$\forall w \in \Sigma^*, \; w \in L(M) \iff w \in L(M'')$

$\forall w \in \Sigma^*$, $\forall q, p \in K'$, $(q, w) \vdash^*_{M'} (p, e) \iff (E(q), w) \vdash^*_{M''} (P, e)$, $p \in P$

$$\underbrace{(s', w) \vdash^*_{M'} (f', e), \; f' \in F'}_{w \in L(M')} \iff \underbrace{(E(s'), w) \vdash^*_{M''} (Q, e), \; f' \in Q}_{w \in L(M'')}$$

<u>Dem</u> → prin inducție după $|w|$

<u>Pas de bază</u>

$|w| = 0$, $w = e$

<u>Ip. inductivă</u>

pp. adev. pt w, $|w| \le K$

Pas de inducție

$\forall w, |w| = k+1.$

$w = va,\ a \in \Sigma,\ v \in \Sigma^*$

$\Rightarrow$ pp $(q, w) \vdash_{M'}^{*} (p, e) \Rightarrow \exists\, r_1, r_2 \in K'$ a.î.

$$(q, va) \vdash_{M'}^{*} (r_1, a) \vdash_{M'} (r_2, e) \vdash_{M'}^{*} (p, e)$$

Cum $(q, va) \vdash_{M'}^{*} (r_1, a) \Rightarrow (q, v) \vdash_{M'}^{*} (r_1, e)$

Dar $|v| = k \overset{\text{lp. ind.}}{\Longrightarrow} (E(q), v) \vdash_{M''}^{*} (R_1, e),\ r_1 \in R_1.$

Cum $(r_1, a) \vdash_{M'} (r_2, e) \Rightarrow (r_1, a, r_2) \in \Delta' \Rightarrow$ din construcția M''

$E(r_2) \subseteq \delta''(R_1, a)$

$(r_2, e) \vdash_{M'}^{*} (p, e) \Rightarrow p \in E(r_2) \Rightarrow p \in \delta''(R_1, a)$

$(R_1, a) \vdash_{M''} (P, e),\ p \in P \Rightarrow (E(q), va) \vdash_{M''}^{*} (R_1, a) \vdash_{M''} (P, e)$

$\Leftarrow$ Pp. $(E(q), va) \vdash^{*}_{M''} (R_1, a) \vdash_{M''} (P, e)$ pt un anumit P a.î. $p \in P$ și

un anumit R_1 a.î. $\delta''(R_1, a) = P$.

Din def. δ'', $\delta''(R_1, a)$ este reuniunea tuturor mulțimilor $E(r_2)$

unde pt o anumită stare $r_1 \in R_1$, (r_1, a, r_2) este o tranziție a lui M'.

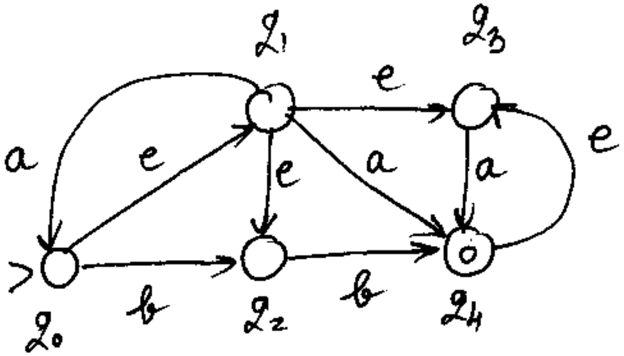
Cum $p \in P = \delta''(R_1, a)$, $\exists\, r_2$ a.î. $p \in E(r_2)$, pt un anumit $r_1 \in R_1$,

(r_1, a, r_2) este o tranziție a lui M'. $\Rightarrow (r_2, e) \vdash^{*}_{M'} (p, e)$ din $E(r_2)$

Din ip. ind. $\Rightarrow (q, v) \vdash^{*}_{M'} (r_1, e) \Rightarrow (q, va) \vdash^{*}_{M'} (r_1, a) \vdash_{M'} (r_2, e) \vdash^{*}_{M'} (p, e)$

q.e.d

ex:



$\delta''(Q, \sigma) = \cup \{ E(p) \mid p \in K,$
$(q, \sigma, p) \in \Delta, q \in Q \}$

$E(q_0) = \{q_0, q_1, q_2, q_3\} = Q_0 = s''$

$E(q_1) = \{q_1, q_2, q_3\}$

$E(q_2) = \{q_2\}$

$E(q_3) = \{q_3\}$

$E(q_4) = \{q_3, q_4\}$.

$\delta''(Q_0, a) = E(q_0) \cup E(q_4) = \{q_0, q_1, q_2, q_3, q_4\} = Q_1$.

$\delta''(Q_0, b) = E(q_2) \cup E(q_4) = \{q_2, q_3, q_4\} = Q_2$

$\delta''(Q_1, a) = Q_1$

$\delta''(Q_1, b) = Q_2$

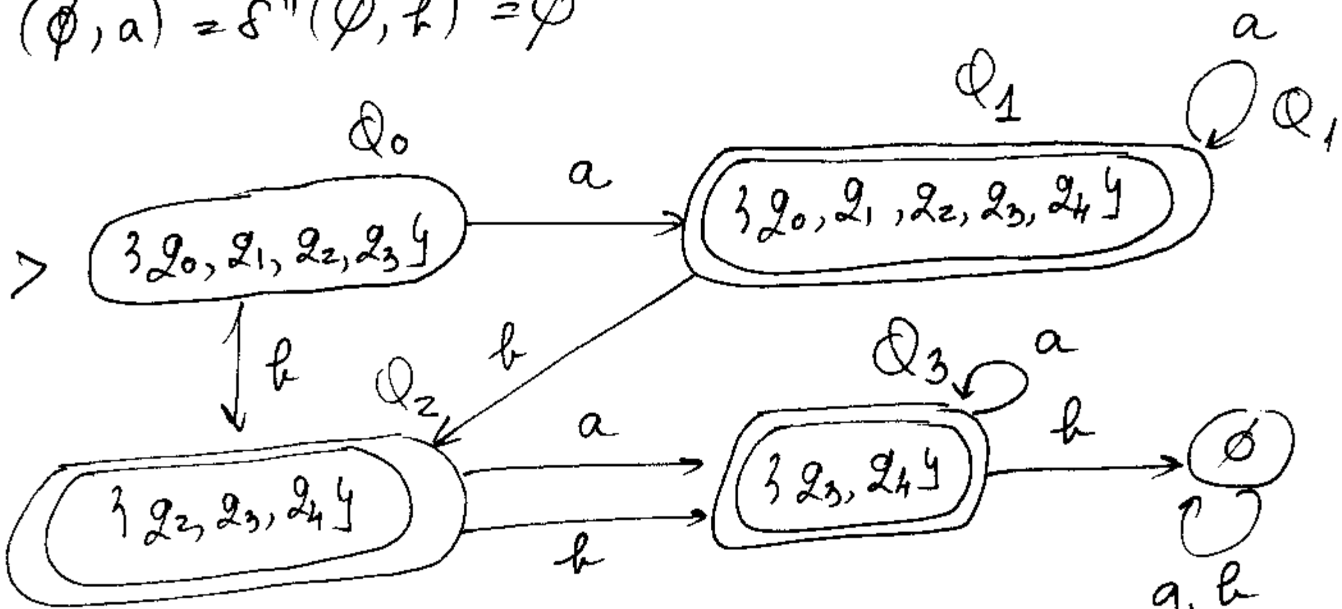
$$\delta''(Q_2, a) = E(q_4) = \{q_3, q_4\} = Q_3$$

$$\delta''(Q_2, b) = E(q_4) = Q_3$$

$$\delta''(Q_3, a) = Q_3$$

$$\delta''(Q_3, b) = \emptyset$$

$$\delta''(\emptyset, a) = \delta''(\emptyset, b) = \emptyset$$



Proprietăți ale limbajelor acceptate de A-Finite

Teorema

Clasa limbajelor acceptate de A-Finite este închisă în raport cu opțiile:

a) reuniune
b) concatenare
c) Kleene Star
d) complementare
e) intersecție

dem.

a) $L_1 = L(M_1)$, $L_2 = L(M_2)$

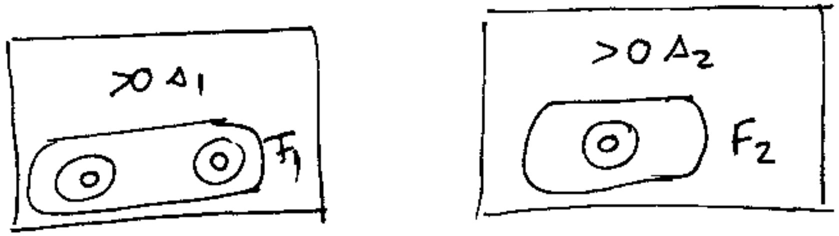
$M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$

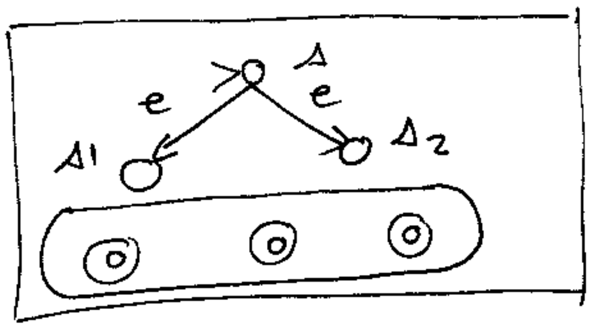
$M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$

$K_1 \cap K_2 = \emptyset$

? M a.î. $L(M) = L(M_1) \cup L(M_2)$

M_1 M_2





$M=(K,\Sigma,\Delta,s,F)$, s este o nouă stare $s\notin K_1 \wedge s\notin K_2$

$K=K_1\cup K_2\cup\{s\}$

$F=F_1\cup F_2$

$\Delta=\Delta_1\cup\Delta_2\cup\{(s,e,s_1),(s,e,s_2)\}$.

$w\in\Sigma^*$, $(s,w)\vdash_M^{*}(q,e)$, $q\in F \iff$

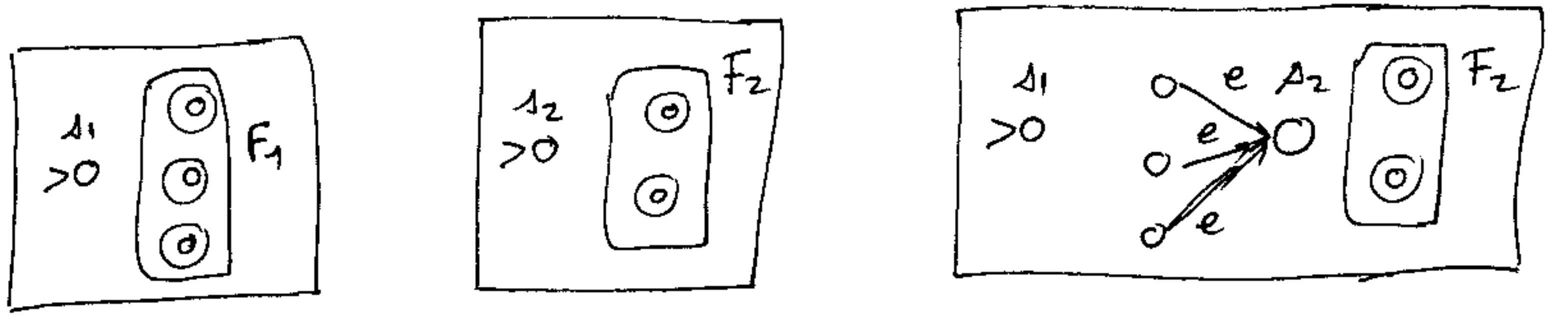
$(s_1,w)\vdash_{M_1}^{*}(q,e)$, $q\in F_1$ sau $(s_2,w)\vdash_{M_2}^{*}(q,e)$, $q\in F_2$

$L(M)=L(M_1)\cup L(M_2)$

f) Concatenare

$M_1, M_2 \rightarrow AFN$

? M ai $L(M) = L(M_1) \cdot L(M_2)$



$M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$

$M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$, $K_1 \cap K_2 = \emptyset$

$M = (K, \Sigma, \Delta, s, F)$

$K = K_1 \cup K_2$

$s = s_1$

$F = F_2$

$\Delta = \Delta_1 \cup \Delta_2 \cup (F_1 \times \{e\} \times \{s_2\})$

$(s, w) \vdash_M^* (q, e)$, $q \in F \iff \exists\, w_1, w_2 \in \Sigma^*$, $\exists\, p \in F_1$ ai

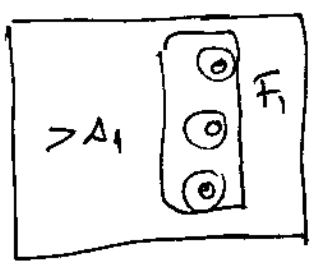
$w = w_1 w_2$ și $(s_1, w_1) \vdash_{M_1}^* (p, e)$ și $(s_2, w_2) \vdash_{M_2}^* (q, e)$

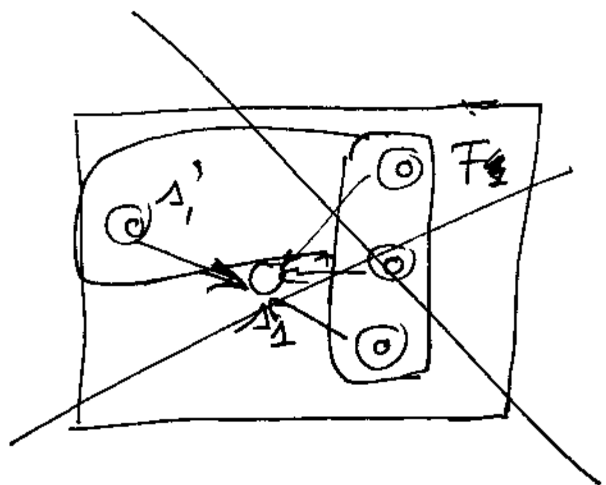
$$L(M) = L(M_1) \cdot L(M_2)$$

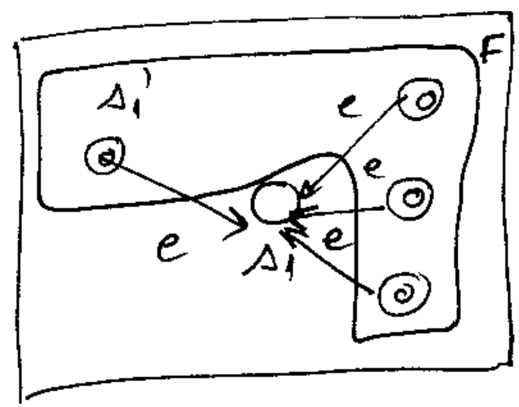
c) <u>Kleene Star</u>

$M_1 \rightarrow$ AFN

? M ai $L(M) = L(M_1)^*$







$M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$

$M = (K, \Sigma, \Delta, s, F)$

$K = K_1 \cup \{s_1'\}$

$s = s_1'$

$F = F_1 \cup \{s_1'\}$

$\Delta = \Delta_1 \cup (F \times \{e\} \times \{s_1\}) \cup \{(s_1', e, s_1)\}$.

$w \in L(M)$, fie $w = e$, fie $w = w_1 \circ \dots \circ w_k$, $k \geq 1$.

$i = 1, .., k$, $\exists f_i \in F$ a.î. $(s_1, w_i) \vdash^{*}_{M_1} (f_i, e)$; $\Rightarrow w \in L(M_1^{*})$.

$w \in L(M_1)^{*}$, atunci fie $w = e$ sau $w = w_1 \cdots w_k$, $w_1, .., w_k \in L(M_1)$

$w = e \Rightarrow w \in L(M)$ pt că s_1' este st. finală

$w \in L(M)$ pt că $f_1, .., f_k \in F_1$

$(s_1', w_1 \cdots w_k) \vdash^{*}_{M} (\cdot\cdot \vdash^{*}_{M} (f_k, e) \Rightarrow L(M) = L(M_1)^{*}$.

d) Complementare

$M = (K, \Sigma, \delta, s, F)$ AFD.

$\Sigma^{*} - L(M)$ acceptat de AFD $\bar{M} = (K, \Sigma, \delta, s, K - F)$

e) Intersecție

L_1, L_2 a.î. $L_1 = L(M_1)$, $L_2 = L(M_2)$

$$L_1 \cap L_2 = \underbrace{\Sigma^* - \underbrace{\left(\underbrace{(\Sigma^* - L_1)}_{AFD} \cup \underbrace{(\Sigma^* - L_2)}_{AFD}\right)}_{\substack{AFN \\ \downarrow \\ AFD}}}_{AFD}.$$

Construcție directă

$M_1 = (K_1, \Sigma, \delta_1, s_1, F_1)$

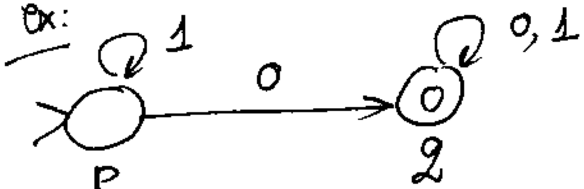
$M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$

$M = (K_1 \times K_2, \Sigma, \delta, (s_1, s_2), F_1 \times F_2)$

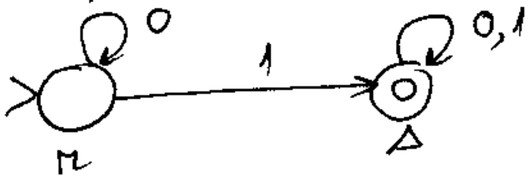
$$\delta((p,q),a) = (\delta_1(p,a), \delta_2(q,a))$$

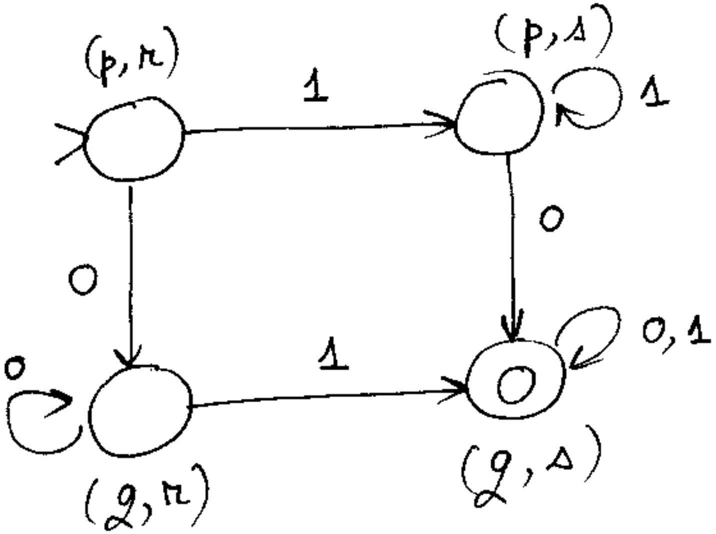
Ex:

M_1:



M_2:





Proprietăți algoritmice

Teorema

Există algoritmi care pot răspunde la urm. întrebări referitoare la L(A.F.):

a) Dat fiind un A.F.(M) și un șir w, este $w \in L(M)$?

b) A.F. M, $L(M) = \emptyset$?

c) A.F. M, $L(M) = \Sigma^*$?

d) A.F. M_1, M_2, $L(M_1) \subseteq L(M_2)$?

e) A.F. M_1, M_2, $L(M_1) = L(M_2)$?

dem. pp. M → AFD.

a) trasez acțiunile lui M pt șirul w, în |w| pași.

b) verificând dc ∃ o secvență de săgeți din st. init într-o st finală

c) construiesc M' a.î. $L(M') = \Sigma^* - L(M)$ și verific $L(M') = \emptyset$

d) $(\Sigma^* - L(M_2)) \cap L(M_1) = \emptyset$