International Business Cycle Accounting

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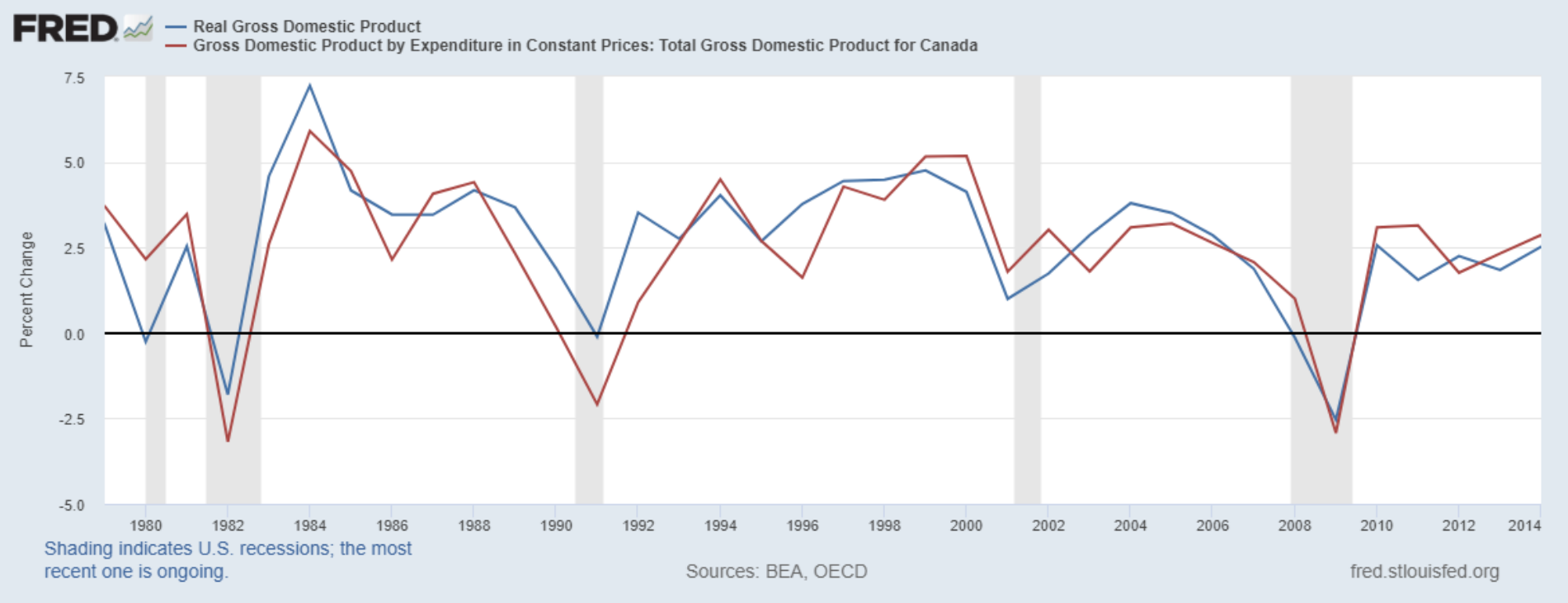
September 1, 2022

- Data show business cycle synchronization across countries.
- What are the promising features to include in a model to generate this synchronization?
- I propose a new approach to account international business cycle named *International Business Cycle Accounting*.
- Application of the method to US and Canada business cycle synchronization study: 2007-2008 crisis.
- Findings:
  - Distortions on labor market in US explain more the decline in Canada output;
  - Decline in US labor is explained mainly by distortions of investment in Canada;

Motivation

Figure 1: US and Canada Real GDP comovement



Economic questions

- What are the main channels of economic cycles transmission across countries?
- Which partners will be more affected by a shock hitting a country ?
- What are the main sectors that facilitate shocks propagation across countries?

Literature review I

Classical approach

- Build a detail model with frictions.
- Calibrate it and see how much it replicates data information.

Recent finding using classical approach

de Soyres (2016) using the classical approach to study ”Trade comovement Puzzle”.

Different approach

Business Cycle Accounting methodology proposed by Chari, Kehoe and McGrattan (2007)

Literature review II

Business cycle Accounting

1. Equivalence result
2. Accounting procedure
   - 2.1 Measure wedges
   - 2.2 Evaluate wedges contribution

Environment : Closed economy

- Government wedge
- Home bias, international finance,...

Methodology Approach I

What I do ?

- Proposition of new approach of analysis in International Macroeconomics: *International Business Cycle Accounting*;
- Application of International Business Accounting to U.S. and Canada to study 2007-2008 crisis.

Contribution

- Diagnostic tool of international business cycle;
- Provide insights that can help to better specify details models in international business cycle studies: *international macroeconomics puzzles*.

Findings

- Theoretically, two channels of business cycle synchronization: *Trade in goods* and *International asset market*.
- The US investment wedge and labor wedge explain respectively 27% and 16% of declines in US output during the crisis;
- For Canada, the main wedges that explain output declines are US labor wedge (26%) and efficiency wedge (14%).

International Business Cycle Accounting I

Step 1: Equivalence Result

- Set up a prototype model with wedges such that a large class of detail models with frictions are equivalent to it.
- A natural choice for the prototype model is international growth model in which we include wedges.
- Wedges look like time varying productivity, labor income taxes, investment taxes, government consumption and taxes on foreign consumption goods and foreign asset.

International Business Cycle Accounting II

Step 2.1: Measure wedges

Calibrate the prototype model parameters and then measure the wedges.

Step 2.2: Evaluate wedges contribution

Feed back the measured wedge values (one at a time or combination) to assess how much of observed movements of output, consumption, labor,investment,... can be attributed to those wedges.

Step 1: Two-country model I

Economic environment

- Two countries specialized each in production of a single good.
- Labor and capital involved in each production process are immobile.
- Only production goods are traded.
- Household can hold assets (debt or lending) from foreign country.
- Wedges resembling taxes consider are: τ_c^i, τ_x^i, τ_l^i, τ_b^i,
- In this canonical model we shut down the role of expectations.

Step 1: Two-country model II

Firms problem for country i

$$\max_{l_t^i, k_t^i} \Pi_t^i = p_t^i y_t^i - w_t^i l_t^i - r_t^i k_t^i \tag{1}$$

Representative country i household problem

- Life time utility

$$max_{c_h^i, c_f^i, x_t^i, b_{t+1}^i, l_t^i} \sum_{t=0}^{\infty} \beta^t U(c_{ht}^i, c_{ft}^i, l_t^i) \tag{2}$$

- Budget constraint

$$p_t^i c_{ht}^i + (1+\tau_{ct}^i) p_t^j c_{ft}^i + (1+\tau_{xt}^i) p_t^i x_{it}^i + b_{t+1}^i = \tag{3}$$
$$(1-\tau_{lt}^i) w_t^i l_t^i + r_t^i k_t^i + \left(1 + (1-\tau_{bt}^i) r_t\right) b_t^i + tr_t^i$$

- Capital law of motions

$$k_{t+1}^i = (1-\delta) k_t^i + x_t^i \tag{4}$$

Step 1: Two-country model III

Proposition 1

- The asset market is important for international finance.

  When $b_t^h = b_t^f = 0$, the net export equals zero each period.

- Proof of proposition 1: ▸ Proof.

Step 1: Two-country model IV

Definition of equilibrium

Given $\{\tau^i_{ct}, \tau^i_{xt}, \tau^i_{lt}, \tau^i_{bt}, z^i_t\}^{\infty}_{t=0}$, a competitive equilibrium is an allocation $\mathcal{A}^i = \left(c^i_{ht}, c^i_{ft}, x^i_t, l^i_t, b^i_t\right)^{\infty}_{t=0}$, a price $\mathcal{P}^i = \left(p^i_t, w^i_t, r^i_t, r_t\right)^{\infty}_{t=0}$ such as,

- $\forall i$, given $\mathcal{P}^i$ and $\mathcal{P}^j$, household of country i, maximizes his lifetime utility subject to budget constraint;
- Given p^i_t, each firm maximizes his profit;
- Government budget constraint is balanced for each country i;

$$g^i_t + \frac{tr^i_t}{p^i_t} = \tau^i_{lt}\frac{w^i_t}{p^i_t}l^i_t + \tau^i_{xt}x^i_t + \tau^i_{ct}\frac{p^j_t}{p^i_t}c^i_{ft} + (\tau^i_{bt}r_t)b^i_t \tag{5}$$

- Goods markets are cleared: $\forall i$,

$$c^i_{ht} + c^j_{ft} + x^i_t + g^i_t = F(k^i_t, l^i_t) \tag{6}$$

- Assets markets are cleared;

$$b^i_t + b^j_t = 0 \tag{7}$$

- Labor markets are cleared: $\forall i$,

$$l^i_t + n^i_t = 1 \tag{8}$$

Step 2.1: Two-country model I

Measure of the wedges

$$y_t^i = F(k_t^i, z_t^i l_t^i) \tag{9}$$

$$u_{2t}(.) = u_{1t}(.)(1 + \tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{10}$$

$$u_{3t}(.) = -u_{1t}(.)(1 - \tau_{lt}^i)z_t^i F_{2t}(.) \tag{11}$$

$$c_{ht}^i + c_{ft}^j + k_{t+1}^i + g_t^i = F(k_t^i, z_t^i l_t^i) + (1 - \delta)k_t^i \tag{12}$$

$$u_{1t}(.)(1 + \tau_{xt}^i) = \tilde{\beta}^i u_{1t+1}(.)\left(F_{1t+1}(.) + (1 - \delta)(1 + \tau_{xt+1}^i)\right) \tag{13}$$

$$1 + (1 - \tau_{bt+1}^i)r_{t+1} = \frac{1}{1 + \tau_{xt}^i}\frac{p_{t+1}^i}{p_t^i}\left(F_{1t+1} + (1 - \delta)(1 + \tau_{xt+1}^i)\right) \tag{14}$$

Step 2.2: Two-country model I

Counterfactual analysis

Given $\tau_{ct}^i, \tau_{lt}^i, \tau_{bt}^i, \tau_{xt}^i, z_t^i, g_t^i, i = h, f$, an equilibrium allocation $\mathcal{A}^i = \left(c_{ht}^i, c_{ft}^i, x_t^i, l_t^i, b_t^i\right)_{t=0}^{\infty}$ i=(h,f) solves the following equations

state in proposition 2.

Proof of proposition 2: ▸ Proof

Step 2.2: Two-country model II

Proposition 2

$$u^i_{3t}(.) = -u^i_{1t}(.)(1-\tau^i_{lt})F^i_{2t}(.) \tag{15}$$

$$u^j_{3t}(.) = -u^j_{1t}(.)(1-\tau^i_{lt})F^j_{2t}(.) \tag{16}$$

$$c^i_{ht} + c^j_{ft} + k^i_{t+1} + g^i_t = F(k^i_t, z^i_t l^i_t) + (1-\delta)k^i_t \tag{17}$$

$$c^j_{ht} + c^i_{ft} + k^j_{t+1} + g^j_t = F(k^j_t, z^j_t l^j_t) + (1-\delta)k^j_t \tag{18}$$

$$u^i_{1t}(.)(1+\tau^i_{xt}) = \beta u^i_{1t+1}(.)\left(F^i_{1t+1}(.) + (1-\delta)(1+\tau^i_{xt+1})\right) \tag{19}$$

$$u^j_{1t}(.)(1+\tau^j_{xt}) = \beta u^j_{1t+1}(.)\left(F^j_{1t+1}(.) + (1-\delta)(1+\tau^j_{xt+1})\right) \tag{20}$$

$$\frac{u^i_{2t}(.)}{u^i_{1t}(.)(1+\tau^i_{ct})} = \frac{u^j_{1t}(.)(1+\tau^j_{ct})}{u^j_{2t}(.)} \tag{21}$$

$$\frac{1}{(1-\tau^i_{bt+1})}\left[\frac{1}{1+\tau^i_{xt}}\left(F^i_{1t+1} + (1-\delta)(1+\tau^i_{xt+1})\right) - 1\right] = \tag{22}$$

$$\frac{1}{(1-\tau^j_{bt+1})}\left[\frac{1}{1+\tau^j_{xt}}\frac{u^i_{2t+1}(.)}{u^i_{1t+1}(.)(1+\tau^i_{ct+1})}\frac{u^i_{1t}(.)(1+\tau^i_{ct})}{u^i_{2t}(.)}\left(F^j_{1t+1} + (1-\delta)(1+\tau^j_{xt+1})\right) - 1\right]$$

Mechanisms

Explanation of the comovement from the model

When a country experiences a shock, it can affect a partner country through:

1. The trade of good: one of the relations between the countries is through the import of foreign good and export a home good for consumption.

2. The financial market: the model allow a country to borrow or lend in this market to insure from a shock.

Step 2 : Accounting procedure in practice

Preliminaries

- Variables $y_t, k_t, l_t, c_{ht}, c_{ft}, p_t, r_t$ will be extract from data for the wedges measurement.
- Functional form for utility(U(.)) and production (F(.)) :
  $u(c_h, c_f, l) = \frac{[(c_h^\theta + c_f^\theta)^{\frac{1}{\theta}}(1-l)^\psi]^{\rho-1}}{\rho - 1}$ and $F(k,l) = k^\alpha l^{1-\alpha}$
- Choose α, β, σ, δ and ρ for each country according to the literature.

Accounting steps

- $\Delta^i_{zt} = z^i_t$, $\Delta^i_{gt} = g^i_t$, $\Delta^i_{lt} = 1 - \tau^i_{lt}$, $\Delta^i_{xt} = \frac{1}{1+\tau^i_{xt}}$ $\Delta^i_{pt} = \frac{1}{1+\tau^i_{ct}}$ and $\Delta^i_{bt} = 1 - \tau^i_{bt}$ are wedges (variables of interest) which are measured.
- Feed back the wedges into the model to evaluate the contribution of each (or combination) of them.

Data

- Application to US and Canada : 2007-2008 synchronization of crisis .
- y_t, x_t, l_t, c_t per capita variables deflated by GDP deflator extract from OECD database;
- c_{ft} is import from counterpart country using Direction Of Trade Statistics (DOTS) database of IMF;
- p_t is backup using exchange rate from OECD database;
- r_t is the interest rate either of Canada or US from OECD database;
- Quarterly time series from 2000:1 to 2014:4.

Parameters calibrated

Table 1: Parameters calibration

Variables	Values
α	0.3333
θ	2.0000
δ	0.0500
ρ	1.0001
ψ	2.5000
β	0.9750
γ_n^{ca}	0.0024
γ_n^{us}	0.0021
γ_z^{ca}	0.0043
γ_z^{us}	0.0047

Description of 2007-2008 crisis

Table 2: Changing in aggregates during the crisis

			Change of aggregates in %		
	Peak	Trough	***Y***	***X***	***L***
Canada	2007Q4	2009Q4	-3.34	-11.76	-5.57
US			-4.44	-21.35	-8.57

▸ Plots for aggregates description

Table 3: Changing in measured wedges from 2007:4 to 2009:4

	Change of wedges in %					
	Z	***L***	***X***	***G***	***P***	***B***
Canada	0.32	-14.22	-7.24	8.01	14.38	-76.94
US	2.00	-9.11	-15.39	8.83	-3.79	-15.85

▸ Plots for wedges description

Evaluation of each wedge contribution using ϕ statistics

Plot

$$\phi_m^S = \frac{1/\sum_t (S_t - S_{mt})^2}{\sum_j \left[1/\sum_t (S_t - S_{jt})^2\right]} \tag{23}$$

$$m = (\Delta^i_{zt}, \Delta^i_{lt}, \Delta^i_{xt}, \Delta^i_{gt}, \Delta^i_{pt}, \Delta^i_{bt})_{(i=Ca,US)}$$

Table 4: ϕ statistics in % for 2007:4 to 2009:4

	Canda Wedges						US Wedges					
	Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
Y Ca	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
X Ca	1.70	3.80	40.02	1.26	2.16	3.72	12.64	4.32	20.48	1.36	4.73	3.80
X US	6.79	8.94	8.36	9.17	5.73	5.97	4.09	2.30	28.96	1.57	9.42	8.70
L Ca	0.76	2.01	10.90	1.13	5.44	7.24	18.52	12.99	5.33	21.33	10.64	3.71
L US	2.70	0.34	26.50	1.89	0.46	8.48	1.80	18.79	25.58	4.00	5.09	4.38

Plots

Comparison with BCA

Take away

- Build an international aggregate prototype model explaining the business cycle sychronization through:
  - Trade in goods;
  - Financial market sharing.
- For the 2008 crisis,
  - The US investment and labor wedges explain mainly the decline observed in US output;
  - The US labor and efficiency wedges explain most the fall observed in Canada output;
  - Results confirm that the crisis has started in US and propagate to Canada.

Next steps I

- **Include expectations in the model** $\Rightarrow$ theoretical and empirical implications.
- **Classify some macro economics details models** in terms of prototype models with specific wedges.
  - Monopolistic competition economies with homogeneous firm like an extension of Krugman (1979,1980) model.
  - Monopolistic competition economies with heterogeneous firm like an extension of Eaton and Kortum (2002), Melitz (2003) or Chaney (2008) models.
  - Models with production linkage (final goods are also intermediate good).
- Research on **Trade Comovement puzzle**(Kose and Yi (2001)
  - Idea : Apply International business cycle to data showing trade comovement.

Next steps II

- Base on promising wedges, study the trade comovement using detail model.
- Using data from 2000 to 2014:4, the accounting procedure suggests that US preference wedge and investment wedge explain mainly US and Canada business cycle synchronization.

Table 5: ϕ statistics in % for 2000 to 2014:4

	Canda Wedges						US Wedges					
	Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
Y Ca	1.26	1.59	4.83	14.20	1.63	0.79	4.26	11.13	14.88	12.69	7.47	25.27
Y US	0.45	0.85	62.59	7.38	1.94	5.43	0.95	1.12	16.71	0.56	1.08	0.94

Thank You!

Proof of Proposition 1

Proposition 1

Without assets trading in the world economy, the net export equals zero every period. We know that the net export is:

$$(X - M)_t^i = p_t^i(y_t^i - c_{ht}^i - x_t^i) - p_t^j c_{ft}^i$$

As the firms are in competitive market, their profit is zero every period such that

$$p_t^i y_t^i = w_t^i l_t^i + r_t^i k_t^i$$

Then

$$(X - M)_t^i = w_t^i l_t^i + r_t^i k_t^i - p_t^i(c_{ht}^i + x_t^i) - p_t^j c_{ft}^i$$

We also know that:

$$tr_t^i = g_t^i = p_t^j \tau_{ct}^i c_{ft}^i + p_t^i \tau_{xt}^i x_t^i + \tau_{lt}^i w_t^i l_t^i \tag{24}$$

Then using the budget constraint 3 (with $b_t^i = 0$) and 24, we obtain $(X - M)_t^i = 0$.

Proof of Proposition 2

Proposition2

Let assume $\mathcal{A}^i = \left(c^i_{ht}, c^i_{ft}, x^i_t, l^i_t, b^i_t\right)_{t=0}^{\infty}$ i=(h,f) solve equations in proposition 2 and let prove there exist price $\mathcal{P}^i = \left(p^i_t, w^i_t, r^i_t, r_t\right)_{t=0}^{\infty}$ such that $\mathcal{A}^i$ and $\mathcal{P}^i$ is a competitive equilibrium.
The proof is straightforward when we make the assumptions:

- Normalize a price:$\forall t, p^h_t = 1$
- Compute $p^f_t = \dfrac{u^h_{2t}(.)}{u^h_{1t}(.)(1+\tau^h_{ct})}$
- Compute $w^i_t = p^i_t F^i_{2t}(.)$
- Compute $r^i_t = p^i_t F_{1t}(.)$

- Using non arbitrage condition, compute

$$r_{t+1} = \frac{1}{(1-\tau^i_{bt+1})}\left(\frac{1}{1+\tau^i_{xt}}\frac{p^i_{t+1}}{p^i_t}(F_{1t+1} + (1-\delta)(1+\tau^i_{xt+1})) - 1\right)$$

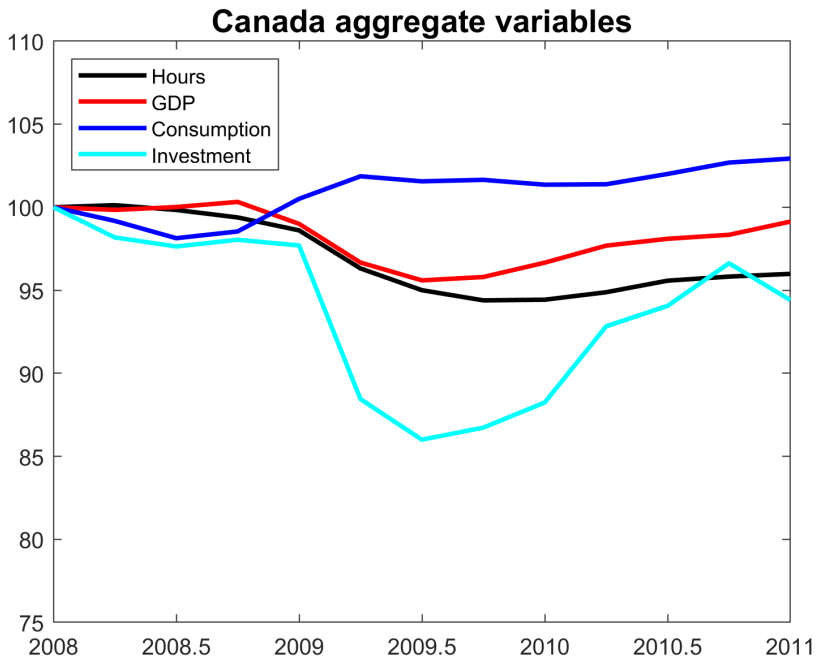
Indeed ,

- From the wage rate and interest rate equations, firms optimize;
- From equations 17 and 18 of proposition 2, resource constraints are satisfied;
- Combining the remind equations and prices we get the FOC of households problems;
- The budgets constraints are satisfied by using them to compute the assets variables.

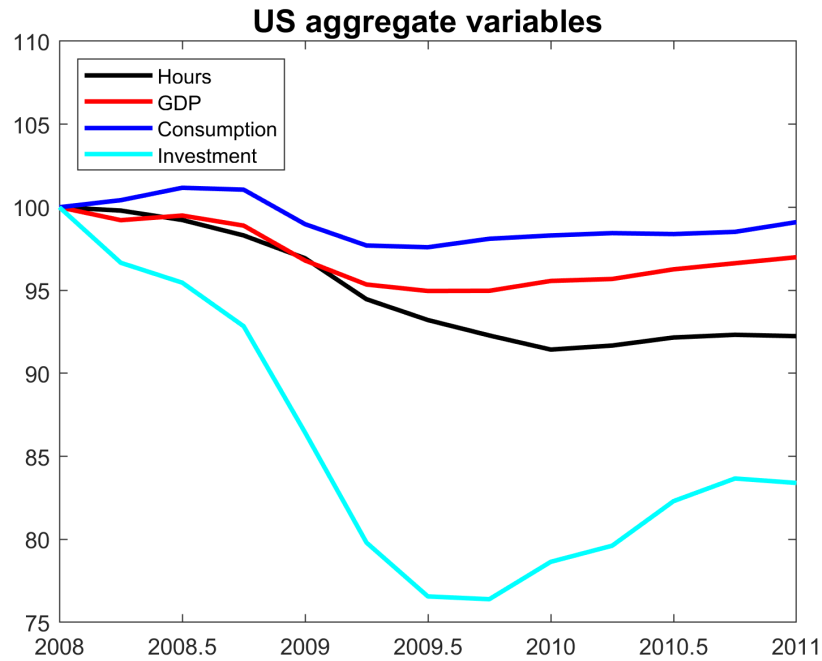
2007-2008 crisis Description

▸ Table for description

Figure 2: Description of the crisis



(a) 2007-2008 crisis Description for Canada



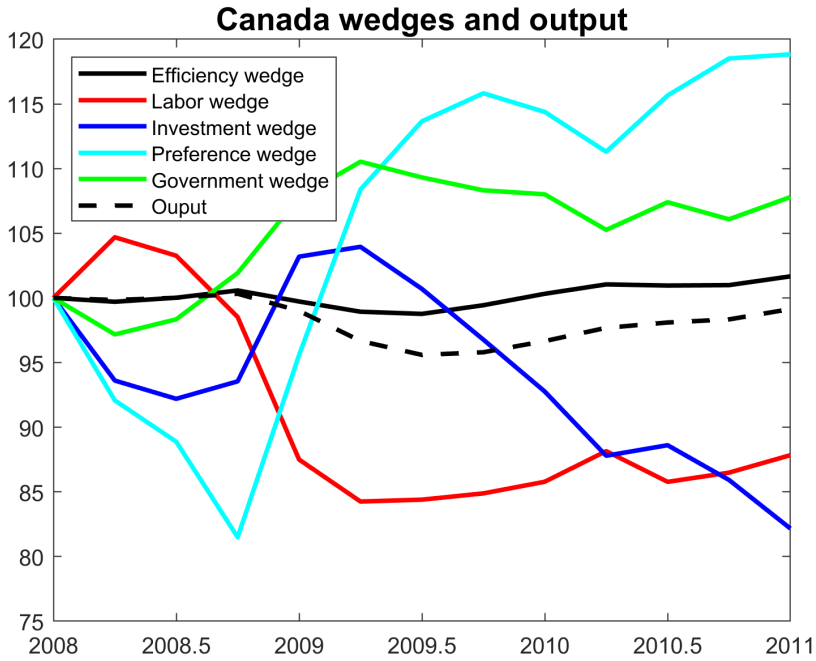
(b) Description of the crisis for US

Source:

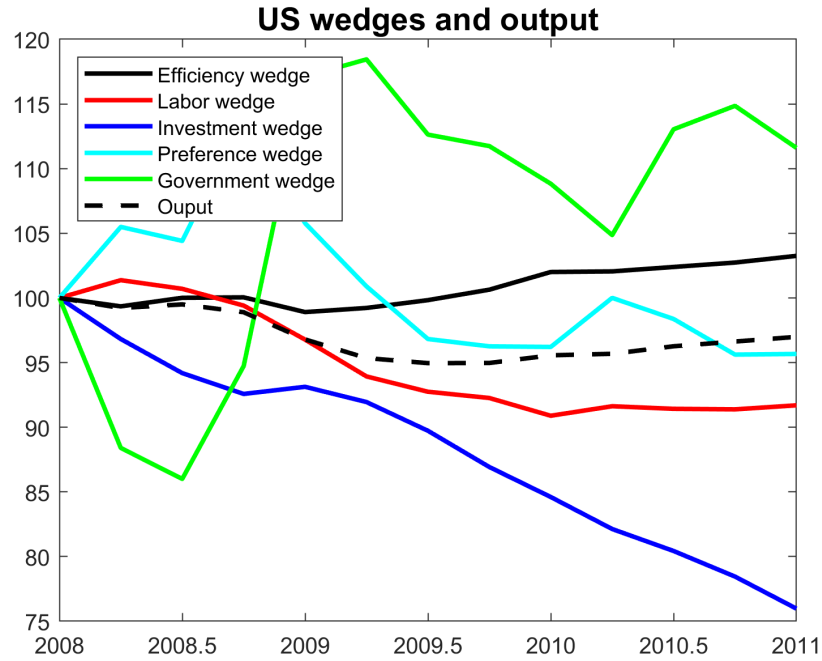
Table for wedges description

Output fluctuation and wedges

Figure 4: Output and wedges



(a) Output and wedges for Canada

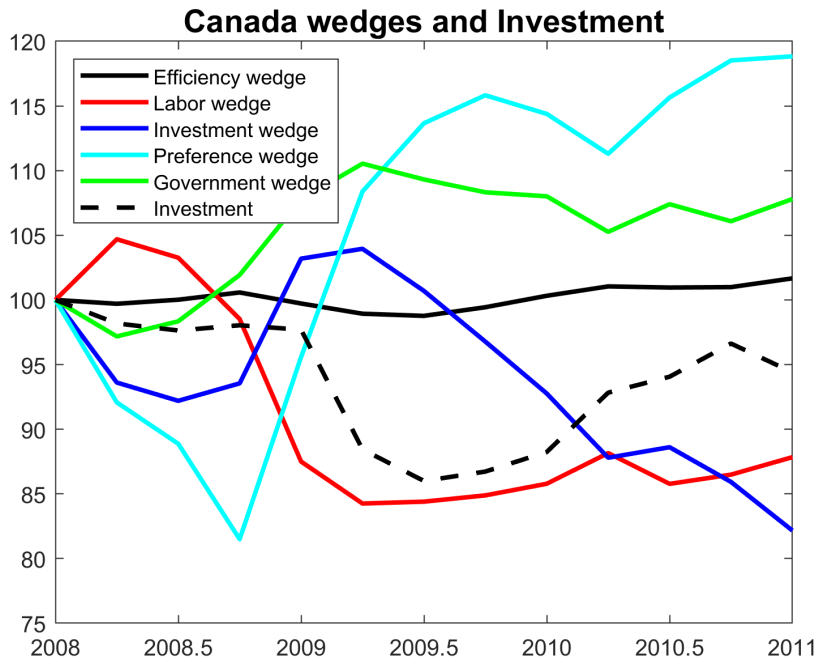


(b) Output and wedges for US

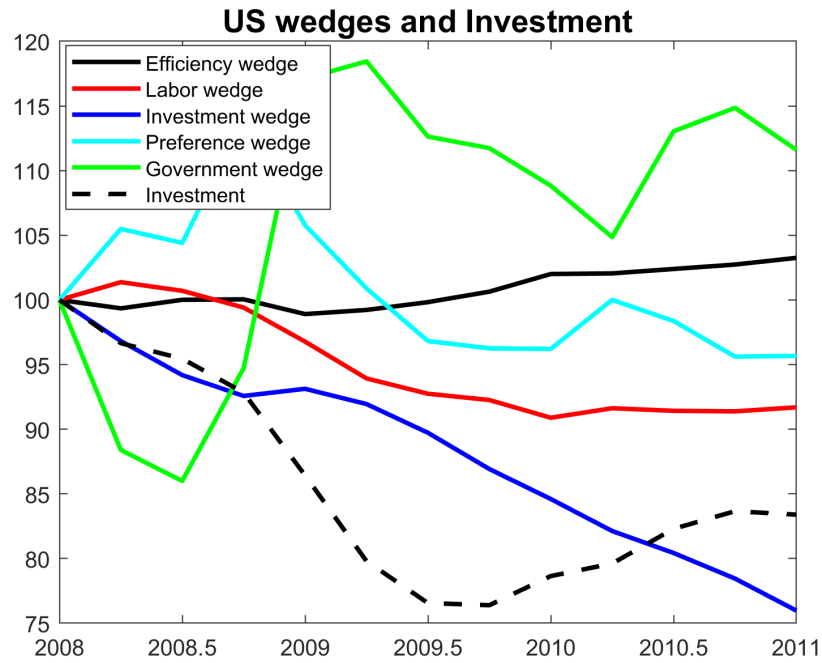
Source:

Investment fluctuation and wedges

Figure 6: Investment and wedges



(a) Investment and wedges for Canada

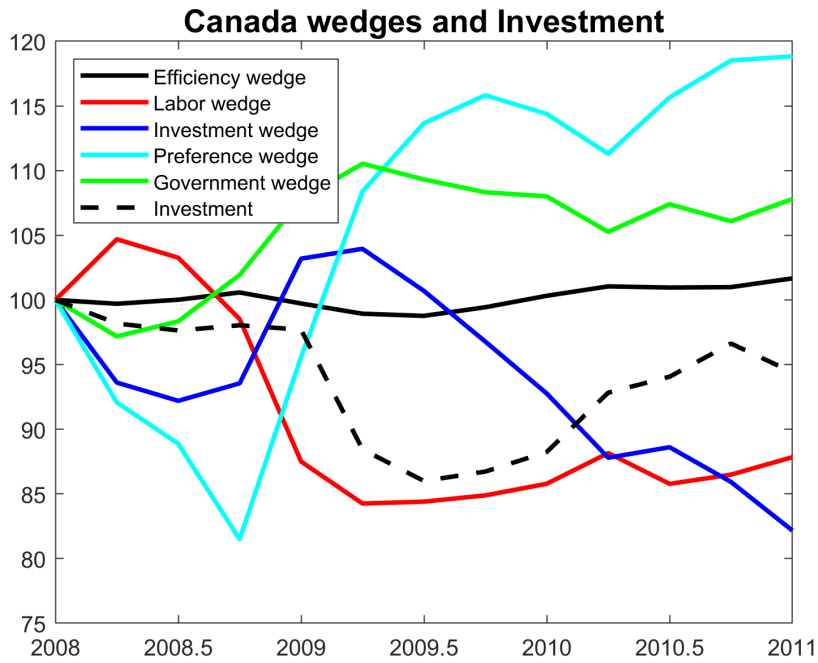


(b) Investment and wedges for US

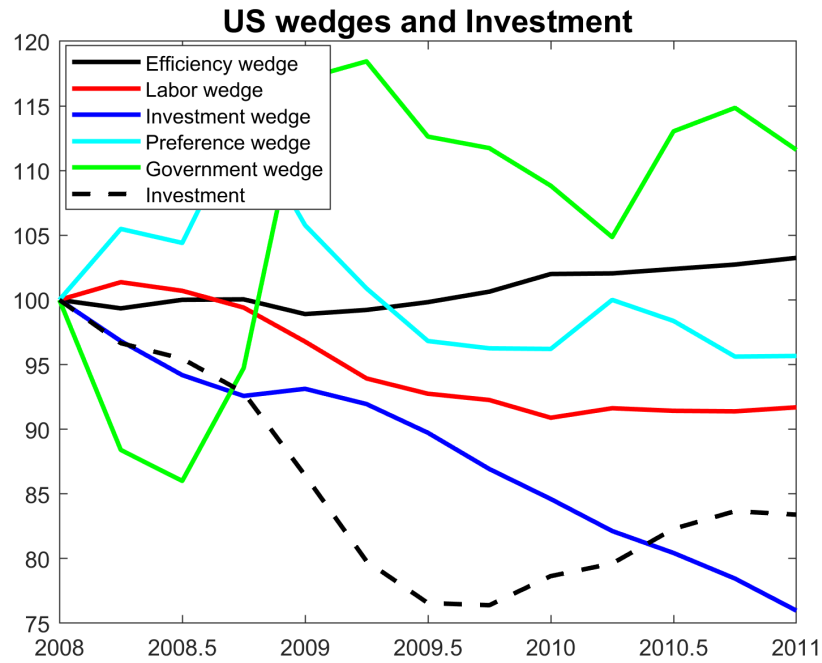
Source:

Labor fluctuation and wedges

Figure 8: Labor and wedges



(a) Labor and wedges for Canada

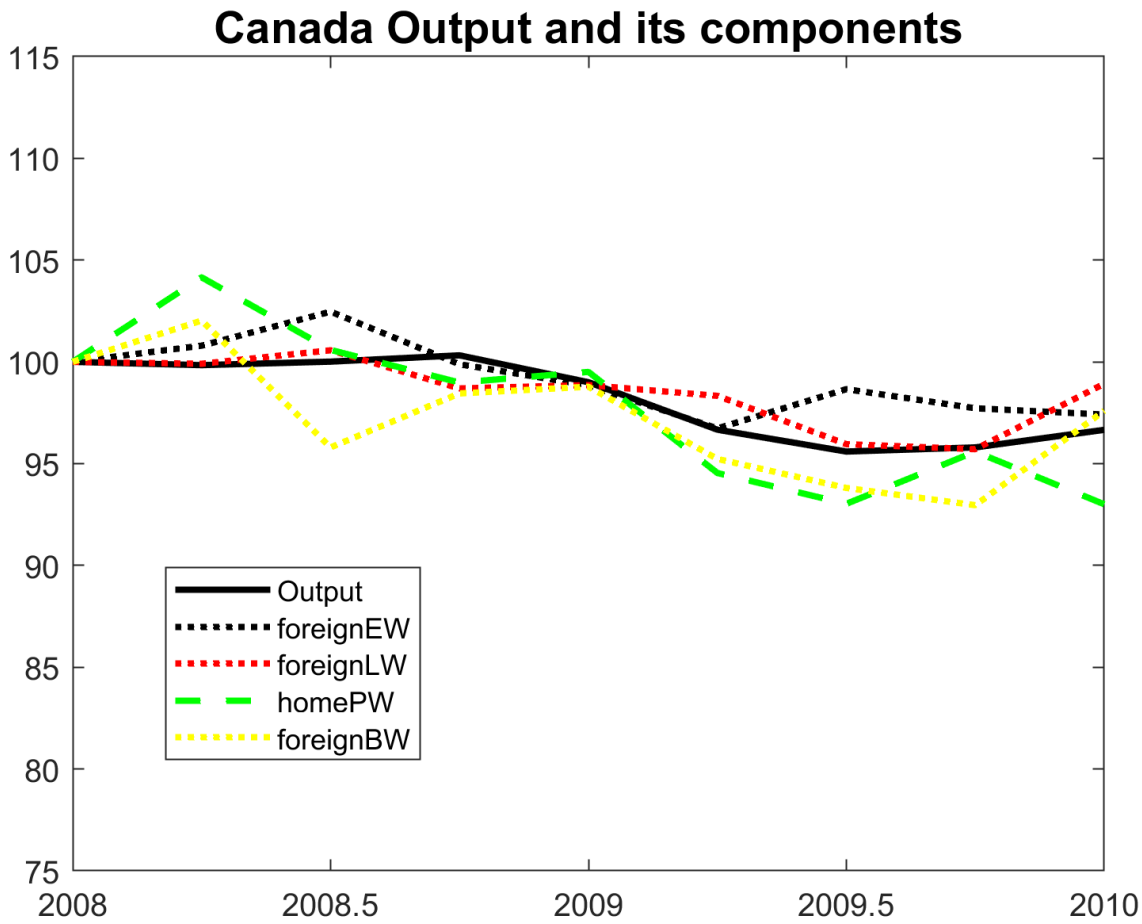


(b) Labor and wedges for US

Source:

▸ Table for ϕ statistics

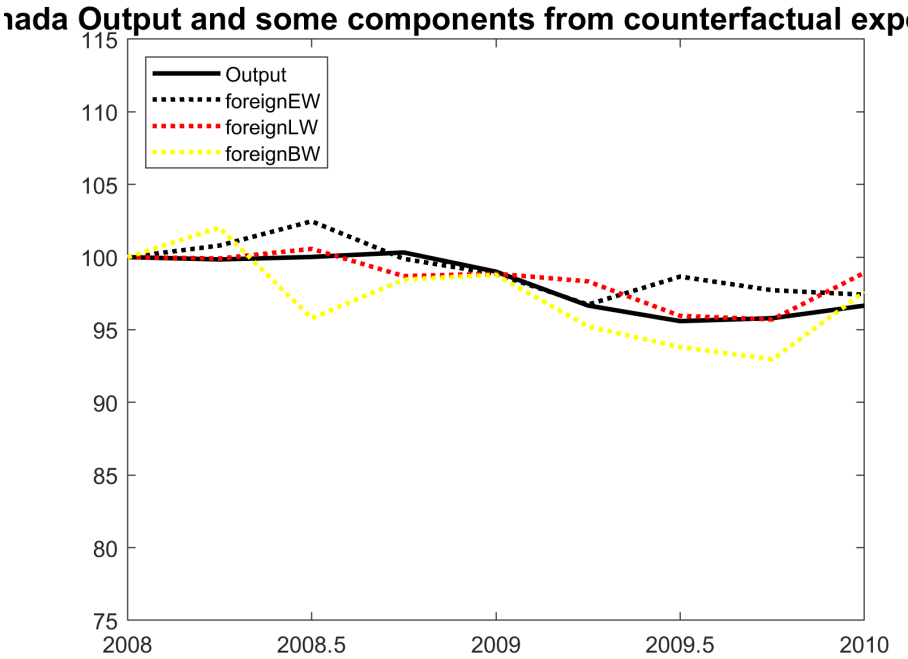
Figure 10: Canada output and its components



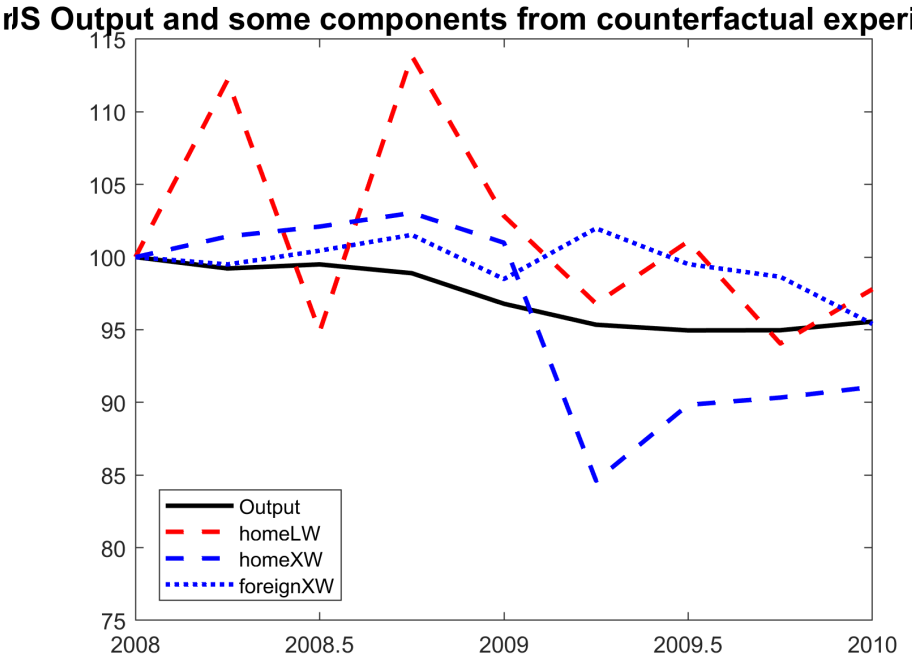
Output variables and main simulated components

▸ Table for ϕ statistics

Figure 11: Outputs and components



(a) Output and components for Canada



(b) Output and components for US

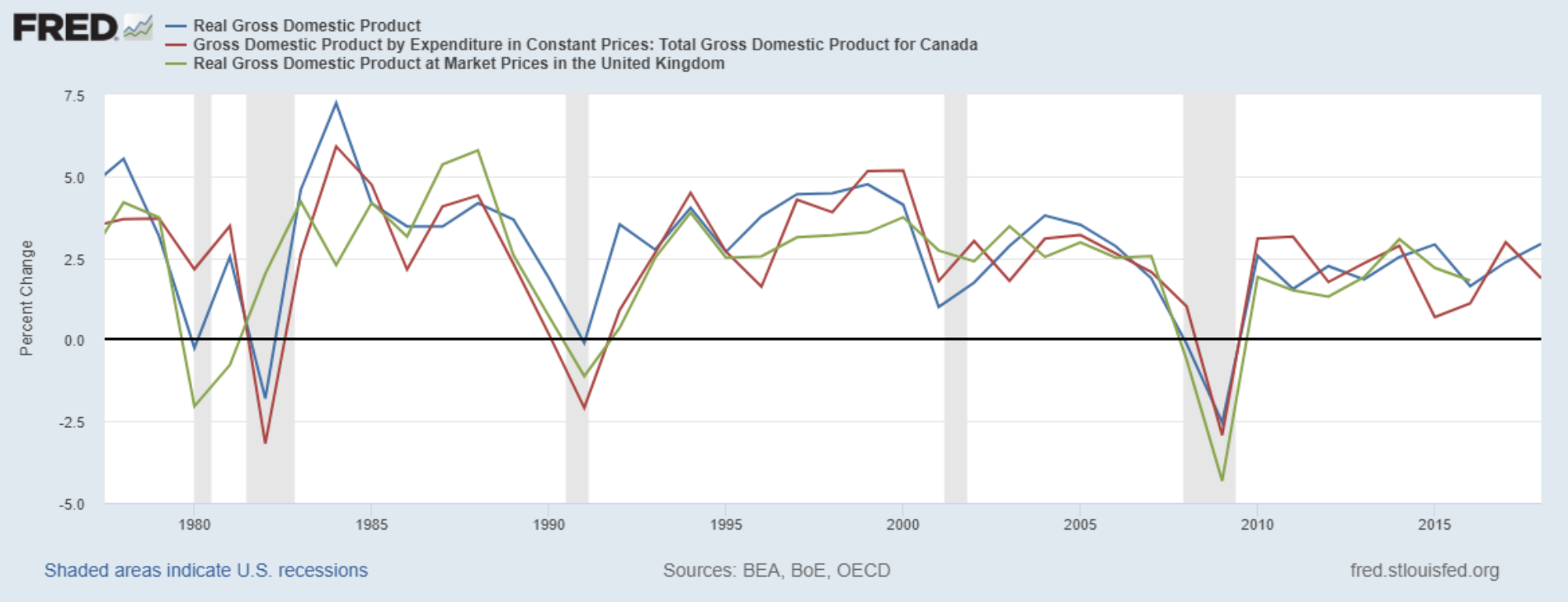
Source:

▸ Table for ϕ statistics

Table 6: ϕ statistics comparison for IBCA and BCA

		Canda Wedges						US Wedges					
		Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
IBCA	**Y Ca**	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
	Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
BCA[1]	**Y Ca**	49.00	13.00	18.00	20.00	-	-	-	-	-	-	-	-
	Y US	-	-	-	-	-	-	16.00	46.00	32.00	6.00	-	-

Figure 13: US, Canada and UK GDP comovement



[1] From Brinca et al. with uncertainty environment