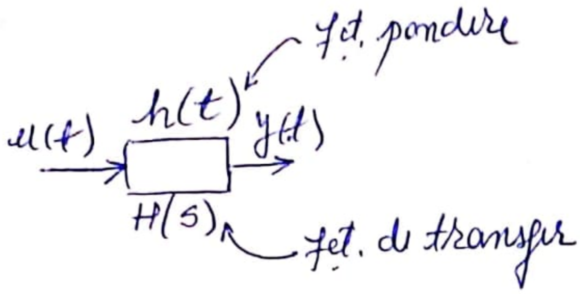
SEMINAR 1 TS

2.10.2019



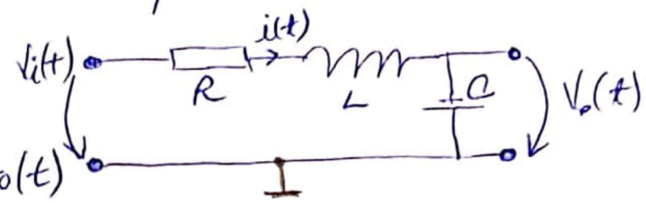
$\mathcal{L}\{h\} = H(s)$

$y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$

$\mathcal{L}\{h * u\} = H(s)U(s)$

$Y = HU$

P. 1.12 → LIVS



a) dep. în timp dintre $v_i(t)$ și $i(t)$, iar apoi dintre $i(t)$ și $v_o(t)$

$v_i(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau$ $\Big/ \mathcal{L}$

$v_o(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$

b.) det. fct. de transfer de la $V_i(s)$ la $V_o(s)$

$V_i(s) = Ri(s) + L(si(s) - i(0)) + \frac{1}{C}\cdot\frac{1}{s}\, i(s)$

$\mathcal{L}\left\{\frac{d^n x}{dt^n}\right\} = s^{(n)}X(s) - X^{(n-1)}(0) - sX^{(n-2)}(0) - \ldots - s^{n-1}X(0)$

$V_o(s) = \frac{1}{sC}\, i(s)$

$H(s) = \frac{V_o(s)}{V_i(s)}\Bigg|_{i(0)=0} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} \longrightarrow H(s) = \underbrace{\frac{1}{LCs^2 + RCs + 1}}_{\text{filtru trece-jos}}$

c) Scrieți sistemul sub forma $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

x → stare sist.
u → intrare
y → măs

$v_i(t) = Ri(t) + L\dot{i} + \frac{1}{C}\int i$

$\underbrace{v_o(t)}_{y} = \frac{1}{C}\underbrace{\int i}_{x}$

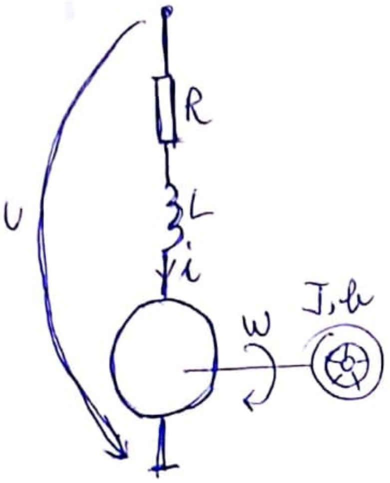
$i = \dot{x}_1 = x_2$

$\dot{i} = \ddot{x}_1 = \dot{x}_2$

$\begin{cases} u = Rx_2 + L\dot{x}_2 + \frac{1}{C}x_1 \\ y = \frac{1}{C}x_1 \\ \dot{x}_1 = x_2 \end{cases} \leftrightarrow \begin{cases} L\dot{x}_2 = \frac{1}{C}x_1 + Rx_2 - u \;/ :(-L) \\ \dot{x}_1 = 0\cdot x_1 + 1\cdot x_2 + 0\cdot u \\ y = \frac{1}{C}x_1 + 0\cdot x_2 + 0\cdot u \end{cases} \leftrightarrow \begin{cases} \dot{x}_1 = 0\cdot x_1 + 1\cdot x_2 + 0\cdot u \\ \dot{x}_2 = \frac{-1}{LC} - \frac{R}{L}x_2 + \frac{1}{L}u \\ y = \frac{1}{C}x_1 + 0\cdot x_2 + 0\cdot u \end{cases}$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix}u \qquad \dot{y} = \begin{bmatrix} \frac{1}{C} & 0 \end{bmatrix}x + \begin{bmatrix} 0 \end{bmatrix}u$

P. 1.13 → L1V5



$$\begin{cases} U - iR - L\frac{di}{dt} = K\omega \\ \mu - \mu_J - \mu_S = J\cdot\underbrace{\frac{d\omega}{dt}}_{\dot\omega} \\ \mu = K_m \cdot i \\ \mu_J = b\omega \end{cases}$$

a) identificați mărimile de intrare (cond și perturb) iar apoi mărimile de ieșire (μ S)

b) scrieți un model de tipul $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

c) Determinați fct. de transfer de la U la ω

a) cond - U   perturbație - μ_S măs - ω

b) $u - U$ $y - \omega$ $d - \mu_S$ $x - ?$ → $x_1 = i$

$x_2 = \omega = y \,/\mathcal{L} \Rightarrow Y(s) = \Omega(s)$

$$\begin{cases} u - x_1 R - L\dot{x}_1 = K x_2 \\ K_m x_1 - b x_2 - d = J\dot{x}_2 \\ y = 0\cdot x_1 + 1\cdot x_2 + 0\cdot u + 0\cdot d \end{cases} \rightarrow u \rightarrow \begin{bmatrix} U \\ \mu_S \end{bmatrix}$$

$$\begin{cases} L\dot{x}_1 = -Rx_1 - Kx_2 + 1\cdot u + 0\cdot d \;/ :L \\ J\dot{x}_2 = K_m x_1 - b x_2 + 0\cdot u - 1\cdot d \;/ :J \\ y = 0\cdot x_1 + 1\cdot x_2 + 0\cdot u + 0\cdot d \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -R/L & -K/L \\ K_m/J & -b/J \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & -1/J \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

SEMINAR 2
TS

16.10.2019

Transformate Laplace și Z

$$y(t) \xrightarrow{\mathcal{L}\{\}} Y(s) \qquad \int_{-\infty}^{\infty} y(t)\, e^{-st}\, dt \quad \nearrow 1(t)$$

$$y(n) \xrightarrow{\mathcal{Z}\{\}} Y(z) \qquad \sum_{k=-\infty}^{\infty} y(k) z^{-k} \quad \searrow 1[n]$$

$$u(t) \rightarrow \boxed{h(t)} \xrightarrow{y(t)}$$

$$y = h * u \xrightarrow{\mathcal{L}} \mathcal{L}\{y\} = \mathcal{L}\{h\}\mathcal{L}\{u\}$$

$$\mathcal{L}\{y^{(n)}\} = s^n Y(s) - \sum_{i=0}^{n-1} s^i y^{(n-1-i)}(0)$$

$$\mathcal{Z}\{y[n+k]\} = z^k Y(z) - z^k \sum_{i=0}^{k-1} y[i] z^{-i}$$

$$\mathcal{L}\{\int_0^t y(\tau)\, d\tau = \frac{1}{s} Y(s)$$

$$\mathcal{Z}\{y[n-k]\} = z^{-k} Y(z)$$

P2.10 → L2V7

a) $\ddot{y} + \dot{y} - 2y = 0 \qquad y(0) = 1 \qquad \dot{y}(0) = 1$

$$\ddot{y} + \dot{y} - 2y = 0 \; / \mathcal{L}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+\alpha}\right\} = e^{-\alpha t} 1(t)$$

$$\mathcal{L}\{\ddot{y}\} = s^2 Y(s) - s y(0) - \dot{y}(0)$$

$$\mathcal{L}\{\dot{y}\} = s Y(s) - y(0)$$

$$s^2 Y(s) - s y(0) - \dot{y}(0) + s Y(s) - y(0) - 2Y(s) = 0$$

$$Y(s)(s^2 + s - 2) = s + 1 + 1$$

$$Y(s) = \frac{s+2}{s^2+s-2} = \frac{\cancel{s+2}}{(s-1)\cancel{(s+2)}} = \frac{1}{s-1} \; / \mathcal{L}^{-1}\{\} \rightarrow y(t) = e^{t} 1(t)$$

$$y(0) = 1 \qquad \dot{y}(0) = 1$$

c*) $\ddot{y} + y = t\, 1(t) / \mathcal{L}\{\} \qquad y(0) = 1 \qquad \dot{y}(0) = 1$

$$s^2 Y(s) - s y(0) - \dot{y}(0) + Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^2+1) = \frac{1}{s^2} + s + 1 = \frac{s^3+s+1}{s^2}$$

$$Y(s) = \frac{s^3+s+1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s^2+1} + \frac{sD}{s^2+1}$$

$B+D=1 \Rightarrow D=1$

$A+C=1 \Rightarrow C=0$

$B=0$

$A=1$

$$y(s)=\frac{1}{s^2}+\frac{s}{s^2+1} \quad / \mathcal{L}^{-1}\{\}$$

$$y(t)=(t+\cos t)\,1(t)$$

$y(0)=1 \qquad y'(t)=(1-\sin t)1(t) \qquad y'(0)=1$

P2.5

c*) $3y[n+2]-4y[n+1]+y[n]=2\cdot 1[n+1] \qquad y[0]=0 \quad y[1]=0$

$$3\left(z^2 y(z)-z^2\left(\underbrace{y(0)}_{=0}z^0+\underbrace{y(1)}_{=0}\frac{1}{z}\right)\right)-4\left(zy(z)-z\underbrace{y(0)}_{=0}z^0\right)+y(z)=$$

$$=2z\,\mathcal{Z}\{1(n)\}-2z\cdot 1(0) \qquad \mathcal{Z}\{1[n]\}=\frac{z}{z-1}$$

$$y(z)(3z^2-4z+1)=\frac{z}{z-1}2z-2z=\frac{2z}{z-1}$$

$$y(z)=\frac{2z}{(3z^2-4z+1)(z-1)}=\frac{2z}{(z-1)^2(3z-1)}=\frac{A}{3z-1}+\frac{B}{z-1}+\frac{C}{(z-1)^2}$$

$$2z=A(z-1)^2+B(3z-1)(z-1)+C(3z-1)$$

$$2z=z^2(A+3B)+z(-2A-B-3B+3C)+A+B-C$$

$A+3B=0 \Rightarrow A=-3B$

$-2A-4B+3C=2$

$A+B-C=0 \Rightarrow -3B+B=C \Rightarrow C=-2B$

$6B-4B-6B=2 \Rightarrow B=\frac{-1}{2}$

$A=\frac{3}{2} \qquad C=1$

$$y(z)=\frac{3}{2}\cdot\frac{1}{3z-1}-\frac{1}{2}\cdot\frac{1}{z-1}+\frac{1}{(z-1)^2}=\frac{1}{z}\left(\frac{3}{2}\frac{z}{3z-1}-\frac{1}{2}\frac{z}{z-1}+\frac{z}{(z-1)^2}\right)$$

$$\frac{3}{2}\frac{z}{3z-1}=\frac{1}{2}\cdot\frac{z}{z-1/3}$$

$$y[n]=\frac{1}{2}\left(\frac{1}{3}\right)^{n-1}1[n-1]-\frac{1}{2}1[n-1]+(n-1)1[n-1]$$

$$y[1]=\frac{1}{2}\cdot 1-\frac{1}{2}\cdot 1+0=\frac{1}{2}-\frac{1}{2}=0$$

$\mathcal{Z}\{1[n]\}=\frac{z}{z-1} \qquad \mathcal{Z}\{n\cdot 1[n]\}=\frac{z}{(z-1)^2} \qquad \mathcal{Z}\{a^n 1[n]\}=\frac{z}{z-a}$

$x(z)\left(z^2-6z+9\right)-z=0$

$\mathcal{Z}\{n\,a$

$x(z)=\frac{3z}{(z-3)^2}\cdot\frac{1}{3}\ /\ \mathcal{Z}^{-1}\{\}$

$x[n]=\frac{1}{3}n\,3^n\,1[n]=n\cdot3^{n-1}\,1[n]$

$y[n]=(n+2)\,3^{n+1}\,1[n+2]$

SEMINAR 3 30.10.2019

TS

Stabilitate și răspuns

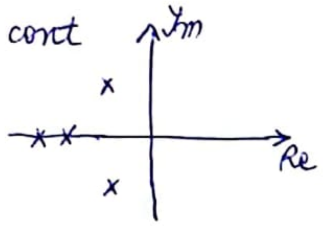
$u \rightarrow \boxed{H} \rightarrow y = h * u$

$\xrightarrow{\mathcal{L}\{\}} H(s) = \frac{B(s)}{A(s)} \rightarrow A(s) = 0 \Rightarrow s_i \xrightarrow{stab} Re(s_i) < 0$

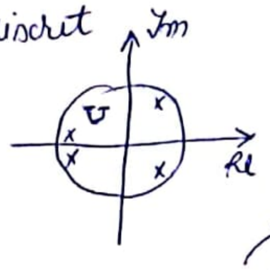
$\xrightarrow{\mathcal{Z}\{\}} H(z) = \frac{B(z)}{A(z)} \rightarrow A(z) = 0 \Rightarrow s_i \xrightarrow{stab} |s_i| < 0$

$\forall u, |u| < t < \infty \rightarrow |y| < B < \infty$

cont



discret



$y = h * u \ / \ \mathcal{L}\{\} \ \rightarrow \ y = \underbrace{H \cdot U}_{Y_F(s)} + \underbrace{Y_L(s)}_{\text{cond. inițiale}} \quad / \ \mathcal{Z}^{-1}\{\}$

Y_L → răspuns liber

$Y_F(s)$ → răspuns forțat

$y(t) = y_f(t) + y_l(t)$

$= y_{perm}(t) + y_{tranz}(t)$

Ex: $y(t) = (te^{-t} + 14 - 3\cos 2t)\,1(t)$ $\begin{cases} y_{tranz}(t) = te^{-t}\,1(t) \\ y_{perm}(t) = (14 - 3\cos 2t)\,1(t) \end{cases}$

TVF în continuu: Dacă sistemul este stabil at. $\lim\limits_{t\to\infty} y(t) = \lim\limits_{s\to 0} s\,Y(s)$

$Y(s) = H(s)\frac{1}{s}$

TVF în discret: Dacă sist. este stabil at. $\lim\limits_{n\to\infty} y[n] = \lim\limits_{z\to 1} (z-1)Y(z)$

$Y(z) = H[z]\frac{z}{z-1}$

Ex1) Se dă ec. dif: $\dot{y} + 2y = \dot{u} + 3u \quad y(0_-) = 1(t)$

a) det. fct. de transfer a procesului

b) analizați stabilitatea procesului

c) calc. răspunsul liber și forțat

d) calc. răsp. total

e) împ. în răsp. permanent și tranz.

f) $y(\infty)$

a) $\dot{y}+2y=\dot{u}+3u \;/\; \mathcal{L}\{\}$ $\qquad u * h - y$

$sY(s)-1+2Y(s)=sU(s)+3U(s)$ $\qquad H=\frac{Y}{U}$

$Y(s)(s+2)=U(s)(s+3)+1$

$H(s)=\frac{Y}{U}=\frac{s+3}{s+2}$ $\quad$ c.i. nule

b) $s_1=-2$ pd $\Rightarrow$ H stabilă

c) $\dot{y}+2y=\delta+3\cdot 1(t) \;/\; \mathcal{L}\{\}$

$sY(s)-1+2Y(s)=1+\frac{3}{s} \Rightarrow Y(s)=\frac{2+\frac{3}{s}}{s+2}=\frac{2s+3}{s(s+2)}$

$Y_F=H(s)U(s)=\frac{s+3}{s+2}\cdot\frac{1}{s}=\frac{s+3}{s(s+2)}$ $\qquad As+2A+3B=s+3$

$\qquad 2A=3 \Rightarrow A=3/2$

$Y_L=Y(s)-Y_F(s)=\frac{2s+3}{s(s+2)}-\frac{s+3}{s(s+2)}=\frac{s}{s+2}$ $\qquad B=-1/2$

$Y_F(s)=\frac{3}{2}\frac{1}{s}-\frac{1}{2}\frac{1}{s+2} \;/\; \mathcal{L}^{-1}\{\} \Rightarrow y_F(t)=\left(\frac{3}{2}-\frac{1}{2}e^{-2t}\right)1(t)$

$Y_L(s)=\frac{1}{s+2} \;/\; \mathcal{L}^{-1}\{\} \Rightarrow y_L(t)=e^{-2t}1(t)$

d) $y(t)=y_F(t)+y_L(t)=\left(\frac{3}{2}+\frac{1}{2}e^{-2t}\right)1(t)$

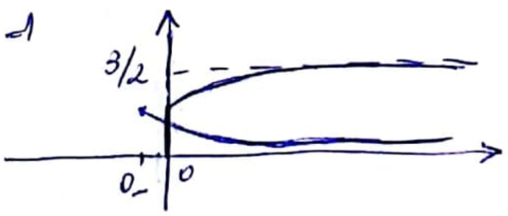
e) $y_t=\frac{1}{2}e^{-2t}\cdot 1(t)$ $\qquad y_p=\frac{3}{2}\cdot 1(t)$

f) $y(\infty)=H(0)=\frac{3}{2}$ (sist. stabil)

Cond. inițiale se verifică pe lilet!

$y(0)=2$ $\qquad y_L(0)=1$

$y(0-)=1$



$H(s)=\frac{s+2}{s+2}=1+\frac{1}{s+2}$

$Y(s)\cdot\frac{1}{s}=\frac{1}{s}+\frac{1}{s(s+2)} \;/\; \mathcal{L}^{-1}\{\} \Rightarrow y(t)=\underbrace{1(t)}_{u(t)}+\ldots$

Ex 2.) $y[n] + \frac{1}{3}y[n-1] = u[n]$ $\quad y[-1] = 1$ $\quad u[n] = 1[n]$

a) $H(z) - ?$ b) stabilitate c) y_L și y_F

d) $y[n]$ e) y_p și y_t f) $y[\infty]$

a) $y[n-1] \stackrel{\text{not}}{=} x[n]$ $\quad y[0] = 1$

$x[n+1] + \frac{1}{3}x[n] = u[n] \;/ z$

$z \cdot X(z) - zX(0) + \frac{1}{3}X(z) = U(z)$

$X(z)\left(z + \frac{1}{3}\right) = U(z) + z$

$X(z) = U(z)\frac{3}{3z+1} + \frac{3z}{3z+1}$

$Y(z) = U(z)\frac{3z}{3z+1} + \frac{3z^2}{3z+1}$

$\Rightarrow H(z) = \frac{3z}{3z+1}$

b) $s_1 = -1/3$

$|s_1| = \frac{1}{3} < 1 \Rightarrow H(z)$ stabil

d) $y[n] = 1[n] - \frac{1}{12}\left(-\frac{1}{3}\right)^{n-1}1[n-1] - \frac{1}{4}1[n-1]$
$+ \left(-\frac{1}{3}\right)^{n+1}1[n+1]$

e) $y_p[n] = 1[n] - \frac{1}{4}1[n-1]$

$y_t[n] = \frac{-1}{12}\left(-\frac{1}{3}\right)^{n-1}1[n-1] + \left(-\frac{1}{3}\right)^{n+1} \cdot 1[n+1]$

f) $y[\infty] \stackrel{TVF}{=} \lim\limits_{z\to 1}(z-1)Y(z)$

$= \lim\limits_{z\to 1}(z-1)\left(\frac{z}{z-1}\cdot\frac{3z}{3z+1} + \frac{3z^2}{3z+1}\right)$

$= \lim\limits_{z\to 1}\frac{3z^2}{3z+1} = \frac{3}{4}$

c) $Y(z) = \underbrace{U(z)\frac{3z}{3z+1}}_{Y_L(z)} + \underbrace{\frac{3z^2}{3z+1}}_{Y_L(z)}$

$Y_L(z) = \frac{3z^2}{3z+1} = z\frac{3z}{3z+1} =$

$= z\frac{z}{z+1/3} = z\frac{z}{z-(-1/3)} \quad / \mathcal{Z}^{-1}\{\}$

$\Rightarrow y[n] = \left(-\frac{1}{3}\right)^{n+1} \cdot 1[n+1]$

$Y_F(z) = \frac{3z}{3z+1}\cdot\frac{z}{z-1} = \left(1 - \frac{1}{3z+1}\right)\frac{z}{z-1} =$

$= \frac{z}{z-1} - \frac{z}{(3z+1)(z-1)} = \frac{z}{z-1} - \left[\frac{A}{3z+1} + \frac{B}{z-1}\right]$

$zA + 3zB + B = z$

$z(A+3B) + (B-A) = z$

$\begin{cases} A = B \\ A + 3B = 4A = 1 \Rightarrow A = \frac{1}{4} = B \end{cases}$

$\Rightarrow Y_F(z) = \frac{z}{z-1} - \frac{1}{4(3z+1)} - \frac{1}{4(z-1)}$

$Y_F(z) = \frac{z}{z-1} - \frac{1}{12z}\cdot\frac{z}{z+\frac{1}{3}} - \frac{1}{4z}\cdot\frac{z}{z-1}$

$\stackrel{\mathcal{Z}^{-1}\{\}}{\Longrightarrow} y_F[n] = 1[n] - \frac{1}{12}\left(-\frac{1}{3}\right)^{n-1}\cdot 1[n-1] - \frac{1}{4}1[n-1]$

Ex3) → Prob. 3.12 (L3/2)

$\ddot{y}+2\dot{y}+4y=4u \quad y(0)=0 \quad \dot{y}(0)=0 \quad u(t)=1(t)$

a) $H(s)$ b) $y(t)$ d) Calc. fct. de transfer în buclă închisă și analizați stabilitatea acesteia

a) $\ddot{y}+2\dot{y}+4y=4u \;/\; \mathcal{L}\{\}$

$s^2Y(s)-sy(0)-\dot{y}(0)+2(sY(s)-y(0))+4Y(s)=4U(s)$

$Y(s)(s^2+2s+4)=4U(s)$

$H(s)=\frac{Y(s)}{U(s)}=\frac{4}{s^2+2s+4}$

b) $Y(s)=\frac{4U(s)}{s^2+2s+4}=\frac{4}{s^3+2s^2+4s}=\frac{4}{s((s+1)^2+(\sqrt{3})^2)}=\frac{{}^{s)}A(s+1)}{(s+1)^2+(\sqrt{3})^2}+\frac{{}^{s)}B}{(s+1)^2+(\sqrt{3})^2}+\frac{{}^{()^2+()^2)}C}{s}$

$4=s^2(A+C)+s(A+B+2C)+4C$

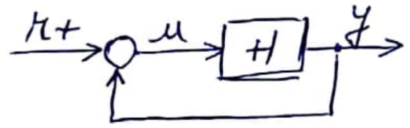
$A+C=0$

$A+B+2C=0$

$4C=4 \Rightarrow C=1 \Rightarrow A=-1 \Rightarrow B=-1$

$Y(s)=\frac{-(s+1)}{(s+1)^2+(\sqrt{3})^2}-\frac{1}{(s+1)^2+(\sqrt{3})^2}+\frac{1}{s} \;/\; \mathcal{L}^{-1}\{\}$

$y(t)=\left(-e^{-t}\cos(\sqrt{3}t)-\frac{1}{\sqrt{3}}e^{-t}\sin(\sqrt{3}t)+1\right)1(t)$

d) 

$u=r-y$

$y=H\cdot u=H(r-y) \Rightarrow y(1+H)=Hr \Rightarrow y=\frac{H}{1+H}r \Rightarrow$

$\Rightarrow T_{yr}=\frac{H}{1+H}=\frac{\frac{4}{s^2+2s+4}}{1+\frac{4}{s^2+2s+4}}=\frac{4}{s^2+2s+4}=\frac{4}{(s+1)^2+(\sqrt{3})^2} \to s_1=-1+\sqrt{7}$

$s_2=-1-\sqrt{7}$

$y=H(y+r) \Rightarrow y=\frac{H}{1-H}r=\frac{\frac{4}{s^2+2s+4}}{1-\frac{4}{s^2+2s+4}}=\frac{4}{s^2+2s}$

I Compensatorul PID

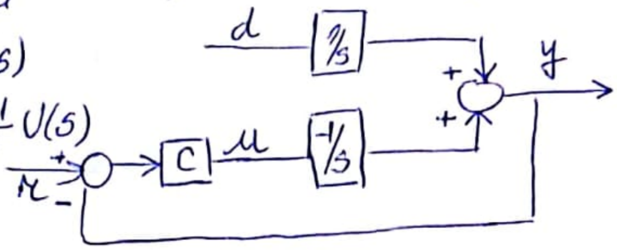
$$C(s) = k_p + \frac{K_i}{s} + K_d s$$

a) Găsiți un regulator de tip P care să asigure $\begin{cases} \sigma = 0 \\ t_t \leq 10 \text{ sec} \\ \varepsilon_{st} = 0 \end{cases}$

b) Găsiți un regulator de tip PI care să asigure aceleași condiții

$$sY(s) = D(s) - U(s)$$

$$Y(s) = \frac{1}{s} D(s) + \frac{-1}{s} U(s)$$



$$y = T_{yr} R + T_{yd} D$$

$$\left.\begin{aligned} y &= \frac{1}{s} D + \frac{-1}{s} U \\ U &= C(R-y) \end{aligned}\right\} \Rightarrow y = \frac{1}{s} D + \frac{-1}{s} C(R-y)$$

$$y = \frac{1}{s} D - \frac{1}{s} CR + \frac{1}{s} Cy$$

$$y\left(1 - \frac{1}{s} C\right) = \frac{1}{s} D + \frac{-1}{s} CR$$

$$y = \frac{1/s}{1 - 1/s\, C} D + \frac{-1/s}{1 - 1/s\, C} R = \frac{1}{s-C} D + \frac{-C}{s-C} R$$

$$T_{yr} = \frac{-C}{s-C} \qquad T_{yd} = \frac{1}{s-C}$$

a) $C(s) = k_p \Rightarrow T_{yr} = \frac{-k_p}{s-k_p} \Rightarrow p = k_p$

$\left.\begin{aligned} & \\ & T_{yd} = \frac{1}{s-k_p} \Rightarrow p = k_p \\ & Re(p) < 0 \end{aligned}\right\} \Rightarrow k_p < 0$

sistem de ord 1 $\left[\begin{aligned} & \sigma = 0 \\ & t_t \simeq 4T \leq 10 \text{ sec} \end{aligned}\right.$

$$T \leq 2.5 \text{ sec}$$

$$\frac{K}{Ts+1} \longrightarrow T = \frac{-1}{k_p} \leq 2.5 \qquad \frac{1}{k_p} \geq -2.5 \rightarrow k_p \leq \frac{-1}{2.5} = -0.4$$

$$\varepsilon_{st} = r(\infty) - y(\infty) = 1 - T_{yr}(0) = 1 - \frac{-k_p}{-k_p} = 0$$

$r(t) = 1(t)$

$d(t) = 0$

$d(t) = 1(t)$

$\varepsilon_{st} = r(\infty) - y(\infty) = 1 - T_{yr}(0) - T_{yd}(0) = 1 - 1 - \frac{-1}{k_p} = \frac{1}{k_p}$

d) $C = k_p + \frac{k_i}{s}$

$T_{yr} = \frac{-C}{s-C} = \frac{-k_p - \frac{k_i}{s}}{s - k_p - \frac{k_i}{s}} = \frac{-k_p s - k_i}{s^2 - k_p s - k_i}$

$T_{yd} = \frac{1}{s-C} = \frac{1}{s - k_p - \frac{k_i}{s}} = \frac{s}{s^2 - k_p s - k_i}$

sist. de ordin 2: $\frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$H = \begin{bmatrix} -k_p & 0 \\ 1 & -k_i \end{bmatrix}$$

$\det \Delta_1 = -k_p > 0 \Rightarrow k_p < 0$
$\det \Delta_2 = k_p k_i > 0$ $\Bigg\} \Rightarrow k_i < 0$

$\sigma = 0 \Rightarrow \xi = 1$

$t_t \leq 10$ sec $\Rightarrow t_t \simeq \frac{4}{\omega_n \xi} = \frac{4}{\omega_n} \leq 10$ $\quad \frac{1}{\omega_n} \leq 2.5 \Rightarrow \omega_n \geq 0.4$

$\chi(s) = s^2 - k_p s - k_i \equiv s^2 + 2\xi\omega_n s + \omega_n^2$

$-k_i = \omega_n^2$
$\omega_n^2 \geq 0.16$ $\Bigg| \Rightarrow -k_i \geq 0.16 \quad k_i \leq -0.16$

$2\xi\omega_n = 2\omega_n = -k_p$

$\omega_n \geq 0.4$

$2\omega_n = -k_p \geq 0.8 \Rightarrow k_p \leq -0.8$

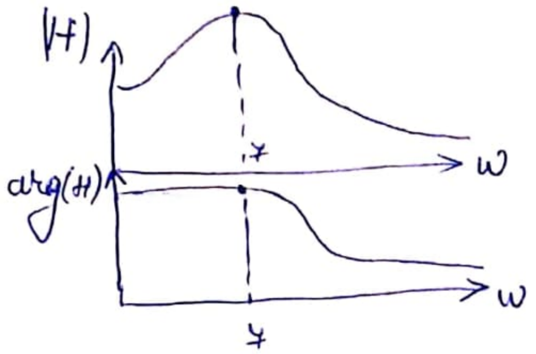
$\varepsilon_{st} = r(\infty) - y(\infty)$

$r(t) = 1(t)$

$d(t) = 1(t)$

$\varepsilon_{st} = 1 - T_{yr}(0) - T_{yd}(0) = 1 - \frac{-k_i}{-k_i} - 0 = 0$

II Diagramele Bode



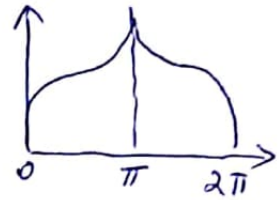
$\xrightarrow{u} \boxed{H} \xrightarrow{y}$

$u(t) = A \sin(zt + \varphi)$

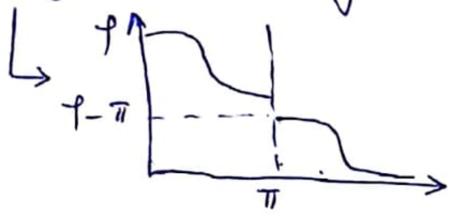
$y(t) = A\,|H(zj)|\sin(zt + \varphi + \arg(H(zj)))$

$H(z)$
$\downarrow$
$z = e^{j\omega} \to \omega \in [0, 2\pi]$

$|H(e^{j\omega})| = |H(e^{j(2\pi+\omega)})| \longrightarrow$



$\arg(H(e^{j\omega})) = \arg(H e^{j(\omega-\pi)}) - \pi$



$y[n] = \frac{1}{2}(u[n] + u[n-1]) \xrightarrow{z\{\}} Y(z) = \frac{1}{2}U(z) + \frac{1}{2}z^{-1}U(z)$

$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{2}(1 + z^{-1}) = \frac{1}{2}z^{-1}(1+z)$

$H(e^{j\omega}) = e^{-j\omega}\frac{1}{2}(1 + e^{j\omega}) = e^{-j\frac{\omega}{2}}\boxed{\frac{1}{2}\left(e^{-j\frac{\omega}{2}} + e^{j\frac{\omega}{2}}\right)} = e^{-j\frac{\omega}{2}}\cos\frac{\omega}{2} \left\langle \begin{matrix} |H| \\ \arg(H) \end{matrix} \right.$

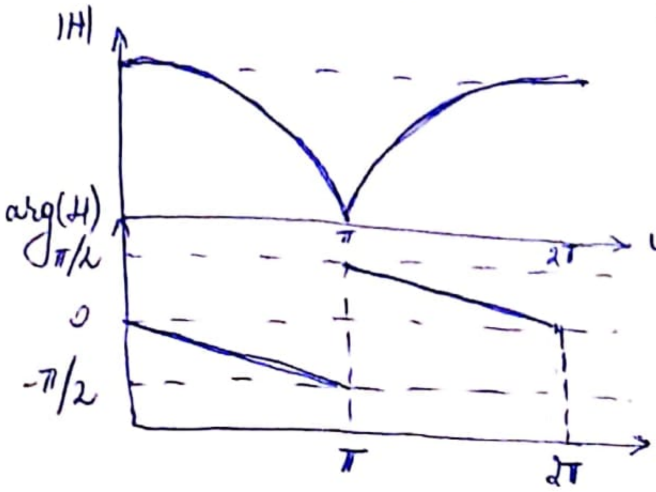
$|H| = |e^{-j\frac{\omega}{2}}\cos\frac{\omega}{2}| = \underbrace{|e^{-j\frac{\omega}{2}}|}_{1}\,|\cos\frac{\omega}{2}| = |\cos\frac{\omega}{2}|$

$\arg(H) = \arg(e^{-j\frac{\omega}{2}}\cos\frac{\omega}{2}) = \arg(e^{-j\frac{\omega}{2}}) + \arg(\cos\frac{\omega}{2}) = -\frac{\omega}{2} + \arg(\cos\frac{\omega}{2})$

$\arg(xy) = \arg x + \arg y$

$\underset{x\in\mathbb{R}}{\arg}(x) = \begin{cases} 0, & x > 0 \\ -\pi, & x < 0 \end{cases}$

$= \begin{cases} -\frac{\omega}{2}, & \omega \in (0, \pi) \\ -\frac{\omega}{2} - \pi, & \omega \in (\pi, 2\pi) \end{cases}$



$\omega \in (0, \pi) \to \frac{\omega}{2} \in (0, \frac{\pi}{2}) \to \cos\frac{\omega}{2} > 0$

$\omega \in (\pi, 2\pi) \to \frac{\omega}{2} \in (\frac{\pi}{2}, \pi) \to \cos\frac{\omega}{2} < 0$

$u[n] = 1[n] = \cos[0n]\,1[n]$

$u[n] = (-1)^n\,1[n] = \cos[n\pi]\,1[n]$

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Diagrame Bode

$z \in \mathbb{C}$ $\to |z| = |\bar{z}| \to |H(e^{j\omega})| = |H(e^{-j\omega})|$

$\to \arg(\bar{z}) = -\arg(z) \to \arg(H(e^{j\omega})) = -\arg(H(e^{-j\omega}))$

$H(z) = H(e^{j\omega})$

Trasarea diagramelor Bode în discret

1. Se scrie $H(e^{j\omega})$ și se formează termeni cu sin și cos
2. Se calculează $|H(e^{j\omega})|$ pentru $\omega \in [0,\pi]$
3. Se calculează $\arg(H(e^{j\omega}))$ pentru $\omega \in [0,\pi]$
4. Se trasează diagrama de amplificare pe $[0,\pi]$ și se simetrizează în raport cu Oy pe $[-\pi, 0)$
5. Se trasează diagrama de fază pe $[0,\pi]$ și se simetrizează cu semn schimbat pe $[-\pi, 0)$

$H(z) = H(e^{j\omega})$

$H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2}) \to H(e^{j\omega}) = \frac{1}{3}(1 + e^{-j\omega} + e^{-2j\omega}) = \frac{1}{3}e^{-j\omega}(e^{j\omega} + 1 + e^{-j\omega})$

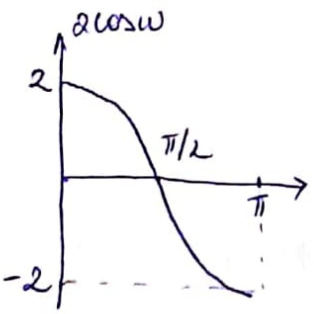
$|e^{jx}| = 1, \forall x \qquad \arg(e^{jx}) = x, \forall x \qquad = \frac{1}{3}e^{-j\omega}(1 + 2\cos\omega)$

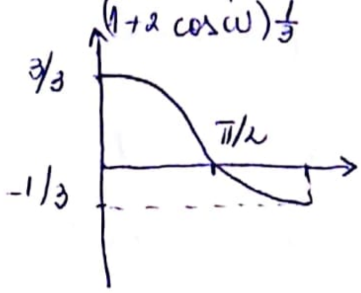
$|H(e^{j\omega})| = \frac{1}{3}|1 + 2\cos\omega|, \ \omega \in [0,\pi]$

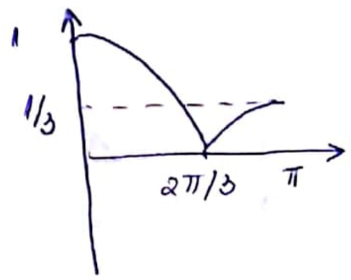
$1 + 2\cos\omega = 0 \to \cos\omega = \frac{-1}{2} \qquad \omega = \frac{2\pi}{3}$

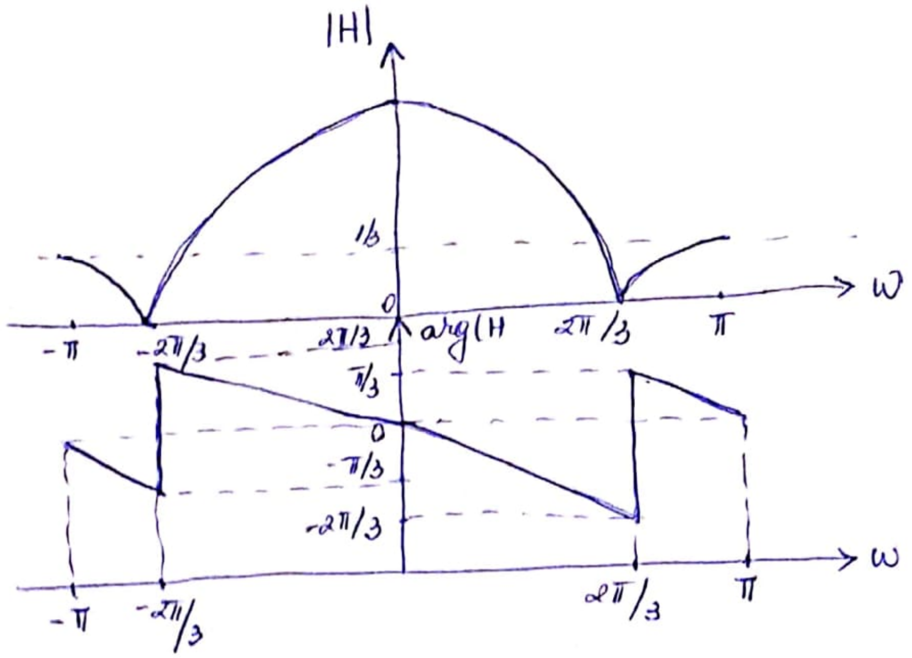
$\arg(H(e^{j\omega})) = \arg\frac{1}{3} + \arg(e^{-j\omega}) + \arg(1 + 2\cos\omega)$

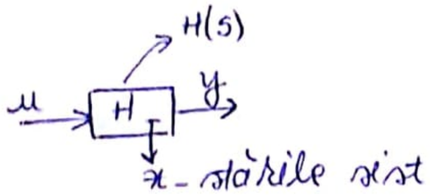
$= 0 + (-\omega) + \begin{cases} 0, & \omega \in [0, \frac{2\pi}{3}) \\ -\pi, & \omega \in (\frac{2\pi}{3}, \pi] \end{cases}$











$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx + Du\end{cases}$$

$$x(t) = e^{At}x_0 + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau$$

$$y(t) = \underbrace{Ce^{At}x_0}_{y_L} + \underbrace{\int_{t_0}^{t} Ce^{A(t-\tau)}Bu(\tau)\,d\tau + Du(t)}_{y_F}$$

$y_F(s) = HU$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$y_F(t) = h(t) * u(t)$

$\mathcal{L}\{e^{At}\} = (sI-A)^{-1}$

$h(t) = Ce^{At}B + D$

$e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$

$\mathcal{L}\{h\} = H(s) = C(sI-A)^{-1}B + D$

e^{At}

$A = TJT^{-1}$ $\quad e^{A} = Te^{J}T^{-1}$

$Tx = \tilde{x} \Rightarrow x = T^{-1}\tilde{x}$

$$T\dot{x} = TAx + TBu \Rightarrow \begin{cases}\dot{\tilde{x}} = TAT^{-1}\tilde{x} + TBu\\ y = CT^{-1}\tilde{x} + Du\end{cases}$$

spectru

$\Lambda(A) = \Lambda(TAT^{-1}) = \Lambda(T^{-1}AT)$

↳ polii sist

$C(sI-A)B + D \equiv CT^{-1}(sI - TAT^{-1})^{-1}TB + D$

Spațiul stărilor

Ex 1. Se dă sistemul $H(s) = \begin{array}{c|cc} & n & m \\ \hline n & A & B \\ p & C & D \end{array} = C(sI-A)^{-1}B + D$ $\begin{cases} n\text{ - nr. stări} \\ m\text{ - nr. intrări} \\ p\text{ - nr. ieșiri} \end{cases}$

$A = \begin{bmatrix} 1 & -1 & -1 \\ -3 & -4 & -3 \\ 4 & 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

a) poli
b) e^{At}
c) $H(s)$

a) $\det(\lambda I - A) = \begin{bmatrix} \lambda-1 & 1 & 1 \\ 3 & \lambda+4 & 3 \\ -4 & -7 & \lambda-6 \end{bmatrix} = (\lambda-1)(\lambda+4)(\lambda-6) - 12 - 21 + 4(\lambda+4) - 3(\lambda-6) + 21(\lambda-1) =$

$= (\lambda^2+3\lambda-4)(\lambda-6) - 33 + 4\lambda + 16 - 3\lambda + 18 + 21\lambda - 21 =$

$= \lambda^3 - 6\lambda^2 + 3\lambda^2 - 18\lambda - 4\lambda + 24 - 20 + 22\lambda =$

$= \lambda^3 - 3\lambda^2 + 4 = \lambda^3 + 1 - 3\lambda^2 + 3 = \lambda^3 + 1 - 3(\lambda^2 - 1) =$

$= (\lambda+1)(\lambda^2 - 4\lambda + 4) = (\lambda+1)(\lambda-2)^2$

poli $\begin{cases} \lambda_1 = -1 \\ \lambda_2 = \lambda_3 = 2 \end{cases}$

b) $e^{At} / \mathcal{L}\{\}$

$(sI-A)^{-1} = \begin{pmatrix} s-1 & 1 & 1 \\ 3 & s+4 & 3 \\ -4 & -7 & s-6 \end{pmatrix}^{-1}$ $(sI-A)^T = \begin{pmatrix} s-1 & 3 & -4 \\ 1 & s+4 & -7 \\ 1 & 3 & s-6 \end{pmatrix}$

$\det(sI-A) = (s+1)(s-2)^2$

$(sI-A)^* = \begin{pmatrix} (s+4)(s-6)+21 & -s-1 & -s-1 \\ -3s+6 & (s-1)(s-6)+4 & -3s+6 \\ 4s-5 & -11+7s & (s-1)(s+4)-3 \end{pmatrix}$

$(sI-A)^{-1} = \frac{1}{(s+1)(s-2)^2} \begin{pmatrix} s^2-2s-3 & -s-1 & -s-1 \\ 6-3s & s^2-7s+10 & -3s+6 \\ 4s-5 & 7s-11 & s^2+3s-7 \end{pmatrix}$

$= \frac{1}{(s+1)(s-2)^2} \begin{pmatrix} (s-3)(s+1) & -(s+1) & -(s+1) \\ -3(s-2) & (s-2)(s-5) & -3(s-2) \\ 4s-5 & 7s-11 & s^2+3s-7 \end{pmatrix}$

$$(sI-A)^{-1} = \begin{pmatrix} \frac{s-3}{(s-2)^2} & \frac{-1}{(s-2)^2} & \frac{-1}{(s-2)^2} \\ \frac{-3}{(s+1)(s-2)} & \frac{s-5}{(s+1)(s-2)} & \frac{-3}{(s+1)(s-2)} \\ \frac{4s-5}{(s+1)(s-2)^2} & \frac{7s-11}{(s+1)(s-2)^2} & \frac{s^2+3s-7}{(s+1)(s-2)^2} \end{pmatrix}$$

$$e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$$

c) $$H(s) = C(sI-A)^{-1}B + D = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \frac{s-3}{(s-2)^2} & \frac{-1}{(s-2)^2} & \frac{-1}{(s-2)^2} \\ \frac{-3}{(s+1)(s-2)} & \frac{s-5}{(s+1)(s-2)} & \frac{-3}{(s+1)(s-2)} \\ \frac{4s-5}{(s+1)(s-2)^2} & \frac{7s-11}{(s+1)(s-2)^2} & \frac{s^2+3s-7}{(s+1)(s-2)^2} \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

4·

$$= \frac{4s-5}{(s+1)(s-2)^2} \quad \frac{7s-11}{(s+1)(s-2)^2}$$

SEMINAR 6 — 11.12.2019

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Proprietăți structurale

1. Controlabilitatea: $\exists\, u$ a.î. $x \to x^*$, $x \in \mathbb{R}^n$

$R = [B \;\; AB \;\; A^2B \;\dots\; A^{n-1}B]$ matrice de controlabilitate
n linii, n×m coloane

$\downarrow$

sist/perechea (A,B) e controlabilă dacă rang $R = n$

Subspațiul controlabil este $Im(R)$

Testul Popov-Belevitch-Hautus (PBH)

per (A,B) e ctrl. dacă $\forall s \in \mathbb{C}$ rang$[sI-A \;\; B] = n$

Stabilizabilitatea: $\forall s \in \mathbb{C}_-$ rang$[sI-A \;\; B] = n$

Descompunere controlabilă $\Rightarrow T = [B(R) \,\vdots\, \text{completare}]^{-1}$

$T \to TAT^{-1}$, TB, CT^{-1} $\Rightarrow H(s) = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \overset{T}{=} \left[\begin{array}{cc|c} \tilde{A}_1 & \tilde{A}_{12} & \tilde{B}_1 \\ 0 & \tilde{A}_2 & 0 \\ \hline \tilde{C}_1 & \tilde{C}_2 & D \end{array}\right] = (\tilde{A}_1, \tilde{B}_1, \tilde{C}_1, D)$

2. Observabilitatea: din y pot să deduc evoluția lui x

$\downarrow$

neobservabilitatea

$Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$ n coloane, n·p linii

$\Rightarrow$ per (C,A) e observabilă dacă rang $Q = n$

$\overset{PBH}{\Longleftrightarrow} \forall s \in \mathbb{C}$ rang$\begin{bmatrix} sI-A \\ C \end{bmatrix} = n$ atunci sist. e obs.

detectabilitate $\Rightarrow$ sist. e detectabil dacă $\forall s \notin \mathbb{C}_-$ rang$\begin{bmatrix} sI-A \\ C \end{bmatrix} = n$

Descompunere observabilă

Subspațiul neobservabil este $Ker(Q)$, $Qx = 0$ $\forall x \neq 0$

$T = [B(Q) \,\vdots\, \text{completare}]^{-1}$

$\downarrow$

$(TAT^{-1}, TB, CT^{-1}, D)$

$H(s) = \left[\begin{array}{cc|c} \tilde{A}_1 & \tilde{A}_{12} & \tilde{B}_1 \\ 0 & \tilde{A}_2 & \tilde{B}_2 \\ \hline 0 & \tilde{C}_2 & D \end{array}\right] = (\tilde{A}_2, \tilde{B}_2, \tilde{C}_2, D)$

realizare ctrl + obsv $\Leftrightarrow$ minimal

$\Lambda(A) \to$ polii sistemului

$A \in R^{n \times n}$
$B \in R^{n \times m}$
$C \in R^{p \times n}$
$D \in R^{p \times m}$

3. Realizabilitatea
- aduc $H(s)$ în formă ireductibilă
- $D = H(\infty)$
- $\tilde{H}(s) = H(s) - D = \dfrac{K_0 + K_1 s + K_2 s^2 + \dots + K_{l-1} s^{l-1}}{\gamma_0 + \gamma_1 s + \gamma_2 s^2 + \dots + \gamma_{l-1} s^{l-1} + s^l}$

$\downarrow$
aducem la cel mai mic multiplu comun la numitor

ex: $\begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \end{bmatrix}$ $\begin{bmatrix} \frac{s+2}{s^2+3s+2} & \frac{s+1}{s^2+3s+2} \end{bmatrix} \rightarrow$ $\gamma_0 = 2$, $\gamma_1 = 3$, $K_0 = [2 \ \ 1]$, $K_1 = [1 \ \ 1]$

Realizarea standard observabilă (RSO)

$$H(s) = \left[\begin{array}{cccccc|c} 0_p & 0_p & 0_p & \dots & 0_p & -\gamma_0 I_p & K_0 \\ I_p & 0_p & 0_p & \dots & 0_p & -\gamma_1 I_p & K_1 \\ 0_p & I_p & 0_p & \dots & 0_p & -\gamma_2 I_p & K_2 \\ 0_p & 0_p & I_p & \dots & 0_p & -\gamma_3 I_p & K_3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0_p & 0_p & 0_p & \dots & I_p & -\gamma_{l-1} I_p & K_{l-1} \\ \hline 0_p & 0_p & 0_p & \dots & 0_p & I_p & D \end{array}\right] \Rightarrow \text{garantat observabilă}$$

Realizarea standard controlabilă (RSC)

$$H(s) = \left[\begin{array}{cccccc|c} 0_m & I_m & 0_m & 0_m & \dots & 0_m & 0_m \\ 0_m & 0_m & I_m & 0_m & \dots & 0_m & 0_m \\ 0_m & 0_m & 0_m & I_m & \dots & 0_m & 0_m \\ \vdots & & & & & \vdots & \vdots \\ 0_m & 0_m & 0_m & 0_m & \dots & I_m & 0_m \\ -\gamma_0 I_m & -\gamma_1 I_m & -\gamma_2 I_m & -\gamma_3 I_m & \dots & -\gamma_{l-1} I_m & I_m \\ \hline K_0 & K_1 & K_2 & K_3 & \dots & K_{l-1} & D \end{array}\right] \Rightarrow \text{garantat controlabilă}$$

Ex: $H(s) = \begin{bmatrix} \frac{s^2+2}{s^2-1} & \frac{s-1}{s^2} & \frac{1}{s^2+s} \end{bmatrix}$

a) RSC și RSO
b) E vreuna din r minimale?

$\downarrow$
$\begin{bmatrix} \frac{s^4+2s^2}{s^4-s^2} & \frac{s^3-s^2-s+1}{s^4-s^2} & \frac{s^2-s}{s^4-s^2} \end{bmatrix}$

$\left.\begin{array}{l} \tilde{H}(s) = H(s) - D \\ D = H(\infty) = [1 \ 0 \ 0] \end{array}\right\} \Rightarrow \tilde{H}(s) = \begin{bmatrix} \frac{3s^2}{s^4-s^2} & \frac{s^3-s^2-s+1}{s^4-s^2} & \frac{s^2-s}{s^4-s^2} \end{bmatrix}$

$\gamma_0 = 0$ $\quad K_0 = [0 \ 1 \ 0]$
$\gamma_1 = 0$ $\quad K_1 = [0 \ -1 \ -1]$
$\gamma_2 = -1$ $\quad K_2 = [3 \ -1 \ 1]$
$\gamma_3 = 0$ $\quad K_3 = [0 \ 1 \ 0]$

RSO:

$$\left[\begin{array}{cccc|ccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$R = [B \ AB \ A^2B \ A^3B]$$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

RSC:

$$\left[\begin{array}{cccc|c} O_3 & I_3 & O_3 & O_3 & O_3 \\ O_3 & O_3 & I_3 & O_3 & O_3 \\ O_3 & O_3 & O_3 & I_3 & O_3 \\ O_3 & O_3 & I_3 & O_3 & I_3 \\ \hline 0\ 1\ 0 & 0\ -1\ -1 & 3\ -1\ 1 & 0\ 1\ 0 & 1\ 0\ 0 \end{array}\right]$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 3 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 1 & -1 & 0 & 3 \end{bmatrix}$$

$$\downarrow$$

$$\det = 9 \neq 0$$

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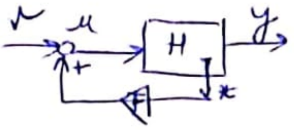
15.01.2019

Compensatorul Kalman



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad u = v + Fx \Rightarrow \begin{cases}\dot{x} = (A+BF)x + Bv \\ y = (C+DF)x + Du\end{cases}$$

Dacă $\exists F$ a.î. $\Lambda(A+BF) \subset \mathbb{C}_-$ at. perechea (A,B) s.n. stabilizabilă

Dacă $\exists F$ a.î. $\Lambda(A+BF) = \Lambda_0 = \overline{\Lambda}_0$ at. (A,B) s.n. controlabilă

controlabil $\Leftrightarrow$ alocabil

Procedura lui ~~Akerman~~ $\Rightarrow m = 1 \Rightarrow (A, b)$ ...

1. $\Lambda_0 \Rightarrow \chi(s) = \prod_{i=1}^{n} (s - \lambda_i)$; $\lambda_i \in \Lambda_0$
2. $R = [b \;\; Ab \;\; A^2 b \; \dots \; A^{n-1} b] \in \mathbb{R}^{n \times n}$
3. $R^T q = 1_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \Rightarrow$ îl aflu pe q

4. $f^T = -q^T \chi(A)$ $\qquad \chi(s) = s^2 + 3s + 2$

$\Downarrow$ $\qquad \chi(A) = A^2 + 3A + 2I$

$\Lambda(A + bf^T) = \Lambda_0$

$\tilde{F} = 0$

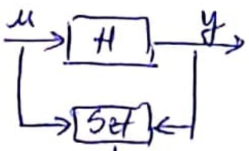
$g = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

$m > 1 \Rightarrow (A, B)$ ctrb $\Rightarrow$ aleg $\tilde{F}$ și g a.î. forma $\begin{cases} \tilde{A} = A + B\tilde{F} \\ b = Bg \end{cases} \Rightarrow (\tilde{A}, b)$ ctrb

Aplică Ackerman pe $(\tilde{A}, b) \Rightarrow f^T$

Atunci $\Lambda(\tilde{A} + bf^T) = \Lambda_0 = \Lambda(A+BF)$ unde $F = \tilde{F} + gf^T$

Estimatoare Luenberger



$\downarrow \hat{x} \rightarrow \lim_{t \to \infty} \hat{x}(t) = x(t)$

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y) \\ \hat{y} = C\hat{x} + Du \end{cases}$$

$\Downarrow$

$\dot{\hat{x}} = (A + LC)\hat{x} + (B + LD)u - Ly$

$\Lambda(A + LC) \subset \mathbb{C}_-$

Scanned by CamScanner

Dacă $\exists L$ a.î. $\Lambda(A+LC) \subset \mathbb{C}_-$ atunci (C,A) s.n. detectabilă

Dacă $\exists L$ a.î. $\Lambda(A+LC) = \Lambda_0 = \bar{\Lambda}_0$ atunci (C,A) s.n. observabilă

Principiul dualității $\Rightarrow$ (A,B) ctrb $\Leftrightarrow$ (B^T, A^T) obsv

(C,A) obsv $\Leftrightarrow$ (A^T, C^T) ctrb

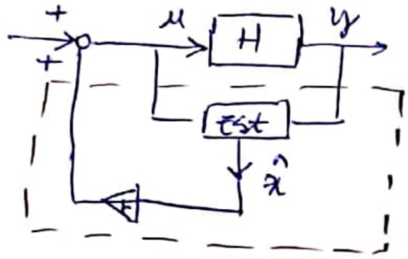
Aplicăm Ackerman pe $(A^T, C^T) \Rightarrow \Lambda(A^T + C^T F) = \Lambda_0$

$\Lambda(A+LC) = \Lambda_0$

$\Lambda(A^T + C^T L^T) = \Lambda_0$

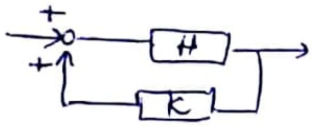
$\Lambda(A^T + C^T F) = \Lambda_0$

$L = F^T$



$\rightarrow$ compensator Kalman

$$\begin{cases} \dot{\hat{x}} = (A+LC)\hat{x} + (B+LD)u - Ly \\ u = F\hat{x} \end{cases}$$



$$\begin{cases} \dot{\hat{x}} = (A + LC + (B+LD)F)\hat{x} - Ly \\ u = F\hat{x} + 0 \cdot y \end{cases}$$

$$K(s) = \left[\begin{array}{c|c} A+BF+LC+LDF & -L \\ \hline F & 0 \end{array}\right]$$

Ex: $G(s) = \dfrac{1}{s^3+s^2+2s+2} \Rightarrow K(s) = ?$

$\ell = 3$: $\gamma_0 = 2$ $\quad k_2 = 0$

$\gamma_1 = 2$ $\quad k_1 = 0$

$\gamma_2 = 1$ $\quad k_0 = 1$

$D = G(\infty) = 0$

$\tilde{G} = G - D = G = \dfrac{1}{s^\ell + \gamma_{\ell-1}s^{\ell-1} + \ldots + \gamma_1 s + \gamma_0}(k_{\ell-1}s^{\ell-1} + \ldots + k_1 s + k_0)$

RSC:

$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2 & -1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\quad C = (1\ 0\ 0)$ $\quad Q = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

rang $Q = 3 \Rightarrow$ obsv & ctrb $\Rightarrow$ min

Aplicăm Ackerman

1) $\Lambda_0 = \{-1, -1, -1\}$ $\quad q = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\chi(s) = (s+1)^3$

2) $R = (B\ AB\ A^2B) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$

3) $R^T q = e_n \Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix} q = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$ $\quad \begin{cases} q_3 = 0 \\ q_2 - q_3 = 0 \Rightarrow q_2 = 0 \\ q_1 - q_2 - q_3 = 1 \Rightarrow q_1 = 1 \end{cases}$

4) $f^T = -q^T \chi(A) = -(1\ 0\ 0)\begin{pmatrix} -1 & 1 & 2 \\ -1 & -5 & -1 \\ 2 & -2 & -1 \end{pmatrix}$

$\Rightarrow f^T = (1\ -1\ -2) = F$

L = - acker(A', C', - ones(1,3))'

Aplicăm ackerman pe (A^T, C^T)

Aplicăm Ackerman

1) $\Lambda_0 = \{-1, -1, -1\}$ $\quad \tilde{A} = A^T \quad \tilde{B} = C^T$

$\chi(s) = (s+1)^3$

2) $R = (\tilde{B} \;\; \tilde{A}\tilde{B} \;\; \tilde{A}^2\tilde{B}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = Q^T$

3) $R^T q = e_n \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} q_1 = 0 \\ q_2 = 0 \\ q_3 = 1 \end{cases} \Rightarrow q = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

4) $f^T = -q^T \chi(\tilde{A})$

$(A^T + I)^3 = ((A+I)^T)^3 = ((A+I)^3)^T = \chi(A)^T$

$f^T = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}^T \begin{pmatrix} -1 & -4 & 2 \\ 1 & -5 & -2 \\ 2 & -1 & -4 \end{pmatrix} = (-2 \;\; 1 \;\; 4) \Rightarrow L = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

$K(s) = \left[\begin{array}{c|c} A + BF + LC + LDF & -L \\ \hline F & 0 \end{array}\right]$

$BF = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}(1 \;\; -1 \;\; -2) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & -2 \end{pmatrix}$ $\qquad LC = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}(1\,0\,0) = \begin{pmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix}$ $\qquad LDF = O_3$

$\Rightarrow K(s) = \left[\begin{array}{ccc|c} -2 & 1 & 0 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & -3 & -3 & -4 \\ \hline 1 & -1 & -2 & 0 \end{array}\right] = \dfrac{11s^2 + 10s - 1}{s^3 + 5s^2 + 8s}$