International Business Cycle Accounting

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Abstract

We develop a method that can give insights to researchers to better specify their quantitative models in international business cycles studies. The guidance comes from the application of accounting procedure that is base on prototype model of international growth model including wedges that captures all the eventual frictions and distortions of markets. For each country, we include -efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge and foreign asset wedge. We then demonstrate the method by applying to US and Canada during the Great recession (2007-2008). We found that the economic downturns in both countries during this period is primary due to US investment wedge, US labor wedge, US efficiency wedge and secondary is due to Canada investment wedge. Those results suggest that the crisis has started in US and has propagated to Canada.

Keywords: Business Cycle, Accounting,...

Mathematics Subject Classification (2010): ...

JEL Classification: ...

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1 Introduction

International macroeconomics realm abounds of puzzles[1]. To solve those puzzles, economics researchers build detailed models in which they add frictions to replicate economic fluctuations observed in data. But they face hard difficulties about which frictions and where to introduce them in the model. We propose in this study a method that could facilitate those choices and we make an application of the method.

Indeed, our method is an extension of the method proposed by Chari et al. (2007) that was designed for closed economies. It has two components: an equivalence result and an accounting procedure. The equivalence result consists in building a prototype model including time-varying wedges that distort the equilibrium of economy (in otherwise in perfect competitive markets) such that a large class of detailed models with frictions are equivalent to this prototype model with some specifics wedges. We consider six wedges for each country in the framework and label them efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge and foreign asset wedge.

The accounting procedure consists in measuring first the wedges. For that purpose, we use the data and the equilibrium conditions of the prototype model. In second, we evaluate the contribution of those wedges by feeding their values back, one at the time or by combination, to the model. Those experiments help to assess how much the fluctuations of output, investment, labor is due to wedges, separately or by combinations.

Here, we focus on the accounting procedure. To better understand this procedure, we apply it for the study of business synchronization of Canada and US during the Great Recession of 2007-2008. We find that the US labor wedge, US efficiency wedge and US investment wedge accounts for the large fall of Canada output and labor with a role for Canada investment wedge in the decline of Canada investment during this period. Concerning the accounting of US economic downturn during this period, it turns out that the US efficiency wedge, US labor wedge and US investment explain the main movements of US output and

[1] anomalies that occur when models predictions or results are different from facts observed in data

US investment while Canada investment wedge plays a role in the decline of US employment.

Our quantitative findings suggest that the financial frictions manifest themselves not only as investment wedge but also as efficiency wedge and labor wedge. US investment wedge, US labor wedge and Us efficiency wedge account mainly for the downturns in both US and Canada. This may imply that the Great Recession has started in US and then propagated to Canada. The trade relation between the two countries is probably the channel of this transmission of business cycle. Indeed, as Canada investment wedge plays a prominent role in the decline of its investment during this period, we guess that the financial shock occurs in US has created some investment distortions in Canada.

The quantitative results also indicate the kind of question our method addressed. One may want to know how much Canada output would fluctuate if the only wedge that fluctuate is the US efficiency wedge. How methodology allow then the study of impacts of foreign shocks on a particular economy. Those topics could not be study using the Business Cycle Accounting of (Chari et al., 2007) as it was designed for closed economy and all interactions with the rest of the world in assess with government wedge.

Our methodology intends to shed light on the mechanisms of business cycle synchronization across countries. It is a diagnostic tool of business cycles of a particular economy. Indeed, this method acts as prism on the light. It distinguishes not only the shocks that affect the aggregate variables fluctuations but also the origin of those shocks (in terms of home shocks or foreign shocks). The method helps then in the study of shocks propagation. The method could also provide insights to researchers in the study of international business cycles. Especially, it can help solving some puzzles in international economics. There exist multiple puzzles in International Macroeconomics: (Obstfeld and Rogoff, 2000) have documented six of them, (Kose and Yi, 2001) documented trade comovement puzzle.

In terms of methodology, the most closely related works to ours are Chari et al. (2007) and Brinca et al. (2016). While the two works used the same method, they differ in application. The former apply the method to US great depression (1929-1939) and the recession of

982 while the later compare the recession of 1982 and the great recession (2008) in OECD countries. However, that paper methodology is designed for a closed economy, thus it differs from mine. In our methodology, we allow a description and a quantification of the relations between countries while (Chari et al., 2007) method summarize all the interactions of a country with the rest of the world in government wedge. Our work is also related to existing literature that study the mechanisms of business cycle transmission across countries. Among those works, Backus et al. (1992), Kose and Yi (2001), Kose and Yi (2006),...

After the current introductory section 1, the rest of paper is organized as follow. In section 2 we present the benchmark prototype model. In section 3 we describe the accounting procedure of our method. The section 4 is dedicated to the description of the application of method and the findings. After that we make a discussion around the results found in section 5. In section 6 we summarize our findings and speculate some directions for further works.

2 Description of the Benchmark Prototype Model

For the matter of simplification, we start by a perfect foresight economy. In this economy we shutdown the role of expectations of agents. In the same order of simplification, we consider a worldwide economy of two countries. Then in our accounting procedure we use a prototype model which is an international growth model with exogenous variables. This economy has six exogenous variables for each country: the *efficiency wedge* A_t , the *labor wedge* $1-\tau_{lt}$, the *investment wedge* $1/(1+\tau_{xt})$, the *government wedge* g_t , the *preference wedge* $1/(1+\tau_{ct})$ and the *asset wedge* $1-\tau_{bt}$.

In this worldwide economy, the consumers of each country we maximize their lifetime utility over per capita home consumption good c^i_{ht}, per capita foreign consumption good c^i_{ft}

and per capita labor l_t^i,

$$\sum_{t=0}^{\infty} \beta^t U(c_{ht}^i, c_{ft}^i, 1 - l_t^i) N_t^i$$

subject to the budget constraint

$$p_t^i c_{ht}^i + p_t^j (1 + \tau_{ct}^i) c_{jt}^i + (1 + \tau_{xt}^i) p_t^i x_t^i + \frac{N_{t+1}^i}{N_t^i} b_{t+1}^i = (1 - \tau_{lt}^i) w_t^i l_t^i + r_t^i k_t^i + (1 + r_t(1 - \tau_{bt}^i)) b_t^i + T r_t^i \tag{1}$$

and the capital law of accumulation

$$\frac{N_{t+1}^i}{N_t^i} k_{t+1}^i = ((1 - \delta) k_t^i + x_t^i) \tag{2}$$

Where for the variable Z_j^i the subscript j indicate the origin country and the subscript we the destination country. k_t^i denotes the per capita capital stock; x_t^i per capita investment, b_t^i per capita foreign asset, w_t^i the wage rate, r_t^i the rental rate on local capital, r_t the rental rate on foreign asset, β the discount factor, δ the depreciation rate of capital, N_t^i the population with $1 + \gamma_n$ growth rate and T_t^i per capita lump sum transfers.

In the production size, the technology used is $A_t^i F(k_t^i, (1 + \gamma)^t l_t^i)$ where $1 + \gamma$ is the rate of labor-augmented technical progress assumed constant. Firms maximize profits given by $A_t^i F(k_t^i, (1 + \gamma)^t l_t^i) - w_t^i l_t^i - r_t^i k_t^i$.

An *equilibrium of the prototype economy* consists of, given wedges (A_t, τ_{lt}, τ_{xt},g_t, τ_{ct} and τ_{bt}), allocation $(c_{ht}, c_{ft}, l_t, k_t, b_t)$ and prices (r_t, rf_t, w_t, p_t) for each country such as these allocations are optimal for consumers and firms and all the market cleared. The goods markets clearance is given by resource constraints: $c_{ht}^i + c_{ft}^j + x_t^i + g_t^i = y_t^i$ for each country i. The asset market clearance states that a lending for a country should equate the borrowing for the other country each period: $b_t^h + b_t^f$.

Also notice that on the point of view of consumers invests in capital is equivalent to

lending on the international market. Based on this remark on could think that the capital and the foreign asset are redundant. But in our economy, this is not the case for two reasons. First, the presence of foreign asset allows the consumer to invest in the local capital and be able to borrow from the abroad. Second, in the point of view of the economy (a social planner for example), the foreign asset plays the role of international finance. In that sens, we a country face a bad shock it could borrow from abroad and in the good time it will payback the debt. The following proposition states the role of foreign in our prototype economy.

Proposition 1 :

When the foreign asset market is nonexistent i.e $b_t^h = b_t^f = 0$, in equilibrium, the trade is balanced each period and there is a lack of international finance. In our framework, there exist then a period that a country incurs a debt from abroad.

Proof :(See appendix C.1)

In our prototype model, the efficiency wedge resembles a time varying technology parameter, the labor, investment, preference, and foreign asset wedges look like tax rates on labor income, investment, foreign consumption good and foreign asset. Those wedges mentioned include more than taxes. Indeed, the labor wedge capture all frictions that induce a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor. The preference wedge measures the gap between the home consumption-foreign consumption marginal rate of substitution through two partner countries. The investment wedge and foreign asset wedge capture all frictions that distort the inter temporal margins. Then the wedges represent the hole distortion to the equilibrium condition of the model: those coming from demand and offer side on all markets.

Notice that we could consider other models where we could change the different wedges but those will capture the same features present in the model, we previous describe. For example, we could add a wedge on the capital, but this will capture the same distortion as the investment wedge.

3 The accounting procedure

The accounting procedure consists first in measuring the different wedges and second evaluating the contribution of those wedges (one at once and by combination).

Measuring the wedges

For this step we use aggregate variable and compute the different wedges using some equations derived from the prototype equilibrium conditions. The wedges are then measured using the following equations $\forall we = h, f$:

$$y_t^i = A_t^i F(k_t^i, (1+\gamma)^t l_t^i) \tag{3}$$

$$u_{c_f t}^i(.) = u_{c_h t}^i(.)(1+\tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{4}$$

$$u_{lt}^i(.) = -u_{c_h t}^i(.)(1-\tau_{lt}^i)(1+\gamma)^t F_{lt}^i \tag{5}$$

$$u_{c_h t}^i(.)(1+\tau_{xt}^i) = \beta^i u_{c_h t+1}^i(.)(F_{kt+1}^i + (1-\delta)(1+\tau_{xt+1}^i)) \tag{6}$$

$$\beta^i \frac{u_{c_h t}^i}{u_{c_h t+1}^i}\frac{p_{t+1}^i}{p_t^i} = \beta^j \frac{u_{c_h t}^j}{u_{c_h t+1}^j}\frac{p_{t+1}^j}{p_t^j} \tag{7}$$

$$c_{ht}^i + c_{ft}^j + x_t^i + g_t^i = y_t^i \tag{8}$$

Where, from here, H_{zt} denotes the derivative of the function with respect to its argument z.

Measuring the contribution of wedges

For this step we conduct different experiments to isolate the marginal effect of some

wedges. For that purpose, we make some wedges fluctuate and shut down the fluctuation of the remaining wedges by setting their values to be constant. For example to evaluate the contribution of country we efficiency wedge, we make this wedge (A_t^i) fluctuates and set the other wedges constants ($\omega_t^i = \omega_1^i$, where ω stands for all of the other wedges in the model) .The insight behind this approach is to compute the aggregate variables by insuring the economy is in equilibrium. The proposition states the conditions of an equilibrium allocation.

Proposition 2

Given $\tau_{ct}^i, \tau_{lt}^i, \tau_{bt}^i, \tau_{xt}^i, i = h, f$, an equilibrium allocation solves the following equations:

$$u_{lt}^i(.) = -u_{c_h t}^i(.)(1 - \tau_{lt}^i)F_{c_f t}^i(.) \tag{9}$$

$$u_{c_h t}^i(.)(1 + \tau_{xt}^i) = \beta u_{c_h t+1}^i(.)\left(F_{c_h t+1}^i(.) + (1 - \delta)(1 + \tau_{xt+1}^i)\right) \tag{10}$$

$$c_{ht}^i + c_{ft}^j + k_{t+1}^i + g_t^i = F(k_t^i, z_t^i l_t^i) + (1 - \delta)k_t^i \tag{11}$$

$$u_{lt}^j(.) = -u_{c_h t}^j(.)(1 - \tau_{lt}^i)F_{c_f t}^j(.) \tag{12}$$

$$u_{c_h t}^j(.)(1 + \tau_{xt}^j) = \beta u_{c_h t+1}^j(.)\left(F_{c_h t+1}^j(.) + (1 - \delta)(1 + \tau_{xt+1}^j)\right) \tag{13}$$

$$c_{ht}^j + c_{ft}^i + k_{t+1}^j + g_t^j = F(k_t^j, z_t^j l_t^j) + (1 - \delta)k_t^j \tag{14}$$

$$\frac{u_{c_f t}^i(.)}{u_{c_h t}^i(.)(1 + \tau_{ct}^i)} = \frac{u_{c_h t}^j(.)(1 + \tau_{ct}^j)}{u_{c_f t}^j(.)} \tag{15}$$

$$\frac{1}{(1 - \tau_{bt+1}^i)}\left(\frac{1}{1 + \tau_{xt}^i}(F_{kt+1}^i + (1 - \delta)(1 + \tau_{xt+1}^i)) - 1\right) = \tag{16}$$

$$\frac{1}{(1-\tau^j_{bt+1})}(\frac{1}{1+\tau^j_{xt}}\frac{u^i_{c_f t+1}(.)}{u^i_{c_h t+1}(.)(1+\tau^i_{ct+1})}\frac{u^i_{c_h t}(.)(1+\tau^i_{ct})}{u^i_{c_f t}(.)}(F^j_{kt+1}+(1-\delta)(1+\tau^j_{xt+1}))-1)$$

Proof:(See appendix C.2)

4 Applying the accounting procedure

In this section we explain our accounting procedure apply to Great Recession of 2007-2008 synchronization for Canada and US.

4.1 Calibration procedure

For the application we use common functional forms in business cycle literature. We opt for a Cobb Douglas form $F(k,l) = k^\alpha l^{1-\alpha}$ for the production function and for the utility function the form $U(c,l) = log(c) + \psi log(1-l)$ with an Armington aggregation for the consumption $\left(c_h^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. The parameters we use are also familiar to business cycle literature. We choose the capital share α to be one-third and the time allocation parameter $\psi = 2.5$. The Armington aggregator coefficient we use in our benchmark model is 2 for the two countries. We then vary this coefficient for the robustness check. We choose the depreciation rate δ, the discount factor β so that, on an annualized basis, depreciation is 5% and the rate of time preference 2.5%. We use the data to compute a country-specific growth of population and technology.

Our benchmark model is an open economy of two countries. For the application of our accounting procedure we choose Canada and US. We collect the aggregate variables need using OECD database. We assume in our model that the households have assess to home consumption good and a foreign consumption good, but the aggregate data does not distinguish them; for each country we have the aggregate consumption good. To overcome this issue, the consider as foreign consumption good, the total import from the partner country. Then the home consumption good is the aggregate consumption good minus the

foreign consumption good. As the world economy is more than two countries and each country trade not only with each other, we let the government wedge be the net export of all other trade partner except Canada and US in the data. Using the same rationale as Chari et al. (2004), we prove that an open economy of $n \geq 3$ is equivalent to an open economy of 2 countries with government wedges for each country.

In order to reconcile the theory and the data, we use per capita variable deflated by GDP deflator. Indeed, our model is a representative agent model, thus using per capita variables both in the model and data make the approximation realistic. The data we use for the measurement of the wedge and the counterfactual experiments are quarterly data for 2000:1 to 2014:4. Those data come mainly from the OECD. In order to estimate the share of foreign consumption good in the aggregate consumption good, we use the import of counterpart partner country using the DOTS (Direction Of Trade Statistics) database of IMF.

4.2 Findings

In this section, we present the results of the accounting procedure performed on US and Canada. We focus on the description of the 2007-2008 crisis and the accounting of this business cycle.

I begin by describing the 2007-2008 crisis; this description is summarized in table A.1. In figure A.1, we see that from 2008:1 to 2009:3, output fell about 3% while investment fell about 12% and labor fell about 6% in Canada. In US, as shown by the figure A.2, output declined about 4% while investment and labor declined respectively about 21% and 9% from 2008:1 to 2009:3.

I then analyze the business accounting results. In table A.2, we see that from 2008:1 to 2009:3, Canada investment wedge fell about 7%, Canada labor wedge fell about 14% and Canada foreign asset wedge worsen dramatically about 77%. While from the same table 1, during this period, US preference wedge declined about 4%, US labor wedge fell about

9% while US investment wedge and US foreign asset wedge dropped respectively about 15% and 16%. As both US wedges and Canada wedges can affect the aggregates variables in both countries, we cannot surely draw a pattern from this analysis. However, we expect that Canada labor wedge, investment wedge, foreign asset wedge and US preference wedge, investment wedge, labor wedge, foreign asset wedge account for the declines of aggregates variables in both countries.

To better assess the contribution of each wedge in the fluctuation of the aggregates we turned on the analysis of wedge contribution experiments. For that purpose we rely on a the ϕ statistic proposed by Brinca et al. (2016). This statistic is the inverse of mean-square error of each wedge. It captures how closed is a component of a variable due a particular wedge to the observe variable in the data. For example, Canada output (Y^{ca}) ϕ statistic due to wedge (m) (Δ_z^{ca}) is compute as follow: $\phi_m^{Y^{ca}} = \dfrac{1/\sum_t \left(Y_t^{ca} - Y_m^{ca}\right)^2}{\sum_j \left[1/\sum_t \left(Y_t^{ca} - Y_j^{ca}\right)^2\right]}$, where $m = (\Delta_{zt}^i, \Delta_{lt}^i, \Delta_{xt}^i, \Delta_{gt}^i, \Delta_{pt}^i, \Delta_{bt}^i)_{(i=Ca,US)}$. We compute similar statics for both countries and for investment and labor. Note that the statistic lies in $[0, 1]$, sums to 1 for all the twelve wedges. More it is closed one more the wedge contributes to the fluctuation of the variable. Thus, when a particular output component fits perfectly, if $y_t - y_{mt} = 0$ for all t, then $\phi_m^Y = 1$.

The Table B.1 summaries the ϕ statistics compute for the period 2007:4-2009:4 for the counterfactual experiments we make. First, the declines of Canada output during this crisis is mainly due to US Labor wedge and US efficiency wedge. Indeed, the ϕ statistic indicates that the US labor wedge accounts for about 26% of the output declines while the US efficiency accounts for about 14% of that declines. The US output movements during this period, however, is due mainly to US investment wedge and US labor wedge as they account respectively for about 27% and 16%. Second, Canada investment fluctuation during this crisis is mainly caused by the Canada investment wedge and US investment with a contribution of about 40% and 21%. In the same time, US investment wedge and US preference wedge account respectively for about 29% and 9% of the fall observed in US investment during the

crisis. Finally, we compute the ϕ statistics to account for the fluctuations of Employment during the Great Recession. It comes out that US efficiency wedge and US labor wedge account for around 19% and 13% of the fall of Canada labor. The decline of US labor is however due to Canada investment wedge (27%) and US investment wedge (26%).

From those results we may conclude that the US labor wedge, efficiency wedge has played the most important role in Canada during the recession with non negligible role US investment wedge and Canada investment wedge for the fluctuation of Canada investment. During the Great Recession, the most important role in US aggregate variables were Us investment wedge, US labor wedge, US efficiency wedge with a little role for Canada investment for the labor decline.

5 Discussions

The theoretical results and quantitative ones are that we have presented omitted the role of expectations. In that sens the interpretation of those results has to be taken with caveats. However, those results confirm the inter-dependence of both Canada and US economies. Indeed, results showed that distortions in US economy (especially US labor wedge, US efficiency wedge, US investment wedge) affect a lot Canada economy. In the same way, distortion in Canada economy (by Canada investment wedge) have an impact on US economy. Another conclusion that arised from the results is the 2007-2008 crisis probably started in US and then propagate to Canada.

Besides, as we mentioned in the introduction Chari et al. proved that an open economy is equivalent to a prototype closed economy with government wedge. what then differs the Business Cycle Accounting proposed by Chari et al. (2007) for closed economy to mine? To answer this question we compare ϕ statistics from our work here and ϕ statistics picked from Brinca et al. (2016) for output. From the results presented in table B.2, it turns out that a particular country business cycle is due mainly to distortions of this country and

the impact of foreign country is through the government wedge. For example, Chari et al. accounting procedure imputes the main Canada output fluctuations to Canada efficiency wedge and Canada investment wedge while our accounting procedure attributes the same output movements to US labor wedge and US efficiency wedge. From this analysis we see that the International Business Cycle Accounting methodology set forth the inter-dependence between countries.

6 Conclusion and extensions

We propose in this work here a method that can give insights to researchers to better specify their quantitative models in international business cycles studies. This method focusses on an accounting procedure base on prototype. This prototype model is an of international growth model including wedges that captures all the eventual frictions and distortions of markets. For each country, we include -efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge and foreign asset wedge. Theoretically, the business co-movements across countries is allow through the trade in goods and the markets sharing include in this model.

To demonstrate our method, we apply it to study Canada and US business cycle synchronization during the Great Recession. It turns out that in both countries, during this period, the primary role in the economic downturns is devoted to US investment wedge, US labor wedge, US efficiency wedge and secondary role is devoted to Canada investment wedge. Those results suggest first that Canada and US are heavily linked and second that the crisis has started in US and has propagated to Canada.

Nevertheless, we must take our results with some caveats because in this framework we omitted the role of expectations. Adding expectations could impact the measurements of wedges in the sense that it explicitly linked all of them. The expectations are also important in agent investment decision making. Thus, a first extension of this work is to add expec-

tations to this current model. A second extension would be to prove the equivalence result of the method. This step with help to classify some existing international macroeconomics models a prototype model with some specific wedges. A further direction of this work could be an investigation of a puzzle, let say trade co-movement puzzle. In practice it will consist of applying our methodology and build a detailed model from the insights of the results and see if it helps solving the puzzle.

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A Figure A: 2007-2008 crisis Description

Figure A.1 – 2007-2008 crisis Description for Canada

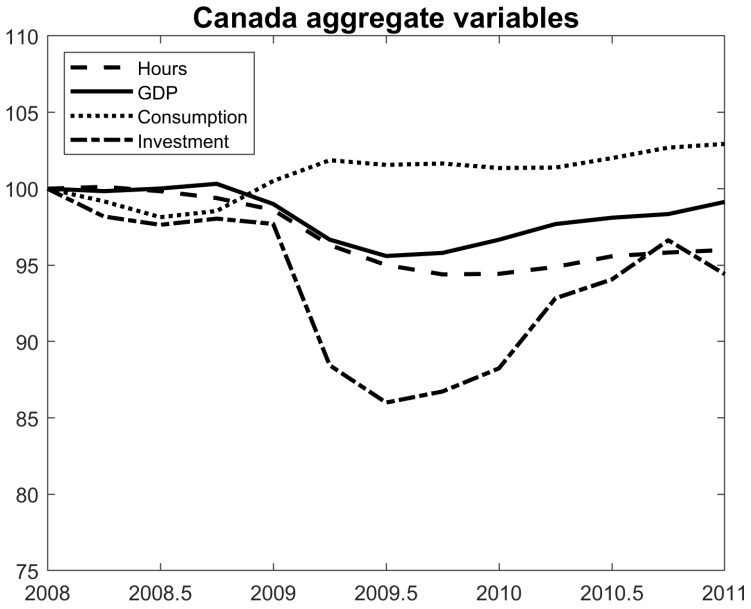
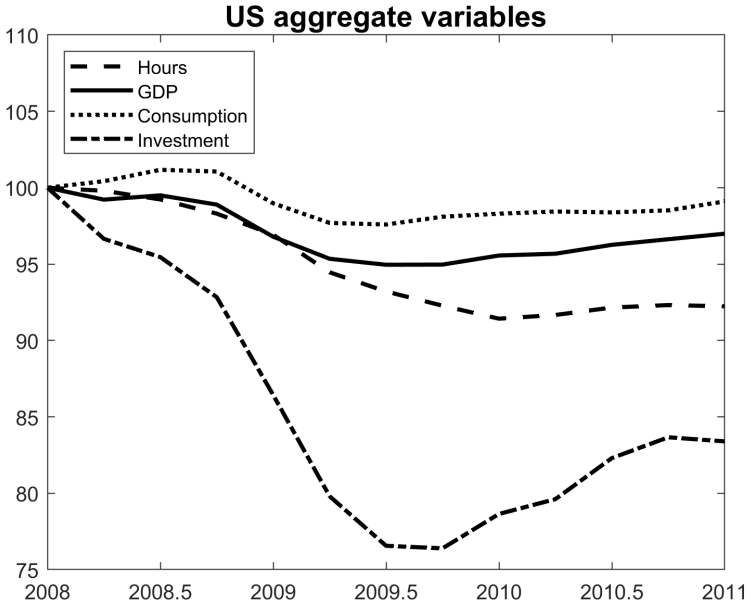


Figure A.2 – 2007-2008 crisis Description for US



A Table A: 2007-2008 crisis Description

Table A.1 – Changing in aggregates during the crisis

			Change of aggregates in %		
	Peak	Through	***Y***	***X***	***L***
Canada	2007Q4	2009Q4	-3.34	-11.76	-5.57
US			-4.44	-21.35	-8.57

Table A.2 – Changing in wedges from 2007:4 to 2009:4

	Change of wedges in %					
	Z	***L***	***X***	***G***	***P***	***B***
Canada	0.32	-14.22	-7.24	8.01	14.38	-76.94
US	2.00	-9.11	-15.39	8.83	-3.79	-15.85

B Table B: Summary statistics

Table B.1 – ϕ statistics in % for 2007:4 to 2009:4

	Canda Wedges						**US Wedges**					
	Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
Y Ca	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
X Ca	1.70	3.80	40.02	1.26	2.16	3.72	12.64	4.32	20.48	1.36	4.73	3.80
X US	6.79	8.94	8.36	9.17	5.73	5.97	4.09	2.30	28.96	1.57	9.42	8.70
L Ca	0.76	2.01	10.90	1.13	5.44	7.24	18.52	12.99	5.33	21.33	10.64	3.71
L US	2.70	0.34	26.50	1.89	0.46	8.48	1.80	18.79	25.58	4.00	5.09	4.38

Table B.2 – ϕ statistics comparison for IBCA and BCA

		Canda Wedges						**US Wedges**					
		Z	***L***	***X***	***G***	***C***	***B***	***Z***	***L***	***X***	***G***	***C***	***B***
IBCA	**Y Ca**	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
	Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
BCA	**Y Ca**	49.00	13.00	18.00	20.00	-	-	-	-	-	-	-	-
	Y US	-	-	-	-	-	-	16.00	46.00	32.00	6.00	-	-

A Data treatment

B Proof of equivalence result

B.1 Equivalent between an open economy with n countries with an open economy with two countries

C Proof of propositions

C.1 Proof of proposition 1

Without assets trading in the world economy, the net export equals zero every period. We know that the net export is:

$$(X - M)_t^i = p_t^i(y_t^i - c_{ht}^i - x_t^i) - p_t^j c_{ft}^i$$

As the firms are in competitive market, their profit is zero every period such that

$$p_t^i y_t^i = w_t^i l_t^i + r_t^i k_t^i$$

Then by replacing the firm revenue into the net export equation we get:

$$(X - M)_t^i = w_t^i l_t^i + r_t^i k_t^i - p_t^i(c_{ht}^i + x_t^i) - p_t^j c_{ft}^i$$

We also know that the government wedge is equal to the transfers to household state by the following equation:

$$tr_t^i = g_t^i = p_t^j \tau_{ct}^i c_{ft}^i + p_t^i \tau_{xt}^i x_t^i + \tau_{lt}^i w_t^i l_t^i \tag{17}$$

Then using the budget constraint 1 (with $b_t^i = 0$) and 17, we obtain $(X - M)_t^i = 0$.

C.2 Proof of proposition 2

Let assume $\mathcal{A}^i = \left(c^i_{ht}, c^i_{ft}, x^i_t, l^i_t, b^i_t\right)_{t=0}^{\infty}$ i=(h,f) solve equations in proposition 2 and let prove there exist price $\mathcal{P}^i = \left(p^i_t, w^i_t, r^i_t, r_t\right)_{t=0}^{\infty}$ such that $\mathcal{A}^i$ and $\mathcal{P}^i$ is a competitive equilibrium. The proof is straightforward when we make the following assumptions:

- Normalize a price: $\forall t, p^h_t = 1$
- Compute $p^f_t = \dfrac{u^h_{2t}(.)}{u^h_{1t}(.)(1+\tau^h_{ct})}$
- Compute $w^i_t = p^i_t F^i_{2t}(.)$
- Compute $r^i_t = p^i_t F_{kt}(.)$
- Using non arbitrage condition, compute $r_{t+1} = \dfrac{1}{(1-\tau^i_{bst+1})}(\dfrac{1}{1+\tau^i_{xt}}\dfrac{p^i_{t+1}}{p^i_t}(F_{kt+1} + (1-\delta)(1+\tau^i_{xt+1})) - 1)$

Indeed ,

- From the wage rate and interest rate equations, firms optimize as shown by equations 18 and 19;
- From equations 11 and 14 of proposition 2, resource constraints are satisfied;
- Combining the remind equations and prices we get the FOC of households problems (equations 20 to 23 for each country i) ;
- The budgets constraints (equations 1 for each country i) are satisfied by using them to compute the assets variables.

First order conditions of firms optimization problems

$$w^i_t = p^i_t F_{lt}(.) \tag{18}$$

$$r_t^i = p_t^i F_{kt}(.) \tag{19}$$

First order conditions of household optimization problems

Using Lagrangian procedure we get the following equations:

$$u_{c_f t}(.) = u_{c_h t}(.)(1 + \tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{20}$$

$$u_{lt}(.) = -u_{c_h t}(.)(1 - \tau_{lt}^i)\frac{w_t^i}{p_t^i} \tag{21}$$

$$u_{c_h t}(.)\frac{1}{p_t^i} = \beta u_{c_h t+1}(.)(1 + (1 - \tau_{bt+1}^i)r_{t+1}))\frac{1}{p_{t+1}^i} \tag{22}$$

$$u_{c_h t}(.)(1 + \tau_{xt}^i) = \beta u_{c_h t+1}(.)(\frac{r_{t+1}^i}{p_{t+1}^i} + (1 - \delta)(1 + \tau_{xt+1}^i)) \tag{23}$$