HW4

December 11, 2018

0.0.1 Problem1

Step1

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        from scipy.optimize import curve_fit
        from scipy.optimize import least_squares, leastsq
        from scipy.optimize import minimize
        import statsmodels.api as sm
        from sklearn.metrics import mean_squared_error
```

```
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/compat/pandas.py:56: FutureW
  from pandas.core import datetools
```

```
In [2]: cmt = pd.read_csv('CMT_Rates.csv', header = 0)
        cmt = cmt.dropna()
```

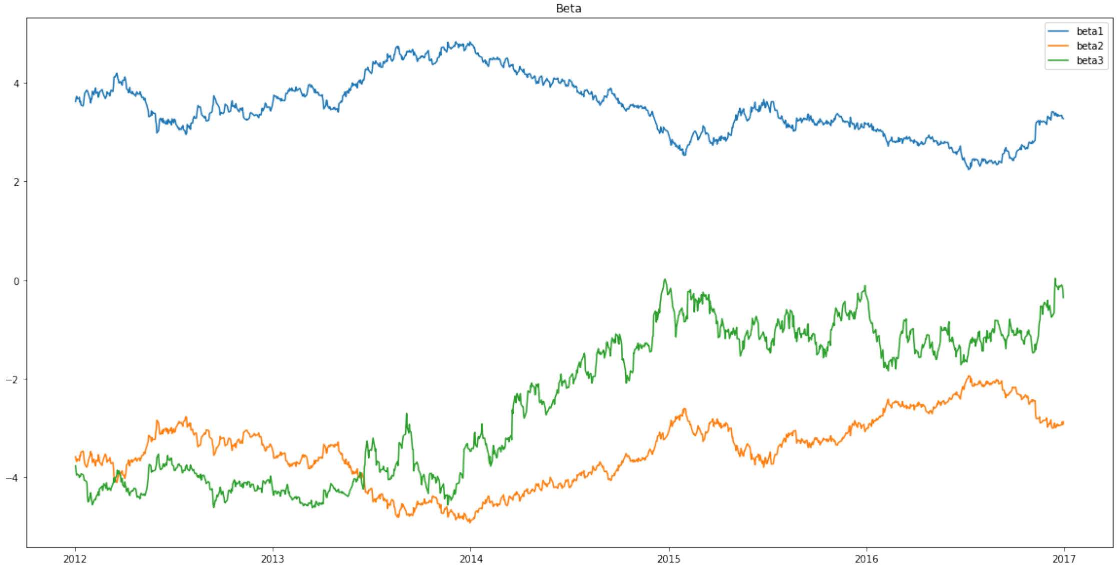
```
In [3]: cmt['Date'] = pd.to_datetime(cmt.Date,format='%y/%m/%d')
        cmt = cmt.set_index('Date')
```

```
In [4]: def Nelson_Siegel(t, b1, b2, b3):
            return b1+b2*((1-np.exp(-0.4*t))/(0.4*t))+b3*((1-np.exp(-0.4*t))/(0.4*t)-np.exp(-0
```

```
In [5]: maturity=[1/12, 3/12, 6/12, 1, 2, 3, 5, 7, 10, 20, 30]
        optimal_parameter = []
        initial_b=[4, -3, 4]
        for i in range(cmt.shape[0]):
            popt, pcov=curve_fit(Nelson_Siegel, maturity, cmt.iloc[i,:])
            optimal_parameter.append([popt[0],popt[1],popt[2]])
        optimal_parameter = pd.DataFrame(optimal_parameter, index = cmt.index)
```

```
In [6]: plt.figure(figsize=(20,10))
        plt.plot(optimal_parameter.iloc[:,0])
        plt.plot(optimal_parameter.iloc[:,1])
        plt.plot(optimal_parameter.iloc[:,2])
        plt.title('Beta')
        plt.legend(['beta1','beta2','beta3'])
```

```
Out[6]: <matplotlib.legend.Legend at 0x10dc2bf28>
```



Step 2

```
In [7]: def model(lam):
            maturity=[1/12, 3/12, 6/12, 1, 2, 3, 5, 7, 10, 20, 30]
            b1 = [1 for i in range(len(maturity))]
            b2 = [(1-np.exp(-lam*tau))/(lam*tau) for tau in maturity]
            b3 = [(1-np.exp(-lam*tau))/(lam*tau)-np.exp(-lam*tau) for tau in maturity]
            beta = []
            x = np.array([b1,b2,b3]).T
            for i in range(len(cmt)):
                y = np.array(cmt.iloc[i,:])
                fitted = sm.OLS(y,x).fit()
                beta.append([fitted.params[0],fitted.params[1],fitted.params[2]])
            beta = np.array(beta)
            return sum(sum((np.dot(beta,x.T)-cmt.values)**2))
```

```
In [8]: res = minimize(model,0.036)
```

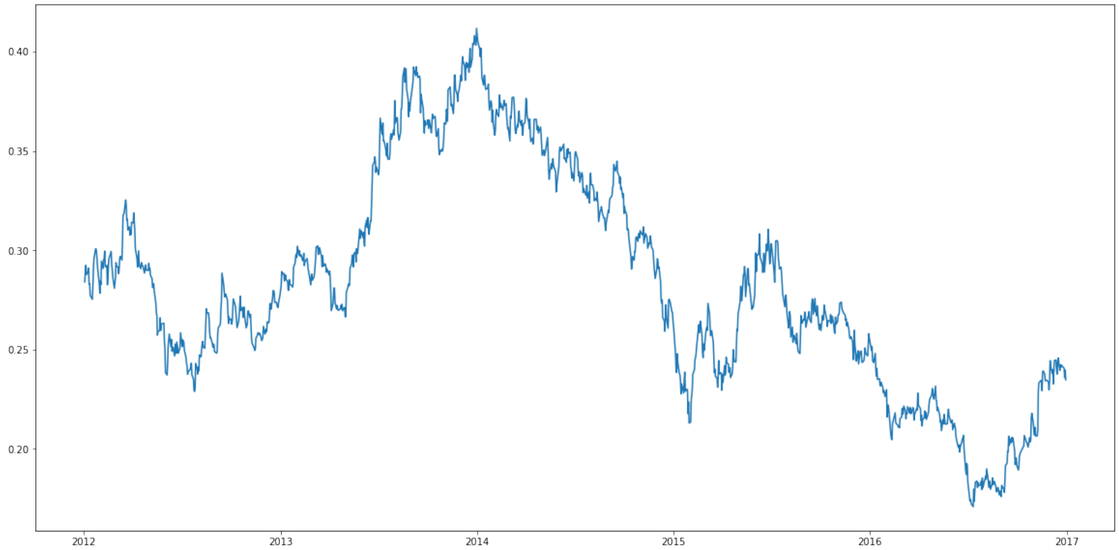
```
In [9]: print ("Optimize Lambda is:",res.x)
        print ("Optimization condition:",res.success)
```

```
Optimize Lambda is: [0.45161825]
Optimization condition: True
```

0.1 Problem2

Step1

```
In [10]: def Nelson_Siegel1(t, b1, b2, b3, lamd):
             return b1+b2*((1-np.exp(-lamd*t))/(0.4*t))+b3*((1-np.exp(-lamd*t))/(lamd*t)-np.ex

In [11]: def model(lam):
             maturity=[1/12, 3/12, 6/12, 1, 2, 3, 5, 7, 10, 20, 30]
             b1 = [1 for i in range(len(maturity))]
             b2 = [(1-np.exp(-lam*tau))/(lam*tau) for tau in maturity]
             b3 = [(1-np.exp(-lam*tau))/(lam*tau)-np.exp(-lam*tau) for tau in maturity]
             x = np.array([b1,b2,b3]).T
             rms = []
             beta = []
             for i in range(len(cmt)):
                 pred = []
                 y = np.array(cmt.iloc[i,:])
                 fitted = sm.OLS(y,x).fit()
                 beta.append([fitted.params[0],fitted.params[1],fitted.params[2]])
                 for t in maturity:
                     pred.append ( Nelson_Siegel1(t, fitted.params[0],fitted.params[1],fitted.
                 pred = np.array(pred)
                 rms.append( np.sqrt(mean_squared_error(cmt.iloc[i,:], pred)))
             rms = np.array(rms)
             beta = np.array(beta)
             return rms, beta

In [12]: lam = res.x

In [13]: rmse, beta = model(lam)
         rmse = pd.DataFrame (rmse, index = cmt.index, columns=['RMSE'] )
         beta = pd.DataFrame(beta, index = cmt.index,columns=['beta1', 'beta2', 'beta3'])

In [14]: plt.figure(figsize=(20,10))
         plt.plot(rmse)

Out[14]: [<matplotlib.lines.Line2D at 0x10d177d30>]
```



```
In [15]: largest_residual20 = rmse.sort_values(by='RMSE', ascending=False).iloc[:20, 0]
         largest_residual20
```

```
Out[15]: Date
         2013-12-31    0.411705
         2013-12-27    0.408053
         2014-01-02    0.405768
         2014-01-03    0.404275
         2013-12-24    0.404112
         2013-12-26    0.403409
         2013-12-30    0.403159
         2014-01-08    0.401735
         2013-12-19    0.401565
         2014-01-06    0.401154
         2013-12-05    0.397451
         2014-01-07    0.397451
         2013-12-18    0.396845
         2013-12-23    0.396413
         2014-01-09    0.396387
         2013-12-06    0.395154
         2013-12-12    0.394554
         2013-12-16    0.394016
         2013-12-09    0.392777
         2013-09-10    0.392322
         Name: RMSE, dtype: float64
```

Step 2

```
In [16]: import statsmodels.tsa.arima_model as ar
In [17]: import datetime
         def getday(date,n=0):
             result_date = date + datetime.timedelta(days=n)
             d = result_date.strftime('%Y-%m-%d')
             return d
```

```
In [18]: def arima_forcast(date_, beta_, forecast_days):
             begin_date = getday(date_, -180)
             pos = np.where(beta.index == date)[0][0]
             forecast = {}
             hl = {}
             for i in range(1,4):
                 model = ar.ARIMA(beta.loc[begin_date:date,'beta'+str(i)], order=(1,0,0))
                 model_fit = model.fit()
                 forecast['beta'+str(i)]= model_fit.forecast(steps=forecast_days)[0]
                 hl['beta'+str(i)] = -np.log(2)/np.log(model_fit.arparams[0])
             index = beta_.ix[(pos+1):(pos+forecast_days+1)].index
             forecast = pd.DataFrame(forecast,index = index)
             hl = pd.DataFrame(hl, index = ['half_life'])
             return forecast, hl
```

```
In [19]: # t+5d forcast
         predicted_beta5 = []
         day_index5 = []
         for date in largest_residual20.index:
             a, b=arima_forcast(date, beta, 5)
             predicted_beta5.append(list(a.iloc[4:].values[0]))
             day_index5.append(a.iloc[4:].index.values[0])
             #print("For day"+str(date), "the fifth predicted value is\n", a.iloc[4,:])
             print("For day"+str(date), "the half life are\n", b)

         predicted_beta5_ = np.array(predicted_beta5)
         predicted_beta5_ = pd.DataFrame(predicted_beta5_, index = day_index5,columns = ['beta
         predicted_beta5_.head()
```

```
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/scipy/signal/signaltools.py:1333: Futur
  out_full[ind] += zi
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/scipy/signal/signaltools.py:1336: Futur
  out = out_full[ind]
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/scipy/signal/signaltools.py:1342: Futur
  zf = out_full[ind]
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/kalmanf/kalmanfilter.py
  if issubdtype(paramsdtype, float):
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/kalmanf/kalmanfilter.py
  elif issubdtype(paramsdtype, complex):
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/kalmanf/kalmanfilter.py
  if issubdtype(paramsdtype, float):
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:11: DeprecationWa
```

```
.ix is deprecated. Please use
.loc for label based indexing or
.iloc for positional indexing

See the documentation here:
http://pandas.pydata.org/pandas-docs/stable/indexing.html#ix-indexer-is-deprecated
  # This is added back by InteractiveShellApp.init_path()


For day2013-12-31 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.361018  10.394945  16.965017
For day2013-12-27 00:00:00 the half life are
                beta1      beta2      beta3
half_life  13.175187  12.074739  15.860641
For day2014-01-02 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.276995  10.320815  15.656381
For day2014-01-03 00:00:00 the half life are
                beta1     beta2     beta3
half_life  10.491494  10.54302  15.99716
For day2013-12-24 00:00:00 the half life are
                beta1      beta2     beta3
half_life  11.560023  10.967468  15.35631
For day2013-12-26 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.278896  11.208453  16.569384
For day2013-12-30 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.996071  10.200774  15.990505
For day2014-01-08 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.740414  10.701328  17.753335
For day2013-12-19 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.534579  10.876356  17.202158
For day2014-01-06 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.536503  10.002894  15.553938
For day2013-12-05 00:00:00 the half life are
                beta1      beta2      beta3
half_life  27.108613  24.894814  14.191517
For day2014-01-07 00:00:00 the half life are
               beta1      beta2    beta3
half_life  9.685462  10.392906  15.6113
For day2013-12-18 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.519357  10.578365  16.295553
```

```
For day2013-12-23 00:00:00 the half life are
              beta1     beta2      beta3
half_life  10.4364  9.602108  15.713081
For day2014-01-09 00:00:00 the half life are
               beta1     beta2      beta3
half_life  9.752426  10.65761  18.403797
For day2013-12-06 00:00:00 the half life are
                beta1      beta2      beta3
half_life  25.990918  24.254205  13.627612
For day2013-12-12 00:00:00 the half life are
                beta1      beta2      beta3
half_life  21.957705  21.375204  13.966335
For day2013-12-16 00:00:00 the half life are
                beta1      beta2      beta3
half_life  17.127394  14.805542  14.562383
For day2013-12-09 00:00:00 the half life are
                beta1      beta2      beta3
half_life  23.415228  21.335849  13.684545
For day2013-09-10 00:00:00 the half life are
                beta1      beta2      beta3
half_life  67.150708  85.906916  33.650859
```

```
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/kalmanf/kalmanfilter.py
  R_mat, T_mat)
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:584: Runtim
  newparams = ((1-np.exp(-params))/
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:585: Runtim
  (1+np.exp(-params))).copy()
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:585: Runtim
  (1+np.exp(-params))).copy()
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:586: Runtim
  tmp = ((1-np.exp(-params))/
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:587: Runtim
  (1+np.exp(-params))).copy()
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/statsmodels/tsa/tsatools.py:587: Runtim
  (1+np.exp(-params))).copy()
```

```
Out[19]:                beta1     beta2     beta3
         2014-01-08  4.691561 -4.729873 -4.101926
         2014-01-06  4.671607 -4.709437 -4.054083
         2014-01-09  4.652669 -4.688409 -4.096298
         2014-01-10  4.655392 -4.689507 -4.046895
         2014-01-02  4.623972 -4.661211 -3.995854
```

```
In [20]: # t+10 forecast
         predicted_beta10 = []
```

```
day_index10 = []
for date in largest_residual20.index:
    a, b=arima_forcast(date, beta, 10)
    predicted_beta10.append(list(a.iloc[9:].values[0]))
    day_index10.append(a.iloc[9:].index.values[0])
    #print("For day"+str(date), "the predicted value is\n", a)
    print("For day"+str(date),"the half life are\n", b)
predicted_beta10_ = np.array(predicted_beta10)
predicted_beta10_ = pd.DataFrame(predicted_beta10_, index = day_index10,columns = ['be
predicted_beta10_
```

```
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:11: DeprecationWa
.ix is deprecated. Please use
.loc for label based indexing or
.iloc for positional indexing

See the documentation here:
http://pandas.pydata.org/pandas-docs/stable/indexing.html#ix-indexer-is-deprecated
  # This is added back by InteractiveShellApp.init_path()



For day2013-12-31 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.361018  10.394945  16.965017
For day2013-12-27 00:00:00 the half life are
                beta1      beta2      beta3
half_life  13.175187  12.074739  15.860641
For day2014-01-02 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.276995  10.320815  15.656381
For day2014-01-03 00:00:00 the half life are
                beta1     beta2     beta3
half_life  10.491494  10.54302  15.99716
For day2013-12-24 00:00:00 the half life are
                beta1      beta2     beta3
half_life  11.560023  10.967468  15.35631
For day2013-12-26 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.278896  11.208453  16.569384
For day2013-12-30 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.996071  10.200774  15.990505
For day2014-01-08 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.740414  10.701328  17.753335
For day2013-12-19 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.534579  10.876356  17.202158
```

```
For day2014-01-06 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.536503  10.002894  15.553938
For day2013-12-05 00:00:00 the half life are
                beta1      beta2      beta3
half_life  27.108613  24.894814  14.191517
For day2014-01-07 00:00:00 the half life are
               beta1      beta2    beta3
half_life  9.685462  10.392906  15.6113
For day2013-12-18 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.519357  10.578365  16.295553
For day2013-12-23 00:00:00 the half life are
              beta1     beta2      beta3
half_life  10.4364  9.602108  15.713081
For day2014-01-09 00:00:00 the half life are
               beta1     beta2      beta3
half_life  9.752426  10.65761  18.403797
For day2013-12-06 00:00:00 the half life are
                beta1      beta2      beta3
half_life  25.990918  24.254205  13.627612
For day2013-12-12 00:00:00 the half life are
                beta1      beta2      beta3
half_life  21.957705  21.375204  13.966335
For day2013-12-16 00:00:00 the half life are
                beta1      beta2      beta3
half_life  17.127394  14.805542  14.562383
For day2013-12-09 00:00:00 the half life are
                beta1      beta2      beta3
half_life  23.415228  21.335849  13.684545
For day2013-09-10 00:00:00 the half life are
                beta1      beta2      beta3
half_life  67.150708  85.906916  33.650859
```

```
Out[20]:                beta1     beta2     beta3
         2014-01-15  4.639352 -4.668162 -4.124158
         2014-01-13  4.624850 -4.650091 -4.090278
         2014-01-16  4.609061 -4.635510 -4.126866
         2014-01-17  4.612223 -4.637737 -4.084395
         2014-01-09  4.582391 -4.606944 -4.047423
         2014-01-10  4.594744 -4.621574 -4.011962
         2014-01-14  4.587770 -4.606372 -4.094529
         2014-01-23  4.580240 -4.630762 -3.869802
         2014-01-06  4.599239 -4.602876 -4.273602
         2014-01-21  4.591311 -4.625101 -4.091238
         2013-12-19  4.663831 -4.639762 -4.630481
         2014-01-22  4.573454 -4.609116 -4.057913
         2014-01-03  4.600793 -4.583699 -4.480525
         2014-01-08  4.540170 -4.556632 -4.051650
         2014-01-24  4.554945 -4.604296 -3.840513
         2013-12-20  4.636423 -4.616461 -4.556093
         2013-12-27  4.620496 -4.614596 -4.463193
         2013-12-31  4.605286 -4.591647 -4.436635
         2013-12-23  4.612346 -4.585948 -4.543267
         2013-09-24  4.548485 -4.648497 -3.643910
```

```
In [22]: # t+1m forecast
         predicted_beta1m = []
         day_index1m = []
         for date in largest_residual20.index:
             a, b=arima_forcast(date, beta, 30)
             predicted_beta1m.append(list(a.iloc[29:].values[0]))
             day_index1m.append(a.iloc[29:].index.values[0])
             print("For day"+str(date),"the half life are\n", b)
         predicted_beta1m_ = np.array(predicted_beta1m)
         predicted_beta1m_ = pd.DataFrame(predicted_beta1m_, index = day_index1m,columns = ['b
         predicted_beta1m_.head()
```

```
/Users/lijinglan/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:11: DeprecationWa
.ix is deprecated. Please use
.loc for label based indexing or
.iloc for positional indexing

See the documentation here:
http://pandas.pydata.org/pandas-docs/stable/indexing.html#ix-indexer-is-deprecated
  # This is added back by InteractiveShellApp.init_path()


For day2013-12-31 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.361018  10.394945  16.965017
For day2013-12-27 00:00:00 the half life are
                beta1      beta2      beta3
half_life  13.175187  12.074739  15.860641
For day2014-01-02 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.276995  10.320815  15.656381
For day2014-01-03 00:00:00 the half life are
                beta1     beta2     beta3
half_life  10.491494  10.54302  15.99716
For day2013-12-24 00:00:00 the half life are
                beta1      beta2     beta3
half_life  11.560023  10.967468  15.35631
For day2013-12-26 00:00:00 the half life are
                beta1      beta2      beta3
```

```
half_life  12.278896  11.208453  16.569384
For day2013-12-30 00:00:00 the half life are
                beta1      beta2      beta3
half_life  10.996071  10.200774  15.990505
For day2014-01-08 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.740414  10.701328  17.753335
For day2013-12-19 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.534579  10.876356  17.202158
For day2014-01-06 00:00:00 the half life are
               beta1      beta2      beta3
half_life  9.536503  10.002894  15.553938
For day2013-12-05 00:00:00 the half life are
                beta1      beta2      beta3
half_life  27.108613  24.894814  14.191517
For day2014-01-07 00:00:00 the half life are
               beta1      beta2    beta3
half_life  9.685462  10.392906  15.6113
For day2013-12-18 00:00:00 the half life are
                beta1      beta2      beta3
half_life  12.519357  10.578365  16.295553
For day2013-12-23 00:00:00 the half life are
              beta1     beta2      beta3
half_life  10.4364  9.602108  15.713081
For day2014-01-09 00:00:00 the half life are
               beta1     beta2      beta3
half_life  9.752426  10.65761  18.403797
For day2013-12-06 00:00:00 the half life are
                beta1      beta2      beta3
half_life  25.990918  24.254205  13.627612
For day2013-12-12 00:00:00 the half life are
                beta1      beta2      beta3
half_life  21.957705  21.375204  13.966335
For day2013-12-16 00:00:00 the half life are
                beta1      beta2      beta3
half_life  17.127394  14.805542  14.562383
For day2013-12-09 00:00:00 the half life are
                beta1      beta2      beta3
half_life  23.415228  21.335849  13.684545
For day2013-09-10 00:00:00 the half life are
                beta1      beta2      beta3
half_life  67.150708  85.906916  33.650859
```

```
Out[22]:                beta1     beta2     beta3
         2014-02-13  4.542405 -4.553309 -4.178923
         2014-02-11  4.523714 -4.528214 -4.176640
```

```
2014-02-14  4.528549 -4.537549 -4.199340
2014-02-18  4.531361 -4.540435 -4.174245
2014-02-07  4.499301 -4.502175 -4.168520
```

Step 3

```
In [23]: maturity=[1/12, 3/12, 6/12, 1, 2, 3, 5, 7, 10, 20, 30]
         b1 = [1 for i in range(len(maturity))]
         b2 = [(1-np.exp(-lam*tau))/(lam*tau) for tau in maturity]
         b3 = [(1-np.exp(-lam*tau))/(lam*tau)-np.exp(-lam*tau) for tau in maturity]
         x = np.array([b1,b2,b3]).T
```

```
In [24]: #calculate the forecast error of predicted value of one day
         def forecastError_RM(cmt_, date_, forecast_days):
             pos = np.where(cmt_.index == date_)[0][0]
             y = cmt.iloc[(pos+forecast_days),:].values
             y_fit = cmt.iloc[pos,:].values
             squared_error = (y-y_fit)**2
             return np.sqrt(squared_error.sum()/11)
```

```
In [25]: #calculate the forecast error of predicted value of one day
         def forecastError_AR(cmt_, factor_loading, date_, predicted_beta, forecast_days):
             pos = np.where(cmt_.index == date_)[0][0]
             y = cmt.iloc[(pos+forecast_days),:].values
             y_fit = np.sum(factor_loading*predicted_beta, axis=1)

             squared_error = (y-y_fit)**2
             return np.sqrt(squared_error.sum()/11)
```

```
In [26]: # t+5d total everage forcast error
         total_error_ar=0
         total_error_rm = 0
         for i, date in enumerate(largest_residual20.index):
             total_error_ar += forecastError_AR(cmt, x, date, predicted_beta5[i], 5)
             total_error_rm += forecastError_RM(cmt, date, 5)
         avg_ar = total_error_ar/20
         avg_rm = total_error_rm/20
         print('The total forcast error for the fifth prediction of ar model is ', avg_ar)
         print('The total forcast error for the fifth prediction of random walk model is ', av
```

```
The total forcast error for the fifth prediction of ar model is  0.12132781604010105
The total forcast error for the fifth prediction of random walk model is  0.05275015463104824
```

```
In [27]: # t+10d total everage forcast error
         total_error_ar=0
         total_error_rm = 0
         for i, date in enumerate(largest_residual20.index):
             total_error_ar += forecastError_AR(cmt, x, date, predicted_beta10[i], 10)
```

```
                 total_error_rm += forecastError_RM(cmt, date, 10)
             avg_ar = total_error_ar/20
             avg_rm = total_error_rm/20
             print('The total forcast error for the fifth prediction of ar model is ', avg_ar)
             print('The total forcast error for the fifth prediction of random walk model is ', avg
```

```
The total forcast error for the fifth prediction of ar model is  0.1348586119031539
The total forcast error for the fifth prediction of random walk model is  0.0943249967135477
```

```
In [28]: # t+1m total everage forcast error
         total_error_ar=0
         total_error_rm = 0
         for i, date in enumerate(largest_residual20.index):
             total_error_ar += forecastError_AR(cmt, x, date, predicted_beta1m[i], 30)
             total_error_rm += forecastError_RM(cmt, date, 30)
         avg_ar = total_error_ar/20
         avg_rm = total_error_rm/20
         print('The total forcast error for the fifth prediction of ar model is ', avg_ar)
         print('The total forcast error for the fifth prediction of random walk model is ', avg
```

```
The total forcast error for the fifth prediction of ar model is  0.11838900243015607
The total forcast error for the fifth prediction of random walk model is  0.15947229619269637
```

From the above results, we could see that when the prediction date is far away from the current day, the random walk model has a poorer performance than the AR model. On the contrary, random walk model have a better performance than the AR model.

```
In [ ]:
```