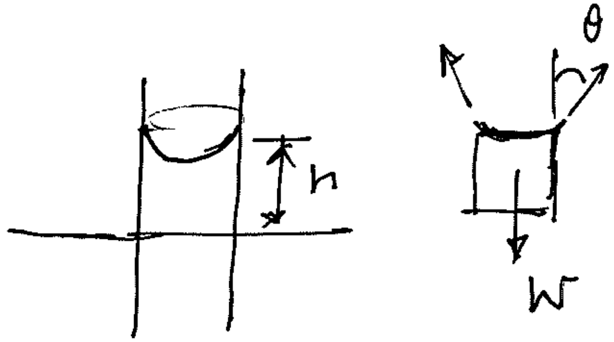
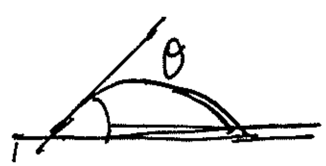
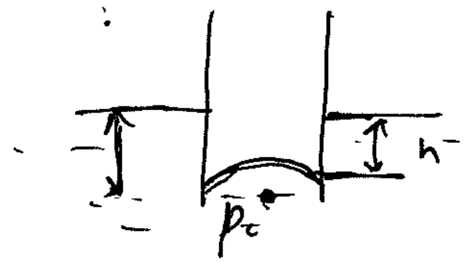
Surface tension σ

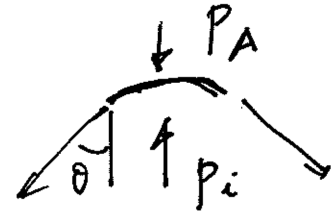


$$(\sigma)\, 2\pi R \cos\theta = W$$
$$= mg$$
$$= \rho A h g$$
$$= \rho \pi R^2 h g$$

$$h = \frac{2\sigma\cos\theta}{\rho R g}$$

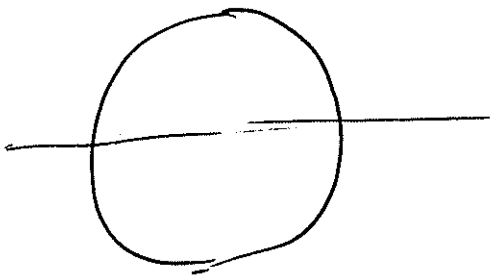


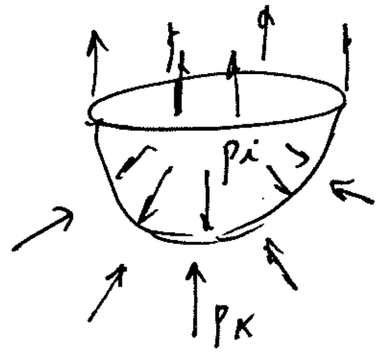




$$p_i A - p_A' A - (\sigma)\, 2\pi R \cos\theta = 0$$
$$(p_A A + A h \rho g) - p_A A = \sigma\, 2\pi R \cos\theta$$
$$A h \rho g = \sigma\, 2\pi R \cos\theta$$
$$h = \frac{2\sigma\cos\theta}{\rho R g}$$

Droplet of radius R





$\Sigma F_y = 0$

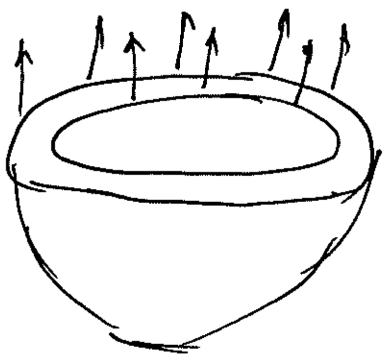
$(\sigma)\, 2\pi R = p_i \cdot \pi R^2 - p_A \pi R^2$

$= \Delta p\, \pi R^2$

$\cancel{\sigma = \frac{2\Delta p}{R}}$

$$\boxed{\Delta p = \frac{2\sigma}{R}}$$

Bubble

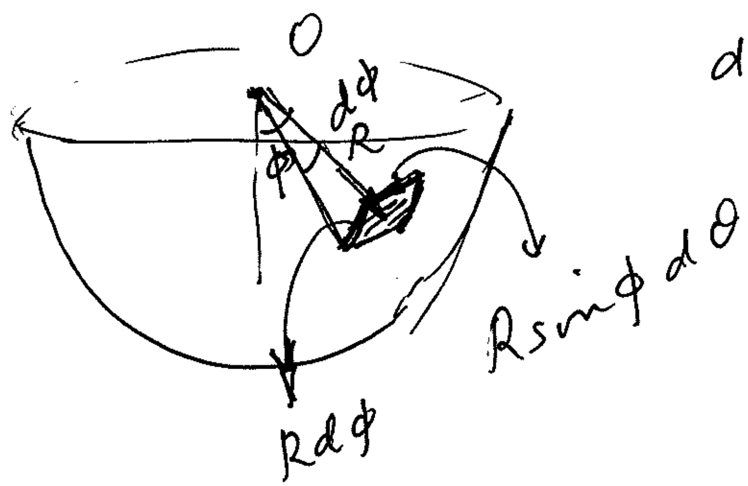


$\Sigma F_y = 0$

$\underset{\text{outside}}{(\sigma)\, 2\pi R} + \underset{\text{inside}}{\sigma\, 2\pi R} = p_i \cdot \pi R^2 - p_A \pi R^2$

$\cancel{\sigma = \frac{4\Delta p}{R}}$

$$\boxed{\Delta p = \frac{4\sigma}{R}}$$



$$dF = p_i\, dA$$

$$dF_y = dF \cos\phi$$

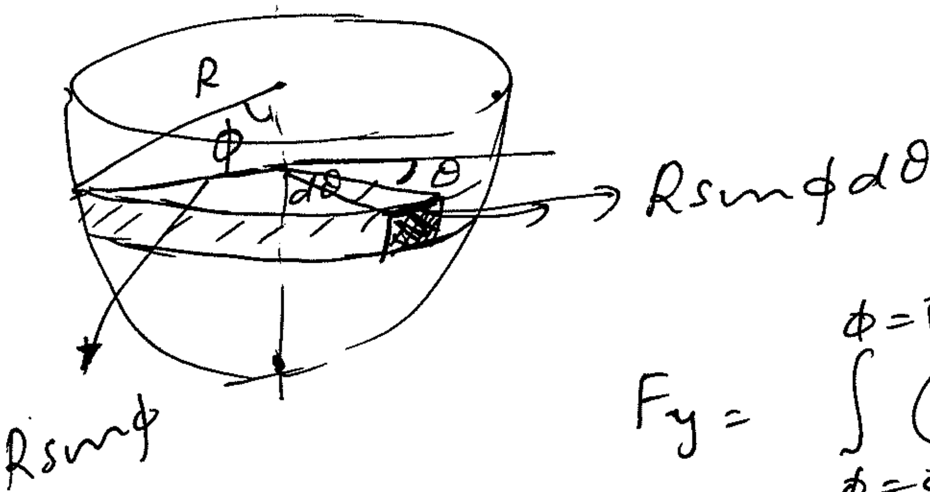
$$dA = R\,d\phi\, R\sin\phi\, d\theta$$

$$dA = R^2 \sin\phi\, d\phi\, d\theta$$

$$dF_y = dF\cos\phi$$

$$= p_i\, dA \cos\phi$$

$$= p_i R^2 \sin\phi \cos\phi\, d\phi\, d\theta$$



$$F_y = \int_{\phi=0}^{\phi=\pi/2} \int_{\theta=0}^{\theta=2\pi} p_i \cdot R^2 \sin\phi \cos\phi\, d\phi\, d\theta$$

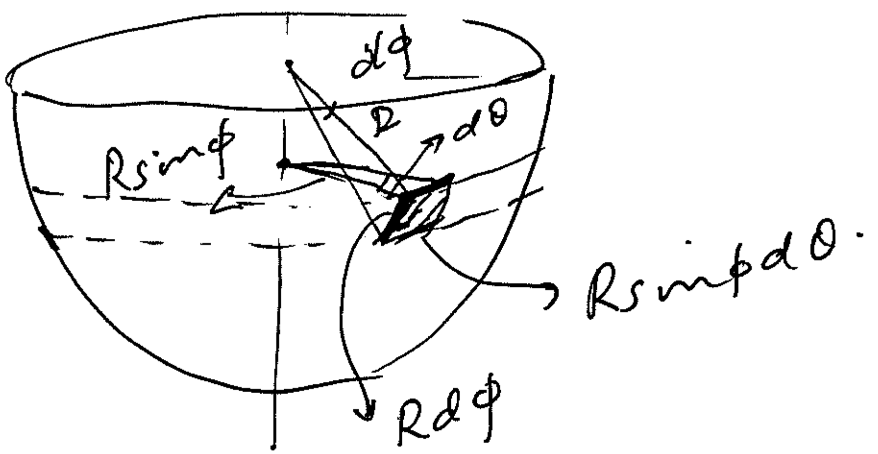
$$F_y = \underbrace{\int_{\phi=0}^{\phi=\pi/2} \left(\sin\phi \cos\phi\, d\phi\right)}_{\int_0^{\pi/2} \frac{\sin 2\phi}{2}\, d\phi} \underbrace{\int_{\theta=0}^{2\pi} d\theta}_{2\pi} \cdot \underbrace{p_i \cdot R^2}_{p_i \cdot R^2}$$

$$= \left. \frac{-\cos 2\phi}{2 \cdot 2} \right|_0^{\pi/2}$$

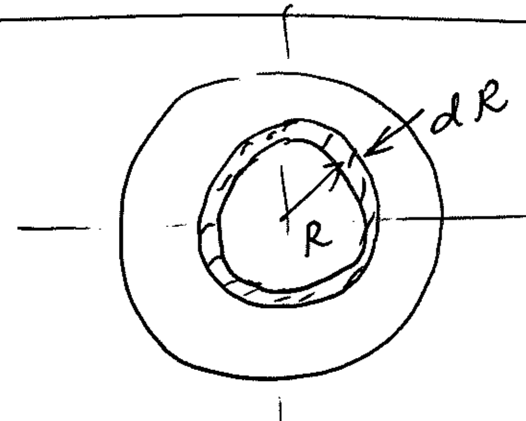
$$F_y = 2\pi \cdot p_i R^2 \frac{1}{4}\left[-\cos\pi - (-\cos 0)\right]$$

$$= 2\pi \cdot p_i R^2 \cdot \frac{1}{4} \cdot 2$$

$$F_y = p_i \pi R^2$$



$dA = R^2 \sin\phi\, d\theta\, d\phi$



$dA = 2\pi R\, dR$

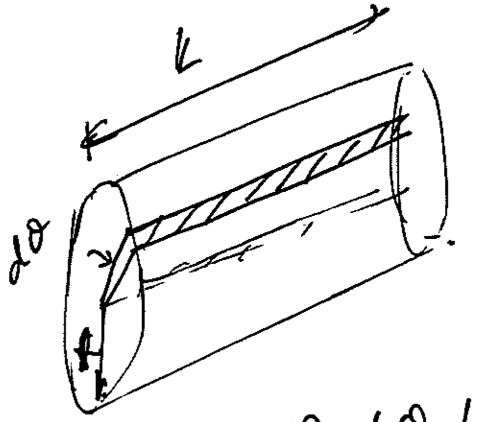
$$dA = \pi(R+dR)^2 - \pi R^2$$

$$= \pi(R^2 + 2R\,dR + dR^2) - \pi R^2$$

$$= \pi(2R\,dR + dR^2)$$

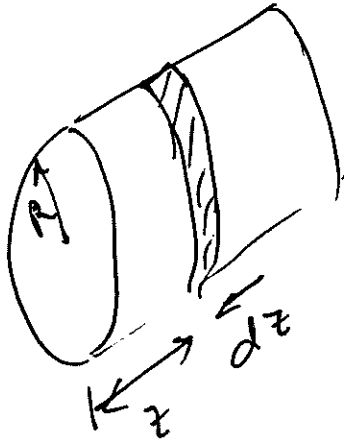
If $dR << R$, $dR^2 << 2R\,dR$

$$dA \approx 2\pi R\, dR$$



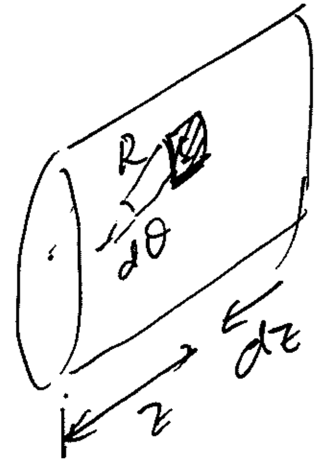
$dA = R\,d\theta\,L$

$A = \int_0^{2\pi} R\,d\theta\,L$



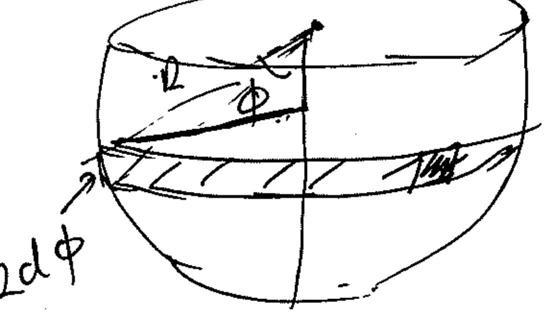
$dA = 2\pi R\,dz$

$A = \int_{z=0}^{z=L} 2\pi R\,dz$



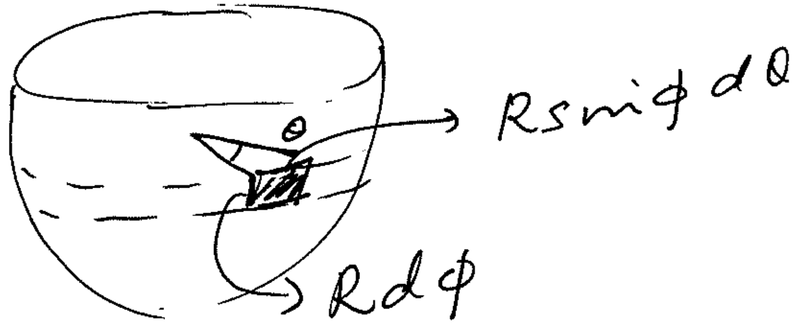
$dA = R\,d\theta\,dz$

$A = \int_{\theta=0}^{\theta=2\pi}\int_{z=0}^{z=L} R\,d\theta\,dz$



$dA = R\,d\phi\,R\sin\phi$

$A = R^2\int_0^{\pi/2} \sin\phi\,d\phi$



$dA = \iint R^2 \sin\phi\,d\theta\,d\phi$

Density: $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{m}{V}$

SI. $[\rho] = kg/m^3$

$$\boxed{\begin{aligned} \rho_{water} &= 1000\ kg/m^3 \\ \gamma_{water} &= 62.4\ lb/ft^3 \end{aligned}}$$

Specific weight (US) $\gamma = \rho g$

Specific gravity SG sp. gr

$= \dfrac{\text{density of a fluid}}{\text{density of water}}$ at same temperature.

EX Sp. gr. of mercury $= \dfrac{\rho_{Hg}}{\rho_{H_2O}} = \dfrac{13600\ kg/m^3}{1000\ kg/m^3}$

$= 13.6$

$\therefore \gamma_{Hg} = (\text{Sp. gr}_{Hg})(\gamma_{H_2O}) = (13.6)(62.4)$

$= 848.64\ lb/ft^3$

Viscosity :

Stationary plate

$$F \sim \frac{VA}{\delta}$$

$$F = \mu \frac{AV}{\delta}$$

$$\frac{F}{A} = \mu \frac{V}{\delta}$$

$$\tau = \mu \frac{V}{\delta}$$

μ is called $\boxed{\text{dynamic viscosity}}$

$$[\tau] = [\mu]\left[\frac{V}{\delta}\right] \Rightarrow Pa = [\mu]\frac{m/s}{m}$$

1) $\boxed{[\mu] = Pas}$

(2) ~~1P = 0.01 Pas~~ (poise)

$\boxed{1P = 0.1\ Pas}$

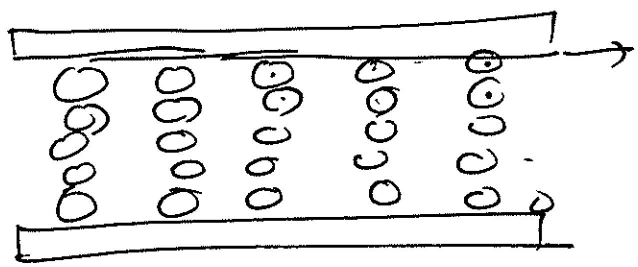
(3) 1cP = 0.01P
(centipoise)

$\boxed{\text{Viscosity of water @ } 20^\circ C = 1\ cP}$

Kinematic viscosity $\boxed{\underset{(\text{nu})}{\nu} \triangleq \frac{\mu}{\rho}}$

Two mechanisms

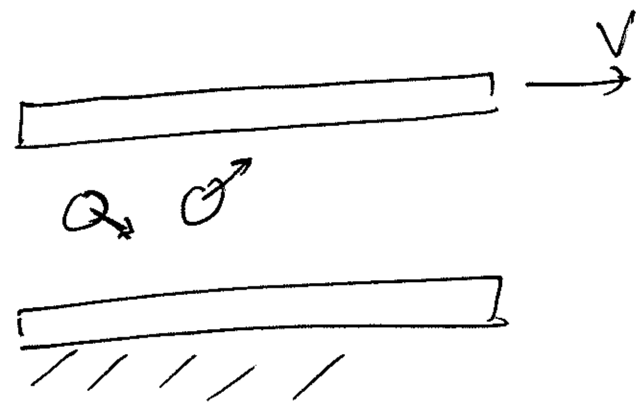
1) Molecular



For liquids, molecular attraction dominates.

As $T \uparrow$, $\mu \downarrow$

(2) Momentum transport



For gases, momentum transport dominates

As $T \uparrow$, $\mu \uparrow$

Fluids that satisfy $F = A\mu \frac{V}{y/\delta}$ $\rightarrow$ Newtonian fluids

$$\tau = \mu \underbrace{\frac{du}{dy}}_{\text{velocity gradient}}$$