

### Problem 5:

- a. Explain the rationale for sparse decomposition algorithms:
- b. Describe & compare results in Figures 1 and 6

- a. The canonical decompositions can be written as:

$$\det(\lambda B - A) = 0$$

Then, we compute extremal generalized eigenvalues using the following forms:

$$\lambda^{max}(A, B) = \max \frac{x^T A x}{x^T B x}$$

Then, we try to maximize the ratio. In the same time, we constrain the cardinality of the coefficient vector  $x$ . Mathematical formula could be described as:

$$\begin{aligned} & \text{maximize } x^T A x / x^T B x \\ & \text{subject to } \text{Card}(x) < k \end{aligned}$$

where  $k > 0$  is a given constant and  $\text{Card}(x)$  is the number of nonzero coefficients in  $x$ . This is sparse decomposition algorithm.

- b. The first figure uses Box-Tiao decomposition while Figures 6 uses sparse canonical decomposition.