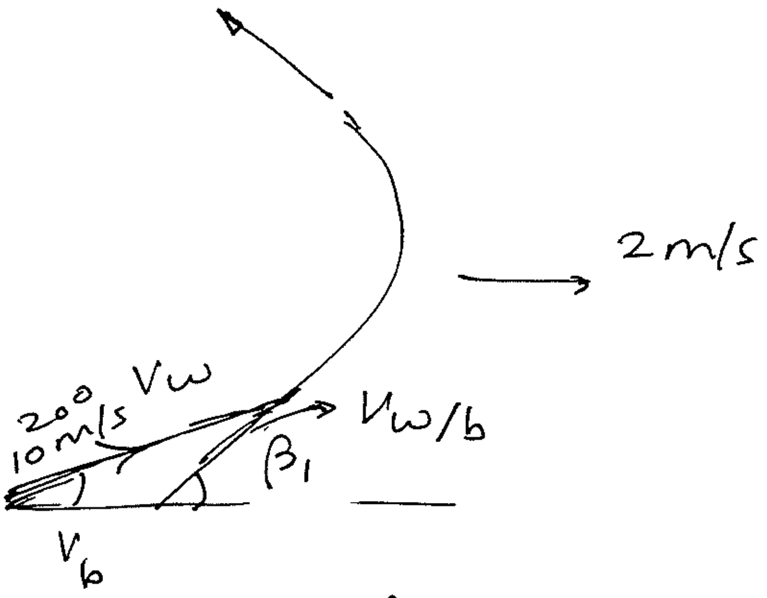
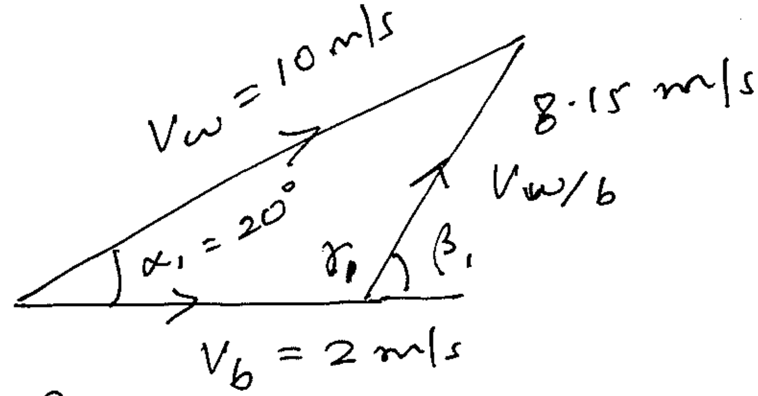
1

$$V_w = V_{w/b} + V_b$$





$$V_{w/b}^2 = V_b^2 + V_w^2 - 2 V_b V_w \cos 20$$

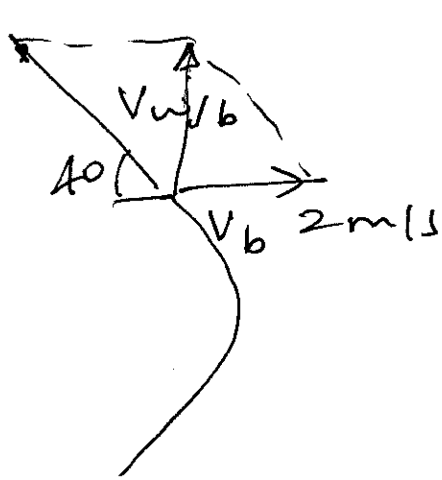
$$= 2^2 + 10^2 - (2)(10)(2)\cos 20$$

(6) $\boxed{V_{w/b} = 8.15 \text{ m/s}}$

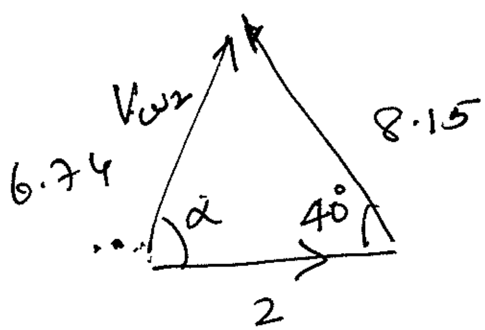
$$\cos \gamma_1 = \frac{2^2 + 8.15^2 - 10^2}{(2)(8.15)(2)} = 155$$

(5) $\boxed{\beta_1 = 180 - 155 = 24.8^\circ}$

(2)



$$\vec{V}_{w/b} = -V_{w/b}\cos 40\,\hat{i} + V_{w/b}\sin 40\,\hat{j}$$
$$= -8.15\cos 40\,\hat{i} + 8.15\sin 40\,\hat{j}$$
$$= -6.\cancel{2}\,\hat{i} + 5.23\,\hat{j}$$



$$V_{w_2} = V_{w_2/b} + V_b$$
$$V_{w_2} = \sqrt{V_{w/b}^2 + V_b^2 - 2 V_{w/b} V_b \cos 40}$$
$$= \sqrt{8.15^2 + 2^2 - (2)(8.15)(2)\cos 40}$$
$$= 6.74 \text{ m/s}$$
$$\alpha = \cos^{-1}\frac{6.74^2 + 2^2 - 8.15^2}{(2)(6.74)(2)} = 129^\circ$$
$$\alpha_2 = 180 - 129 = 51^\circ$$

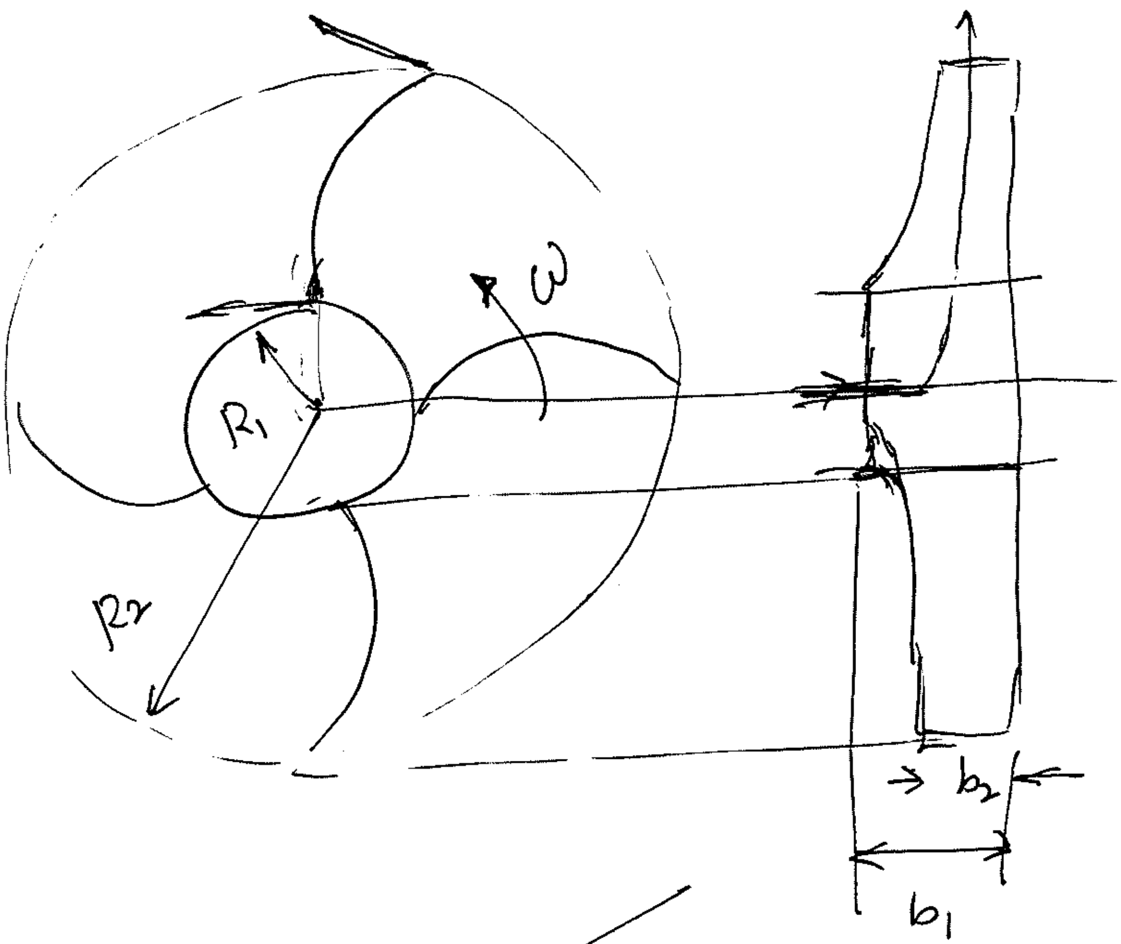
$$\dot{m} = \rho A V_r = (10^3)\,\frac{\pi}{4}\times(0.04)^2 \times 8.15 = 10.23 \text{ kg/s}$$

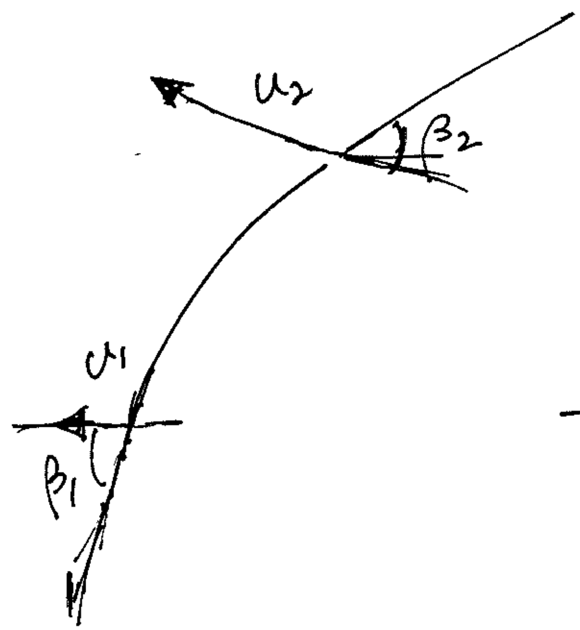
Horizontal: $\dot{m} V_{in_x} - \dot{m} V_{out_x} - F_x = 0 \Rightarrow F_x = (10.23)[10\cos 20 - 6.74\cos 129]$
$= 139.5$ N

Vertical: $\dot{m} V_{in_y} - \dot{m} V_{out_y} - F_y = 0 \Rightarrow F_y = (10.23)[10\sin 20 - 6.74\sin 129]$
$= -18.6$ N

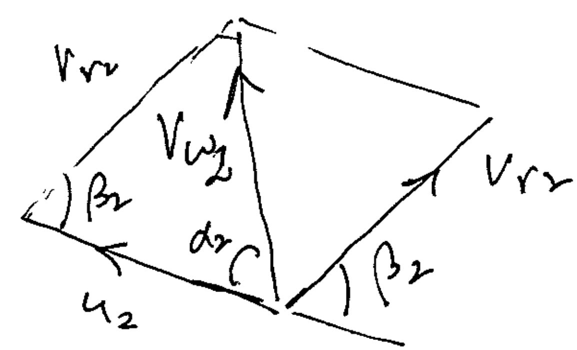
$u_1 = R_1 \omega$

$u_2 = R_2 \omega$

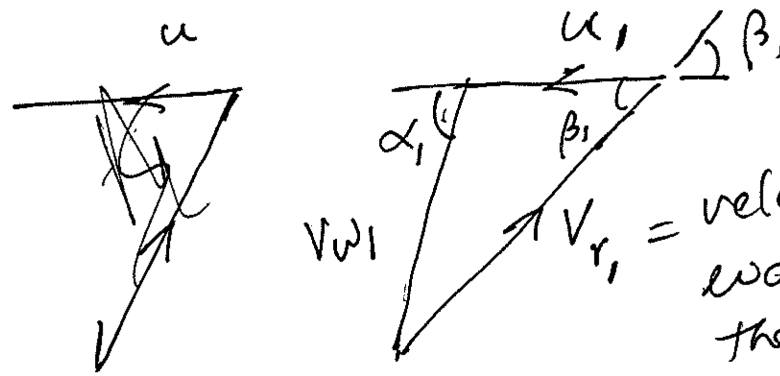




Outlet triangle

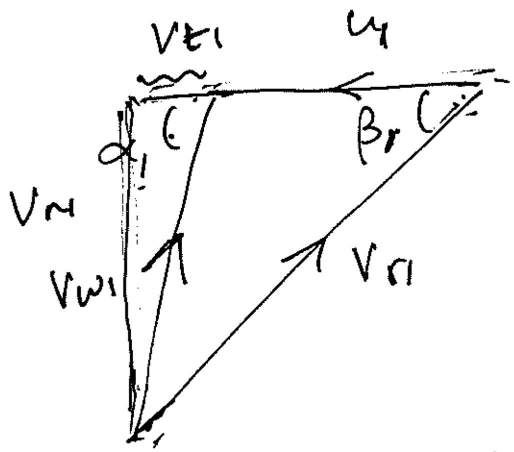


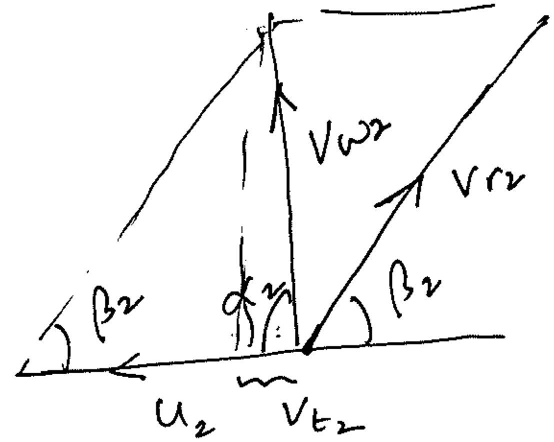
Inlet Triangle



V_{r_1} = velocity of the ~~jet~~ water relative to the blade

Inlet





$V_{w_1} =$ $V_{t_1}, V_{n_1} =$ tangential & normal components of V_{w_1} $\leftarrow$ V_{t_2}, V_{n_2}

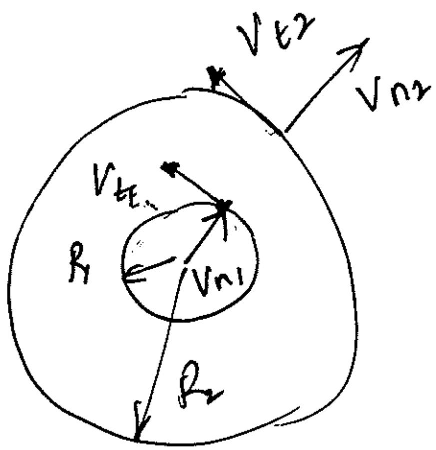
$V_{n_1} = V_{w_1} \sin\alpha_1 = V_{r_1} \sin\beta_1$

$V_{t_1} = = V_{r_1} \cos\beta_1 - u_1$

$V_{n_2} = V_{r_2} \sin\beta_2 = V_{w_2} \sin\alpha_2$

$V_{t_2} = u_2 - V_{r_2} \cos\beta_2$

$Q = AV = 2\pi R_1 b_1 V_{n_1} = 2\pi R_2 b_2 V_{n_2}$



Conservation of Angular Momentum

$$\dot{H}_{in} - \dot{H}_{out} + \Sigma T = \frac{dH_{cv}}{dt}$$

$$\dot{H} = \dot{m} r_1 V_{t_1} - \dot{m} r_2 V_{t_2} + T = 0$$

$$\boxed{T = \dot{m}(r_2 V_{t_2} - r_1 V_{t_1})}$$

Power $P = T\omega$

$$= \dot{m}\,\omega\,(r_2 V_{t_2} - r_1 V_{t_1})$$

$$= \dot{m}\left[V_{t_2}(r_2\omega) - (r_1\omega)V_{t_1}\right]$$

$$\boxed{P = \dot{m}\left[V_{t_2} u_2 - V_{t_1} u_1\right]}$$

Euler's Turbomachinery Equations.

$$\boxed{w = \frac{P}{\dot{m}} = \left(V_{t_2} u_2 - V_{t_1} u_1\right)}$$

work done per unit mass

$$\boxed{h_{pump} = \frac{w}{g} = \frac{P}{\dot{m} g} \cancel{f} = \frac{V_{t_2} u_2 - V_{t_1} u_1}{g}}$$

← Pump head.

Special Case: $\alpha_1 = 90 \Rightarrow$ radial entry $\Rightarrow V_{t_1} = 0$

$$\boxed{h_{pump} = \frac{V_{t_2} u_2}{g}}$$

Assume Radial entry

$$h_{pump} = \frac{V_{t2}\, u_2}{g} = \frac{u_2}{g}\left(u_2 - V_{r2}\cos\beta_2\right)$$

$$= \frac{u_2}{g}\left(u_2 - \frac{V_{n2}}{\sin\beta_2}\cos\beta_2\right)$$

$$= \frac{u_2}{g}\left(u_2 - \frac{Q}{2\pi R_2 b_2}\frac{\cos\beta_2}{\sin\beta_2}\right)$$

$$= \frac{u_2^2}{g} - Q\left[\frac{u_2}{2\pi R_2 b_2}\cdot\frac{\cot\beta_2}{g}\right]$$

Let $a = \frac{u_2^2}{g}$, $b = \frac{u_2}{2\pi R_2 b_2}\frac{\cot\beta_2}{g}$

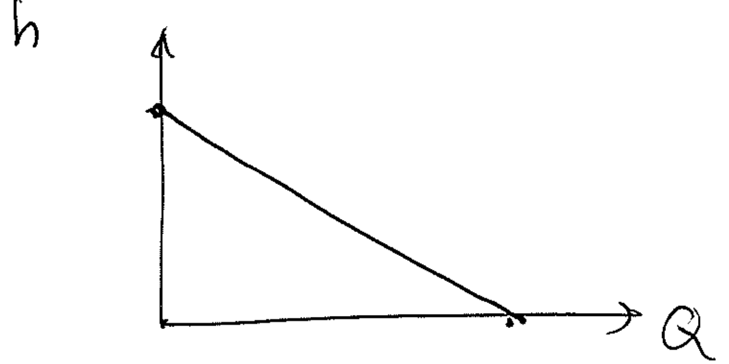
$$\boxed{h_{pump} = a - bQ}$$

When $Q = 0$ $h_{pump} = a = \frac{u_2^2}{g}$

Shut-off head.

$Q = \frac{a}{b}$ $h_{pump} = 0$

Maximum flow-rate



Parameter	Inlet	Outlet
Radius (mm)	175	500
Blade width (b mm)	50	30
Blade angle β	65	70

$N = 750$ rpm.

$Q = 0.75\ m^3/s$

Theoretical head?

$P = ?$

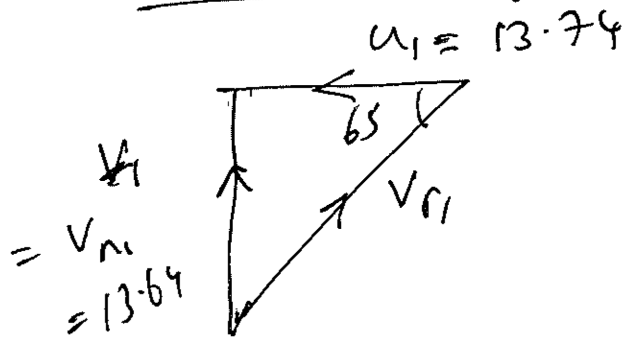
$\omega = (750)\frac{2\pi}{60} = 78.54$ rad/s

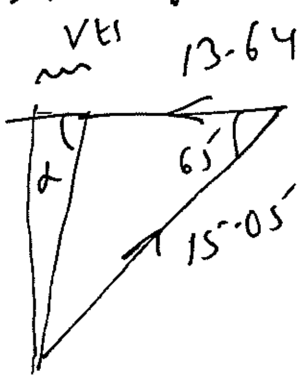
$u_1 = (78.54)(0.175) = 13.74$ m/s

$u_2 = (78.54)(0.5) = 39.27$ m/s

~~Assume Radial entry~~

Inlet triangle





$$V_{n1} = \frac{Q}{2\pi R_1 b_1} = \frac{0.75}{(2\pi)(0.175)(0.05)} = 13.64 \text{ m/s}$$

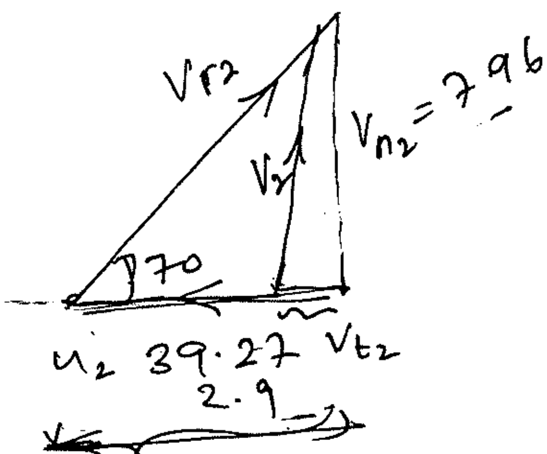
$$V_{r1} = \frac{V_{n1}}{\sin\beta_1} = \frac{13.64}{\sin 65} = 15.05 \text{ m/s}$$

$$V_{t1} = u_1 - V_{r1}\cos\beta_1$$

$$= 13.64 - 15.05\cos 65$$

$$\approx 7.28 \text{ m/s}$$

Outlet triangle



$$V_{r2} = \frac{V_{n2}}{\tan 70}$$

$$Q = 2\pi R_2 b_2 V_{n2}$$

$$V_{n2} = \frac{Q}{2\pi R_2 b_2} = \frac{0.75}{(2\pi)(0.5)(0.03)} = 7.96 \text{ m/s}$$

$$V_{t2} = u_2 - \underbrace{V_{r_2}\cos\beta_2}$$
$$= 39.27 - 2.9$$
$$= 36.37 \text{ m/s}$$

$$h_{pump} = \frac{V_{t_2}u_2 - V_{t_1}u_1}{g} = \frac{(36.37)(39.27) - (13.74)\,7.28}{9.81}$$
$$= 135 \text{ m}$$

$$\text{Power} = \dot{m}\, g\, h_{pump}$$
$$= \rho Q g h_{pump} = (10^3)(0.75)(9.81)(135)$$
$$= 996 \text{ KW}$$

$$\text{Torque} = \dot{m}(r_2 V_{t_2} - r_1 V_{t_1}) = (10^3)(0.75)[(0.5)(36.37) - (0.375)\,7.28]$$
$$= 12.7 \text{ KN-m}$$