

11. (10p) Se consideră tipurile de date $\text{LIST}\langle N \rangle$ și $\text{TREE}\langle N \rangle$, pentru care avem definiți constructorii:

$[] : \rightarrow \text{LIST}\langle N \rangle$
 $\text{cons}(a, l) : N \times \text{LIST}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$
 $\text{leaf} : \rightarrow \text{TREE}\langle N \rangle$
 $\text{node}(t1, a, t2) : \text{TREE}\langle N \rangle \times N \times \text{TREE}\langle N \rangle \rightarrow \text{TREE}\langle N \rangle$

și axiomele:

insert: $N \times \text{TREE}\langle N \rangle \rightarrow \text{BOOLEAN}$

(I1) $\text{insert}(a, \text{leaf}) = \text{node}(\text{leaf}, a, \text{leaf})$
(I2) $\text{insert}(a, \text{node}(t1, x, t2)) =$
 $(a < x) ? \text{node}(\text{insert}(a, t1), x, t2)$
 : $\text{node}(t1, x, \text{insert}(a, t2))$

preorder: $\text{TREE}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$

(P1) $\text{preorder}(\text{leaf}) = []$
(P2) $\text{preorder}(\text{node}(t1, x, t2)) =$
 $\text{cons}(x, \text{append}(\text{preorder}(t1), \text{preorder}(t2)))$

member: $N \times \text{LIST}\langle N \rangle \rightarrow \text{BOOLEAN}$

(M1) $\text{member}(e, []) = \text{false}$
(M2) $\text{member}(e, \text{cons}(a, x)) = (e ==$
 $a)$
 $|| \text{member}(e, x)$

append: $\text{LIST}\langle N \rangle \times \text{LIST}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$

(A1) $\text{append}([], l2) = l2$
(A2) $\text{append}(\text{cons}(a, x), l2) =$
 $\text{cons}(a, \text{append}(x,$
 $l2))$

Verificați prin inducție structurală că următoarea proprietate este adevărată:

$P(t) = (\text{member}(a, \text{preorder}(\text{insert}(a, t))) == \text{true}) \forall t \in \text{TREE}\langle N \rangle$

11. (10p) Se consideră tipurile de date $\text{LIST}\langle N \rangle$ și $\text{TREE}\langle N \rangle$, pentru care avem definiți constructorii:

$[] : \rightarrow \text{LIST}\langle N \rangle$

$\text{cons}(a, x) : N \times \text{LIST}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$

$\text{leaf} : \rightarrow \text{TREE}\langle N \rangle$

$\text{node}(t1, a, t2) : \text{TREE}\langle N \rangle \times N \times \text{TREE}\langle N \rangle \rightarrow \text{TREE}\langle N \rangle$

și axiomele:

memberT: $N \times \text{TREE}\langle N \rangle \rightarrow \text{BOOLEAN}$

(MT1) memberT(e, leaf) = false

(MT2) memberT(e, node(t1, x, t2)) =
(e == x) || memberT(e, t1)
|| memberT(e, t2)

flatten: $\text{TREE}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$

(F1) flatten(leaf) = []

(F2) flatten(node(t1, x, t2)) =
append(flatten(t1), cons(x, flatten(t2)))

member: $N \times \text{LIST}\langle N \rangle \rightarrow \text{BOOLEAN}$

(M1) member(e, []) = false

(M2) member(e, cons(a, x)) = (e == a)
|| member(e, x)

append(l1, l2): $\text{LIST}\langle N \rangle \times \text{LIST}\langle N \rangle \rightarrow \text{LIST}\langle N \rangle$

(A1) append([], l2) = l2

(A2) append(cons(a, x), l2) =
cons(a, append(x, l2))

Verificați prin inducție structurală că următoarea proprietate este adevărată:

$P(t) = (\text{memberT}(e, t) \rightarrow \text{member}(e, \text{flatten}(t))) \forall t \in \text{TREE}\langle N \rangle$