Teoria Sistemelor

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Examen

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Problema 1.

$$H(s) = \frac{s^2+1}{s^2+3s+2}$$

1. $A(s) = s^2 + 3s + 2 \Rightarrow H = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$\Delta_1 = |3| = 3 > 0$

$\Delta_2 = \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = 6 > 0$

$\Rightarrow$ $A(s)$ este polinom Hurwitz $\Rightarrow$ Are toți polii în semiplanul complex stâng $\Rightarrow$ sistemul este stabil.

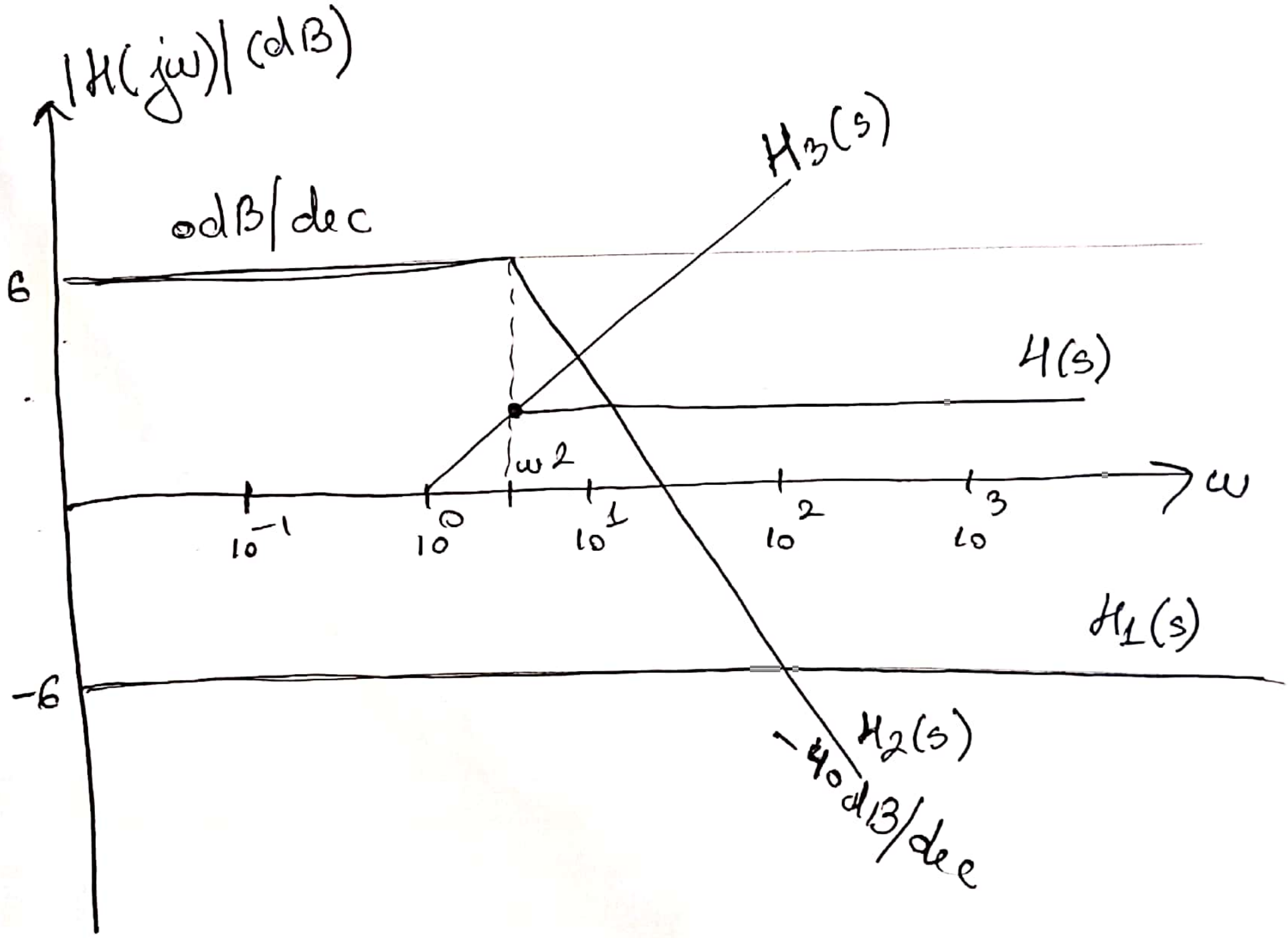
②. $H(s) = \frac{s^2+1}{s^2+3s+2} = \frac{1}{2} \cdot \frac{2}{s^2+3s+2} \cdot (s^2+1)$

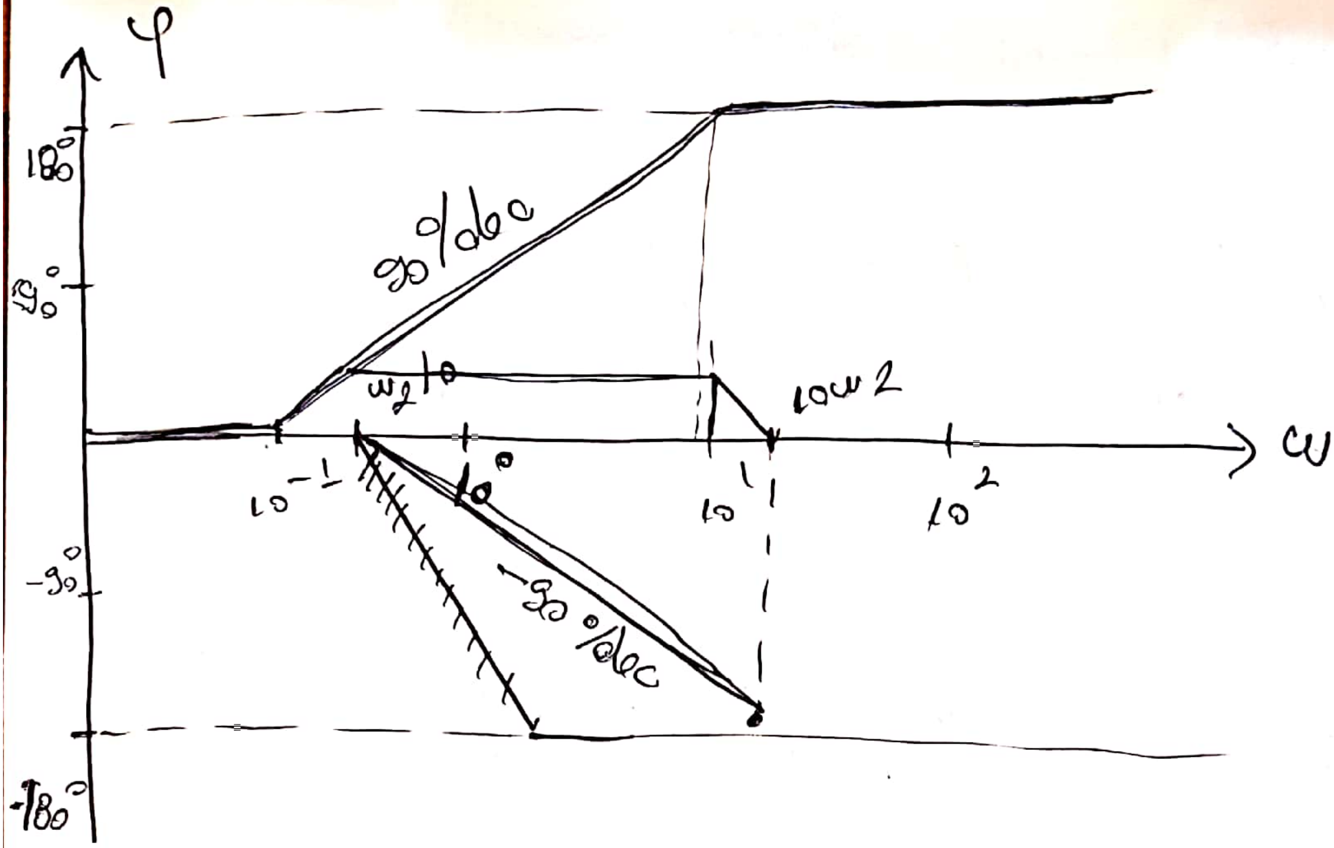
$H_1(s) = \frac{1}{2} \Rightarrow 0\ dB/dec$; $20\lg\frac{1}{2} = 20\cdot(-0{,}3) = 6dB$

$H_2(s) = \frac{2}{s^2+3s+2} \Rightarrow -40\ dB/dec$; $40\lg\sqrt{2} = 40\lg 2^{1/2} =$

$= 20\lg 2 = 20\cdot 0{,}3 = 6\ dB$; $\omega_2 = \sqrt{2} = 1{,}41$

$H_3(s) = s^2+1 \Rightarrow 40\ dB/dec$; $20\lg 1 = 0dB$; $\omega_3 = 1$





φ
180°
90°
90°/dec
$\omega_2/10$
$10\omega_2$
ω
10^{-1}
10^0
10^1
10^2
-90°
-90°/dec
-180°

③. $u(t) = \sin t \cdot 1(t)$

$U(s) = \mathcal{L}\{\sin t\}(s) = \dfrac{1}{s^2+1}$

$y(t) = h(t) * u(t)$

$$Y(s) = H(s) \cdot U(s) = \frac{s^2+1}{s^2+3s+2} \cdot \frac{1}{s^2+1} = \frac{1}{s^2+3s+2}$$

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}(t) = e^{-t} - e^{-2t}$$

$$\lim_{t \to \infty} y(t) = 0 \Rightarrow y_{permanent} = 0$$

$$y_{tranz} = e^{-t} - e^{-2t}$$

Calcul RAPID

Teorema Valorii Finale (TVF) $\Rightarrow \lim_{t \to \infty} y(t) = 0 \Rightarrow y_{perm.} = 0.$

④. R.S.O.

$$D = H(\infty) = \lim_{s \to \infty} H(s) = \lim_{s \to \infty} \frac{s^2+1}{s^2+3s+2} = 1$$

$$\tilde{H}(s) = \frac{-3s-1}{s^2+3s+2} \Rightarrow \quad \gamma_0 = 2, \quad \gamma_1 = 3$$

$K_0 = -1$

$K_1 = -3$

$$H(s) = \left[\begin{array}{cc|c} 0 & -\gamma_0 & K_0 \\ 1 & -\gamma_1 & K_1 \\ \hline 0 & 1 & D \end{array}\right] = \begin{bmatrix} 0 & -2 & -1 \\ 1 & -3 & -3 \\ 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$; $B = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$; $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$; $D = 1$

⑤. (A, B, C, D) observabilă $\underline{I}$. (din R.S.O)

$R = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ -3 & 8 \end{bmatrix} \Rightarrow$

$A \cdot B = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$\Rightarrow \det R = \begin{vmatrix} -1 & 6 \\ -3 & 8 \end{vmatrix} = -8 + 18 = 10 \neq 0 \Rightarrow$

$\Rightarrow$ rang $R = 3 \Rightarrow (A, B)$ controlabilă $\underline{II}$.

$\left.\begin{matrix} \underline{I}. \\ \underline{II}. \end{matrix}\right| \Rightarrow (A, B, C, D)$ realizare minimală

⑥. $\underline{I}$. $\Lambda(A+BF) \in \mathbb{C}^- \Rightarrow$ alocare pt. (A, B)

$\underline{II}$. $\Lambda(A+LC) \in \mathbb{C}^- \Rightarrow$ alocare pt. (A^T, C^T)

cazul $m = 1$

$\underline{I}$. Alegem $\Lambda(A+BF) = \{-1, -1\}$

Matricea de controlabilitate este

$R = \begin{bmatrix} B & A \cdot B \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ -3 & 8 \end{bmatrix}$

$R^T \cdot q = e_2$; unde $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $q = [q_1\ q_2]^T$

$$\Rightarrow \begin{bmatrix} -1 & -3 \\ 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} -g_1 - 3g_2 = 0 \quad \Big| \cdot 6 \\ 6g_1 + 8g_2 = 1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -6g_1 - 18g_2 = 0 \\ 6g_1 + 8g_2 = 1 \end{cases}$$

$$(+) \Rightarrow 10g_2 = -1 \Rightarrow g_2 = -\frac{1}{10}$$

$$-g_1 - 3g_2 = 0 \Rightarrow g_1 = -3g_2 \Rightarrow g_1 = \frac{3}{10}$$

$$g = \begin{bmatrix} 3/10 \\ -1/10 \end{bmatrix}$$

$$f^T = -g^T \cdot \mathcal{X}(A) = \begin{bmatrix} -\frac{3}{10} & \frac{1}{10} \end{bmatrix} \cdot (A + I_2)^2 =$$

$$= \begin{bmatrix} -\frac{3}{10} & \frac{1}{10} \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

$$A + BF = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & \frac{6}{5} \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & -\frac{8}{5} \\ \frac{2}{5} & -\frac{9}{5} \end{bmatrix}$$

$$P_{A+BF}(\lambda) = \begin{bmatrix} \lambda+1/5 & -8/5 \\ \frac{2}{5} & \lambda+\frac{9}{5} \end{bmatrix} = \lambda^2 + 2\lambda + \frac{9}{25} + \frac{16}{25} =$$

$$= \lambda^2 + 2\lambda + 1 = (\lambda+1)^2 \Rightarrow$$ Se verifică alocarea făcută

$$\Lambda(A+BF) = \{-1, -1\}$$

II. Alegem $A+LC$ a.î. $\Lambda(A+LC) \in \mathbb{C}^-$

Alocare pt. (A^T, C^T).

Fie $\Lambda(A^T + C^T \cdot E) = \{-1, -1\}$

$R = \text{ctrb}(A^T, C^T)$ matr. de controlabilitate

$$R = \begin{bmatrix} C^T & A^T \cdot C^T \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$A^T \cdot C^T = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$R^T \cdot g = e_2$; unde $g = [g_1 \; g_2]^T$; $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} g_2 = 0 \\ g_1 - 3g_2 = 1 \end{cases} \Rightarrow g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$f^T = -g^T \cdot \chi(A^T) = [-1 \;\; 0] \cdot (A^T + I_2)^2 =$$

$$= [-1 \;\; 0] \cdot \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}^2 = [-1 \;\; -1] \cdot \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} = [1 \;\; 1]$$

$$L^T = [1 \;\; 1]$$

$$LC = \begin{bmatrix}1\\1\end{bmatrix}\cdot\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}$$

$$LDF = \begin{bmatrix}1\\1\end{bmatrix}\cdot\begin{bmatrix}1\end{bmatrix}\cdot\begin{bmatrix}1/5 & -2/5\end{bmatrix} = \begin{bmatrix}1/5 & -2/5\\1/5 & -2/5\end{bmatrix}$$

$$A + BF = \begin{bmatrix}-1/5 & -8/5\\2/5 & -9/5\end{bmatrix}$$

$$A + BF + LC + LDF = \begin{bmatrix}-\frac{1}{5} & -\frac{8}{5}\\ \frac{2}{5} & -\frac{9}{5}\end{bmatrix} + \begin{bmatrix}0 & 1\\0 & 1\end{bmatrix} + \begin{bmatrix}\frac{1}{5} & -\frac{2}{5}\\ \frac{1}{5} & -\frac{2}{5}\end{bmatrix} =$$

$$= \begin{bmatrix}0 & -1\\ \frac{3}{5} & -\frac{6}{5}\end{bmatrix}$$

$$K = \left[\begin{array}{c|c} A + BF + LC + LDF & -L\\ \hline F & 0\end{array}\right]$$

$$K = \left[\begin{array}{cc|c} 0 & -1 & -1\\ \frac{3}{5} & -\frac{6}{5} & -1\\ \hline \frac{1}{5} & -\frac{2}{5} & 1\end{array}\right]$$

$\downarrow$

Compensatorul Kalman care stabilizează sistemul.

Problema 2.

$$y(n) = \frac{1}{2}\left[u(n) - u(n-1)\right] \quad \Big| \; Z\{\cdot\}$$

$$Y(z) = \frac{1}{2}\left(U(z) - \frac{1}{z}U(z)\right) = \frac{U(z)}{2}\left(1 - \frac{1}{z}\right)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{2}\left(1 - \frac{1}{z}\right)$$

$$H(e^{j\omega}) = \frac{1}{2}\left(1 - e^{-j\omega}\right) =$$

$$= \frac{e^{-j\omega/2}}{2}\left(e^{j\omega/2} - e^{-j\omega/2}\right) =$$

$$= j \cdot e^{-j\omega/2} \cdot \frac{e^{j\omega/2} - e^{-j\omega/2}}{2j} =$$

$$= j \cdot e^{-j\omega/2} \cdot \sin\frac{\omega}{2} = e^{j\frac{\pi}{2}} \cdot e^{-j\frac{\omega}{2}} \cdot \sin\frac{\omega}{2}$$

Amplitudine $= |H(e^{j\omega})| = \sin\frac{\omega}{2} > 0 \; ; \; \omega \in [0, \pi]$

Faza $= \arg(H(e^{j\omega})) = \frac{\pi}{2} - \frac{\omega}{2} = \frac{\pi - \omega}{2}$

$\varphi = \frac{\pi - \omega}{2}$ fct. lin. $\Rightarrow$ faza liniară

$$y(t) = u(t) \cdot |H(j\omega)| \cdot e^{j \cdot \arg(H(j\omega))}$$

$$y(n) = u(n) \cdot \sin\frac{\omega}{2} \cdot e^{j\left(\frac{\pi - \omega}{2}\right)}$$

$u(n) = 1 = e^{j \cdot \omega \cdot n}$; $\omega = 0 \Rightarrow y(n) = 0$ pt. $u(n) = 1$