# Internet Appendix
# Factors and Risk Premia in Individual International Stock Returns

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In this Internet Appendix, we provide additional details and results that complement those in the main text. Appendix A describes the number of stocks kept for each country using a direct application of the Gagliardini et al. (2016) (GOS) methodology without due modifications and our new methodology. Appendix B details our data construction. Appendix C presents a Monte Carlo experiment to examine the performance of the diagnostic criterion of Gagliardini et al. (2019a) for identifying a remaining factor structure in finite samples.

We repeat our main empirical analysis for mixed regional models, instead of mixed world models, in Appendix D. Appendix E presents formal asset pricing test results for all models. Finally, Appendix F presents results on beta determinants for the mixed regional four- and $q$-factor models.

# A Number of stocks kept

In this section, we report the number of stocks kept in the estimation of different models. We report in Table 1 for each country the number of stocks kept using our new methodology and using the GOS methodology. We report the number of stocks for the mixed world four-factor model, the mixed world $q$-factor model, the mixed regional four-factor model, and the mixed regional $q$-factor model.

In the first-pass time-series regression, the GOS methodology removes a stock if its time-series is too short (less than 60 months) or if the condition number of its regressor matrix is larger than 15. The same criteria are applied in the new methodology. However, we remove some regressors to avoid removing too many stocks. Precisely, if the condition number for a stock is larger than 15, we iteratively remove instruments for factor exposures until it is smaller than 15. When removing an instrument for a factor exposure, we make sure to adjust the instruments used for the intercept such that no-arbitrage conditions are respected. If we remove all instruments for factor exposures and the condition number is still too large, we remove the stock.

Table 1 shows that we cannot apply the GOS methodology in our international setting. Indeed, the large number of parameters needed to model time-varying factor exposures and risk premia and the limited number of available stocks in some countries result in too few or even no stocks kept. In contrast, our new methodology keeps a sufficient number of stocks in all countries to estimate the different models.

**Table 1**
**Number of stocks kept in estimations**

| Country | *New methodology* | | | | *GOS methodology* | | | |
|---|---|---|---|---|---|---|---|---|
| | World | | Regional | | World | | Regional | |
| | four-factor *(i)* | *q*-factor *(ii)* | four-factor *(ii)* | *q*-factor *(iv)* | four-factor *(v)* | *q*-factor *(vi)* | four-factor *(vii)* | *q*-factor *(viii)* |
| Argentina | 15 | 22 | 16 | 22 | 0 | 8 | 3 | 11 |
| Australia | 420 | 407 | 443 | 434 | 0 | 96 | 13 | 102 |
| Austria | 11 | 11 | 13 | 11 | 0 | 3 | 0 | 3 |
| Belgium | 14 | 18 | 16 | 19 | 0 | 0 | 0 | 1 |
| Brazil | 28 | 49 | 29 | 48 | 0 | 6 | 4 | 13 |
| Canada | 370 | 253 | 371 | 258 | 15 | 64 | 19 | 74 |
| Chile | 26 | 49 | 28 | 52 | 0 | 11 | 4 | 23 |
| China | 751 | 1, 185 | 773 | 1, 208 | 22 | 400 | 113 | 468 |
| Denmark | 48 | 47 | 48 | 47 | 2 | 18 | 3 | 20 |
| Finland | 23 | 31 | 25 | 31 | 0 | 8 | 0 | 9 |
| France | 165 | 189 | 174 | 195 | 1 | 48 | 2 | 56 |
| Germany | 161 | 149 | 170 | 160 | 0 | 47 | 0 | 50 |
| Hong Kong | 363 | 507 | 381 | 512 | 3 | 131 | 43 | 129 |
| India | 459 | 689 | 488 | 715 | 10 | 135 | 89 | 179 |
| Indonesia | 69 | 121 | 72 | 127 | 2 | 36 | 21 | 51 |
| Ireland | 10 | 5 | 10 | 5 | 0 | 2 | 0 | 2 |
| Israel | 53 | 86 | 54 | 87 | 0 | 20 | 1 | 25 |
| Italy | 35 | 33 | 40 | 35 | 0 | 7 | 0 | 11 |
| Japan | 377 | 495 | 395 | 520 | 10 | 171 | 35 | 183 |
| Jordan | 6 | 25 | 8 | 28 | 0 | 3 | 0 | 0 |
| Malaysia | 179 | 228 | 195 | 243 | 0 | 43 | 32 | 40 |
| Mexico | 24 | 23 | 24 | 24 | 0 | 10 | 6 | 11 |
| Morocco | 3 | 13 | 3 | 15 | 0 | 1 | 0 | 0 |
| Netherlands | 37 | 34 | 38 | 33 | 0 | 9 | 0 | 9 |
| New Zealand | 16 | 20 | 17 | 22 | 0 | 3 | 1 | 4 |
| Norway | 46 | 35 | 49 | 35 | 0 | 13 | 0 | 13 |
| Oman | 9 | 10 | 10 | 12 | 0 | 1 | 0 | 1 |
| Pakistan | 53 | 99 | 53 | 103 | 2 | 24 | 11 | 34 |
| Peru | 8 | 14 | 10 | 18 | 0 | 2 | 2 | 4 |
| Philippines | 33 | 47 | 33 | 46 | 0 | 9 | 4 | 11 |
| Poland | 91 | 117 | 91 | 122 | 0 | 24 | 14 | 39 |
| Portugal | 11 | 11 | 10 | 11 | 0 | 5 | 0 | 7 |
| Russia | 3 | 17 | 2 | 18 | 0 | 1 | 0 | 2 |
| Saudi Arabia | 23 | 33 | 24 | 32 | 0 | 9 | 0 | 10 |
| Singapore | 136 | 205 | 148 | 201 | 0 | 34 | 5 | 36 |

| | *New methodology* | | | | *GOS methodology* | | | |
|---|---|---|---|---|---|---|---|---|
| Country | World | | Regional | | World | | Regional | |
| | four-factor *(i)* | $q$-factor *(ii)* | four-factor *(ii)* | $q$-factor *(iv)* | four-factor *(v)* | $q$-factor *(vi)* | four-factor *(vii)* | $q$-factor *(viii)* |
| *Continued ...* | | | | | | | | |
| South Africa | 66 | 66 | 73 | 72 | 0 | 22 | 19 | 31 |
| South Korea | 273 | 326 | 292 | 325 | 8 | 108 | 82 | 154 |
| Spain | 23 | 25 | 24 | 26 | 0 | 7 | 1 | 9 |
| Sri Lanka | 13 | 25 | 15 | 23 | 0 | 1 | 0 | 1 |
| Sweden | 74 | 72 | 76 | 73 | 0 | 12 | 1 | 16 |
| Switzerland | 27 | 30 | 29 | 31 | 0 | 8 | 1 | 9 |
| Taiwan | 194 | 205 | 200 | 220 | 2 | 17 | 13 | 20 |
| Thailand | 80 | 128 | 87 | 129 | 0 | 40 | 21 | 61 |
| Turkey | 35 | 83 | 39 | 92 | 0 | 19 | 0 | 28 |
| U.A.E. | 10 | 14 | 10 | 16 | 0 | 2 | 0 | 6 |
| U.K. | 353 | 315 | 366 | 316 | 11 | 88 | 14 | 91 |
| U.S. | 813 | 824 | 824 | 836 | 50 | 226 | 58 | 259 |

We report for each country the number of stocks available for each model. Columns *(i)* to *(iv)* report the number of stocks kept using our new methodology, which extends the GOS methodology by automatically selecting the instruments for each stock while ensuring that no-arbitrage conditions are respected. Columns *(v)* to *(viii)* report on the number of stocks kept using the GOS methodology. In both cases, we keep a stock only if its time series is long enough (60 months) and the condition number of the regressor matrix used in the first-pass time-series regression is low enough (below 15), see Appendix A in the main text for details. In the new methodology, for each stock we iteratively remove instruments for factor exposures until the condition number is below 15, and do not keep the stock if all instruments for factor exposures are removed. In the GOS methodology, we keep all instruments and remove the stock if the condition number is above 15.

# B Equity data construction

Our objective is to build a database of common stocks traded on major stock exchanges. We examine the pros and cons of using Datastream versus Compustat Global/xpressfeed. Given the longer time series found on CRSP for US stocks, we focus on non-US countries.

Our main conclusions are as follows. Datastream has longer time series for some but not all stocks. However, it contains many errors. Compustat has fewer data errors, the history of SEDOLs and ISINs, and the type of daily quote, which to our knowledge is not available on Datastream (only the current identifiers are available).

The following steps describe how we construct the data for each country. By visual inspection of value- and equal-weighted indexes, we investigate each discrepancy. In some cases, we can confirm a mistake in Datastream (Compustat) by using data from Compustat (Datastream). For example, a spike in the total return index on Datastream is identified and removed by looking at the total return index on Compustat. In other cases, we can not conclude which of the two databases has an error and further check on Bloomberg and/or MSCI.

Given the advantages listed above, we use data from Compustat/xpressfeed in this paper. We describe the filters and error corrections we use for each of the two databases in the following steps. Therefore, this guide can be used for research based on Datastream or Compustat data.

1. **Stock Universe:**

   - Datastream: We retrieve all securities which are classified as equity (*instrument_type* = *'Equity'*).
   - Compustat: We retrieve all securities which are classified as common or ordinary shares (*tpci* = *'0'*).

2. **Major Stock Exchanges:** We keep only stocks listed on a country's major stock exchange. We define the major stock exchange as the one with the highest number of listed stocks. In most cases, the choice is obvious. However, we include more than one stock exchange in a few countries.

3. **Refining the common stock universe:** Securities are misclassified in both databases. We apply the following additional filter on the security name:

    - Datastream: We apply the name and industry filters as in Griffin et al. (2010). We add "BDR" to the list of keywords to remove Brazilean Depositary Receipts. We also use additional keyword filters used by Lee (2011): "AFV" in Belgium due to their preferential tax treatment, "INC.FD." in Canada because they are income trusts, and "RSP" in Italy due to their nonvoting provisions.
    - Compustat: We remove non-common stocks based on the presence of the same keywords in their issue description (*dsci*).

4. **Preliminary cleaning of times series:**

    - Compustat: We use only days for which a price (*prccd*) is available with a price code status (*prcstd*) either equal to 3 (high, low and close prices) or 10 (prices as reported). We also include price code status 4 (bid, ask, average/last volume close) for Canadian issues because Compustat historically delivered prices as the average of the bid/ask pricing for U.S. and Canadian issues.
    - Datastream: We use only days for which the unadjusted price (*UP*) is available. Datastream does not provide any indication as to the type of quote it provides. In many cases, total return indexes (*RI*) continue after the price stops quoting. Datastream repeats the last price after a stock stops. For each stock, we verify each day if the rest of the time series is the same price and remove the rest of the time series in such a case. This procedure does not capture cases in which a stock stops quoting for a few months and then starts again. In this case, we get a series of zero returns.

      At this stage, indexes built from Datastream have longer time series for many countries compared to Compustat indexes. This is especially the case for some developed countries whose indexes start in the early 1970s whereas all non-North American data on Compustat starts in the early 1980s. However, many unexplained spikes in Datastream time series come from days for which only the price is available. We can match several of these cases to Compustat data and confirm that they correspond to a price standard (*prcstd*) equal to 5 (no price is available, the last price is carried forward). Unfortunately, we cannot match these cases with Compustat data in the pre-1980s period. Therefore, we keep only quotes for which either the volume, low, or high is available as a sign of real market activity. This filter solves many of the initial discrepancies between the two data providers.

5. **Controlling for spikes that are reversed:**

    - Datastream: Following Ince and Porter (2006), we control for extreme daily returns that are reversed the following day. If the total return over two consecutive days is below 50% and any of the two daily total return is above 100%, we remove both daily observations.
    - Compustat: No filters are applied.

6. **Computing monthly returns:** We build monthly returns by using the last available total return index value during the previous month and the last available value in the current month.

    - Datastream: We use the total return index ($RI$). We convert the local total return index to U.S. dollars and keep nine decimals such that monthly returns are not impacted by rounding (using the function *DPL#(X(RI) U$,9)*).
    - Compustat: We build total return indexes using prices (*prccd*), adjustment factors (*ajexdi*), quotation units (*qunit*), exchange rates (*exratd*), and total return factors (*trfd*). We follow Shumway (1997) and apply a $-30\%$ delisting return when delisting is performance related (using the delisting reason *dlrsni*).

7. **Computing market capitalizations:** We build monthly lagged market capitalizations by using the last available market capitalization during the previous month.

    - Datastream: We use the market value ($MV$) converted to U.S. dollars.

- Compustat: We build market capitalization by multiplying the number of shares by prices (*prccd*). For non-North American stocks, we use the current number of shares outstanding (*cshoc*). For North-American stocks, we use the last report number of shares outstanding (*cshoi*).

8. **Manual data corrections:** We investigate and identify in Table 2 errors for Compustat data not captured by the filters above.

   In unreported figures available upon request, we plot for each country the returns of the value-weighted and equal-weighted market portfolios as well as the number of stocks over time using both databases.

| gvkey/iid | Error |
|---|---|
| 202192/01W, 203051/01W, 207206/01W, 208514/01W | In January 1992 in Argentina, there are four stocks for which the transition from the old currency code ARA to ARS creates 10,000+ returns. We remove them for this month. |
| 203579/01W, 205247/01W | Before January 1992 in Argentina, these two stocks' USD market capitalization are off by a factor 10. We multiply the market capitalization by 0.1. |
| 029178/01W | This Argentinean stock's market cap is too large and erratic, and there are some holes. Its data on Datastream starts on January 1992. We start in October 1990 after the last hole when the market capitalization is not erratic. |
| 208536/01W | The adjustment factor *ajexdi* does not adjust for the 0.0513-to-1 stock split on May $20^{th}$, 2015. We remove the stock for this month. |
| 030581/01W | Before February 1992, this stock in Brazil has extreme market values. |
| All stocks in Brazil | In January 1989, the 1-to-1,000 change from the Cruzado to the Cruzado novo is not reflected in Compustat's exchange rate table (nor is the one in 1986). We divide returns by 1,000. |
| 206477/01W | There is an error in the adjustment factor (*ajexdi*) from 01/09/2007 to 20/3/2007, it should be 1 instead of 10, verified on Bloomberg. |
| 208194/02W 203187/01W 229956/02W 208200/01W 203462/01W 203682/01W 208603/01W 208366/01W 209409/01W | Spike for these Chinese stocks in March and June 1993. Spike for 203187/01W in June 1993 is confirmed with Bloomberg (but return of 700% happens in July). Datastream show missing infrequent returns for these months. We check all large returns on June 1993 with Bloomberg and we can confirm all but one. We multiply the return in March by 10 and divide by 10 in June. |
| 213573/01W | In February 2002 in Estonia, we replace the $25^{th}$ return with the $21^{st}$, Datastream ends on the $21^{st}$. We set R = 0.0111301630700127 / 0.0645498918825071 - 1. |
| 103255/01W, 210759/01W, 240641/01W | There are errors caused by the change of currency to the Euro for these three European stocks. We remove them for January 1999. |
| All stocks in Iceland | For Iceland, the currency plummets on Oct $8^{th}$, 2008 and doubles on February $2^{nd}$, 2009. We cannot find this plunge on Bloomberg nor on Yahoo. We use Datastream exchange rates, namely, FX rate 0.009452, 0.008440, 0.006994, 0.008246, 0.008773, and 0.008778 for the month of September 2008 through February 2009. |
| 200503/01W | Spike in price creates a return of 15. This Peruvian stock is not on Datastream and it starts in 1996 on Bloomberg. We remove it for December 1992. |
| All Peruvian stocks | In January 1992, the 1,000,000-to-1 change described below (from Wikipedia) is not reflected on CSXF. "Because of the bad state of economy and hyperinflation in the late 1980s the government was forced to abandon the inti and introduce the sol as the country's new currency. The currency was put into use on July 1, 1991 (by Law No. 25,295) to replace the inti at a rate of 1 sol to 1,000,000 intis. Coins denominated in the new unit were introduced on October 1, 1991 and the first banknotes on November 13, 1991. Hitherto, the sol has retained a low inflation rate of 1.5%, the lowest inflation rate ever in both Latin and South America. Since the new currency was put into effect, it has managed to maintain a stable exchange rate between 2.2 and 3.66 per United States dollar." We divide returns by 1,000,000. |

| | |
|---|---|
| 201673/01W | In July 1998, this New Zealand stock has the same price as on Datastream, but its adjustment factor (*ajexdi*) and total return factor (*trfd*) create a huge difference compared to Datastream. We remove it for this month. |
| 206463/03W | Moscow City Telephone Network Co has random 1000x spikes in the price time series, it would take too many corrections to solve the problem. We remove the complete time series. |
| 284439/01W | In January 2005, there is an error in the adjustment factor (*ajexdi*) when the currency changed. Other stocks' prices (*prccd*) and *ajexdi* adjust. This stock *prccd* adjusts, but not its *ajexdi*. We remove it for this month. |
| 217719/01W | In February and March 1994, there is an error for this Colombian stock (verified with Datastream) and remove it for those two months. |
| 185208/01C | This Canadian stock is delisted on January $1^{st}$ 2017, there is a spike in the price on December $30^{th}$, 2016, and the time series ends on December $2^{nd}$, 2016, on Bloomberg. We remove it for December 2016. CSXF is also missing the total return adjustment for the 100-to-1 conversion on November $1^{st}$, 2013, which creates a 100+ % return. We remove it for November 2013. |
| 202022/01W | This Chilean stock has erratic and infrequent quotes before January 2004. There are price spikes on days with unavailable volumes, but classified as "prices as reported" (*prcstd*=10). There are no quotes on these days on Bloomberg. We remove infrequent returns before January 2004. |
| 149822/01C | The number of shares outstanding (*cshoc*) is off by a factor 100 for the last two days of June 2004. We then correct the number of shares. |

**Table 2**
We report in this table the manual data corrections to data on Compustat/xpressfeed.

# C Monte Carlo simulation

In this section, we run a Monte Carlo simulation to see how the GOS2 diagnostic criterion performs when the number of stocks and the time-series dimension is similar to what we have in our equity sample. The diagnostic criterion makes the right decision when the number of stocks and time-series dimension tends to infinity, so we want to ensure it also performs well in finite sample.

We follow a setup close to the one in the Online Appendix 7 of Gagliardini et al. (2019b). Our Monte Carlo simulation follows these steps:

1. We simulate the model

$$r_{i,t} = \sum_{k=1}^{K} \beta_{i,k} f_{k,t} + \epsilon_{i,t}, \quad \text{for } i = 1, ..., n, \text{ and } t = 1, ..., T, \tag{1}$$

   where

$$\epsilon_{i,t} = \sqrt{\frac{1}{1+2Jb^2}} \left( \upsilon_{i,t} + \sum_{j=\max(i-J,1)}^{i-1} b\upsilon_{j,t} + \sum_{j=i+1}^{\min(i+J,n)} b\upsilon_{j,t} \right). \tag{2}$$

   The factor exposures $\beta_{i,k}$, factors $f_{k,t}$, and innovations $\upsilon_{i,t}$ are drawn from standardized normal distributions. The parameters $J$ and $b$ control the magnitude of cross-sectional dependence across adjacent stocks.

2. We create an unbalanced panel using these steps. For each stock $i$,

   (a) we select randomly a starting point $T_i^{Start}$;

   (b) we simulate a time-series length by generating a uniform variable, $u_i$, distributed over the $T_{\min} = 60$ to $T$ interval;

   (c) we keep only the returns, $r_{i,t}$, in the $T_i^{Start}$ to $T_i^{Start} + \lfloor u_i \rfloor - 1$ interval, where $\lfloor u_i \rfloor$ denotes the integer part of $u_i$.

3. We estimate the model

$$r_{i,t} = \sum_{k=1}^{\tilde{K}} \beta_{i,k} f_{k,t} + \varepsilon_{i,t}, \quad \text{for } i = 1, ..., n, \text{ and } t = T_i^{Start}, ..., T_i^{Start} + \lfloor u_i \rfloor - 1, \tag{3}$$

where $\tilde{K}$ is not necessarily equal to $K$. We compute the GOS2 diagnostic criterion, $\zeta$, on the residuals $\hat{\varepsilon}_{i,t}$.

4. We repeat Steps 1-3 $S = 1,000$ times and compute the proportion of $\zeta$ with negative values.

Table 3 contains the proportions of negative $\zeta$s across different simulation setups. We use the average time-series length across our sample of countries, $T = 260$. We consider cases with a small number of stocks ($n = 150$) in Panels A and B, and cases with a number of stocks close to the average number of stocks across countries in Panels C and D ($n = 1,000$). Panels A and C differ from Panels B and D by their specification on innovation cross-correlations. Panels A and C have uncorrelated error terms (corresponding to an exact factor structure), whereas we use $J = 10$ and $b = 0.2$ in Panels B and D to allow for some cross-correlation.

We simulate models with $K = 1, 2, 3, 4, 5$ factors and report results in each row. We estimate models with $\tilde{K} = 1, 2, 3, 4, 5, 6$ factors (reported in columns). When $\tilde{K} < K$, the estimated model omits important factors and the diagnostic criterion should detect a factor structure in the residuals. In the case where the model is correctly specified, $\tilde{K} = K$, or we include irrelevant factors, $\tilde{K} > K$, the diagnostic criterion should not detect any remaining factors in the residuals. Therefore, each panel in Table 3 should have zeros below the diagonal and a high proportion on the diagonal (correctly specified) and above the diagonal (overspecified).

We find in Panels A, C, and D that the diagnostic criterion makes the right decision, either detecting at least one omitted factor when there is at least one or not detecting at least one factor when there are none, in all simulations. In Panel B, where the number of stocks is low ($n = 150$) and residuals are cross-correlated, we find that the diagnostic criterion does not always detect that the model is correctly specified (see diagonal elements) or overspecified (see above the diagonal). However, all proportions are above 99%, indicating a high success rate of the diagnostic criterion even when the cross-sectional dimension is small.

**Table 3**
**Proportion of negative diagnostic criteria in Monte Carlo simulations**

| True number of factors $K$ | Number of factors $\tilde{K}$ in estimation 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| *Panel A: $n = 150$, $T = 260$, $J = 0$, and $b = 0$* | | | | | | |
| 1 | 100 | 100 | 100 | 100 | 100 | 100 |
| 2 | 0 | 100 | 100 | 100 | 100 | 100 |
| 3 | 0 | 0 | 100 | 100 | 100 | 100 |
| 4 | 0 | 0 | 0 | 100 | 100 | 100 |
| 5 | 0 | 0 | 0 | 0 | 100 | 100 |
| *Panel B: $n = 150$, $T = 260$, $J = 10$, and $b = 0.2$* | | | | | | |
| 1 | 100 | 100 | 99.80 | 99.80 | 99.70 | 99.60 |
| 2 | 0 | 99.60 | 99.70 | 99.70 | 99.70 | 99.70 |
| 3 | 0 | 0 | 99.90 | 99.90 | 99.90 | 99.90 |
| 4 | 0 | 0 | 0 | 99.60 | 99.60 | 99.70 |
| 5 | 0 | 0 | 0 | 0 | 99.80 | 99.80 |
| *Panel C: $n = 1000$, $T = 260$, $J = 0$, and $b = 0$* | | | | | | |
| 1 | 100 | 100 | 100 | 100 | 100 | 100 |
| 2 | 0 | 100 | 100 | 100 | 100 | 100 |
| 3 | 0 | 0 | 100 | 100 | 100 | 100 |
| 4 | 0 | 0 | 0 | 100 | 100 | 100 |
| 5 | 0 | 0 | 0 | 0 | 100 | 100 |
| *Panel D: $n = 1000$, $T = 260$, $J = 10$, and $b = 0.2$* | | | | | | |
| 1 | 100 | 100 | 100 | 100 | 100 | 100 |
| 2 | 0 | 100 | 100 | 100 | 100 | 100 |
| 3 | 0 | 0 | 100 | 100 | 100 | 100 |
| 4 | 0 | 0 | 0 | 100 | 100 | 100 |
| 5 | 0 | 0 | 0 | 0 | 100 | 100 |

We report the proportion (in %) of Monte Carlo simulations for which the diagnostic criterion is negative, indicating that there are no remaining factors in the residuals. The first column reports the true number of factors, $K$, in the model. In each of the other columns, we report on models estimated with different numbers of factors, $\tilde{K}$. Panel A and B report on the case with a small number of stocks, $n = 150$. Panel A has uncorrelated innovations, whereas Panel B allows for some cross-correlations. Panels C and D replicate the first two panels but with a larger number of stocks, $n = 1000$. In all panels, we simulate unbalanced panels with $T = 260$ periods. We use 1,000 Monte Carlo simulations.

## D Estimation results for mixed regional models

We provide in this section estimation results for the mixed regional models, which complement those for the mixed world models in the main text.

Table 4 presents the average risk premia and proportions of significant risk premia across countries and models, as well as the proportion of significant $\alpha$s in factor spanning tests. Table 4 has the same structure as Table 2 in the main text. All results are similar to those obtained from mixed world models, except that average risk premia for the excess country market factors are smaller. However, they remain significant in a large proportion of countries.

Using the mixed regional models, we decompose the expected return of equal-weighted portfolios into contributions from pricing errors and factor risk premia. Figure 1 reports results for the mixed regional four-factor model and Figure 2 reports on the mixed regional $q$-factor model. These figures are comparable to Figures 4-5 in the main text.

**Table 4**
**Risk premium estimates in mixed regional models**

| Factor | Region | Average risk premia per month (%) | | Proportion of significant risk premia (%) | |
|---|---|---|---|---|---|
| | | Four-factor *(i)* | *q*-factor *(ii)* | Four-factor *(iii)* | *q*-factor *(iv)* |
| Market | DM | 0.67 | 0.58 | 70 | 65 |
| | EM | 1.15 | 0.43 | 54 | 75 |
| Excess country market | DM | −0.20 | −0.47 | 65 | 74 |
| | EM | −0.28 | 0.27 | 50 | 50 |
| Size | DM | 0.24 | 0.48 | 78 | 65 |
| | EM | 1.43 | 0.00 | 75 | 63 |
| Value | DM | 0.09 | | 61 | |
| | EM | 0.85 | | 54 | |
| Momentum | DM | 0.09 | | 48 | |
| | EM | −1.24 | | 46 | |
| Profitability | DM | | 0.35 | | 61 |
| | EM | | −0.11 | | 63 |
| Investment | DM | | 0.07 | | 61 |
| | EM | | 0.15 | | 58 |

We report on the average risk premium estimates for each factor in columns *(i)* and *(ii)* and their significance at the 5% level in columns *(iii)* and *(iv)*. Because the common instruments, except the constant, are standardized to have a mean of zero, the average risk premium corresponds to the parameter for the constant, $\Lambda_0$. We report the average risk premium across DMs and EMs for the mixed regional four-factor model in columns *(i)* and *(iii)* and for the mixed regional $q$-factor model in columns *(ii)* and *(iv)*.

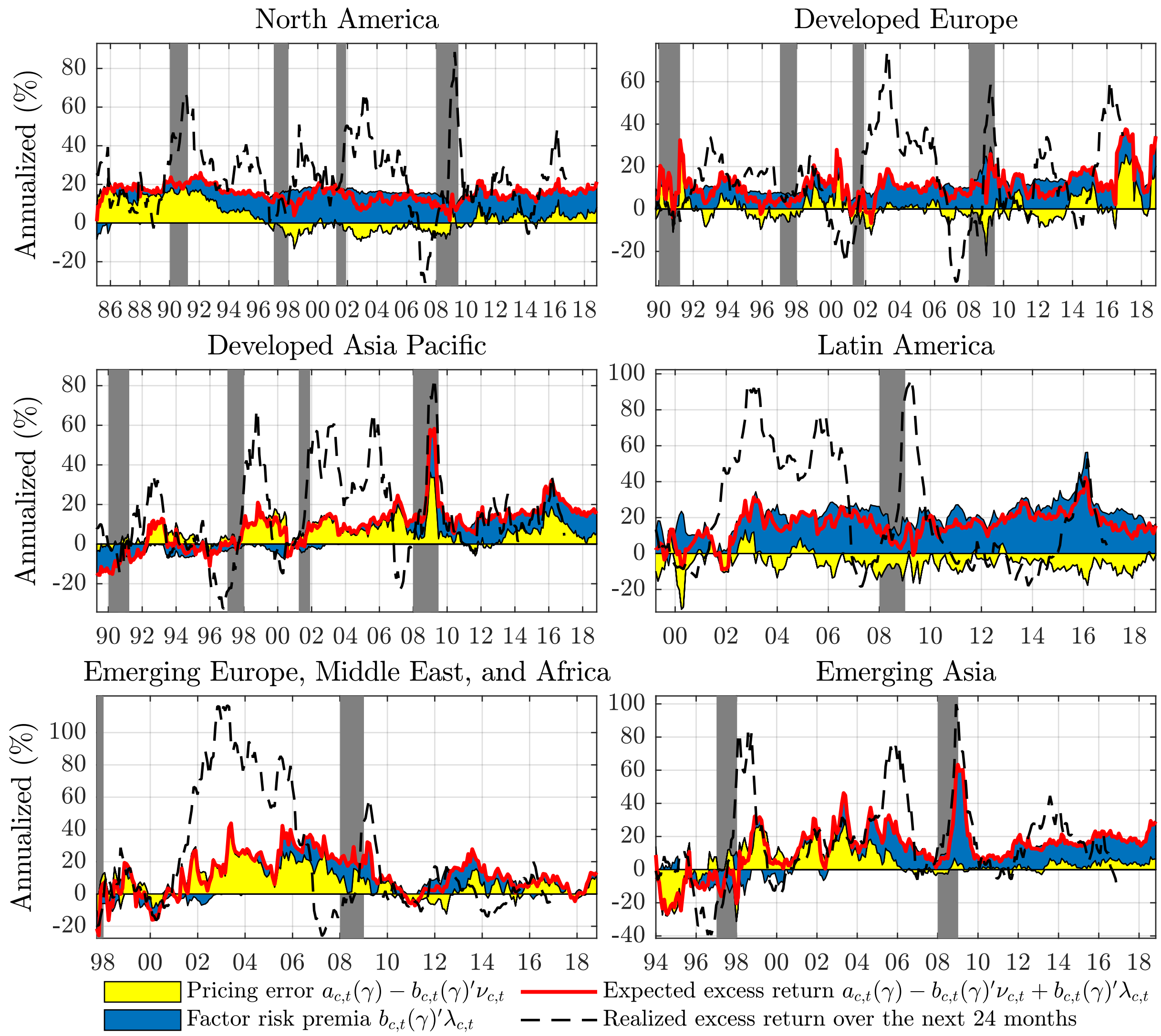


**Figure 1. Decomposition of expected returns in the mixed regional four-factor**
Each month, we compute the pricing errors, $a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}$, and sum of factor risk premia, $b_{c,t}(\gamma)'\lambda_{c,t}$, across all available stocks. We report the equal-weighted average pricing error in yellow, factor risk premium in blue, and total expected return using a red line. We report using a dashed line the forward 24-month equal-weighted average excess return. We report results for each of the three DM regions and EM regions. All returns are monthly and in USD. We use recession dates from the NBER for the U.S. and the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region, which is equal to one when at least half of the countries in the region are in a recession. We report in each figure gray areas to denote recession periods (across all DM regions for DMs and all EM regions for EMs).

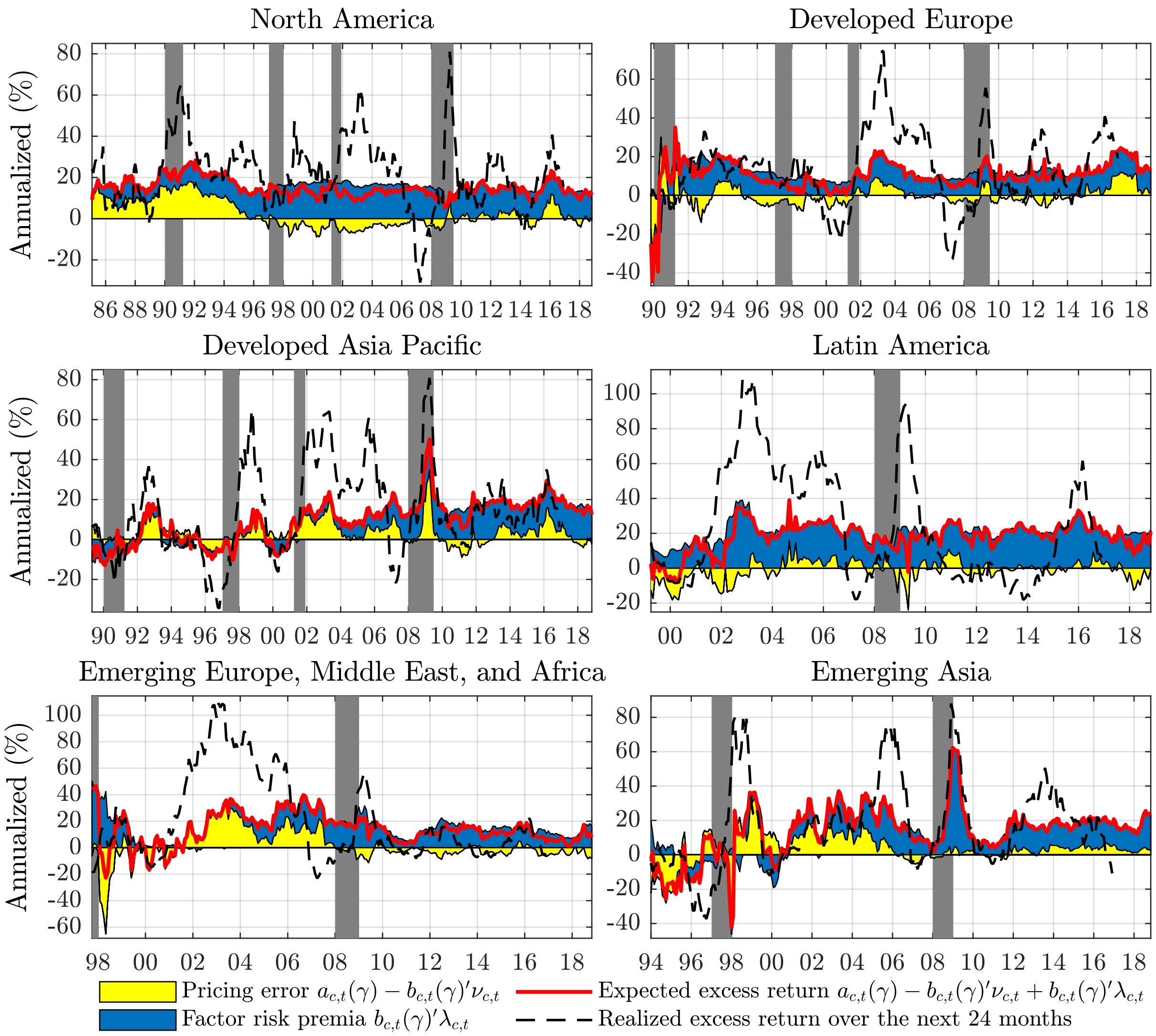


**Figure 2. Decomposition of expected returns in the mixed regional $q$-factor**
Each month, we compute the pricing errors, $a_{c,t}(\gamma) - b_{c,t}(\gamma)'\nu_{c,t}$, and sum of factor risk premia, $b_{c,t}(\gamma)'\lambda_{c,t}$, across all available stocks. We report the equal-weighted average pricing error in yellow, factor risk premium in blue, and total expected return using a red line. We report using a dashed line the forward 24-month equal-weighted average excess return. We report results for each of the three DM regions and EM regions. All returns are monthly and in USD. We use recession dates from the NBER for the U.S. and the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region, which is equal to one when at least half of the countries in the region are in a recession. We report in each figure gray areas to denote recession periods (across all DM regions for DMs and all EM regions for EMs).

## E Asset pricing tests

We report in this section formal asset pricing tests for the mixed four- and $q$-factor models. Table 5 reports the proportion of non-rejection of different models across regions. We do not reject a model when the diagnostic criterion for the factor structure is negative and the $p$-value for the asset pricing restrictions is above the significance level. We use a significance level of 5% using a Bonferroni correction, $5\%/N$, where $N$ is the number of countries in the region. In columns (i) and (iii), we report on the test for the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$. In columns (ii) and (iv), we report on the test for the asset pricing restrictions with tradable factors, $a_{c,t}(\gamma) = 0$. We report on mixed world models in columns (i) and (ii) and mixed regional models in columns (iii) and (iv). Panel A contains results for the mixed four-factor models with world (regional) market, size, value, and momentum factors and a country excess market factor. Panel B contains results for the mixed $q$-factor models with world (regional) market, size, profitability, and investment factors and a country excess market factor.

We find that models are rejected in a large proportion of countries. The traded factor restrictions is not favored in the data as the proportion of non-rejections in columns *(ii)* and *(iv)* are lower than proportions in columns *(i)* and *(iii)* where we do not assume traded factors.

**Table 5**
**Do asset pricing restrictions hold?**

| Region | *World Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ *(i)* | $H_0 : a_{c,t}(\gamma) = 0$ *(ii)* | *Regional Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ *(iii)* | $H_0 : a_{c,t}(\gamma) = 0$ *(iv)* |
|---|---|---|---|---|
| *Panel A: mixed four-factor* | | | | |
| Developed Markets | 22% | 0% | 17% | 0% |
| Emerging Markets | 38% | 4% | 25% | 0% |
| North America | 0% | 0% | 0% | 0% |
| Developed Europe | 25% | 0% | 19% | 0% |
| Developed Asia Pacific | 20% | 0% | 0% | 0% |
| Latin America | 80% | 20% | 20% | 0% |
| E.E., M.-E. and A. | 44% | 0% | 44% | 0% |
| Emerging Asia | 10% | 0% | 10% | 0% |
| *Panel B: mixed q-factor* | | | | |
| Developed Markets | 17% | 9% | 26% | 4% |
| Emerging Markets | 8% | 0% | 21% | 4% |
| North America | 0% | 0% | 0% | 0% |
| Developed Europe | 19% | 6% | 25% | 6% |
| Developed Asia Pacific | 20% | 20% | 20% | 0% |
| Latin America | 20% | 0% | 0% | 0% |
| E.E., M.-E. and A. | 11% | 0% | 33% | 11% |
| Emerging Asia | 0% | 0% | 0% | 0% |

We report for different models and different regions the proportion of countries for which the model is not rejected. A model is not rejected when the diagnostic criterion for the factor structure is negative and the $p$-value for the asset pricing restrictions is above the significance level. We use a significance level of 5% using a Bonferroni correction, $5\%/N$, where $N$ is the number of countries in the region. In columns *(i)* and *(iii)*, we report on the test for the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$. In columns *(ii)* and *(iv)*, we report on the test for the asset pricing restrictions with tradable factors, $a_{c,t}(\gamma) = 0$. We report on mixed world models in columns *(i)* and *(ii)* and mixed regional models in columns *(iii)* and *(iv)*. Panel A contains results for the mixed four-factor models with world (regional) market, size, value, and momentum factors and a country excess market factor. Panel B contains results for the mixed $q$-factor models with world (regional) market, size, profitability, and investment factors and a country excess market factor.

# F Which instruments are important for time-varying factor exposures in regional models?

In this section, we provide the median coefficients for each factor-instrument interaction for the mixed regional four-factor model in Table 6 and for the mixed regional $q$-factor model in Table 7. These tables have the same structure as Tables 3-4 in the main text.

**Table 6**
**Which instruments drive time-variations in factor exposures in the mixed regional four-factor model?**

| Factor | Region | Constant *(i)* | Country dividend yield *(ii)* | Size *(iii)* | Value *(iv)* | Momentum *(v)* |
|---|---|---|---|---|---|---|
| Market | DM | 0.96 | 0.01 | −0.15 | −0.01 | 0.10 |
| | | (100.00) | (53.29) | (55.15) | (59.31) | (72.80) |
| | EM | 0.93 | 0.00 | −0.13 | 0.19 | 0.07 |
| | | (100.00) | (54.41) | (49.98) | (52.00) | (67.88) |
| Excess country market | DM | 0.93 | 0.08 | 0.17 | 0.12 | −0.03 |
| | | (100.00) | (60.01) | (61.70) | (62.83) | (76.51) |
| | EM | 1.04 | −0.02 | −0.07 | 0.11 | −0.03 |
| | | (100.00) | (57.90) | (51.96) | (54.95) | (70.10) |
| Size | DM | 0.81 | 0.01 | −1.05 | 0.40 | −0.03 |
| | | (100.00) | (53.75) | (58.48) | (60.36) | (74.04) |
| | EM | 0.66 | −0.06 | −0.88 | 0.29 | 0.05 |
| | | (100.00) | (56.47) | (54.56) | (56.58) | (70.87) |
| Value | DM | 0.06 | −0.07 | −0.59 | 0.98 | −0.13 |
| | | (100.00) | (31.26) | (44.15) | (48.75) | (52.75) |
| | EM | 0.04 | −0.01 | −0.17 | 0.52 | −0.16 |
| | | (100.00) | (52.31) | (52.97) | (52.93) | (62.64) |
| Momentum | DM | 0.08 | −0.00 | −0.39 | −0.07 | 0.54 |
| | | (100.00) | (36.39) | (42.89) | (45.55) | (58.67) |
| | EM | 0.04 | −0.07 | 0.31 | 0.02 | 0.41 |
| | | (100.00) | (49.86) | (53.94) | (56.66) | (64.58) |
| Median time-series $R^2$ (%) | DM | 23.94 | | | | |
| | EM | 39.27 | | | | |

We report the median coefficient value for each factor-instrument interaction in the time-series regressions for the mixed regional four-factor model. For each factor in the first column, we report the median coefficient for each instrument in columns *(i)* to *(v)* across all stocks in developed markets and across all stocks in emerging markets. Below each coefficient median, we report in parentheses the proportion (in %) of stocks for which the regressor is selected by our methodology. Finally, we report in the last rows the median time-series regression $R^2$s.

**Table 7**
**Which instruments drive time-variations in factor exposures in the mixed regional $q$-factor model?**

| Factor | Region | Constant *(i)* | Country dividend yield *(ii)* | Size *(iii)* | Profitability *(iv)* | Investment *(v)* |
|---|---|---|---|---|---|---|
| Market | DM | 0.91 | 0.02 | −0.26 | −0.07 | 0.02 |
| | | (100.00) | (57.05) | (59.08) | (60.29) | (68.78) |
| | EM | 0.93 | 0.01 | −0.21 | 0.01 | 0.00 |
| | | (100.00) | (60.08) | (60.35) | (61.62) | (65.28) |
| Excess country market | DM | 0.81 | 0.07 | −0.14 | 0.07 | −0.01 |
| | | (100.00) | (58.75) | (63.65) | (65.29) | (73.29) |
| | EM | 1.02 | −0.01 | −0.05 | 0.00 | 0.01 |
| | | (100.00) | (62.67) | (60.00) | (64.37) | (68.95) |
| Size | DM | 0.74 | −0.03 | −1.68 | 0.02 | −0.00 |
| | | (100.00) | (58.34) | (62.08) | (63.85) | (73.40) |
| | EM | 0.59 | −0.02 | −1.20 | −0.07 | −0.09 |
| | | (100.00) | (63.15) | (64.45) | (67.28) | (71.46) |
| Profitability | DM | −0.06 | −0.06 | −0.07 | 0.61 | 0.03 |
| | | (100.00) | (52.54) | (61.88) | (63.80) | (71.14) |
| | EM | −0.02 | 0.05 | 0.67 | 0.49 | −0.06 |
| | | (100.00) | (66.90) | (66.01) | (70.67) | (74.74) |
| Investment | DM | 0.04 | −0.08 | −1.02 | 0.15 | −0.36 |
| | | (100.00) | (53.13) | (61.39) | (62.62) | (68.60) |
| | EM | 0.09 | −0.05 | −0.83 | 0.05 | −0.21 |
| | | (100.00) | (61.86) | (66.15) | (68.54) | (73.75) |
| Median time-series $R^2$ (%) | DM | 23.17 | | | | |
| | EM | 38.44 | | | | |

We report the median coefficient value for each factor-instrument interaction in the time-series regressions for the mixed regional $q$-factor model. For each factor in the first column, we report the median coefficient for each instrument in columns *(i)* to *(v)* across all stocks in developed markets and across all stocks in emerging markets. Below each coefficient median, we report in parentheses the proportion (in %) of stocks for which the regressor is selected by our methodology. Finally, we report in the last rows the median time-series regression $R^2$s.

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