}$$
$$= 2\pi R_1 b_1 V_{n_1}$$
$$= 2\pi R_2 b_2 V_{n_2}$$

$$\dot{m} = \rho \dot{\forall}$$
$$= \frac{\dot{\forall}}{v}$$



$u_1 = \omega R_1$ $\qquad$ $u_2 = \omega R_2$

β_1, β_2 are the blade angles at inlet and outlet relative to the tangent of the circle at the blade

V_{r_1}, V_{r_2} are the relative velocity vectors at the inlet and outlet

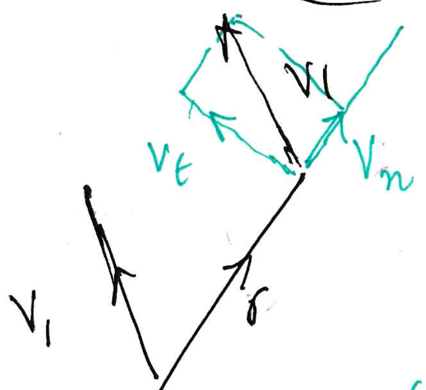
V_1 = ~~vel~~ absolute velocity at inlet

$= V_{r_1} + u_1$

V_2 = absolute velocity at exit

$= V_{r_2} + u_2$

α_1, α_2 = angle made by inlet and outlet velocity vectors with the tangent to the circle at the inlet and outlet respectively.



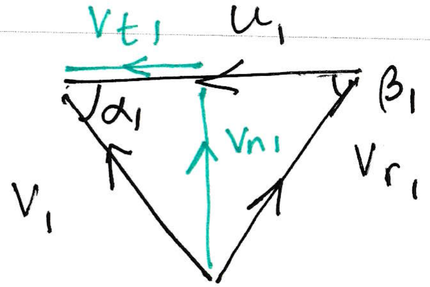
$$r \times mv = r \times m(\underline{V}_t + \underline{V}_n)$$
$$= m(r \times V_t) + (r \times \cancel{V_n})n$$
$$0$$
$$= m r V_t$$

Conservation of angular momentum (steady state)

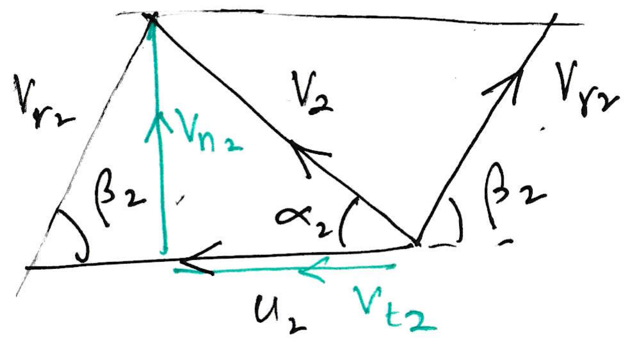
$$\dot{H}_1 - \dot{H}_0 + \sum T = 0 \Rightarrow T = \dot{H}_0 - \dot{H}_i = \dot{m} r_2 V_{t_2} - \dot{m} r_1 V_{t_1}$$

$$H_i = r \times mv = r m V_t$$

Inlet



Outlet



$$V_{n_1} = V_1 \sin\alpha_1 = V_{r_1} \sin\beta_1 \Rightarrow V_{r_1} = \frac{V_{n_1}}{\sin\beta_1}$$

$$V_{t_1} = u_1 - V_{r_1}\cos\beta_1$$

$$= V_1 \cos\alpha_1$$

$$u_1 - \left(\frac{V_{n_1}}{\sin\beta_1}\right)\cos\beta_1$$

$$V_{t_1} = u_1 - V_{n_1}\cot\beta_1$$

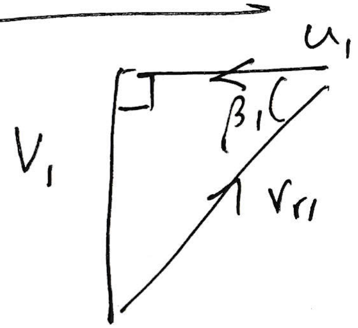
$$V_{n_2} = V_2 \sin\alpha_2 = V_{r_2}\sin\beta_2 \Rightarrow V_{r_2} = \frac{V_{n_2}}{\sin\beta_2}$$

$$V_{t_2} = V_2\cos\alpha_2 = u_2 - V_{r_2}\cos\beta_2$$

$$= u_2 - \frac{V_{n2}}{\sin\beta_2}\cos\beta_2$$

$$= u_2 - V_{n_2}\cot\beta_2$$

Special Case: $V_{t_1} = 0$ (Radial entry)



$$\boxed{\alpha_1 = 90^\circ}$$

$$T = \dot{m} r_2 V_{t_2} - \dot{m} r_1 V_{t_1}$$
$$= \dot{m}\left[r_2 V_{t_2} - r_1 V_{t_1}\right]$$

$$P = T\omega$$
$$= \dot{m}\left[\omega r_2 V_{t_2} - \omega r_1 V_{t_1}\right]$$
$$= \dot{m}\left[u_2 V_{t_2} - u_1 V_{t_1}\right]$$

$$\boxed{w = \frac{P}{\dot{m}} = u_2 V_{t_2} - u_1 V_{t_1}}$$ ← Euler's Turbomachines equation.

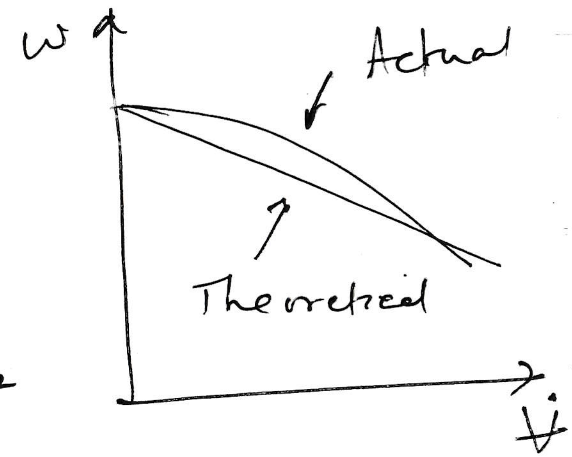
work done per unit mass

<u>Special Case</u>: $V_{t_1} = 0$

$$w = u_2 V_{t_2}$$
$$= u_2\left(u_2 - V_{n_2}\cot\beta_2\right)$$
$$= u_2^2 - u_2 V_{n_2}\cot\beta_2$$

Since $\dot{V} = 2\pi R_2 b_2 V_{n_2} \Rightarrow V_{n_2} = \dfrac{\dot{V}}{2\pi R_2 b_2}$

$$\boxed{w = u_2^2 - \frac{u_2 \cot\beta_2}{2\pi R_2 b_2}\dot{V}}$$



Thermodynamics

~~k~~ Energy in + work done = Energy out

$$\underbrace{h_1 + \frac{V_1^2}{2}}_{h_{01}} + w = \underbrace{h_2 + \frac{V_2^2}{2}}_{h_{02}}$$

w ← work done from Euler's equation.

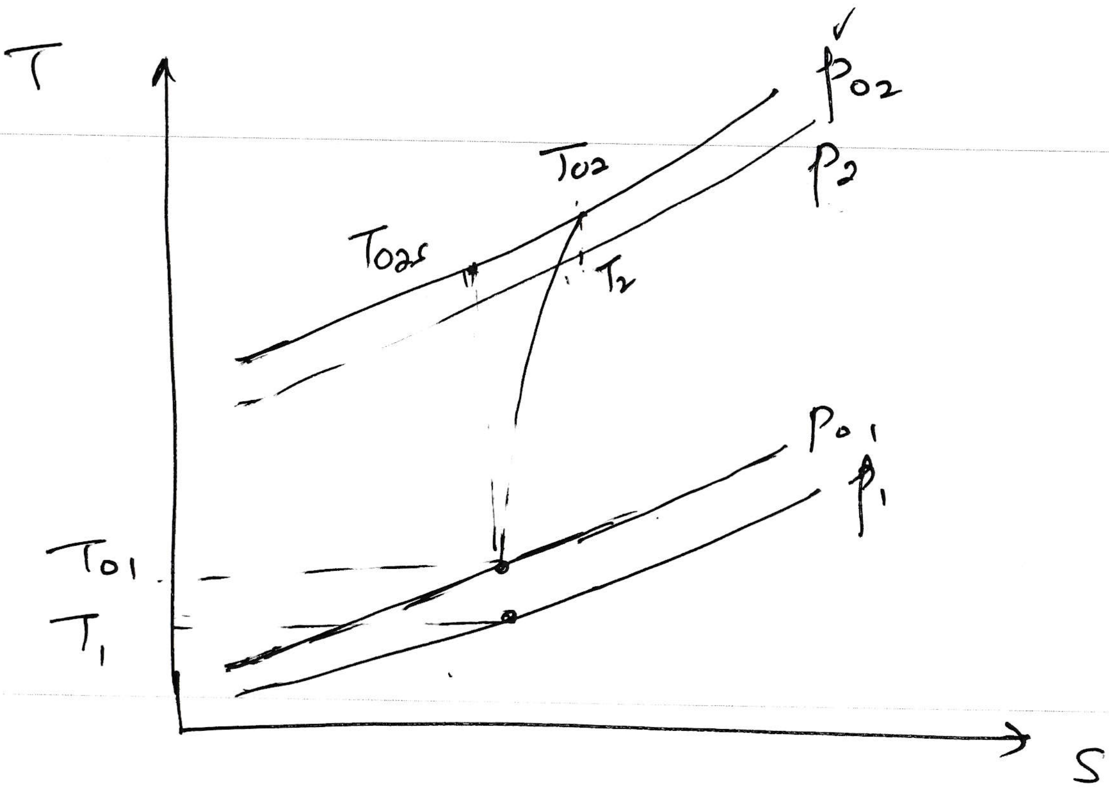
h_{01} Stagnation enthalpy at inlet

h_{02} Stagnation enthalpy ~~at~~ at exit

$$h_{01} + w = h_{02} \Rightarrow w = h_{02} - h_{01}$$

$$= C_p(T_{02} - T_{01})$$

$$\Rightarrow T_{02} = T_{01} + \frac{w}{C_p}$$



stagnation temp at inlet

$$h_{01} = h_1 + \frac{V_1^2}{2} \Rightarrow C_p T_{01} = C_p T_1 + \frac{V_1^2}{2} \Rightarrow T_{01} = T_1 + \frac{V_1^2}{2C_p}$$

must be J/kg K

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1}$$

$$T_2 = T_{02} - \frac{V_2^2}{2C_p}$$

$$\frac{p_2}{p_{02}} = \left(\frac{T_2}{T_{02}}\right)^{\gamma/\gamma-1}$$

$$T_{02} = T_{01} + \frac{w}{C_p}$$

$$\eta = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$ ← find T_{02s}

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$