Balance

Mass Balance

$$\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{gen} = \frac{d\,m_{cv}}{dt}$$

For steady state: $\dot{m}_{in} = \dot{m}_{out}$

with no generation

$$\dot{m}_{in} = \rho A V$$

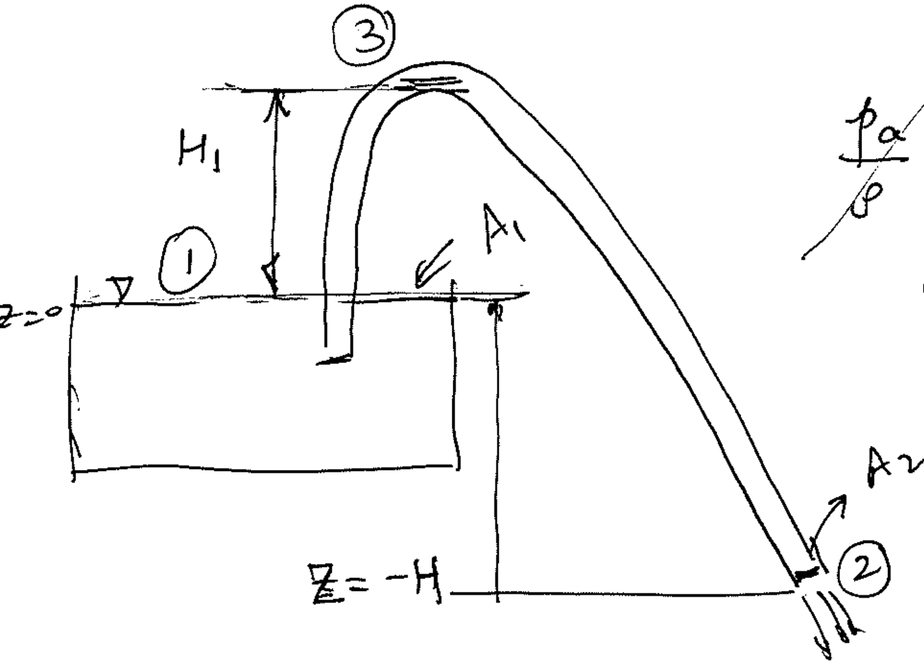
Energy Balance:

$$\dot{m}_i e_i - \dot{m}_o e_o + \cancel{\Sigma}\left(\dot{Q} + \dot{W}\right) + \dot{E}_{gen} = \frac{dE_{cv}}{dt}$$

$$\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) - \left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \frac{\dot{W}_{non\text{-}flow}}{\dot{m}} - \frac{\mathcal{F}}{\dot{m}} = 0$$

(Steady state)

$$\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \frac{\dot{W}_{non\text{-}flow}}{\dot{m}} - \frac{\mathcal{F}}{\dot{m}} = \left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right)$$



(2)

$$\cancel{\frac{p_a}{\rho}} + \frac{V_1^2}{2} + g(0) + 0 - 0 = \cancel{\frac{p_a}{\rho}} + \frac{V_2^2}{2} + g(-H)$$

Mass Balance: $A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2}{A_1} V_2$$

Since $A_1 >> A_2$,

$$\frac{A_2}{A_1} << 1$$

$$\Rightarrow \quad V_1 << V_2$$

Hence $V_1 \approx 0$.

$$\frac{V_2^2}{2} - gH = 0 \quad \Longrightarrow \quad V_2 = \sqrt{2gH}$$

$\underline{1-3}$: $$\frac{p_a}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{p_3}{\rho} + \frac{V_3^2}{2} + gH_1$$

$V_1 \approx 0$

$$\frac{p_a - p_3}{\rho} = \frac{V_3^2}{2} + gH_1$$

$$= \frac{2gH}{2} + gH_1$$

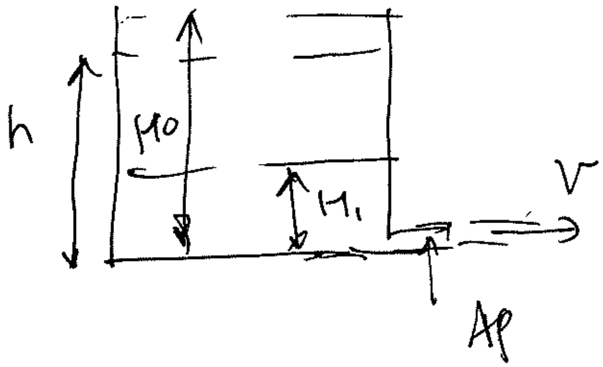
$$= g(H + H_1)$$

$$\frac{p_a}{\rho} - g(H+H_1) = \frac{p_3}{\rho}$$

Vapor pressure

$p_{3\,min} = p_{vapor}$ @ temperature of fluid

(4)



$V = \sqrt{2gH}$

Initial ht. of fluid is $= H_0$

How long does it take for the fluid to drop to H_1?

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt}$$

$\dot{m}_{in} = 0$

$$\dot{m}_{out} = \rho A_p V = \rho A_p \sqrt{2gh}$$

$$m_{cv} = \rho A_t h$$

$$\frac{dm_{cv}}{dt} = \rho A_t \frac{dh}{dt}$$

$$-\rho A_p \sqrt{2gh} = \rho A_t \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{\cancel{\rho} A_p}{A_t} \sqrt{2g}\sqrt{h}$$

$$\frac{dh}{\sqrt{h}} = -\frac{A_p\sqrt{2g}}{A_t} t$$

$$\left.\frac{h^{1/2}}{1/2}\right|_{H_0}^{H} = -\frac{A_p}{A_t}\sqrt{2g}\, t$$

$$H_1^{1/2} - H_0^{1/2} = -\frac{A_p}{A_t} \cdot \frac{\sqrt{2g}}{2} t$$

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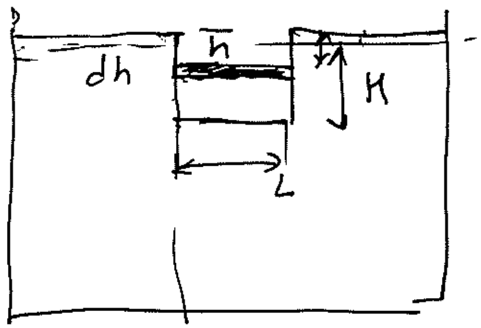

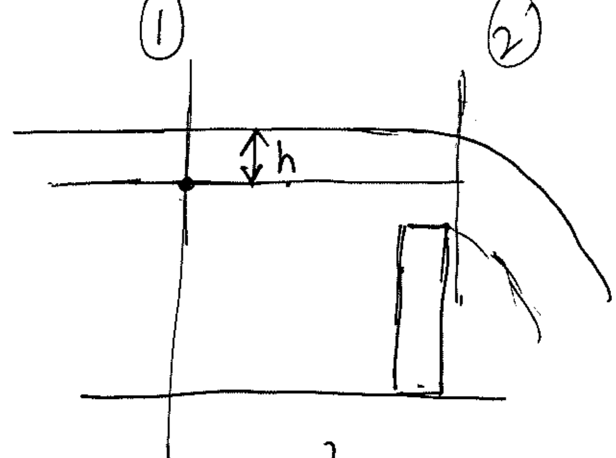

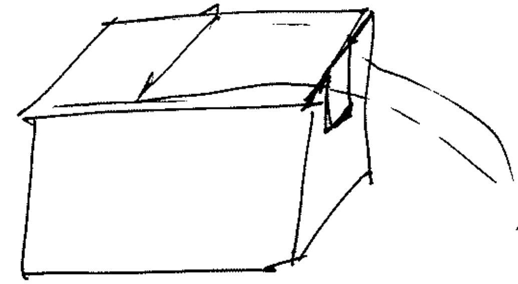

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$\frac{\rho g h}{\rho} + g h = 0 + \frac{V_2^2}{2} + g h \Rightarrow V_2 = \sqrt{2gh}$$

$$dQ = v_2\, dA$$
$$= v_2\, dh\, L$$

$$dQ = \sqrt{2gh}\, dh\, L$$

$$Q = \int_0^H \sqrt{2gh}\, dh\, L = \sqrt{2g}\, L \int_0^H h^{1/2}\, dh$$
$$= \sqrt{2g}\, L\, \frac{H^{3/2}}{3/2}$$
$$= \frac{2L}{3}\sqrt{2g}\, H^{3/2}$$

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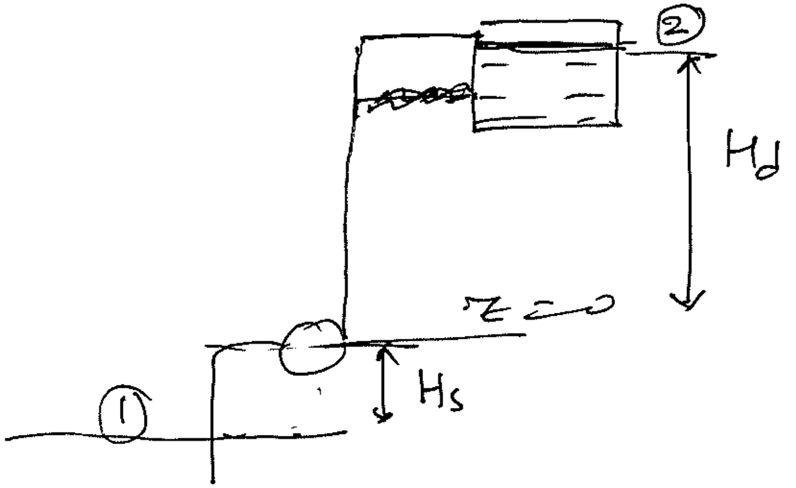


$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \frac{\dot{W}_{nf}}{\dot{m}} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$p_1 = p_2 = p_a$

$V_1 \approx V_2 \approx 0$ $\qquad z_2 = +H_d$

$\frac{\dot{W}_{nf}}{\dot{m}} = g(z_2 - z_1)$ $\qquad z_1 = -H_s$

$= g(H_d + H_s)$

$\dot{W}_{nf} = \dot{m}\, g\,(H_d + H_s)$

$H_d = 10\text{ m}$

$H_s = 1\text{ m}$

$Q = 10\text{ liter/s}$

$1\text{ m}^3 = 10^3\text{ liter}$

$1\text{ liter} = 10^{-3}\text{ m}^3$

$\dot{W}_{nf} = (\rho Q)\, g\,(H_d + H_s)$

$= (10^3)(10 \times 10^{-3})(9.81)(10+1)$

$\approx 1.1\text{ kW}$

Losses : → Major → flow through ducts/ pipes

→ Minor loss → associated with bends, joints, fittings, valves, contraction / expansion

Losses $\sim \frac{V^2}{2}$

Major loss $\quad (g h_{major}) = f \frac{L}{D}\left(\frac{V^2}{2}\right)$

f → Darcy - Weisbach friction factor

Minor loss $\quad g h_{minor} = K \frac{V^2}{2}$

K → minor loss coefficients.

Flow in pipes $\longrightarrow$ Laminar $\qquad 0 < Re < 2000$

Transition.

Turbulent $\qquad Re > 4000$

Reynold's number $$Re = \frac{\rho V D}{\mu}$$

$$= \frac{VD}{\nu}$$ (since $\nu = \mu/\rho$ kinematic viscosity)

Ref

Friction factor

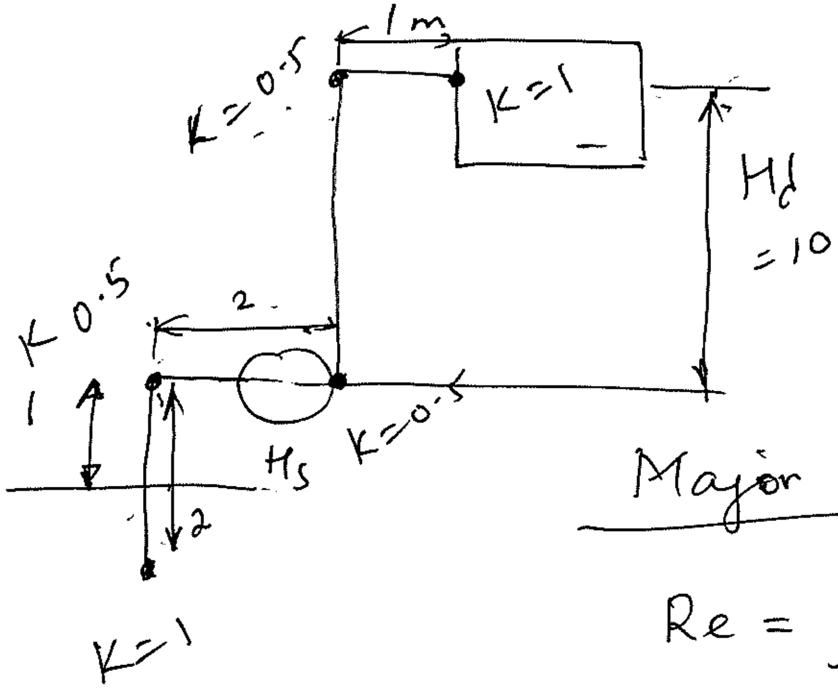
$f = \frac{64}{Re}$ for laminar flow.

Colebrook's equation $$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\epsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right]$$

Approximation: $$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

D = Diameter of pipe

ϵ = Surface roughness



$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + gz_1 + \frac{\dot{W}_{nf}}{\dot{m}} - (gh) = \cancel{\frac{P_2}{\rho}} + \cancel{\frac{V_2^2}{2}} + gz_2$$

$$\frac{\hat{\dot{W}}_{nf}}{\dot{m}} = (gh) + g(z_2 - z_1)$$

$Q = 10$ liter/s

$D = 25$ mm

$\epsilon = 1\ \mu m$

$\nu = 10^{-6}\ m^2/s$

$A = \frac{\pi}{4} \times (0.025)^2$

$= 4.91 \times 10^{-4} m^2$

Major Loss $(gh)_{major} = f \frac{L}{D} \frac{V^2}{2}$

$$Re = \rho \frac{VD}{\mu} = \frac{VD}{\nu}$$

$$Q = AV \Rightarrow V = \frac{Q}{A} = \frac{10 \times 10^{-3}}{4.91 \times 10^{-4}} = 20.4 \frac{m}{s}$$

$$Re = \frac{(20.4)(25 \times 10^{-3})}{10^{-6}} = 509296 > 2300$$

$\Rightarrow$ turbulent

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\frac{1 \times 10^{-6}}{25 \times 10^{-3}}}{3.7} + \frac{5.74}{509296^{0.9}}\right)\right]^2}$$

$= 0.021$

Major loss $= (0.021)\dfrac{(15)}{(25\times10^{-3})}\dfrac{(20.4)^2}{2} = 2621.8$

Minor loss $= K\dfrac{V^2}{2} = (1+0.5+0.5+0.5+1)\dfrac{(20.4)^2}{2}$

$= (3.5)\dfrac{(20.4)^2}{2}$

$= 728.3$

$\underbrace{(gh)}$ $= 2621.8 + 728.3$

$= 3350.1$

$\dfrac{\dot{W}_{nf}}{\dot{m}} = (3350.1) + (9.81)(11)$

$= 3458$

$\dot{W}_{nf} = (\dot{m})(3458)$

$= (10^3)(10\times10^{-3})(3458)$

$= 34.58$ KW.

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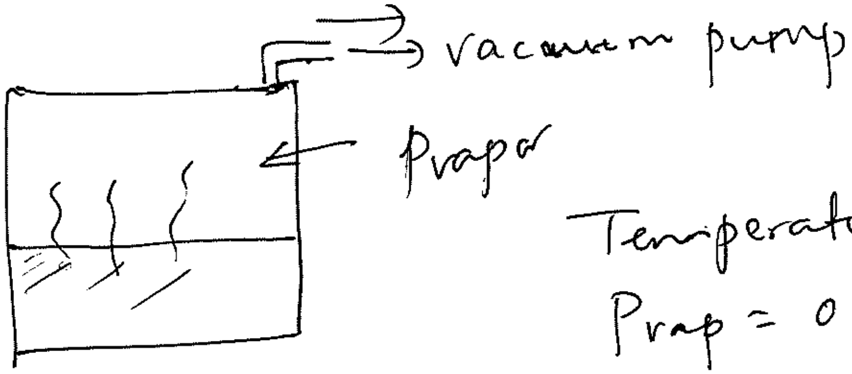

Temperature of water = $30^\circ c$

$P_{vap} = 0.04246$ bar = 4.246 kPa (abs)

$$\frac{P_a}{\rho} - g(H + H_1) = \frac{P_3}{\rho}$$

Let $H = 1$ m.

$$101.3 - (9.81)(1 + H_1) = 4.246$$

$$\frac{101.3 - 4.246}{9.81} = H_1 + 1 \Rightarrow H_1 = 9.1\,m$$