International Business Cycle Accounting

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Abstract

I have developed a method that can provide insights to researchers to better specify their quantitative models in international business cycle studies. The guidance comes from the application of an accounting procedure based on a prototype model of international growth that includes wedges capturing all the potential frictions and distortions of markets. For each country, I include an efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge, and foreign asset wedge. I then demonstrate the method by applying it to the US and Canada during the Great Recession (2007-2008). I found that the economic downturns in both countries during this period were primarily due to the US investment wedge, US labor wedge, and US efficiency wedge, with the Canada investment wedge playing a secondary role. These results suggest that the crisis originated in the US and was propagated to Canada.

Keywords: *International business cycle; Great Recession; shocks propagation; business cycle accounting*
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1 Introduction

International macroeconomics is a realm full of puzzles[1]. To solve these puzzles, economics researchers build detailed models in which they add frictions to replicate economic fluctuations observed in data. However, they face significant difficulties about which frictions to introduce and where in the model. In this study, I propose a method that could facilitate those choices, and I apply the method.

Indeed, my method is an extension of the method proposed by Chari, Kehoe, and McGrattan (2007) that was designed for closed economies. It has two components: an equivalence result and an accounting procedure. The equivalence result consists of building a prototype model, including time-varying wedges that distort the equilibrium of the economy (otherwise in perfectly competitive markets), such that a large class of detailed models with frictions is equivalent to this prototype model with some specific wedges. I consider six wedges for each country in my framework and label them efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge, and foreign asset wedge.

The accounting procedure consists of measuring the wedges first. For that purpose, we use the data and the equilibrium conditions of the prototype model. Second, we evaluate the contribution of those wedges by feeding their values back, one at a time or by combination, to the model. These experiments help assess how much the fluctuations of output, investment, and labor are due to wedges, separately or in combination.

Here, I focus on the accounting procedure. To better understand this procedure, I apply it to the study of business synchronization of Canada and the US during the Great Recession of 2007-2008. I find that the US labor wedge, US efficiency wedge, and US investment wedge account for the large fall in Canada's output and labor, with a role for Canada's investment wedge in the decline of Canada's investment during this period. Concerning the accounting of the US economic downturns during this period, it turns out that the US efficiency wedge, the US labor wedge, and the US investment wedge explain the main movements of the US output and the US investment while the Canada investment wedge plays a role in the decline of US employment.

My quantitative findings suggest that the financial frictions manifest themselves not only as investment wedges but also as efficiency wedges and labor wedges. The US investment wedge, US labor wedge, and the US efficiency wedge account mainly for the downturns in both US and Canada. This may imply that the Great Recession started in the US and then propagated to Canada. The trade

[1] Anomalies that occur when model predictions or results differ from the facts observed in data

relationship between the two countries is probably the channel of this transmission of the business cycle. Indeed, the prominent role of the US investment wedge in the decline of Canadian investment during this period suggests that the financial shock that occurred in the US has created some investment distortions in Canada.

The quantitative results also indicate the kind of question our method addressed. One may want to know how much Canada's output would fluctuate if the only wedge that fluctuates is the US efficiency wedge. This methodology allows the study of the impacts of foreign shocks on a particular economy. These topics could not be studied using the Business Cycle Accounting of (Chari et al., 2007) as it was designed for a closed economy, and all interactions with the rest of the world are assessed with the government wedge.

My methodology intends to shed light on the mechanisms of business cycle synchronization across countries. It is a diagnostic tool for the business cycles of a particular economy. Indeed, this method acts as a prism for light. It distinguishes not only the shocks that affect the fluctuations of the aggregate variable but also the origin of those shocks (in terms of home shocks or foreign shocks). The method helps then in the study of shock propagation. The method could also provide insights to researchers in the study of international business cycles. Especially, it can help solve some puzzles in international economics. There exist multiple puzzles in International Macroeconomics: (Obstfeld & Rogoff, 2000) have documented six of them, (Kose & Yi, 2001) documented trade comovement puzzle.

In terms of methodology, the most closely related works to mine are Chari et al. (2007) and Brinca, Chari, Kehoe, and McGrattan (2016). While the two works use the same method, they differ in application. The former applies the method to the US great depression (1929-1939) and the recession of 1982 while the latter compares the recession of 1982 and the great recession (2008) in OECD countries. Moreover, that paper's methodology is designed for a closed economy, thus it differs from mine. In my methodology, I allow a description and a quantification of the relations between countries while (Chari et al., 2007) method summarize all the interactions of a country with the rest of the world into the government wedge. My work is also related to existing literature that studies the mechanisms of business cycle transmission across countries. Among those works we have Backus, Kehoe, and Kydland (1992), Kose and Yi (2001), Kose and Yi (2006),...

After the current introductory section 1, the rest of the paper is organized as follows. In section 2 I present the benchmark prototype model. In section 3 I describe the accounting procedure of my method. The section 4 is dedicated to the description of the application of the method and the findings. After that, I make a discussion around the results found in section 5. The section 6 summarizes my

findings and suggests some directions for further work.

2 Description of the Benchmark Prototype Model

For the matter of simplification, I start with a perfect foresight economy. In this economy, I shut down the role of expectations of agents. In the same order of simplification, I consider the worldwide economy to be constituted of two countries. Then in my accounting procedure, I use a prototype model which is an international growth model with exogenous variables. This economy has six exogenous variables for each country: the *efficiency wedge* A_t, the *labor wedge* $1-\tau_{lt}$, the *investment wedge* $1/(1+\tau_{xt})$, the *government wedge* g_t, the *preference wedge* $1/(1+\tau_{ct})$ and the *asset wedge* $1-\tau_{bt}$.

In this worldwide economy, the consumers of each country maximize their lifetime utility (equation 2) over per capita home consumption good c^i_{ht}, per capita foreign consumption good c^i_{ft} and per capita labor l^i_t subject to their budget constraint (equation 2), and the capital law of accumulation (equation 2)

$$\sum_{t=0}^{\infty}\beta^t U(c^i_{ht}, c^i_{ft}, 1-l^i_t)N^i_t$$

$$p^i_t c^i_{ht} + p^j_t(1+\tau^i_{ct})c^i_{jt} + (1+\tau^i_{xt})p^i_t x^i_t + \frac{N^i_{t+1}}{N^i_t}b^i_{t+1} = \tag{1}$$
$$(1-\tau^i_{lt})w^i_t l^i_t + r^i_t k^i_t + (1+r_t(1-\tau^i_{bt}))b^i_t + Tr^i_t$$

$$\frac{N^i_{t+1}}{N^i_t}k^i_{t+1} = ((1-\delta)k^i_t + x^i_t) \tag{2}$$

Where for the variable Z^i_j the subscript j indicate the origin country and the subscript we the destination country. k^i_t denotes the per capita capital stock; x^i_t per capita investment, b^i_t per capita foreign asset, w^i_t the wage rate, r^i_t the rental rate on local capital, r_t the rental rate on foreign asset, β the discount factor, δ the depreciation rate of capital, N^i_t the population with $1+\gamma_n$ growth rate and T^i_t per capita lump sum transfers.

In the production size, the technology used is $A^i_t F(k^i_t, (1+\gamma)^t l^i_t)$ where $1+\gamma$ is the rate of labor-augmented technical progress assumed constant. Firms maximize profits given by $A^i_t F(k^i_t, (1+\gamma)^t l^i_t) - w^i_t l^i_t - r^i_t k^i_t$.

An *equilibrium of the prototype economy* consists of, given wedges (A_t, τ_{lt}, τ_{xt},g_t, τ_{ct} and τ_{bt}), allocation $(c_{ht}, c_{ft}, l_t, k_t, b_t)$ and prices (r_t, rf_t, w_t, p_t) for each country such as these allocations are optimal for consumers and firms and all the market cleared. The goods markets clearance is given by resource constraints: $c^i_{ht} + c^j_{ft} + x^i_t + g^i_t = y^i_t$ for each country i. The asset market clearance states that lending for a country

should equate the borrowing for the other country each period: $b_t^h + b_t^f$.

Also, notice that from the point of view of consumers, investing in capital is equivalent to lending on the international market. Based on this observation, one might think that capital and foreign assets are redundant. However, in our economy, this is not the case for two reasons. First, the presence of foreign assets allows consumers to invest in local capital and also borrow from abroad. Second, from the perspective of the economy as a whole (for example, a social planner), foreign assets play the role of international finance. In this sense, if a country faces a bad shock, it can borrow from abroad, and in good times, it can pay back the debt. The following proposition describes the role of foreign assets in our prototype economy.

Proposition 1: When the foreign asset market is nonexistent, i.e., $b_t^h = b_t^f = 0$, in equilibrium, trade is balanced each period, and there is a lack of international finance. In our framework, there exists a period during which a country incurs debt from abroad.

Proof: (See Appendix C.1)

In our prototype model, the efficiency wedge resembles a time-varying technology parameter. The labor, investment, preference, and foreign asset wedges look like tax rates on labor income, investment, foreign consumption goods, and foreign assets. These wedges include more than taxes. Indeed, the labor wedge captures all frictions that induce a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor. The preference wedge measures the gap between the home consumption-foreign consumption marginal rate of substitution through two partner countries. The investment wedge and foreign asset wedge capture all frictions that distort the intertemporal margins. Then the wedges represent the whole distortion to the equilibrium condition of the model: those coming from the demand and offer side on all markets.

Note that we could consider other models where we could change the different wedges, but those models would capture the same features present in the model we previously described. For example, we could add a wedge on capital, but this would capture the same distortion as the investment wedge.

3 The accounting procedure

The accounting procedure consists first in measuring the different wedges and second in evaluating the contribution of those wedges (one at a time and in combination).

Measuring the wedges

For this step, we use aggregate variables and compute the different wedges using some equations derived from the prototype equilibrium conditions. The wedges are then measured using the following equations $\forall we = h, f$:

$$y_t^i = A_t^i F(k_t^i, (1+\gamma)^t l_t^i) \tag{3}$$

$$u_{c_f t}^i(.) = u_{c_h t}^i(.)(1+\tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{4}$$

$$u_{lt}^i(.) = -u_{c_h t}^i(.)(1-\tau_{lt}^i)(1+\gamma)^t F_{lt}^i \tag{5}$$

$$u_{c_h t}^i(.)(1+\tau_{xt}^i) = \beta^i u_{c_h t+1}^i(.)(F_{kt+1}^i + (1-\delta)(1+\tau_{xt+1}^i)) \tag{6}$$

$$\beta^i \frac{u_{c_h t}^i}{u_{c_h t+1}^i}\frac{p_{t+1}^i}{p_t^i} = \beta^j \frac{u_{c_h t}^j}{u_{c_h t+1}^j}\frac{p_{t+1}^j}{p_t^j} \tag{7}$$

$$c_{ht}^i + c_{ft}^j + x_t^i + g_t^i = y_t^i \tag{8}$$

Where, from here, H_{zt} denotes the derivative of the function with respect to its argument z.

Measuring the contribution of wedges

For this step, we conduct different experiments to isolate the marginal effect of some wedges. For that purpose, we make some wedges fluctuate and shut down the fluctuation of the remaining wedges by setting their values to be constant. For example, to evaluate the contribution of the country efficiency wedge, we make this wedge (A_t^i) fluctuate and set the other wedges constants ($\omega_t^i = \omega_1^i$, where ω stands for all of the other wedges in the model). The insight behind this approach is to compute the aggregate variables by ensuring the economy is in equilibrium. The proposition states the conditions of an equilibrium allocation.

Proposition 2 Given $\tau_{ct}^i, \tau_{lt}^i, \tau_{bt}^i, \tau_{xt}^i, i = h, f$, an equilibrium allocation solves the following equations:

$$u_{lt}^i(.) = -u_{c_h t}^i(.)(1-\tau_{lt}^i)F_{c_f t}^i(.) \tag{9}$$

$$u_{c_h t}^i(.)(1+\tau_{xt}^i) = \beta u_{c_h t+1}^i(.)\left(F_{c_h t+1}^i(.) + (1-\delta)(1+\tau_{xt+1}^i)\right) \tag{10}$$

$$c_{ht}^i + c_{ft}^j + k_{t+1}^i + g_t^i = F(k_t^i, z_t^i l_t^i) + (1-\delta)k_t^i \tag{11}$$

$$u_{lt}^j(.) = -u_{c_h t}^j(.)(1-\tau_{lt}^i)F_{c_f t}^j(.) \tag{12}$$

$$u_{c_h t}^j(.)(1+\tau_{xt}^j) = \beta u_{c_h t+1}^j(.)\left(F_{c_h t+1}^j(.) + (1-\delta)(1+\tau_{xt+1}^j)\right) \tag{13}$$

$$c^j_{ht} + c^i_{ft} + k^j_{t+1} + g^j_t = F(k^j_t, z^j_t l^j_t) + (1-\delta)k^j_t \tag{14}$$

$$\frac{u^i_{c_f t}(.)}{u^i_{c_h t}(.)(1+\tau^i_{ct})} = \frac{u^j_{c_h t}(.)(1+\tau^j_{ct})}{u^j_{c_f t}(.)} \tag{15}$$

$$\frac{1}{(1-\tau^i_{bt+1})}\left(\frac{1}{1+\tau^i_{xt}}(F^i_{kt+1} + (1-\delta)(1+\tau^i_{xt+1})) - 1\right) = \tag{16}$$
$$\frac{1}{(1-\tau^j_{bt+1})}\left(\frac{1}{1+\tau^j_{xt}}\frac{u^i_{c_f t+1}(.)}{u^i_{c_h t+1}(.)(1+\tau^i_{ct+1})}\frac{u^i_{c_h t}(.)(1+\tau^i_{ct})}{u^i_{c_f t}(.)}(F^j_{kt+1} + (1-\delta)(1+\tau^j_{xt+1})) - 1\right)$$

Proof:(See appendix C.2)

4 Applying the accounting procedure

In this section, we explain our accounting procedure applied to the Great Recession of 2007-2008 synchronization for Canada and the US.

4.1 Calibration procedure

For the application, we use common functional forms in business cycle literature. We opt for a Cobb-Douglas form $F(k,l) = k^\alpha l^{1-\alpha}$ for the production function and for the utility function the form $U(c,l) = \log(c) + \psi\log(1-l)$ with an Armington aggregation for the consumption $\left(c_h^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. The parameters we use are also familiar to business cycle literature. We choose the capital share α to be one-third and the time allocation parameter $\psi = 2.5$. The Armington aggregator coefficient we use in our benchmark model is 2 for the two countries. We then vary this coefficient for the robustness check. We choose the depreciation rate δ and the discount factor β so that, on an annualized basis, depreciation is 5% and the rate of time preference 2.5%. We use the data to compute country-specific growth of population and technology.

Our benchmark model is an open economy of two countries. For the application of our accounting procedure, we choose Canada and the US. We collect the aggregate variables needed using the OECD database. We assume in our model that households have access to home consumption goods and a foreign consumption good, but the aggregate data does not distinguish between them; for each country, we have the aggregate consumption good. To overcome this issue, we consider the total import from the partner country as the foreign consumption good. Then, the home consumption good is the aggregate consumption good minus the foreign consumption good. As the world economy consists of more than two countries

and each country trades not only with each other, we let the government wedge be the net export of all other trade partners except Canada and the US in the data. Using the same rationale as Chari, Kehoe, and McGrattan (2004), we prove that an open economy of $n \geqslant 3$ is equivalent to an open economy of 2 countries with government wedges for each country.

In order to reconcile the theory and the data, we use per capita variables deflated by the GDP deflator. Indeed, our model is a representative agent model, thus using per capita variables both in the model and data makes the approximation realistic. The data we use for the measurement of the wedge and the counterfactual experiments are quarterly data for 2000:1 to 2014:4. These data mainly come from the OECD. In order to estimate the share of foreign consumption goods in the aggregate consumption goods, we use the import of the counterpart partner country using the DOTS (Direction Of Trade Statistics) database of the IMF.

4.2 Findings

In this section, we present the results of the accounting procedure performed on the US and Canada. We focus on the description of the 2007-2008 crisis and the accounting of this business cycle.

I begin by describing the 2007-2008 crisis; this description is summarized in Table A.1. In Figure A.1, we see that from 2008:1 to 2009:3, output fell about 3%, while investment fell about 12%, and labor fell about 6% in Canada. In the US, as shown by Figure A.2, output declined about 4%, while investment and labor declined by about 21% and 9%, respectively, from 2008:1 to 2009:3.

I then analyze the business accounting results. In Table A.2, we see that from 2008:1 to 2009:3, the Canada investment wedge fell about 7%, the Canada labor wedge fell about 14%, and the Canada foreign asset wedge worsened dramatically about 77%. While from the same Table 1, during this period, the US preference wedge declined about 4%, the US labor wedge fell about 9%, while the US investment wedge and US foreign asset wedge dropped by about 15% and 16%, respectively. As both US wedges and Canada wedges can affect the aggregate variables in both countries, we cannot surely draw a pattern from this analysis. However, we expect that the Canada labor wedge, investment wedge, foreign asset wedge, and the US preference wedge, investment wedge, labor wedge, and foreign asset wedge account for the declines of aggregate variables in both countries.

To better assess the contribution of each wedge in the fluctuation of the aggregates, we turned to the analysis of wedge contribution experiments. For that purpose, we rely on the ϕ statistic proposed by Brinca et al. (2016). This statistic is

the inverse of the mean-square error of each wedge. It captures how close a component of a variable due to a particular wedge is to the observed variable in the data. For example, the Canada output (Y^{ca}) ϕ statistic due to the wedge (m) ($\Delta^{ca}z$) is computed as follows: $\phi^{Y^{ca}}m = \frac{1/\sum_t (Y^{ca}t - Y^{ca}m)^2}{\sum_j \left[1/\sum_t (Y^{ca}t - Y^{ca}j)^2\right]}$,
where $m = (\Delta^i zt, \quad \Delta^i lt, \quad \Delta^i_{xt}, \quad \Delta^i_{gt}, \quad \Delta^i_{pt}, \quad \Delta^i_{bt})(i = Ca, US)$. We compute similar statistics for both countries and for investment and labor. Note that the statistic lies in $[0, 1]$, which sums to 1 for all the twelve wedges. The more it is close to one, the more the wedge contributes to the fluctuation of the variable. Thus, when a particular output component fits perfectly if $y_t - ymt = 0$ for all t, then $\phi^Y_m = 1$.

Table B.1 summarizes the ϕ statistics computed for the period 2007:4-2009:4 for the counterfactual experiments we conducted. First, the decline in Canada's output during this crisis was mainly due to the US Labor wedge and the US efficiency wedge. Indeed, the ϕ statistic indicates that the US labor wedge accounts for about 26% of the output decline while the US efficiency wedge accounts for about 14% of that decline. The US output movements during this period, however, were mainly due to the US investment wedge and US labor wedge as they account for about 27% and 16%, respectively.

Second, the fluctuation in Canada's investment during this crisis was mainly caused by the Canada investment wedge and the US investment wedge, with a contribution of about 40% and 21%, respectively. At the same time, the US investment wedge and US preference wedge account for about 29% and 9% of the fall observed in US investment during the crisis.

Finally, we computed the ϕ statistics to account for the fluctuations in employment during the Great Recession. It turns out that the US efficiency wedge and US labor wedge account for around 19% and 13%, respectively, of the fall in Canada's labor. However, the decline in US labor is mainly due to the Canada investment wedge (27%) and US investment wedge (26%).

From these results, we can conclude that the US labor wedge and efficiency wedge played the most important role in Canada during the recession, with a non-negligible role for the US investment wedge and Canada investment wedge in the fluctuation of Canada's investment. During the Great Recession, the most important role in US aggregate variables was played by the US investment wedge, US labor wedge, and US efficiency wedge, with a small role for Canada's investment in the labor decline.

5 Discussions

The theoretical and quantitative results presented in this study omit the role of expectations, so the interpretation of these results should be taken with some caution. However, the findings confirm the interdependence of the Canadian and US economies. The results indicate that distortions in the US economy, especially the US labor wedge, US efficiency wedge, and US investment wedge, have a significant impact on the Canadian economy. Similarly, distortions in the Canadian economy, specifically the Canada investment wedge, have an impact on the US economy. Another conclusion that emerged from the results is that the 2007-2008 crisis probably originated in the US and then spread to Canada.

In addition, as we mentioned in the introduction, Chari et al. demonstrated that an open economy is equivalent to a prototype closed economy with a government wedge. So what differentiates the Business Cycle Accounting proposed by Chari et al. (2007) for a closed economy from ours? To answer this question, we compare the ϕ statistics from our study and those obtained from Brinca et al. (2016) for output. The results presented in Table B.2 indicate that a particular country's business cycle is mainly due to distortions in that country, and the impact of foreign countries is through the government wedge. For example, Chari et al.'s accounting procedure attributes most of the fluctuations in Canadian output to the Canada efficiency wedge and Canada investment wedge, while our accounting procedure attributes the same output movements to the US labor wedge and US efficiency wedge. This analysis shows that the International Business Cycle Accounting methodology highlights the interdependence between countries.

6 Conclusion and extensions

We propose in this work a method that can provide insights for researchers to better specify their quantitative models in international business cycle studies. Our method focuses on an accounting procedure based on a prototype model of international growth that includes wedges to capture potential frictions and distortions in markets. For each country, we include an efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge, and foreign asset wedge. Theoretically, the business co-movements across countries are allowed through the trade in goods and the market sharing included in this model.

To demonstrate our method, we apply it to study the synchronization of Canada and US business cycles during the Great Recession. It turns out that, during this period, the primary role in the economic downturns in both countries is attributed

to the US investment wedge, the US labor wedge, and the US efficiency wedge, with a secondary role attributed to the Canada investment wedge. These results suggest that Canada and the US are heavily linked and that the crisis started in the US and propagated to Canada.

Nevertheless, we must consider our results with some caveats because we omitted the role of expectations in this framework. Adding expectations could impact the measurements of wedges as it explicitly links all of them. Expectations are also essential in investment decision-making by agents. Thus, a first extension of this work would be to add expectations to the current model. A second extension would be to prove the equivalence result of the method, which would help classify some existing international macroeconomic models as a prototype model with specific wedges. A further direction of this work could be to investigate a puzzle, such as the trade co-movement puzzle. In practice, this would involve applying our methodology, building a detailed model from the insights of the results, and seeing if it helps solve the puzzle.

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A Figure A: 2007-2008 crisis Description

Figure A.1: 2007-2008 crisis Description for Canada

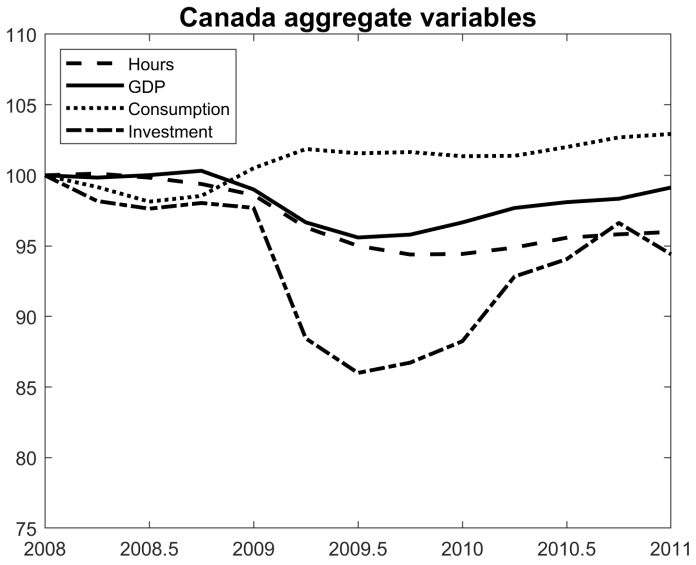
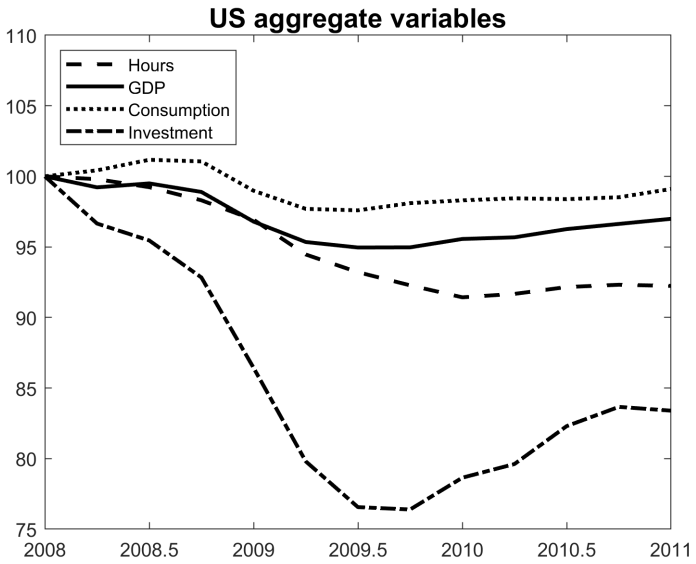


Figure A.2: 2007-2008 crisis Description for US



A Table A: 2007-2008 crisis Description

Table A.1: Changing in aggregates during the crisis

			Change of aggregates in %		
	Peak	Through	Y	X	L
Canada	2007Q4	2009Q4	-3.34	-11.76	-5.57
US			-4.44	-21.35	-8.57

Table A.2: Changing in wedges from 2007:4 to 2009:4

	Change of wedges in %					
	Z	L	X	G	P	B
Canada	0.32	-14.22	-7.24	8.01	14.38	-76.94
US	2.00	-9.11	-15.39	8.83	-3.79	-15.85

B Table B: Summary statistics

Table B.1: ϕ statistics in % for 2007:4 to 2009:4

	Canda Wedges						US Wedges					
	Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
Y Ca	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
X Ca	1.70	3.80	40.02	1.26	2.16	3.72	12.64	4.32	20.48	1.36	4.73	3.80
X US	6.79	8.94	8.36	9.17	5.73	5.97	4.09	2.30	28.96	1.57	9.42	8.70
L Ca	0.76	2.01	10.90	1.13	5.44	7.24	18.52	12.99	5.33	21.33	10.64	3.71
L US	2.70	0.34	26.50	1.89	0.46	8.48	1.80	18.79	25.58	4.00	5.09	4.38

Table B.2: ϕ statistics comparison for IBCA and BCA

		Canda Wedges						US Wedges					
		Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
IBCA	**Y Ca**	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
	Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
BCA	**Y Ca**	49.00	13.00	18.00	20.00	-	-	-	-	-	-	-	-
	Y US	-	-	-	-	-	-	16.00	46.00	32.00	6.00	-	-

A Data treatment

B Proof of equivalence result

B.1 Equivalent between an open economy with n countries with an open economy with two countries

C Proof of propositions

C.1 Proof of proposition 1

Without assets trading in the world economy, the net export equals zero every period. We know that the net export is:

$$(X-M)_t^i = p_t^i(y_t^i - c_{ht}^i - x_t^i) - p_t^j c_{ft}^i$$

As the firms are in competitive market, their profit is zero every period such that

$$p_t^i y_t^i = w_t^i l_t^i + r_t^i k_t^i$$

Then by replacing the firm revenue into the net export equation we get:

$$(X-M)_t^i = w_t^i l_t^i + r_t^i k_t^i - p_t^i(c_{ht}^i + x_t^i) - p_t^j c_{ft}^i$$

We also know that the government wedge is equal to the transfers to household state by the following equation:

$$tr_t^i = g_t^i = p_t^j \tau_{ct}^i c_{ft}^i + p_t^i \tau_{xt}^i x_t^i + \tau_{lt}^i w_t^i l_t^i \tag{17}$$

Then using the budget constraint 2 (with $b_t^i = 0$) and 17, we obtain $(X-M)_t^i = 0$.

C.2 Proof of proposition 2

Let assume $\mathcal{A}^i = \left(c_{ht}^i, c_{ft}^i, x_t^i, l_t^i, b_t^i\right)_{t=0}^{\infty}$ i=(h,f) solve equations in proposition 2 and let prove there exist price $\mathcal{P}^i = (p_t^i, w_t^i, r_t^i, r_t)_{t=0}^{\infty}$ such that $\mathcal{A}^i$ and $\mathcal{P}^i$ is a competitive equilibrium.
The proof is straightforward when we make the following assumptions:

- Normalize a price: $\forall t, p_t^h = 1$
- Compute $p_t^f = \frac{u_{2t}^h(.)}{u_{1t}^h(.)(1+\tau_{ct}^h)}$

- Compute $w_t^i = p_t^i F_{2t}^i(.)$
- Compute $r_t^i = p_t^i F_{kt}(.)$
- Using non arbitrage condition, compute $r_{t+1} = \frac{1}{(1-\tau_{bst+1}^i)}(\frac{1}{1+\tau_{xt}^i}\frac{p_{t+1}^i}{p_t^i}(F_{kt+1} + (1-\delta)(1+\tau_{xt+1}^i)) - 1)$

Indeed ,

- From the wage rate and interest rate equations, firms optimize as shown by equations 18 and 19;
- From equations 11 and 14 of proposition 2, resource constraints are satisfied;
- Combining the remind equations and prices we get the FOC of households problems (equations 20 to 23 for each country i) ;
- The budget constraints (equations 2 for each country i) are satisfied by using them to compute the assets variables.

First order conditions of firms optimization problems

$$w_t^i = p_t^i F_{lt}(.) \tag{18}$$

$$r_t^i = p_t^i F_{kt}(.) \tag{19}$$

First order conditions of household optimization problems

Using the Lagrangian procedure we get the following equations:

$$u_{c_f t}(.) = u_{c_h t}(.)(1+\tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{20}$$

$$u_{lt}(.) = -u_{c_h t}(.)(1-\tau_{lt}^i)\frac{w_t^i}{p_t^i} \tag{21}$$

$$u_{c_h t}(.)\frac{1}{p_t^i} = \beta u_{c_h t+1}(.)(1+(1-\tau_{bt+1}^i)r_{t+1}))\frac{1}{p_{t+1}^i} \tag{22}$$

$$u_{c_h t}(.)(1+\tau_{xt}^i) = \beta u_{c_h t+1}(.)(\frac{r_{t+1}^i}{p_{t+1}^i} + (1-\delta)(1+\tau_{xt+1}^i)) \tag{23}$$