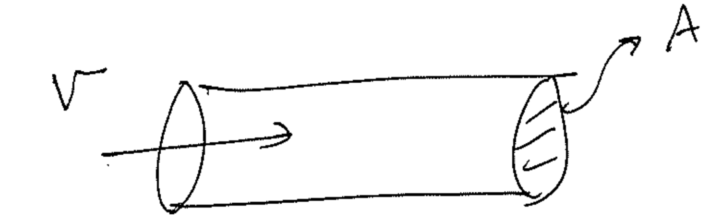
(1)



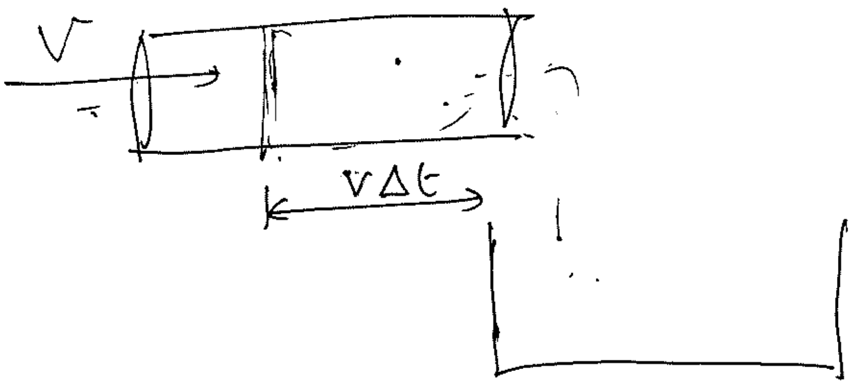
Mass flow rate $\hat{m} = \rho Q$

$\uparrow$ volumetric flow rate

$$\hat{m} = \rho A V$$

$\underline{Q = AV}$

Wait for time Δt. How much fluid is collected?

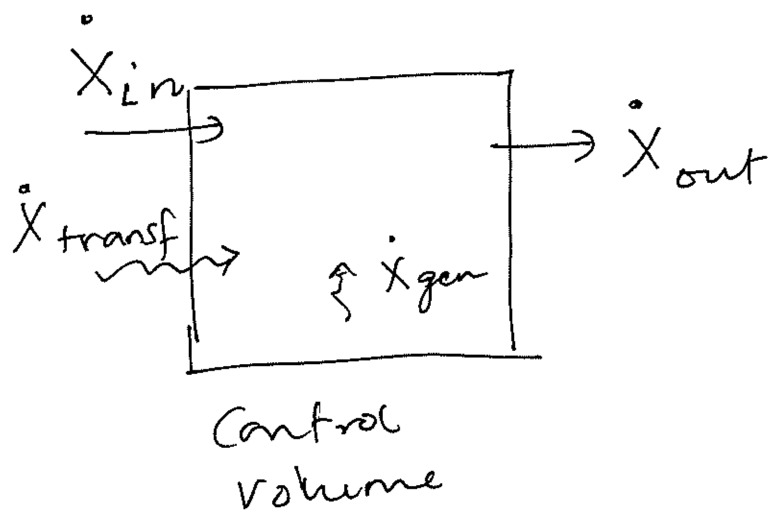


$\cancel{\hat{m} = \rho}$ $\Delta m = \rho (V \Delta t) A$

$$\frac{\Delta m}{\Delta t} = \rho V A$$

$$\boxed{\hat{m} = \rho V A}$$

Balances



$\dot{X}_{in}$ = rate at which quantity X enters the volume

$\dot{X}_{transferred}$ = transferred at boundary.

$\dot{X}_{gen}$ = rate at which X is generated in the control volume.

$$\dot{X}_{in} + \dot{X}_{transf} + \dot{X}_{gen} - \dot{X}_{out} = \frac{dX_{cv}}{dt}$$

X : mass ✓
energy ✓
linear momentum ✓
angular momentum ✓
entropy
chemical species

Mass balance:

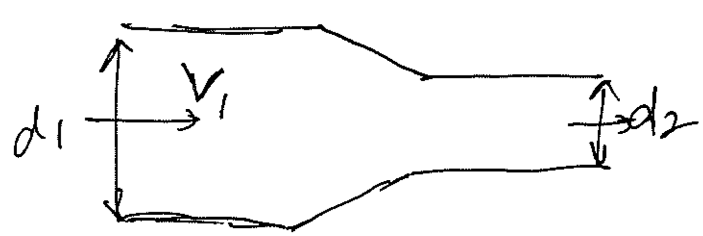
$$\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{gen} = \frac{dm_{cv}}{dt}$$

Steady state: $\frac{dm_{cv}}{dt} = 0 \Rightarrow m_{cv} = \text{constant}$

Assume: $\dot{m}_{gen} = 0$

$$\dot{m}_{in} = \dot{m}_{out}$$

EX:



$$\dot{m}_{in} = \dot{m}_{out}$$

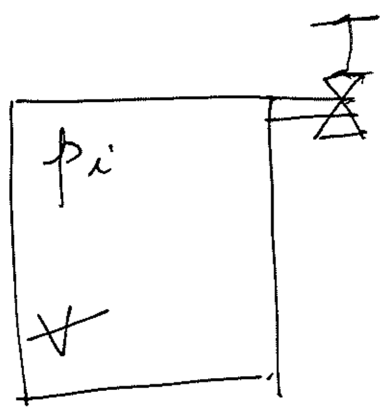
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Assume incompressible flow

$$\rho_1 = \rho_2$$

$$A_1 V_1 = A_2 V_2$$

EX



Tank has gas at initial pressure and temperature p_i & T_i. The valve is opened for t seconds. What's the resulting pressure of air in the tank?

$$\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{gen} = \frac{dm_{cv}}{dt}$$

$\dot{m}_{in} = 0 \qquad \dot{m}_{out} = \rho Q \qquad \dot{m}_{gen} = 0 \qquad m_{cv} = \frac{p\forall}{RT}$

$$\frac{d}{dt}\left(\frac{p\forall}{RT}\right) = -\rho Q$$

(Assume isothermal)

$$\frac{\forall}{RT}\frac{d}{dt}p = -\frac{p}{RT}Q$$

$$\int_{p_i}^{p(t)} \frac{dp}{p} = -\int_0^t \frac{Q}{\forall}\,dt$$

$$\ln p\Big|_{p_i}^{p(t)} = -\frac{Q}{\forall}t \quad \Rightarrow \quad \boxed{p(t) = p_i \cdot e^{-Q/\forall\, t}}$$

$$\ln p(t) - \ln p_i(t)$$

$$\ln \frac{p(t)}{p_i(t)} = -\frac{Q}{\forall}t$$

$\frac{Q}{\forall}$ = rate

Energy Balance

$$\dot{E}_{in} - \dot{E}_{out} + \underbrace{\dot{E}_{transf}}_{\dot{Q} \quad \dot{W}} + \dot{E}_{gen} = \frac{dE_{CV}}{dt}$$

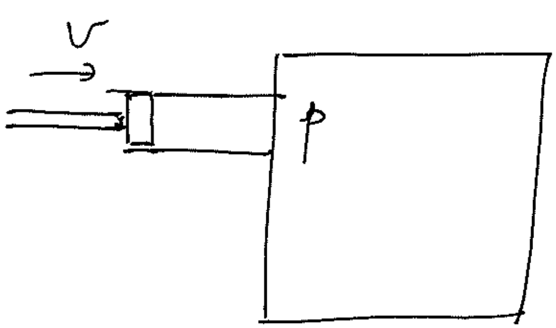
$$\dot{E}_{in} = \dot{m}_{in} e_i \quad (e_i = \text{specific energy})$$

$$= \dot{m}_i \left[\frac{1}{2} V_i^2 + g z_i + u_i \right]$$

$$\dot{E}_{out} = \dot{m}_o \left[\frac{1}{2} V_o^2 + g z_o + u_o \right]$$

$$E_{CV} = \cancel{\dot{m}_{cv}} \left[\quad m_{cv} \left[\frac{1}{2} V^2 + g z + u \right] \right.$$

$$\dot{W} = \dot{W}_{flow} + \dot{W}_{non\text{-}flow}$$



$W_{flow} = P \Delta V$ → change in volume

$= \cancel{PQ} \; P(Q \Delta t)$ → volumetric flow rate)

$$\frac{W_{flow}}{\Delta t} = \dot{W}_{flow} = PQ$$

$$= P \frac{\dot{m}}{\rho}$$

$(\dot{m} = \rho Q$

$\Rightarrow Q = \frac{\dot{m}}{\rho}$

Assume steady state : $\frac{dE_{cv}}{dt} = 0$, $\hat{E}_{gen} = 0$

$$\dot{m}_i \left[\frac{1}{2} V_i^2 + g z_i + u_i \right] - \dot{m}_o \left[\frac{1}{2} V_o^2 + g z_o + u_o \right] + \dot{Q}$$
$$+ \dot{m}_i \frac{P_i}{\rho_i} - \dot{m}_o \frac{P_o}{\rho_o} + \dot{W}_{non\text{-}flow} = 0$$

$$\dot{m}_i \left[u_i + \frac{P_i}{\rho_i} + \frac{1}{2} V_i^2 + g z_i \right] - \dot{m}_o \left[u_o + \frac{P_o}{\rho_o} + \frac{1}{2} V_o^2 + g z_o \right]$$
$$+ \dot{Q} + \dot{W}_{non\text{-}flow} = 0$$

$$\dot{m}_i \left[\frac{P_i}{\rho_i} + \frac{1}{2} V_i^2 + g z_i \right] - \dot{m}_o \left[\frac{P_o}{\rho_o} + \frac{1}{2} V_o^2 + g z_o \right] + \left(\dot{Q} + \dot{m}_i u_i - \dot{m}_o u_o \right)$$
$$+ W_{non\text{-}flow} = 0$$

$$\dot{Q} + \dot{m}_i u_i - \dot{m}_o u_o \triangleq -\mathcal{F} \quad \text{(friction heating)}$$

$$\dot{m}_i \left[\frac{P_i}{\rho_i} + \frac{1}{2} V_i^2 + g z_i \right] - \dot{m}_o \left[\frac{P_o}{\rho_o} + \frac{1}{2} V_o^2 + g z_o \right] + \hat{W}_{nonflow} - \mathcal{F} = 0$$

Let $\dot{m}_i = \dot{m}_o = \dot{m}$. Divide by $\dot{m}$

$$\left(\frac{P_i}{\rho_i} + \frac{1}{2}v_i^2 + g z_i\right) - \left(\frac{P_o}{\rho_o} + \frac{1}{2}v_o^2 + g z_o\right) + \frac{\dot{W}}{\dot{m}} - \frac{\mathcal{F}}{\dot{m}} = 0$$

Energy equation

with $\dot{W} = 0$ & $\mathcal{F} = 0$

$$\frac{P_i}{\rho_i} + \frac{1}{2}v_i^2 + g z_i = \frac{P_o}{\rho_o} + \frac{1}{2}v_o^2 + g z_o$$

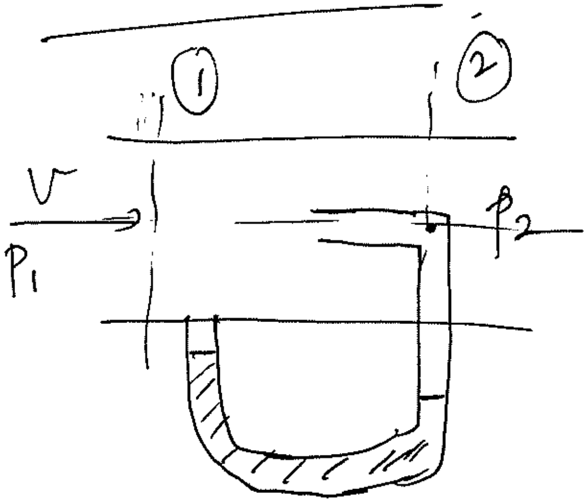
← Bernoulli's equation.

$$\left(\frac{P_i}{\rho_i} + \frac{1}{2}v_i^2 + g z_i\right) + \frac{\dot{W}}{\dot{m}} - \frac{\mathcal{F}}{\dot{m}} = \frac{P_o}{\rho_o} + \frac{1}{2}v_o^2 + g z_o$$

<u>Head form</u> : Divide by g

$$\left(\underbrace{\frac{P_i}{\rho g}}_{\text{Pressure head}} + \underbrace{\frac{1}{2}\frac{v_i^2}{g}}_{\text{Velocity head}} + \underbrace{z_i}_{\text{elevation}}\right) + \underbrace{\frac{\dot{W}}{\dot{m}g}}_{\text{turbine/pump head}} - \underbrace{\frac{\mathcal{F}}{\dot{m}g}}_{\text{head loss}} = \frac{P_o}{\rho g} + \frac{1}{2}\frac{v_o^2}{g} + z_o$$

Pitot tube



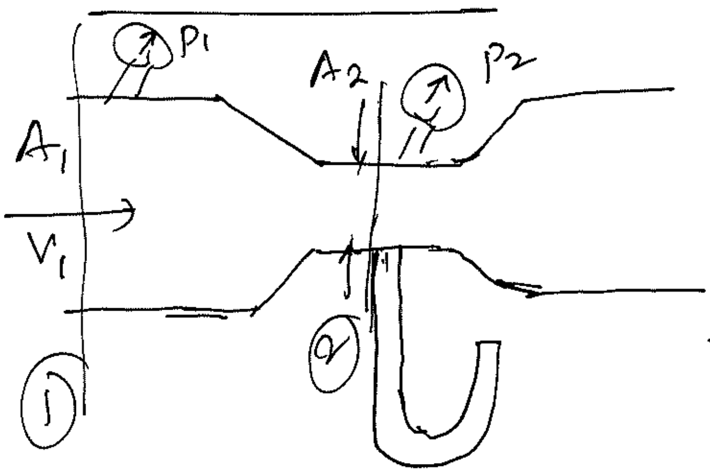
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}$$

$$V_2 = 0$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} \Rightarrow$$

$$V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

Venturi meter



$V_2 > V_1$

$P_1 > P_2$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} \leftarrow \text{Energy}$$

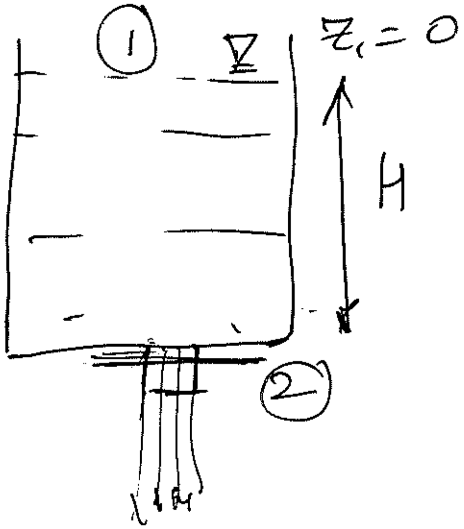
Mass Balance : $Q = A_1 V_1 = A_2 V_2$

$V_1 = Q/A_1$, $V_2 = Q/A_2$

$$\frac{P_1}{\rho} + \frac{Q^2}{2A_1^2} = \frac{P_2}{\rho} + \frac{Q^2}{2A_2^2}$$

$$Q^2\left[\frac{1}{2A_2^2} - \frac{1}{2A_1^2}\right] = \frac{P_1 - P_2}{\rho}$$

$$Q = \sqrt{\frac{P_1 - P_2}{\rho\left(1/2A_2^2 - 1/2A_1^2\right)}}$$



$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$\cancel{\frac{p_a}{\rho}} + \frac{V_1^2}{2} + g(0) = \cancel{\frac{p_a}{\rho}} + \frac{V_2^2}{2} + g(-H)$$

$$\frac{V_1^2}{2} = \frac{V_2^2}{2} - gH$$

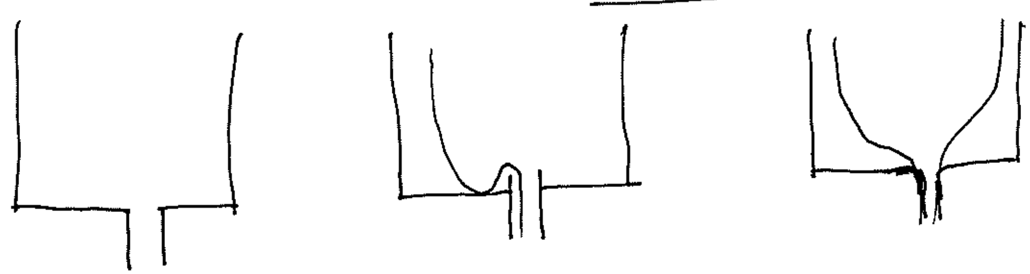
Mass balance: $A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{A_2}{A_1} V_2$

If $A_2 << A_1$ $\quad V_1 << V_2 \Rightarrow V_1^2 << V_2^2$

$\Rightarrow V_1^2 \approx 0$

$$0 = \frac{V_2^2}{2} - gH$$

$$\Rightarrow \boxed{V_2 = \sqrt{2gH}}$$ ← Torricelli's equation



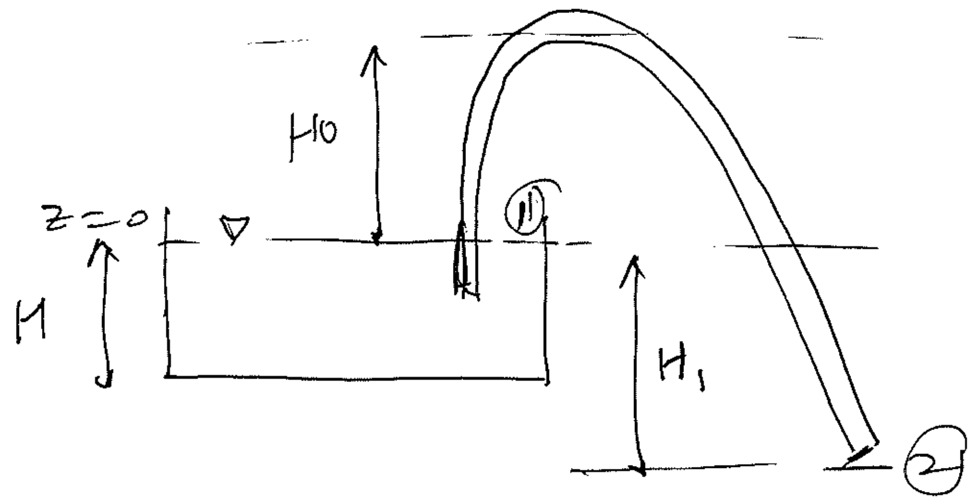
Define $C_D \triangleq \frac{V_2}{\sqrt{2gH}}$

Coefficient of discharge

experimental

theory velocity

Siphon



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$\frac{P_a}{\rho} + 0 + 0 = \frac{P_a}{\rho} + \frac{V_2^2}{2} - g(H_1)$$

$(V_1 \simeq 0)$

$$\Rightarrow \quad V_2 = \sqrt{2 g H_1}$$