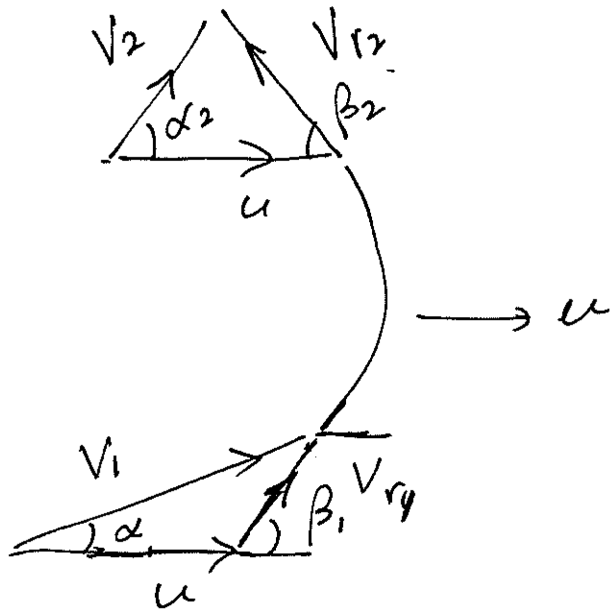


$$V_1 = V_{1b} + u_b$$
$$V_1 = V_{r1} + u$$

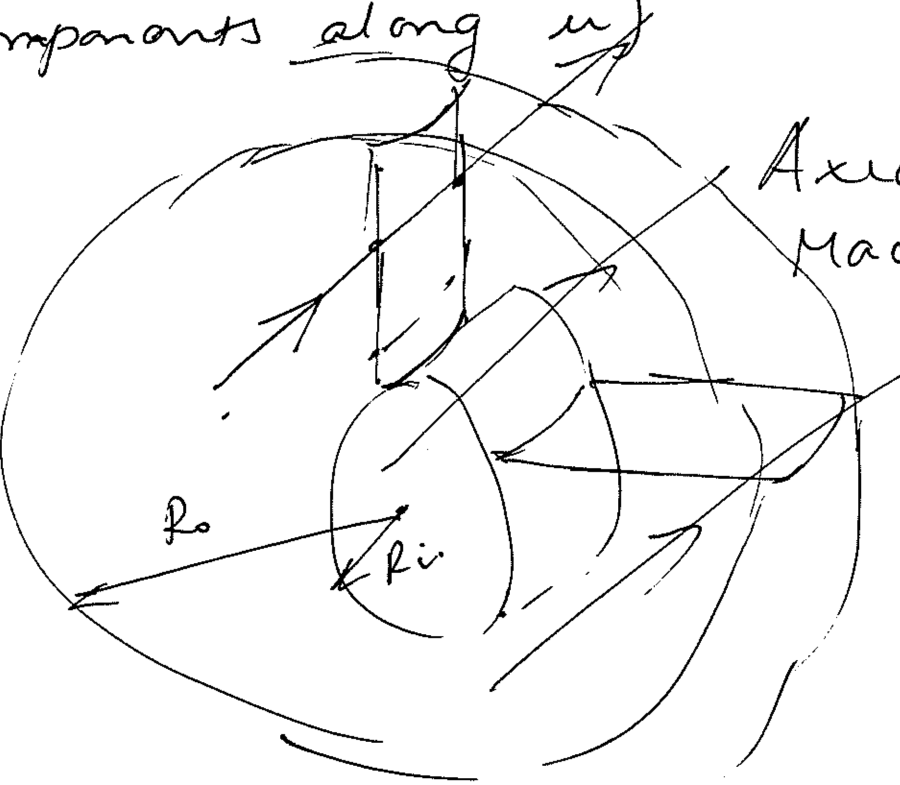
Σ X: Momentum

$$\dot{L}_{in} - \dot{L}_{out} + \sum F = 0$$

$$\dot{m} V_{t_1} - \dot{m} V_{t_2} + F = 0$$

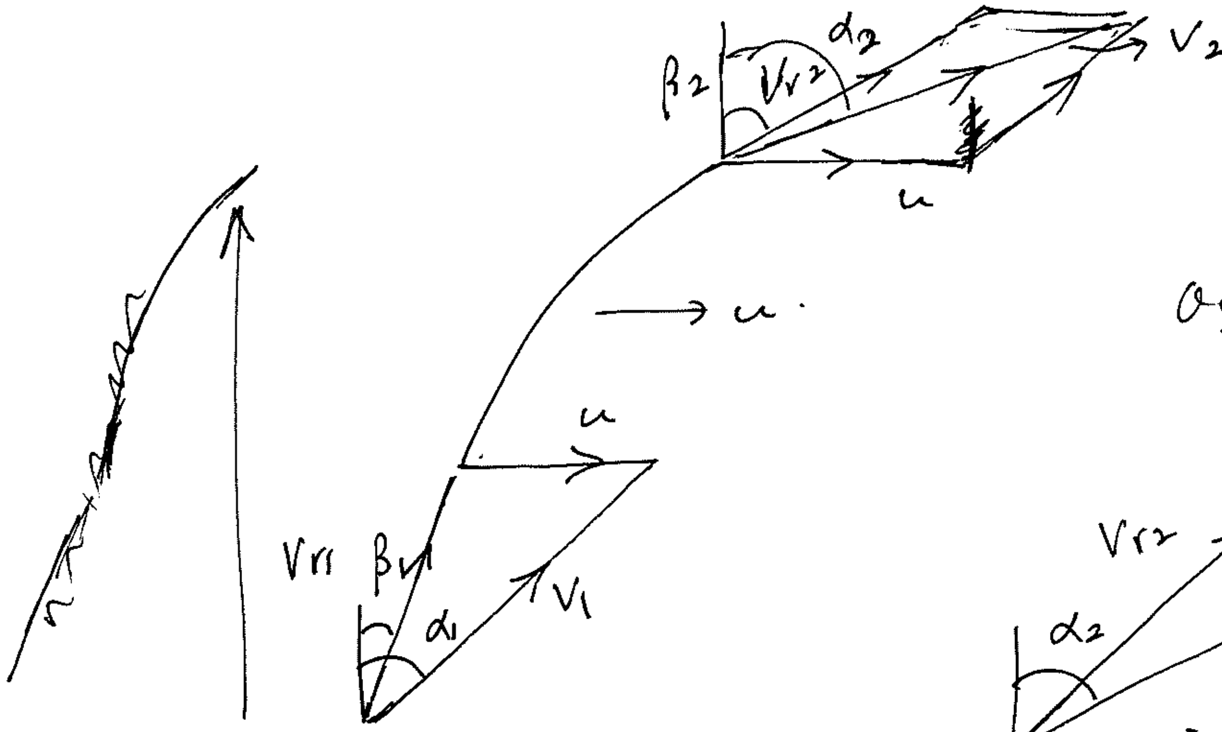
Tangential components of V_1 & V_2
(ie components along u)

Centrifugal/
Radial
Machines

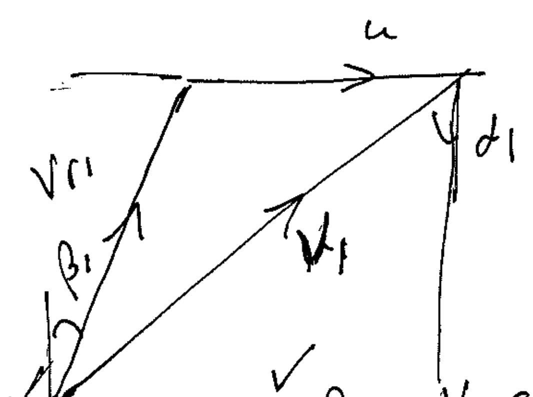


Axial
Machines

$$r = \frac{R_i + R_o}{2}$$



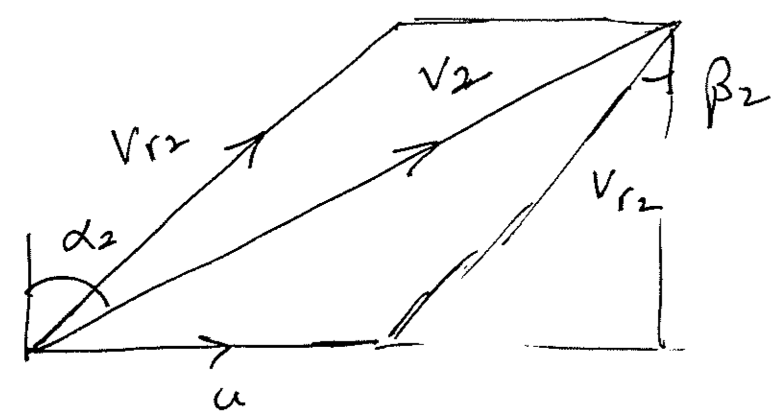
Inlet



$V_a = V_{r_1} \cos\beta_1 = V_1 \cos\alpha_1$

$V_{t_1} = V_{r_1} \sin\beta_1 + u$

Outlet



$V_a = V_{r_2} \cos\beta_2 = V_2 \cos\alpha_2$

$V_{t_2} = V_{r_2} \sin\beta_2 + u$

Conservation of Angular Momentum

$\dot{H}_{in} - \dot{H}_{out} + T = 0$

$\dot{m} V_{t_1} r - \dot{m} V_{t_2} r + T = 0$

$T = \dot{m}(V_{t_2} - V_{t_1}) r$

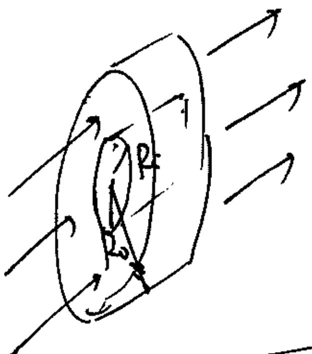
$= \dot{m}(V_{r_2}\sin\beta_2 + u - V_{r_1}\sin\beta_1 - u) r$

$T = \dot{m}(V_{r_2}\sin\beta_2 - V_{r_1}\sin\beta_1) r$

$$V_{r_1} = \frac{V_a}{\cos\beta_1} \qquad V_{r_2} = \frac{V_a}{\cos\beta_2}$$

$$T = \dot{m}\left(V_a \cancel{\cos} \frac{\sin\beta_2}{\cos\beta_2} - V_a \frac{\sin\beta_1}{\cos\beta_1} \right) r$$

$$\boxed{T = \dot{m}\, V_a (\tan\beta_2 - \tan\beta_1)\, r}$$



$$\dot{m}_{front} = \dot{m}_{back}$$

$$\cancel{\rho} \cancel{A}_{fr} V_{a_{fr}} = \cancel{\rho} \cancel{A}_{back} V_{a_{back}}$$

$$V_{a_{fr}} = V_{a_{back}}$$

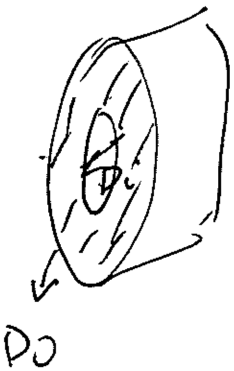
$$\boxed{P = \cancel{\dot{m}}\, T\omega = \dot{m}\, V_a (\tan\beta_2 - \tan\beta_1)\, u}$$

Work done per unit mass $w = \frac{P}{\dot{m}} = V_a u (\tan\beta_2 - \tan\beta_1)$

$$\boxed{\text{Head} = \frac{w}{g} = \frac{V_a u (\tan\beta_2 - \tan\beta_1)}{g}}$$

Ex: $\dot{m} = 10\,kg/s$ air $\quad N = 1800$ rpm.

$\beta_1 = 30^\circ \quad \beta_2 = 60^\circ \quad D_i = 0.8\,m \quad D_o = 1.6\,m.$



Mean Diameter $= \dfrac{0.8 + 1.6}{2} = 1.2\,m.$

$u = R\omega = \left(\dfrac{1.2}{2}\right)(1800)\dfrac{2\pi}{60} = 113.1\,m/s$

$V_a = Q \quad Q = V_a \dfrac{\pi}{4}(D_o^2 - D_i^2)$

$\rho = 1.25\,\dfrac{kg}{m^3}$

$Q = \dfrac{\dot{m}}{\rho} = \dfrac{10}{1.25} = 8\,m^3/s$

$8 = V_a \dfrac{\pi}{4}(1.6^2 - 0.8^2) \Rightarrow V_a = 5.31\,m/s$

$T = \dot{m} V_a (\tan\beta_2 - \tan\beta_1) r$

$= (10)(5.31)(0.6)(\tan 60 - \tan 30) = 36.8$ N-m.

Power $= T\omega = (36.8)(188.5) = \boxed{6.94\,kW.}$

pressure rise across machine (blade)

Pressure ~~pr~~ rise.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + w = \frac{p_2}{\rho} + \frac{V_2^2}{2}$$

$$\Delta p = \frac{p_2 - p_1}{\rho} = \frac{V_1^2}{2} + w - \frac{V_2^2}{2}$$

$$w = V_a u (\tan\beta_2 - \tan\beta_1)$$

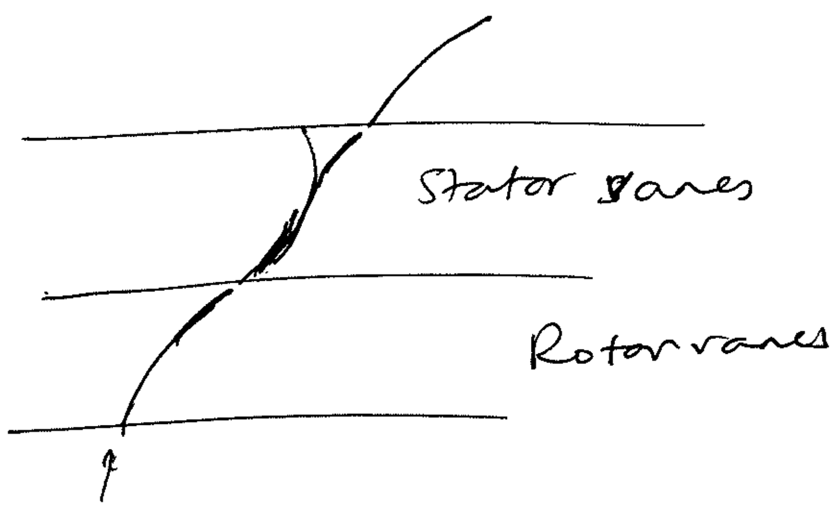
$$= (5.31)(113)(\tan 60 - \tan 30)$$

$$= 692.8 \ \cancel{kJ/k} \ J/kg$$

$$\frac{\Delta p}{\rho} = 6\cancel{9}92.8 + \frac{116.2^2}{2} - \frac{122.2^2}{2}$$

$$= 692.8 - 715.2$$

$$= -22.4 \ Pa$$



$V_a = 5.3\cancel{1}$

$\beta_1 = \cancel{1}30°$

$$V_{r_1} = \frac{V_a}{\cos\beta_1} = 6.13 \text{ m/s}$$

$$V_{t_1} = u + V_{r_1}\sin\beta_1$$

$$= 113 + 6.13 \sin 30$$

$$= 116.1 \text{ m/s}$$

$$V_1 = \sqrt{V_{t_1}^2 + V_a^2} = 116.2 \text{ m/s}$$

$$V_{r_2} = \frac{V_a}{\cos 60} = \frac{6.13}{2} = 3.07 \text{ m/s}$$

$$V_{t_2} = u + V_{r_2} \cancel{\cos} \sin\beta_2$$

$$= 113 + \cancel{3.07} \sin 60$$

$$=$$

$$V_{r_2} = \frac{V_a}{\cos 60} = \frac{5.31}{\cos 60} = 10.62 \text{ m/s}$$

$$V_{t_2} = 113 + 10.62 \sin 60$$

$$= 122.2 \text{ m/s}$$

$$V_2 = \sqrt{122.2^2 + 5.31^2} \approx 122.2$$

Scaling laws

$u = R\omega$

$V_a \sim u$

$(N = \text{rpm})$

$$Q = V_a \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$

$$= R\omega \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$

$$\boxed{Q \sim D^3 N}$$

$$\Delta p \sim w = V_a u(\tan\beta_1 - \tan\beta_2)$$

$$\Delta p \sim (DN)(DN)$$

$$\boxed{\Delta p \sim D^2 N^2}$$

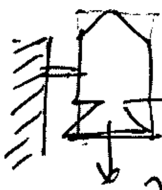
$$\text{Power} = \dot{m}\, V_a (\tan\beta_2 - \tan\beta_1)\, u$$

$$= \rho Q\, V_a (\tan\beta_2 - \tan\beta_1)\, u.$$

$$\text{Power} \sim D^3 N\, (DN)\, (DN)$$

$$\boxed{\text{Power} \sim D^5 N^3}$$

<u>Rockets</u>:



$\dot{m}_e$

v_e

<u>Momentum Balance</u>:

$$\mathring{L}_{in} - \mathring{L}_{out} + \sum F = 0$$

$$\mathring{L}_{in} = 0$$

$$\dot{L}_{out} = -\dot{m}_e v_e$$

$$F = F_{stand} - mg + (p_e - p_a) A_e$$

p_e → pressure at exit of nozzle

p_a → atmospheric pressure

A_e ← nozzle area.

$$-\dot{m}_e v_e + F_{stand} - mg + (p_e - p_a) A_e = 0$$

$$F_{stand} = \dot{m}_e v_e + mg - (p_e - p_a) A_e$$

If ~~mg can be ignored~~

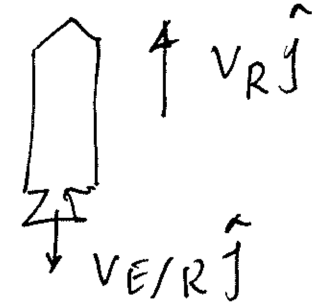
$$F_{thrust} = (F_{stand} - mg)$$

$$= \dot{m}_e v_e - (p_e - p_a) A_e$$

$$\frac{F_{thrust}}{\dot{m}_e g} = \text{Specific impulse} \longrightarrow \text{Units (in seconds)}$$

Rocket in motion

$$\dot{L}_{in} - \dot{L}_{out} + \Sigma F = \frac{d L_{Rocket}}{dt}$$



$$\dot{L}_{in} = 0$$

$$\dot{L}_{out} = \dot{m}_e v_e =$$

$$= \dot{m}_e (-V_{E/R} + V_R)$$

$$V_E = V_{E/R} + V_R$$

$$= -V_{E/R} \hat{j} + V_R \hat{j}$$

$$= (-V_{E/R} + V_R) \hat{j}$$

$$F = mg$$

$$m(t) = M_R + m_f(t)$$

$$= (M_R + m_{fo} - \dot{m}_e t)$$

$$L_{Rocket} = m(t) V_R(t)$$

$$\frac{d L_{Rocket}}{dt} = \frac{dm(t)}{dt} V_R(t) + m(t) \frac{d V_R}{dt}$$

$$= -\dot{m}_e V_R(t) + m(t) \frac{d V_R}{dt}$$

$$0 - \dot{m}_e(-V_{E/R} + V_R) - mg = -\dot{m}_e V_R(t) + m(t) \frac{d V_R}{dt}$$

$$\dot{m}_e V_{E/R} - mg = m \frac{d V_R}{dt}$$

$$\frac{\dot{m}_e V_{E/R}}{m} - g = \frac{d V_R}{dt}$$

$$\frac{dv_R}{dt} = \frac{\dot{m}_e v_{E/R}}{M_R + m_{f0} - \dot{m}_e t} - g$$

$$v_R = \dot{m}_e v_{ER} \left. \frac{\ln(M_R + m_{f0} - \dot{m}_e t)}{-\dot{m}_e} \right|_0^t - gt$$

$$= -v_{E/R}\left[\ln(M_R + m_{f0} - \dot{m}_e t) - \ln(M_R + m_{f0})\right] - gt$$

$$\boxed{v_R(t) = -v_{E/R} \ln\left(1 - \frac{\dot{m}_e t}{M_R + m_{f0}}\right) - gt}$$

Time taken for fuel to burn up: $m_f(t) = m_{f0} - (\dot{m}_e)(t) = 0$

$$\Rightarrow \boxed{t = \frac{m_{f0}}{\dot{m}_e}}$$

$$h(t) = \int_0^t v_R(t)\, dt$$