$$inorder(t) = reverse\left(inorder\left(mirror(t)\right)\right)$$

Cazul de baza: $t = leaf$

$$inorder(leaf) = reverse\left(inorder\left(mirror(leaf)\right)\right) \Leftrightarrow$$

$$[] = reverse\left(inorder(leaf)\right) = reverse([]) = []$$

Ipoteza de inductie: Presupunem ca

$$inorder(t_1) = reverse\left(inorder\left(mirror(t_1)\right)\right) (II1)$$

$$inorder(t_2) = reverse\left(inorder\left(mirror(t_2)\right)\right) (II2)$$

Pasul de inductie: $t = node(t_1, a, t_2)$

Prelucrez separat membrul stang si respectiv drept al egalitatii de demonstrat.

$$MS = inorder\left(node(t_1, a, t_2)\right) = append\left(inorder(t_1), a: inorder(t_2)\right)$$

$$MD = reverse\left(inorder\left(mirror\left(node(t_1, a, t_2)\right)\right)\right) =$$

$$reverse\left(inorder\left(node\left(mirror(t_2), a, mirror(t_1)\right)\right)\right) =$$

$$reverse\left(append\left(inorder\left(mirror(t_2)\right), a: inorder\left(mirror(t_1)\right)\right)\right)$$

Pentru membrul stang, aplic ipotezele de inductie (II1 si II2):

$$MS = append\left(reverse\left(inorder\left(mirror(t_1)\right)\right), a: reverse\left(inorder\left(mirror(t_2)\right)\right)\right)$$

Notez $L_1 = inorder\left(mirror(t_1)\right)$ si $L_2 = inorder\left(mirror(t_2)\right)$ si observam ca MS si MD devin:

$$MS = append\left(reverse(L_1), a: reverse(L_2)\right)$$

$$MD = reverse\left(append(L_2, a: L_1)\right)$$

Aratam ca MS = MD, tot prin inductie structurala. Observam ca ajuns la o proprietate care se refera doar la liste (am scapat de arbori).

Pentru aceasta noua proprietate, facem inductie dupa L_2. Am ales L_2 deoarece la inductia pentru L_1, in cazul de baza $L_1 = []$ ajungeam la o noua proprietate de demonstrat (tot prin inductie), in care era implicat doar L_2, ceea ce mi-a sugerat ca e mai simplu sa fac inductia dupa L_2 de la inceput. Este posibil sa iasa si inductia dupa L_1 aproximativ la fel de usor.

Cazul de baza: $L_2 = []$. Avem de aratat:

$$append(reverse(L_1), a{:}[]) = reverse(append([], a{:}L_1)) \Leftrightarrow$$

$$append(reverse(L_1), a{:}[]) = reverse(a{:}L_1) = append(reverse(L_1), a{:}[])$$

Ipoteza de inductie (II): $append\big(reverse(L_1), a{:}reverse(L_2)\big) = \ reverse\big(append(L_2, a{:}L_1)\big)$

Pasul de inductie: $L_2 = b : L_2$

$$MS = append\big(reverse(L_1), a{:}reverse(b{:}L_2)\big)$$

Folosesc proprietatea $a{:}reverse(L) = reverse(append(L), a{:}[])$ (se demonstreaza tot prin inductie, e destul de usor). Mentionez ca aceasta a fost proprietatea la care am ajuns cand am incercat sa fac inductie dupa L_1 in loc de L_2, de aceea stiam ca e adevarata si ca o pot folosi. MS devine, aplicand proprietatea pentru $L = b{:}L_2$ in al doilea membru al parantezei:

$$MS = append\left(reverse(L_1), reverse\big(append(b{:}L_2, a{:}[])\big)\right) =$$

$$append\left(reverse(L_1), reverse\big(b{:}append(L_2, a{:}[])\big)\right) =$$

$$append\big(reverse(L_1), append(reverse(L_2, a{:}[]), b{:}[])\big)$$

Acum folosesc proprietatea suplimentara data in enunt, cu $l_1 = \ reverse(L_1)$, $l_2 = \ reverse(L_2, a{:}[])$, $l_3 = b{:}[]$.

$$MS = append\Big(append\big(reverse(L_1), reverse(L_2, a{:}[])\big), b{:}[]\Big)$$

Acum folosesc proprietatea $reverse(L_2, a{:}[]) = a{:}reverse(L_2)$. Folosesc aceasta proprietate pentru a transforma MS in asa fel incat sa pot aplica ipoteza de inductie.

$$MS = append\Big(append\big(reverse(L_1), a{:}reverse(L_2)\big), b{:}[]\Big)$$

Acum aplic II si obtin:

$$MS = append\Big(reverse\big(append(L_2, a{:}L_1)\big), b{:}[]\Big)$$

Manipulez acum MD:

$$MD = reverse\big(append(b{:}L_2, a{:}L_1)\big) = reverse\big(b{:}append(L_2, a{:}L_1)\big) =$$

$$append(reverse\big(append(L_2, a{:}L_1)\big), b{:}[])$$

Observam ca am ajuns la acelasi lucru ca si in cazul MS, ceea ce incheie rezolvarea.