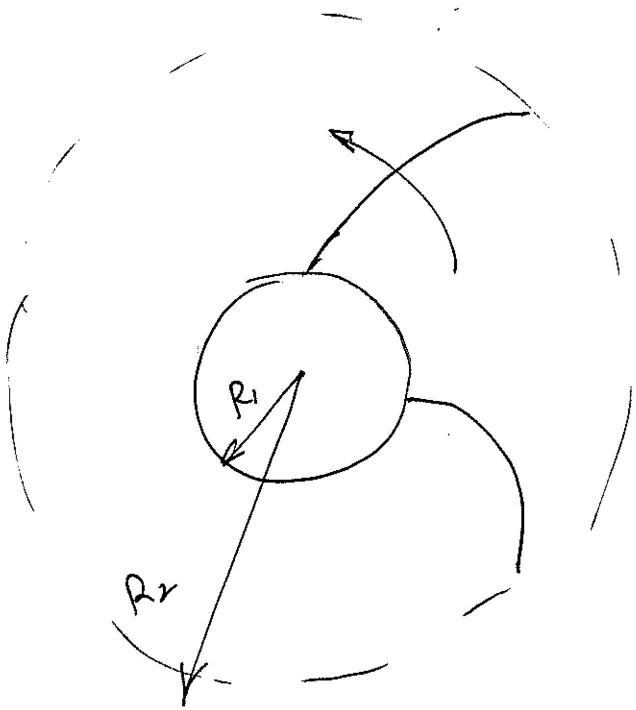
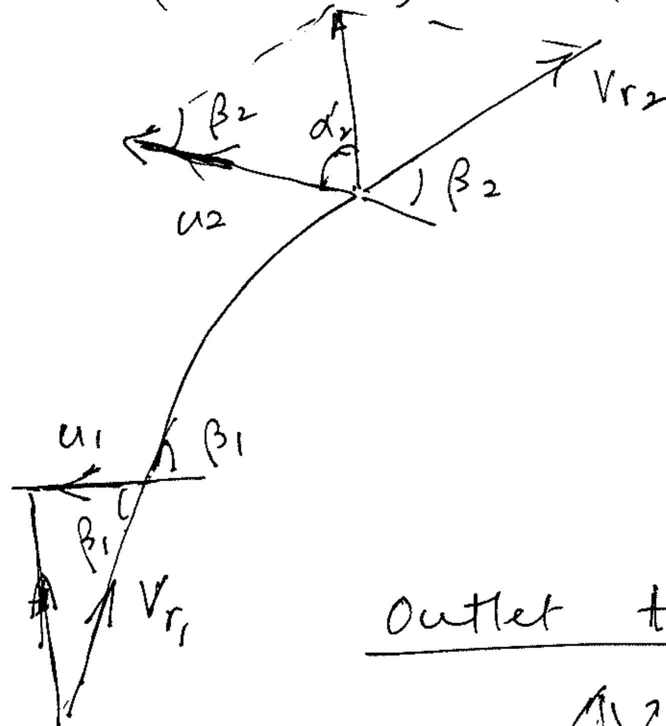
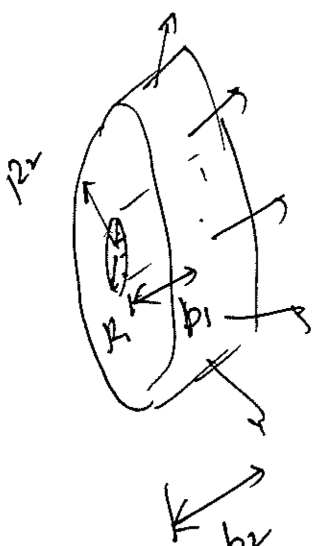
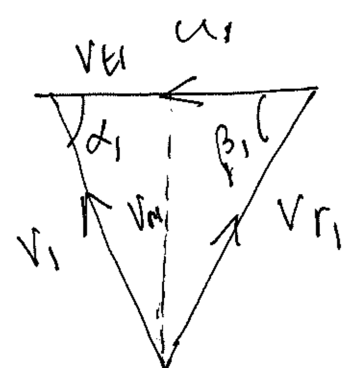
$$Q = AV = (2\pi R_1 b_1) V_{n_1} = (2\pi R_2 b_2) V_{n_2}$$







Inlet triangle

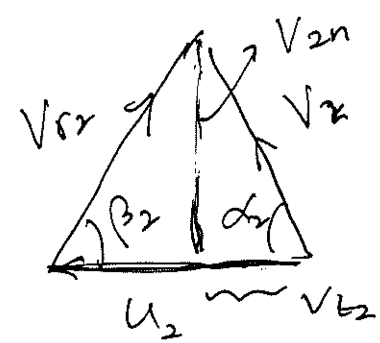


$$V_{n_1} = V_1 \sin\alpha_1 = V_{r_1} \sin\beta_1$$

$$V_{t_1} = u_1 - V_{r_1}\cos\beta_1$$

$$= u_1 - V_{n_1}\cot\beta_1$$

Outlet triangle



$$V_{2n} = V_2 \sin\alpha_2 = V_{r_2} \sin\beta_2$$

$$V_{t_2} = u_2 - V_{r_2}\cos\beta_2$$

$$= u_2 - V_{n_2}\cot\beta_2$$

Angular momentum balance

$$\dot{H}_{in} - \dot{H}_{out} + T = 0$$

$$\dot{m} V_{t_1} r_1 - \dot{m} V_{t_2} r_2 + T = 0 \Rightarrow \boxed{T = \dot{m}\left(V_{t_2} r_2 - V_{t_1} r_1\right)}$$

Power: $P = T\omega$

$$= \dot{m}\left(V_{t_2} r_2 \omega - V_{t_1} r_1 \omega\right) \qquad u_1 = R_1 \omega, \quad u_2 = R_2 \omega$$

$$\boxed{P = \dot{m}\left(V_{t_2} u_2 - V_{t_1} u_1\right)}$$

Work per kg $\boxed{w = \frac{P}{\dot{m}} = V_{t_2} u_2 - V_{t_1} u_1}$

Pump head $h_p = \frac{w}{g} = \frac{P}{\dot{m} g} = \frac{V_{t_2} u_2 - V_{t_1} u_1}{g}$

Special case: Radial entry: $\alpha_1 = 90^\circ \Rightarrow V_1 = V_{n1}$, $V_{t1} = 0$

$$h_p = \frac{V_{t_2} u_2}{g} = \frac{\left(u_2 - V_{n2}\cot\beta\right) u_2}{g} = \frac{u_2^2}{g} - \frac{V_{n_2} \cot\beta_2 u_2}{g}$$

$$= \frac{u_2^2}{g} - Q\left[\frac{1}{2\pi R_2 b_2} \cot\beta_2 \frac{u_2}{g}\right]$$

$$\boxed{H_0 = u^2/g \qquad A = \frac{1}{2\pi R_2 b_2} \cot\beta_2 \frac{u_2}{g}}$$

Pump curve $\boxed{h_p = H_0 - AQ}$

$Q = 0.5\ m^3/s$

$R_1 = 2.5$ cm

$R_2 = 18$ cm

$N = 1800$ rpm

$b = 1$ cm

$\beta_2 = 75^\circ$

$Q = 0.5 = (2\pi R_1 b_1) V_{n_1} = (2\pi R_2 b_2) V_{n_2}$

$= (2\pi)(0.025)(0.01) V_n = (2\pi)(0.18)(0.01) V_{n2}$

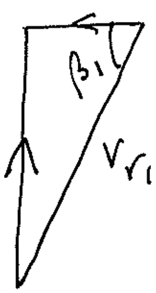
$V_{n_1} = 318.3$ m/s $\qquad V_{n2} = \cancel{44.2} \; 44.2$ m/s

$N = 1800$ rpm. $= 188.5$ rad/s

$u_1 = R_1 \omega_1 = (0.025)\,188.5$
$= 4.7$ m/s

$u_2 = R_2 \omega_2 = (0.18)188.5$
$= 33.9$ m/s

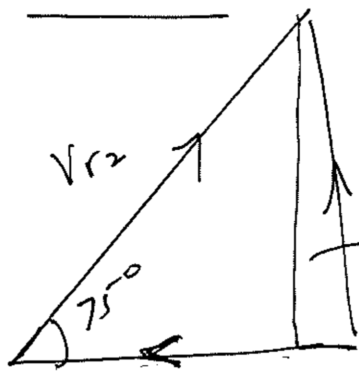
Inlet triangle



$= V_{r_1} \approx 318.3$

$\beta_1 = 89.15^\circ$

outlet



$V_{t2} = \cancel{44.2}\; 33.9 - 11.8 = 22.1$ m/s

Torque $= \dot{m}\,(V_{t2}\, r_2) = (0.5)(10)^3 [(22.1)(0.18)] = 1.98$ kN-m

Power $= T\omega = (1.98)(188.5) = 374$ kW

work per kg $\dfrac{P}{\dot{m}} = \dfrac{374 \times 10^3}{(.5)\times 10^3} = 748$ J/kg

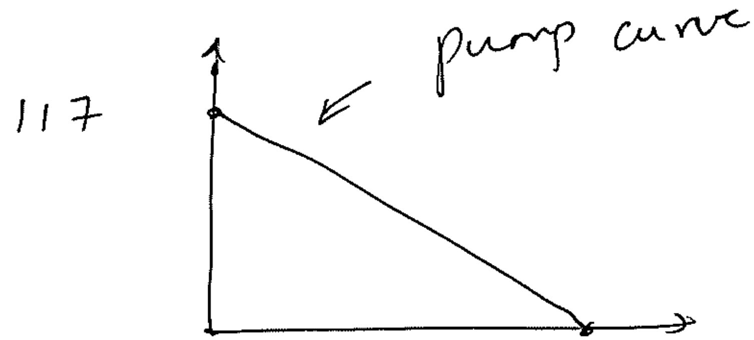
Head $= \dfrac{P}{\dot{m} g} = 76.2$ m

$$H = H_0 - AQ$$

$$H_0 = \frac{u_2^2}{g} = \frac{(33.9)^2}{9.81} = 117\ m$$

$$A = \frac{\cancel{Q}}{2\pi R_2 b_2} \cot\beta_2 \frac{u_2}{g} = \frac{1}{(2\pi)(0.18)(0.01)} \frac{1}{\tan 75} \frac{33.9}{9.81} = 81.9$$

$$\boxed{H = 117 - 81.9\,Q}$$



$$Q_{max} = \frac{117}{81.9} = 1.43\ m^3/s$$

Pressure at inlet to pump:

$$\left(\frac{p_a}{\rho} + \underbrace{\frac{V_1^2}{2}}_{=0} + g\underbrace{z_1}_{=-5}\right) - gh_L = \frac{p_s}{\rho} + \frac{V_s^2}{2} + g\underbrace{z_2}_{=0}$$

$L = 8\,m$

$K = 5.$

$$\cancel{p_a} - \quad \frac{p_a}{\rho} - \frac{p_s}{\rho} = \frac{V_s^2}{2} + gh_L - gz_1$$

$$V_s = \frac{Q}{A} = \frac{0.611}{(\pi/4)(0.5)^2} = 3.1\,m/s$$

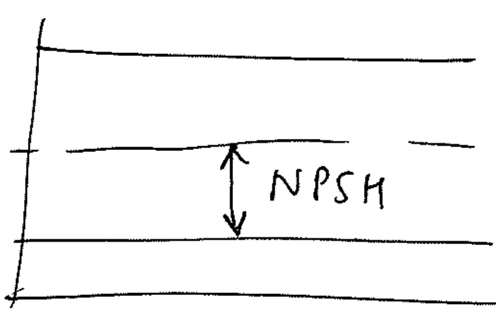
$$h_L = \left(f\frac{L}{D} + K\right)\frac{V_s^2}{2g} = \left[0.03 \times \frac{8}{0.5} + 5\right]\frac{(3.1)^2}{2g} = \frac{\cancel{11.9}}{\cancel{9.81}} \quad 2.68$$

$$\frac{p_a - p_s}{\rho} = \frac{(3.1)^2}{2} + (9.81)\overset{2.68}{(\cancel{11.9})} - (9.81)(-5)$$

$$= \cancel{80.1} \quad 80.1$$

$$p_a - p_s = 80.1\,kPa$$

$$\boxed{p_s = 101.3 - 80.1 = 20.2\,kPa}$$



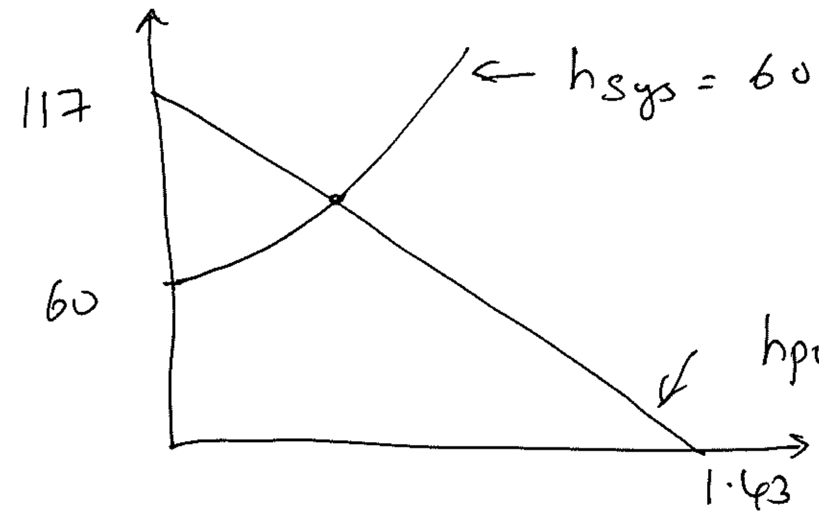
101.3 Pa

$p_s = 20.2\,kPa$

$p_v = 4.25\,kPa$

Net positive suction head (available) $= \dfrac{(20.2 - 4.25)}{\rho g}$

$$= \boxed{1.63\,m}$$



<u>Operating point:</u>

$$117 - 81.9Q = 60 + 19.6Q^2$$

$$\Rightarrow \quad 19.6Q^2 + 81.9Q - 57 = 0$$

$$Q = \frac{-81.9 \pm \sqrt{(81.9)^2 + (4)(19.6)(57)}}{(2)(19.6)}$$

$$= 0.611 \text{ m}^3/\text{s}$$

head $= 117 - (0.611)\,81.9 = 67$ m

Power $= \cancel{\rho Q}\ \dot{m} g h$

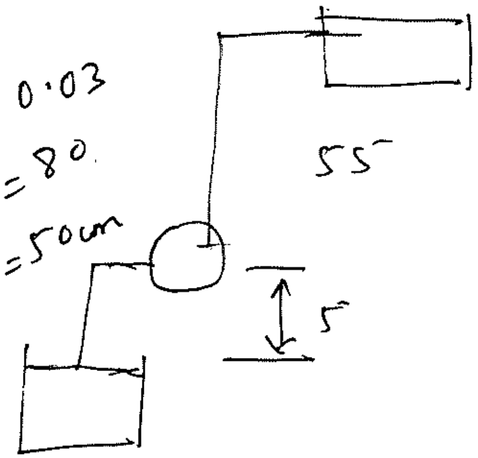
$= \rho Q g h = (10^3)(0.611)(9.81)\,67$

$= 401$ kW

$f = 0.03$

$L = 80$

$D = 50\,cm$



$K_m = 10$

$$\frac{\cancel{p_1}}{\cancel{\rho}} + \frac{V_1^2}{2} + g z_1 + g h_p - g h_L = \frac{\cancel{p_2}}{\cancel{\rho}} + \frac{V_2^2}{2} + g z_2$$

$$p_1 = p_2 = p_a =$$

$$V_1 \approx V_2 \approx 0 \quad (\text{tanks are large})$$

$$g z_1 + g h_p - g h_L = g z_2$$

$$h_p = \underbrace{(z_2 - z_1)}_{H_s = 60} + h_L$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} + K \frac{V^2}{2g} = f \frac{L}{D} \cancel{+ \frac{V^2}{}} \left(f \frac{L}{D} + K \right) \frac{V^2}{2g}$$

$$= \left(f \frac{L}{D} + k \right) \frac{Q^2}{A^2 2g}$$

$$= b Q^2$$

$$b = \left(f \frac{L}{D} + k \right) \frac{1}{(A^2)(2g)}$$

$$= \left(0.03 \frac{80}{0.5} + 10 \right) \frac{1}{(2)(9.81)(0.196)^2}$$

$$= 19.6$$

$$A = \frac{\pi}{4} D^2$$

$$= (\pi/4)(0.5)^2$$

$$= 0.196$$

$$\boxed{h_p = 60 + 19.6\, Q^2}$$

Pump scaling laws

$b \sim D$

$V_{n2} = R_2 \omega_2 \sim DN$

$$Q = 2\pi R_2 b_2 V_{n2}$$

$$Q \sim D\,D\,D\,N$$

$$\boxed{Q \sim D^3 N}$$

$1800\text{ rpm} \longrightarrow 0.611\ m^3/s$

$? \;—\; 0.5\ m^3/s$

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1} \Rightarrow \frac{0.5}{0.611} = \frac{N_2}{1800} \rightarrow \boxed{N_2 = 1473\text{ rpm}}$$

$$\cancel{h \sim} \quad h = \frac{V_{t2} u_2}{g} \qquad h \sim (DN)\,DN$$

Since $V_{t2} \approx DN$

$u_2 \sim DN$

$$\boxed{h \sim D^2 N^2} \longrightarrow \frac{h_2}{h_1} = \frac{N_2^2}{N_1^2}$$

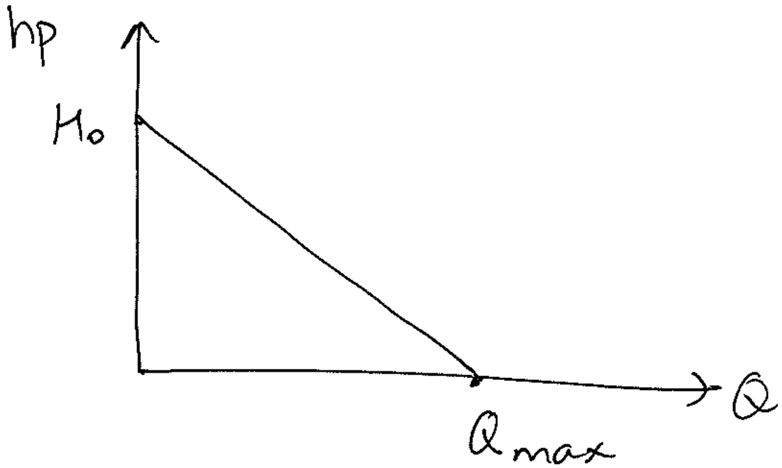
$$\frac{h_2}{67} = \left(\frac{1473}{1800}\right)^2$$

$$\boxed{h_2 = 44.8\ m}$$

$$P = \rho Q g H$$

$$P \sim D^3 N\, D^2 N^2 \Rightarrow \boxed{P \sim D^5 N^3}$$

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 \Rightarrow P_2 = \left(\frac{1473}{1800}\right)^3 401 = \boxed{219.8\ kW}$$



H_0 = Shut-off head

$$Q_{max} = \frac{H_0}{A}$$