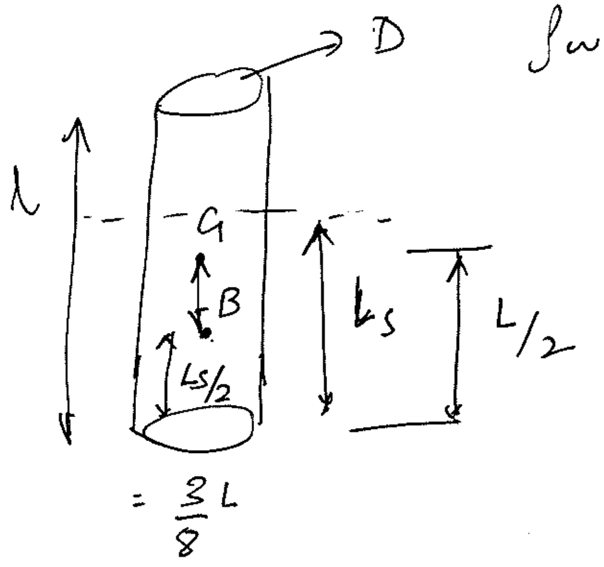
1



$\rho_{wood} = 0.6 \qquad \rho_{oil} = 0.8$

$\Sigma F_y = 0: \qquad W_{log} = F_{buoyancy}$

$$\rho_{wood} \frac{\pi D^2}{4} L = \rho_{oil} \left(\frac{\pi}{4} D^2\right) L_s$$

$$L_s = \frac{\rho_{wood}}{\rho_{oil}} L = \frac{0.6}{0.8} L = \frac{3}{4} L$$

$$BG = L/2 - 3/8\, L = L/8$$

$$GM = \frac{I_c}{\forall} - BG$$

$\forall$ → submerged volume

Greatest length for stability

$$\boxed{L < \sqrt{\frac{2}{3}}\, D}$$

GM > 0 for stability -

$$\frac{I_c}{\forall} - BG > 0 \Rightarrow \frac{\frac{\pi}{64} D^4}{\frac{\pi}{4} D^2 L_s} - \frac{L}{8} > 0$$

$$= \frac{D^2}{16 L_s} - \frac{L}{8} > 0 \Rightarrow \frac{D^2}{\cancel{16}\,4 \cdot \frac{3}{\cancel{4}} L} - \frac{L}{8} > 0$$

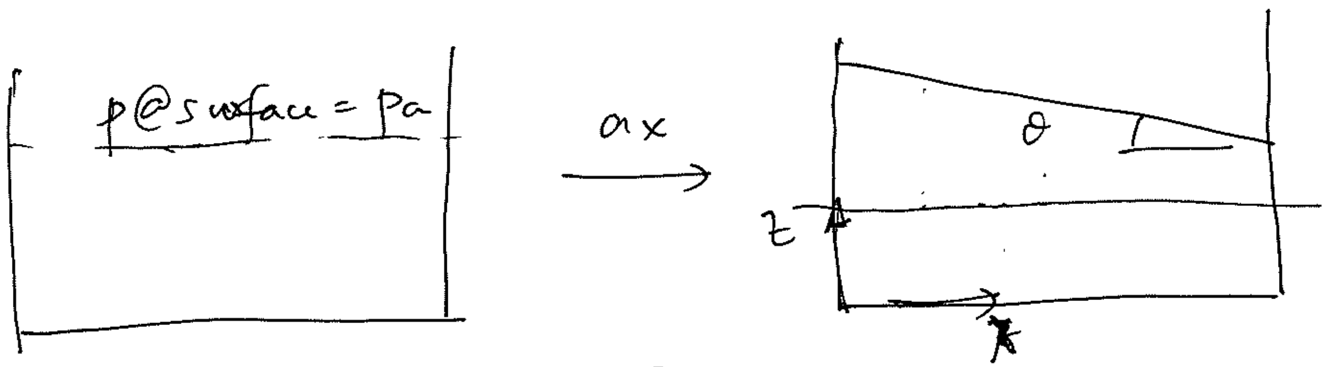
$$\Rightarrow D^2 > \frac{3L^2}{\cancel{2}} \Rightarrow D > \frac{\sqrt{3}}{\sqrt{2}} L$$

Accelerating Containers

$$\frac{\partial p}{\partial x} = -\rho a_x$$

$$\frac{\partial p}{\partial y} = -\rho a_y$$

$$\frac{\partial p}{\partial z} = -\rho (a_z + g)$$



$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz$$

Along the open surface $dp = 0$ ($\because p$ is atmospheric)

$$dp = 0 \Rightarrow \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz = 0 \Rightarrow \frac{dz}{dx} = \frac{-\partial p/\partial x}{\partial p/\partial z}$$

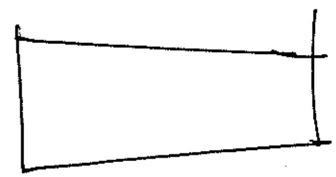
slope of surface

$$\frac{dz}{dx} = -\frac{-\rho a_x}{-\rho(a_z+g)} \Rightarrow \frac{dz}{dx} = -\frac{a_x}{a_z+g}$$

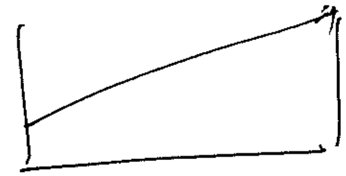
Let $a_z = 0$.

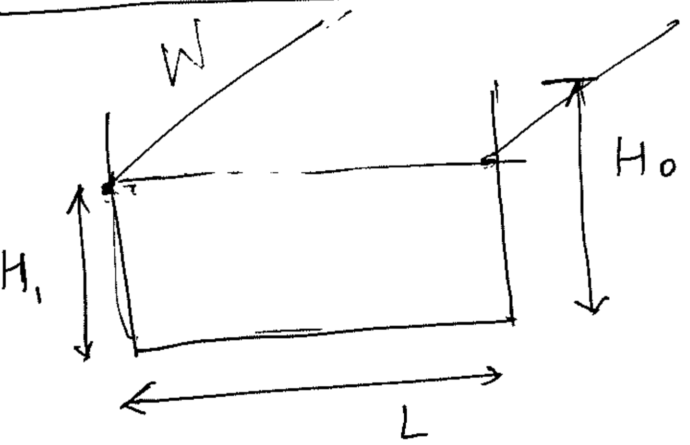
$$\frac{dz}{dx} = \frac{-a_x}{g}$$

$a_x > 0$

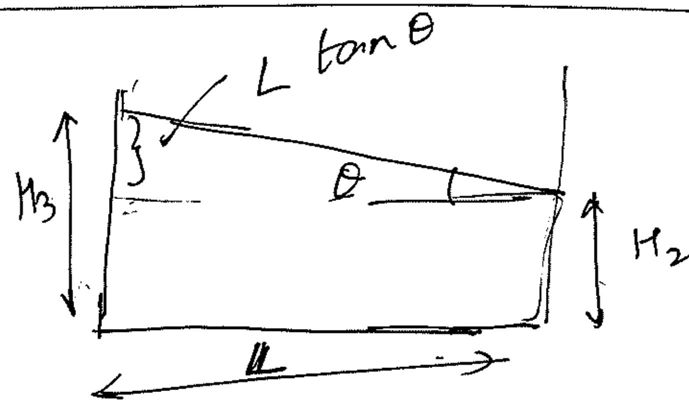


$a_x < 0$





$a_x \longrightarrow$



$$\frac{dz}{dx} = \tan\theta = \frac{a_x}{g}$$

$$\text{Volume} = W H_1 L$$

$$=$$

$$\text{Volume} = W\left(L H_2 + \frac{1}{2}(H_3 - H_2) L\right)$$

$$= W L \frac{H_2 + H_3}{2}$$

$$H_1 = \frac{H_2 + H_3}{2} \Rightarrow H_2 + H_3 = 2H_1$$

$$H_3 - H_2 = L \tan\theta$$

$$H_3 = \frac{2H_1 + L\tan\theta}{2}, \quad H_2 = \frac{2H_1 - L\tan\theta}{2}$$

$$= \frac{2H_1 + L\frac{a_x}{g}}{2} \qquad H_2 = \frac{2H_1 - \frac{L a_x}{g}}{2}$$

$$dp = \frac{\partial p}{\partial x}\,dx + \frac{\partial p}{\partial z}\,dz$$

$$\int dp = \int \frac{\partial p}{\partial x}\,dx + \int \frac{\partial p}{\partial z}\,dz$$

$$p = \int -\rho a_x\,dx + \int -\rho(a_z+g)\,dz$$

Assuming a_x, a_z are constant

$$= -\rho a_x x - \rho(a_z+g)z + C$$

At $(\cancel{x_0, z_0})$ $(0, H_1)$ $p = p_a$

or $(0, H_1)$

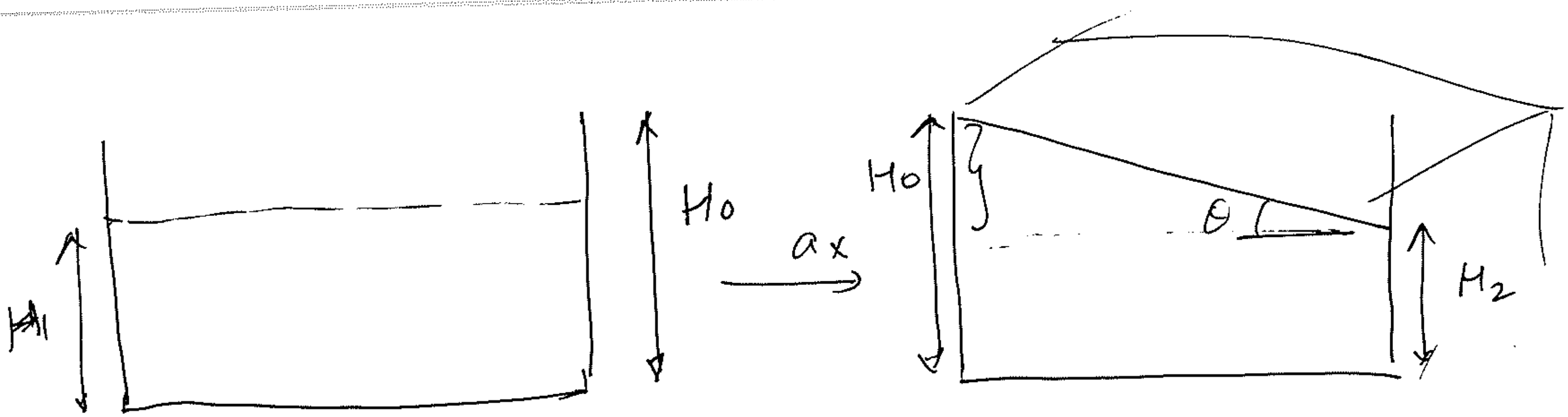
$$p_a = -\rho a_x(0) - \rho(a_z+g)H_1 + C$$

$$C = p_a + \rho(a_z+g)H_1$$

$$p = -\rho a_x x - \rho(a_z+g)(z-H_1) + p_a$$

$$\boxed{p - p_a = -\rho a_x x - \rho(a_z+g)(z-H_1)}$$

(4)



$$\forall = L H_1 W = \frac{1}{2} L W (H_0 + H_2)$$

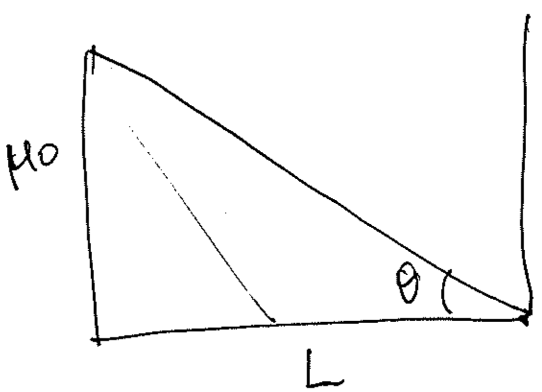
$$H_1 = \frac{1}{2}(H_0 + H_2) \Rightarrow H_2 = 2H_1 - H_0$$

$$\tan\theta = \frac{H_0 - H_2}{L}$$

$$= \frac{H_0 - (2H_1 - H_0)}{L} = \frac{2(H_0 - H_1)}{L}$$

$$\frac{a_x}{g} = \frac{2(H_0 - H_1)}{L} \Rightarrow a_x = 2\left(\frac{H_0 - H_1}{L}\right) g$$

$$a_x > \frac{2(H_0 - H_1)}{L} g$$

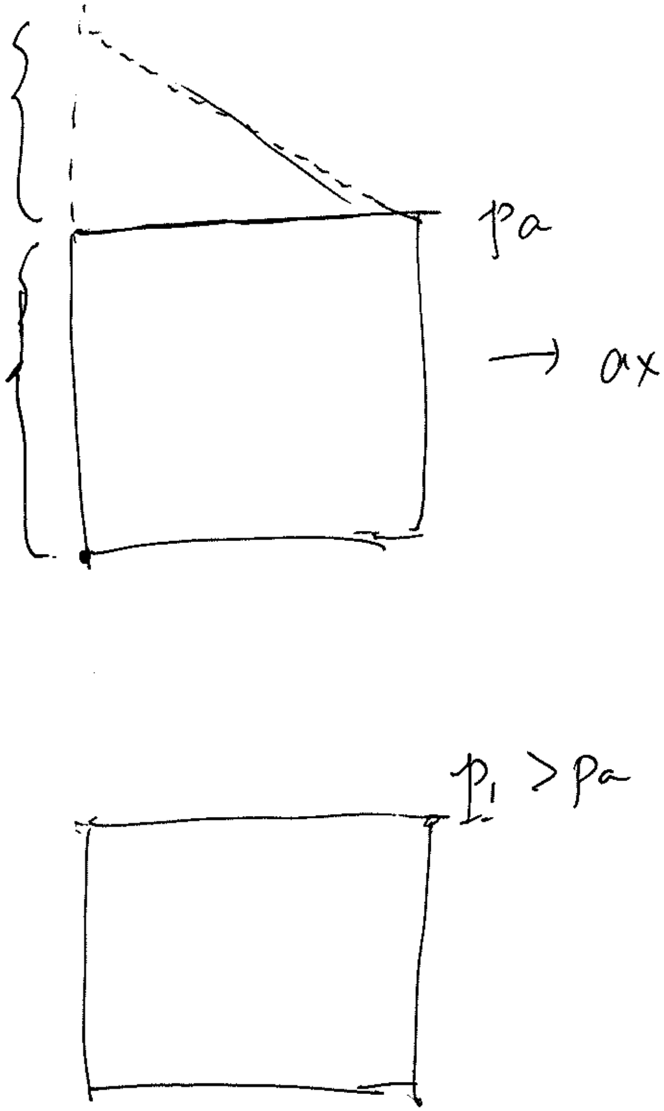


$$\tan\theta = \frac{H_0}{L}$$

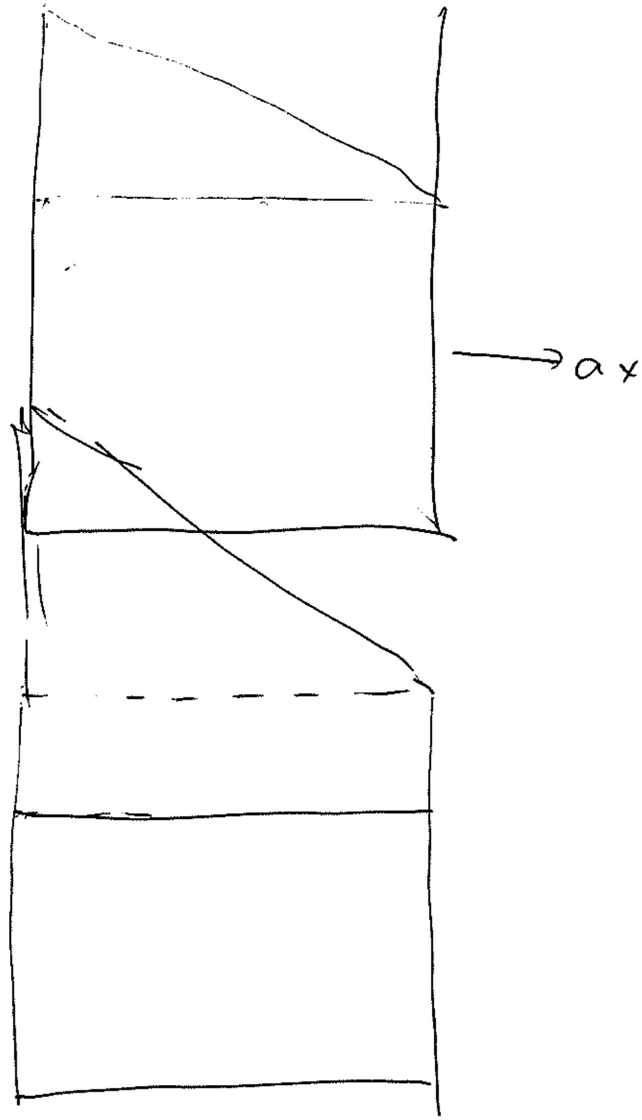
$$\frac{a_x}{g} = \frac{H_0}{L} \Rightarrow a_x = \frac{g H_0}{L}$$

Volume of water spilled

$$(L H_1 W) - \frac{1}{2} L H_0 W$$

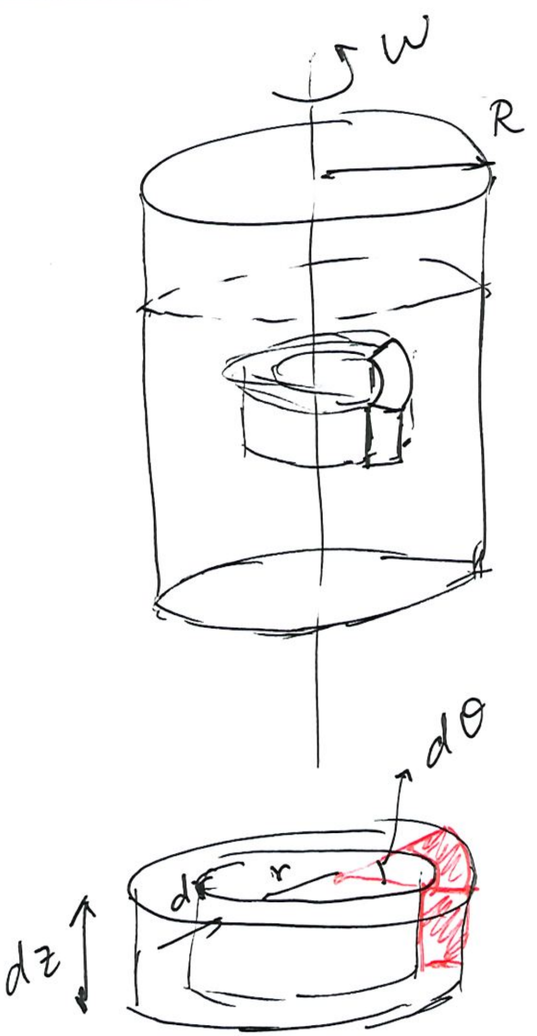


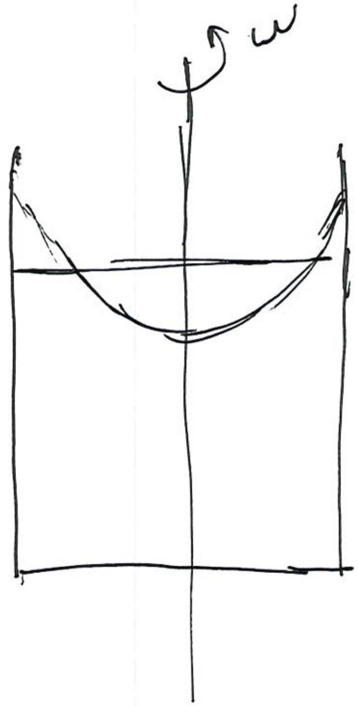
pa
→ ax
$p_1 > p_a$



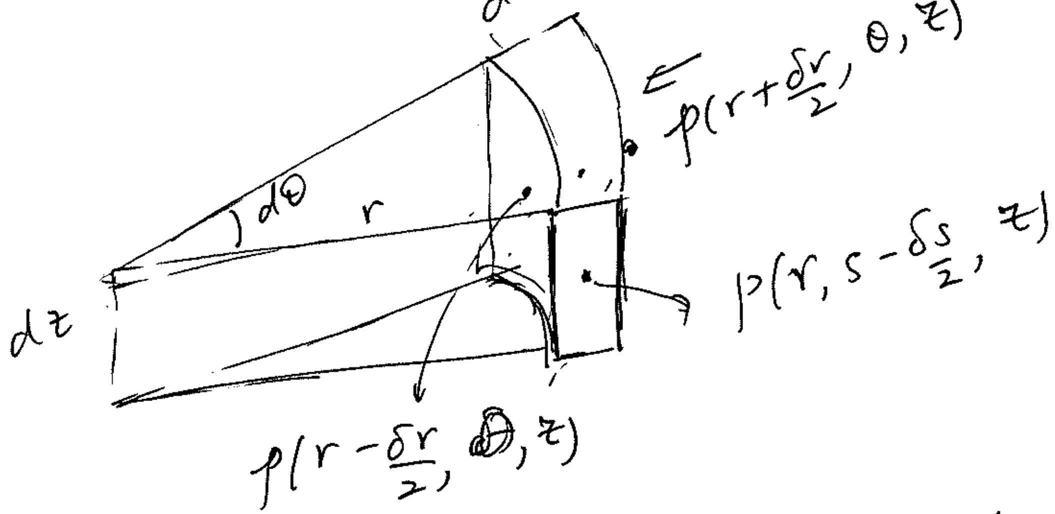
→ ax

6





Pressure at center of element is $p(r, \theta, z)$



$$\Sigma F_r = m a_r \Rightarrow p(r-\frac{\delta r}{2}; \theta, z)\, dz\, ds - p(r+\frac{\delta r}{2}, \theta, z)\, dz\, ds = -(\rho\, \delta r\, \delta z\, \delta s)\, \omega^2 r$$

$$\left(p(r,\theta,z) - \frac{\partial p}{\partial r}\frac{\delta r}{2}\right) dz\, ds - \left(p(r,\theta,z) + \frac{\partial p}{\partial r}\frac{\delta r}{2}\right) dz\, ds = -\rho\, \delta r\, \delta z\, \delta s\, \omega^2 r$$

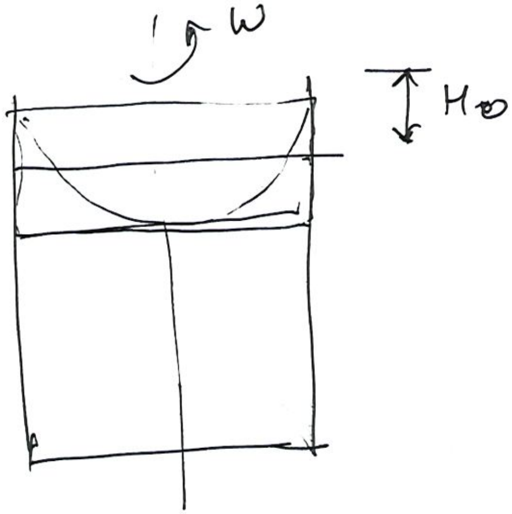
$$\boxed{-\frac{\partial p}{\partial r} = -\rho \omega^2 r}$$

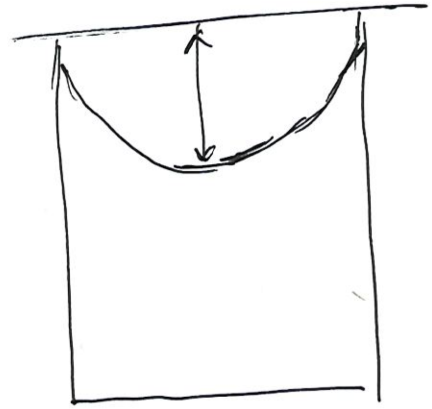
$$\Sigma F_\theta = m a_\theta \Rightarrow p(r, s-\frac{\delta s}{2}, z)\, dr\, dz - p(r, s+\frac{\delta s}{2}, z)\, dr\, dz = \rho\, \delta r\, \delta z\, \delta s\, a_\theta$$

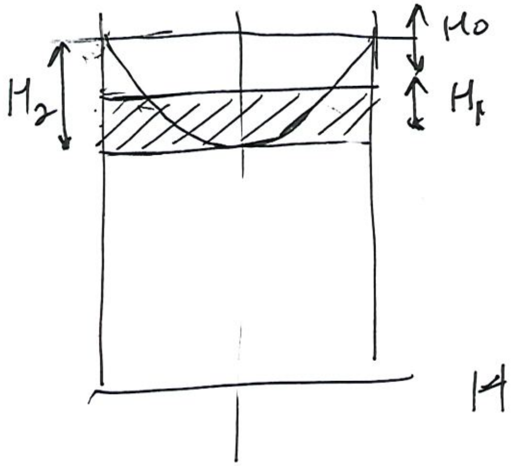
$$\Rightarrow \boxed{-\frac{\partial p}{\partial s} = \rho a_\theta}$$

$$\Sigma F_z = m a_z \Rightarrow p(r, s, z-\frac{\delta z}{2})\, \delta s\, \delta r - p(r, s, z+\frac{\delta z}{2})\, \delta s\, \delta r - \rho\, \delta r\, \delta z\, \delta s\, g = \rho\, \delta r\, \delta z\, \delta s\, a_z$$

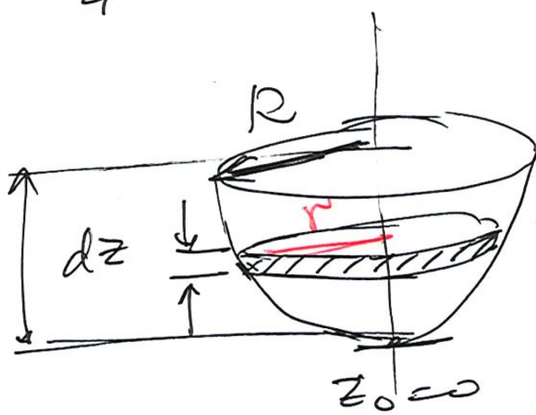
$$-\frac{\partial p}{\partial z} - \rho g = \rho a_z \Rightarrow \boxed{-\frac{\partial p}{\partial z} = +\rho(a_z + g)}$$







$\frac{\pi}{4} D^2 H_1$



$$z = \frac{\omega^2}{2g} r^2 \Rightarrow r^2 = \frac{z}{\omega^2/2g}$$

$$d\forall = \pi r^2 \, dz$$

$$\forall = \int \pi r^2 \, dz$$

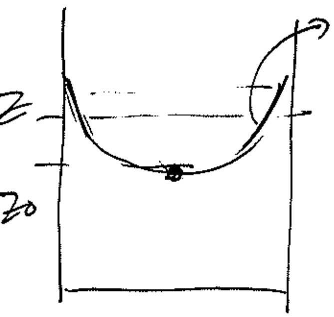
$$= \int_0^H \frac{\pi}{(\omega^2/2g)} z \, dz$$

$$= \frac{\pi}{\frac{\omega^2}{2g}} \frac{H^2}{2}$$

$$\forall = \pi \frac{H}{(\omega^2/2g)} \frac{H}{2}$$

$$\forall = \pi R^2 \frac{H}{2}$$

$\pi R^2 H$ = volume of circumscribing cylinder



free surface

$p = p_a$

$\Rightarrow dp = 0$

$$dp = \frac{\partial p}{\partial r}\,dr + \frac{\partial p}{\partial z}\,dz = 0$$

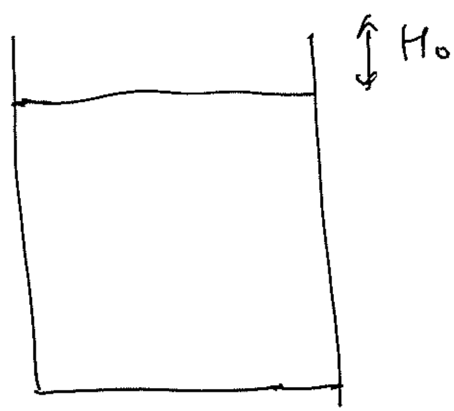
ω = constant $\qquad a_z = \cancel{g}\ 0$

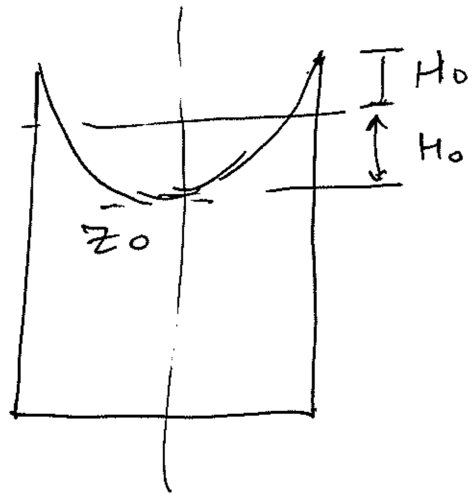
$$\frac{\partial p}{\partial r}\,dr + \frac{\partial p}{\partial z}\,dz = 0 \Longrightarrow \frac{dz}{dr} = -\frac{\partial p/\partial r}{\partial p/\partial z}$$

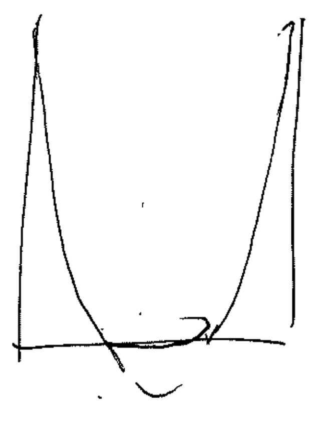
$$\Rightarrow \frac{dz}{dr} = -\frac{\rho\omega^2 r}{\cancel{g} - \rho g}$$

$$\Rightarrow \frac{dz}{dr} = \frac{\omega^2 r}{g}$$

$$\Rightarrow \int_{z_0}^{z} dz = \int_0^r \frac{\omega^2 r}{g}\,dr \Rightarrow \underbrace{z - z_0 = \frac{\omega^2 r^2}{2g}}_{\text{Paraboloid of revolution}}$$







$$z - z_0 = \frac{\omega^2 r^2}{2g}$$

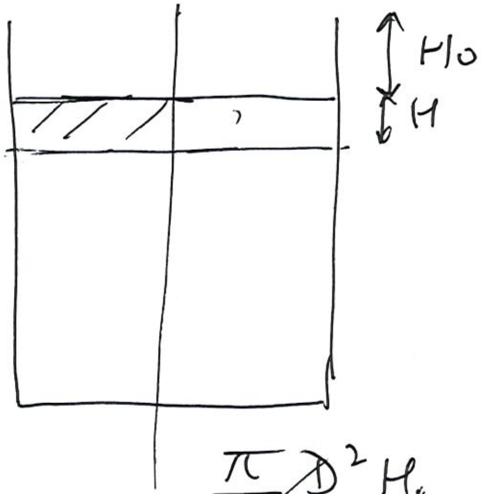
When $r = R$,

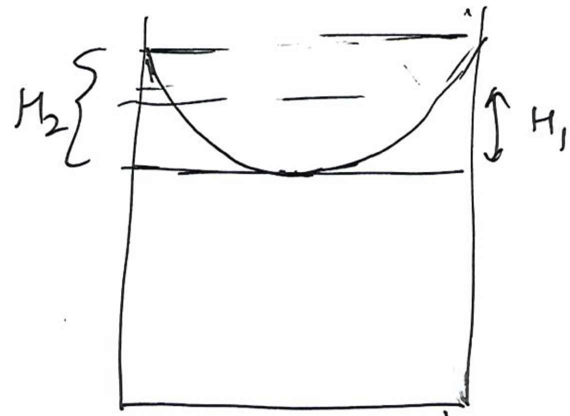
$$z - z_0 = 2H$$

$$2H = \frac{\omega^2 R^2}{2g}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4gH}{R^2}}$$

If $\omega > \sqrt{\frac{4gH}{R^2}}$, fluid spills.



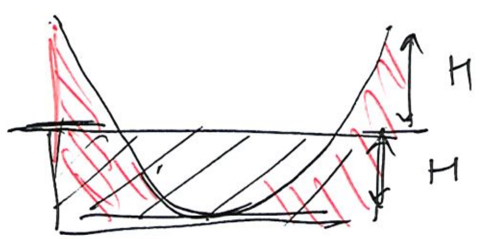


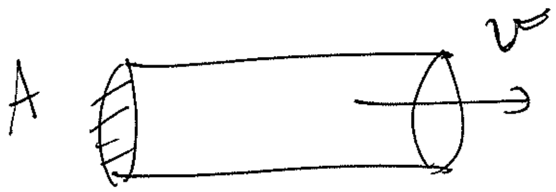
$\frac{\pi}{4} D^2 H_1$

volume of cylinder (before rotation)

$\frac{1}{2}\left(\frac{\pi}{4} D^2 H_2\right) \Rightarrow H_1 = \frac{H_2}{2}$

Volume of paraboloid (after rotation)





Volumetric flow rate

$$\dot{V} = Av$$

$$\boxed{Q = Av}$$

volumetric flow rate

$[Q] = m^3/s$, liters/s

$$1 \text{ liter} = \frac{1}{1000} m^3$$

$(1 m^3 = 1000 L)$

$[Q]$ = gallons per minute (gpm)
cubic feet per minute (cfm)