## Properties of fluids.

1) Density = $\dfrac{\text{mass}}{\text{Volume}}$ $\quad \rho = \dfrac{m}{\forall}$

$\rho_{H_2O} = 1000 \text{ kg/m}^3 = 1 \text{ kg/Liter}$

$\rho_{Hg} = 13.6 \times 10^3 \text{ kg/m}^3$

$\rho_{gasoline} = 720 - 750 \text{ kg/m}^3$

**For gases:** use ideal gas law.

$P\forall = n\bar{R}T$ $\quad$ ($n$ = # of moles, $\bar{R} = 8.314$ kJ/kmol K)

$= \dfrac{m}{MW}\bar{R}T$

MW: molar mass

$= m\left(\dfrac{\bar{R}}{MW}\right)T$

$= mRT$

where $R \triangleq \bar{R}/MW$

R: gas constant specific to gas

$\bar{R}$: universal gas constant

$\Rightarrow P\forall = mRT$

$\Rightarrow P = \dfrac{m}{\forall}RT$

$\Rightarrow \boxed{P = \rho RT}$

Ex: Air @ 30°C. $MW_{air} \simeq 29$

$R = \dfrac{8.314}{29} \simeq 0.287$ kJ/kg K

$P = 101.3$ kPa

$\rho = \dfrac{P}{RT} = \dfrac{101.3 \times 10^3}{(287)(303)} = 1.165 \text{ kg/m}^3$

Bulk Modulus: Compressibility.

$$\boxed{K \triangleq \frac{-dp}{(d\forall/\forall)}}$$

Speed of sound in medium (fluid) $\Rightarrow$ $\boxed{c = \sqrt{\frac{dp}{d\rho}}}$

$$m = \rho\forall \Rightarrow \underbrace{dm}_{0} = \rho d\forall + d\rho\,\forall$$

$$\Rightarrow \rho d\forall + d\rho\forall = 0$$

$$\Rightarrow d\rho/\rho = -d\forall/\forall$$

$$\boxed{K = \frac{-dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho}}$$

Ex: $K_{H_2O} = 2.1\ GPa$ $\quad c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.1\times 10^9}{10^3}} = 1450\ m/s$

Air: Assume isothermal process: $c = \sqrt{\frac{dp}{d\rho}}$

$P = \rho R T \Rightarrow dp = d\rho(RT)$

$c$ $\Rightarrow \frac{dp}{d\rho} = RT$

$c = \sqrt{RT} = \sqrt{(287)(303)} = 294\,m/s$

Assume adiabatic

$P\forall^{\gamma} = c_1 \Rightarrow P = C_2 \rho^{\cancel{\gamma}}$

$P\left(\frac{m}{\rho}\right)^{\gamma} = c_1 \Rightarrow P = \rho^{\gamma}\left(\frac{c_1}{m^{\gamma}}\right)$

$\boxed{P = c_2 \rho^{\gamma}}$

$\cancel{d\rho}\; dp = c_2 \gamma \rho^{\gamma-1} d\rho$

$\frac{dp}{d\rho} = \frac{\gamma(c_2\rho^{\gamma})}{\rho}$

$= \gamma \frac{P}{\rho}$

$= \gamma RT$

$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\gamma RT}$

$= \sqrt{(1.4)(287)(303)}$

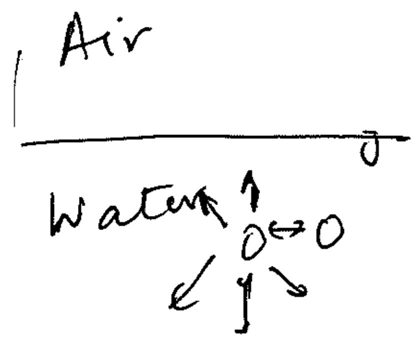
$\boxed{c = 348.9\,m/s}$

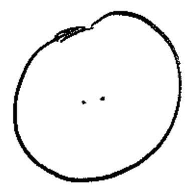
Mach number $Ma \triangleq \frac{v}{c}$

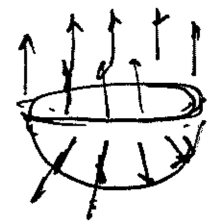
If $Ma < 0.3 \sim 0.4$, assume medium is incompressible

If $Ma > 1 \Rightarrow$ supersonic flow.

# Surface Tension:





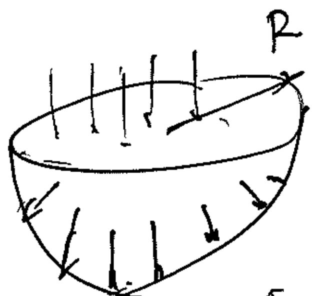
→ surface tension force

$\sigma$ = surface tension

has units of

$F/L$

$[\sigma] = N/m$



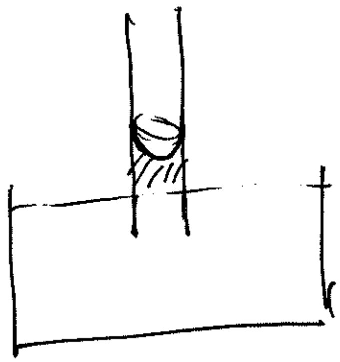

Force due to pressure: $= p\pi R^2$

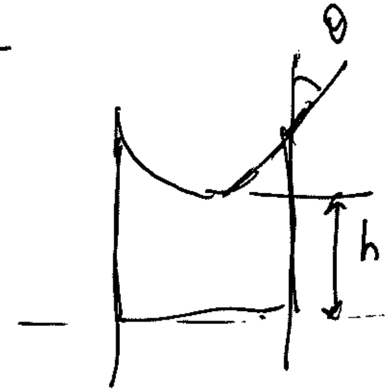
Force due to surface tension: $(\sigma)(2\pi R)$

Equating $\quad p\pi R^2 = \sigma 2\pi R$

$$\Rightarrow \boxed{p = \frac{2\sigma}{R}}$$

## Capillary Action






Upward force due to surface tension

$$(\sigma\, 2\pi R) \cos\theta$$

Downward force due to gravity

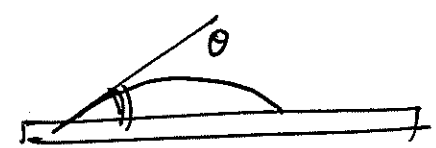
$$= mg$$
$$= \rho \forall g$$
$$= \rho \pi R^2 h g$$

Balancing:

$$\sigma\, 2\pi R \cos\theta = \rho \pi R^2 h g$$

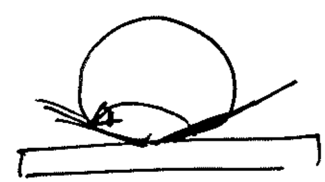
$$\boxed{h = \frac{2\sigma \cos\theta}{\rho R g}}$$

θ is called "wetting angle"




Water on glass

$0 < \theta < 90$



Water on oily surface

$90 < \theta < 180$