Complete Root Locus Analysis Report

Generated by Root Locus Analyzer

July 21, 2025

1 System Transfer Function

The analyzed open-loop transfer function is:

$$G(s) = \frac{s^2 + 4.0 \cdot s + 5.0}{s^4 + 9.0 \cdot s^3 + 31.0 \cdot s^2 + 59.0 \cdot s + 60.0}$$

2 Poles and Zeros

System Poles:

$$p_1 = -4.0$$

$$p_2 = -3.0$$

$$p_3 = -1.0 + 2.0 \cdot j$$

$$p_4 = -1.0 - 2.0 \cdot j$$

System Zeros:

$$z_1 = -2.0 + 1.0 \cdot j$$

$$z_2 = -2.0 - 1.0 \cdot j$$

Number of branches ($\#r$) = Number of poles ($\#p$) = 4.

3 Detailed Analysis

3.1 Asymptote Analysis

With $\#p = 4$ and $\#z = 2$, we have $q = \#p - \#z = 2$ asymptotes.

$$\sigma_a = \frac{\sum p_i - \sum z_i}{\#p - \#z}$$

$$\rightarrow \sigma_a = \tfrac{-9.0-(-4.0)}{2} = -2.5000$$

The angles are given by $\theta_{a,k} = \frac{(2k+1)180°}{q}$:

$$k = 0: \quad \theta_{a,0} = \frac{(2 \cdot 0 + 1)180°}{2} = 90.0000°$$

$$k = 1: \quad \theta_{a,1} = \frac{(2 \cdot 1 + 1)180°}{2} = 270.0000°$$

3.2 Departure and Arrival Angles

These angles indicate the direction of the locus as it leaves a complex pole or arrives at a complex zero.

3.2.1 Angles of Departure

$$\theta_{p1} = 127.8750°$$

$$\theta_{p2} = 232.1250°$$

3.2.2 Angles of Arrival

$$\theta_{z1} = 135.0000°$$

$$\theta_{z2} = 225.0000°$$

3.3 Breakaway/Break-in Points

These points are found by solving $\frac{dK}{ds} = 0$. This yields the following polynomial equation:

$$-2.0 \cdot s^5 - 21.0 \cdot s^4 - 92.0 \cdot s^3 - 200.0 \cdot s^2 - 190.0 \cdot s - 55.0 = 0$$

The roots of this equation are the potential points:

$$s = -3.445$$

$$s = -1.471$$

$$s = -0.495$$

$$s = -2.544 - 2.118 \cdot j$$

$$s = -2.544 + 2.118 \cdot j$$

3.4 Imaginary Axis Crossing

The Routh-Hurwitz criterion is applied to the characteristic polynomial $1 + KG(s) = 0$.

$$5.0 \cdot K + s^4 + 9.0 \cdot s^3 + s^2 \cdot (K + 31.0) + s \cdot (4.0 \cdot K + 59.0) + 60.0 = 0$$

The Routh Table is:

s^4	1.0	$K + 31.0$	$5.0 \cdot K + 60.0$
s^3	9.0	$4.0 \cdot K + 59.0$	0
s^2	$0.5556 \cdot K + 24.44$	$5.0 \cdot K + 60.0$	0
s^1	$\frac{2.222 \cdot K^2 + 85.56 \cdot K + 902.2}{0.5556 \cdot K + 24.44}$	0	0
s^0	$5.0 \cdot K + 60.0$	0	0

No positive critical gain K was found.

4 Root Locus Plot

5 Angle Calculation Visualizations

The following plots detail the calculation for each departure and arrival angle.

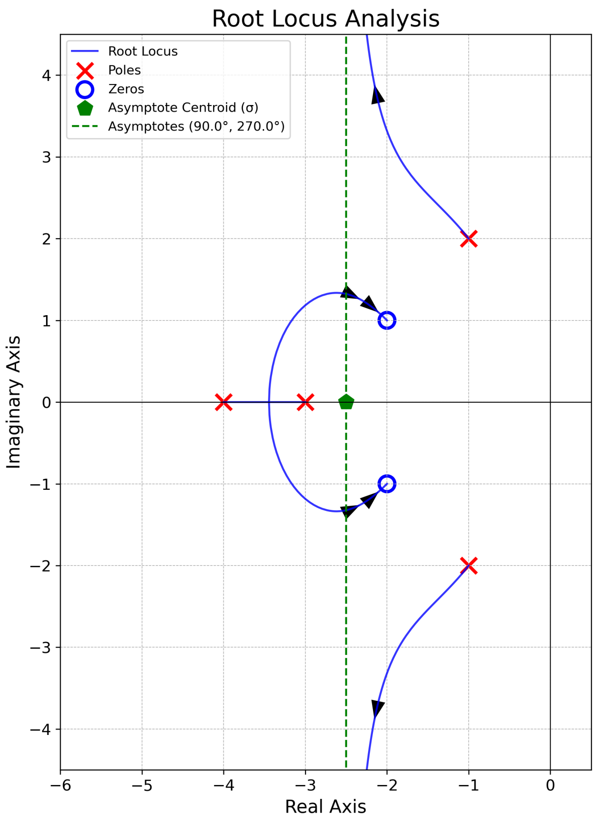
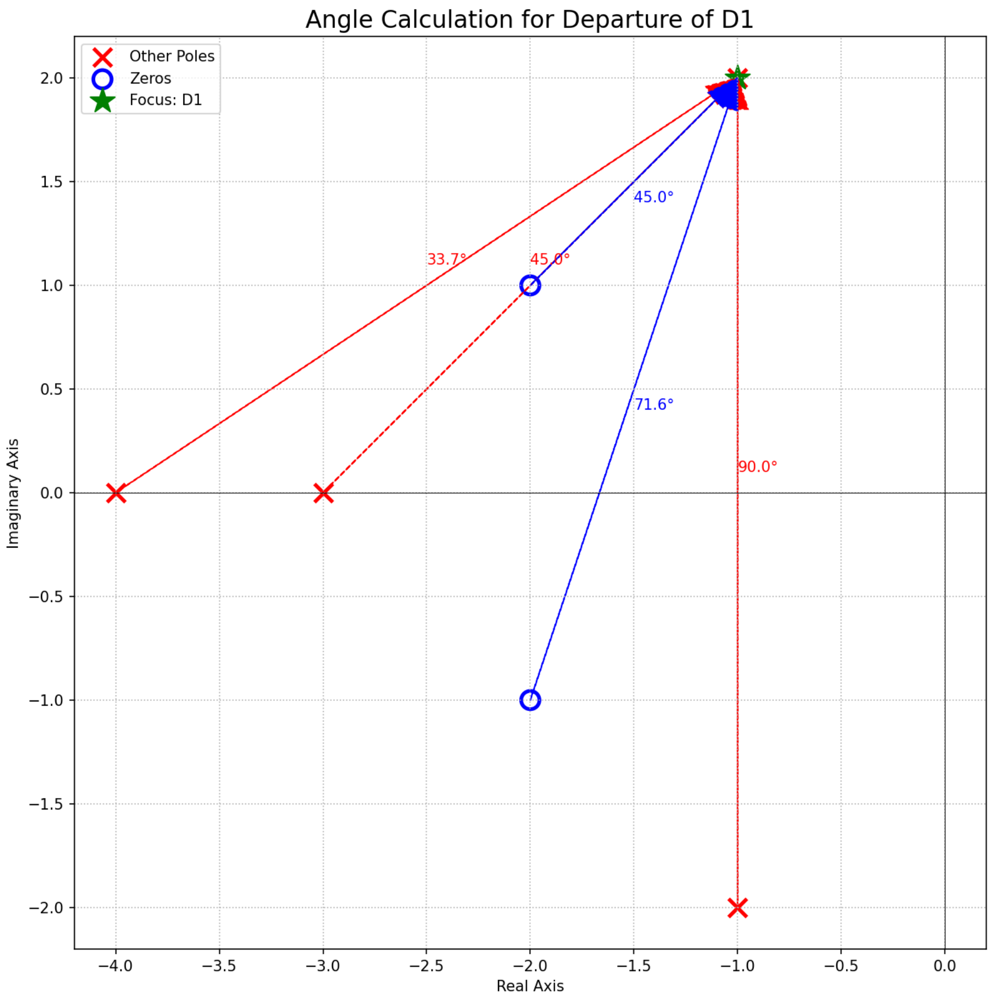


Figure 1: Complete Root Locus plot including poles, zeros, and asymptotes.



$$\theta_{p1} = 180° - (\sum\theta_p - \sum\theta_z)$$

$$\theta_{p1} = 180° - (168.7° - 116.6°) = 127.87°$$

Figure 2: Visualization of the angle calculation for the departure of pole p1.

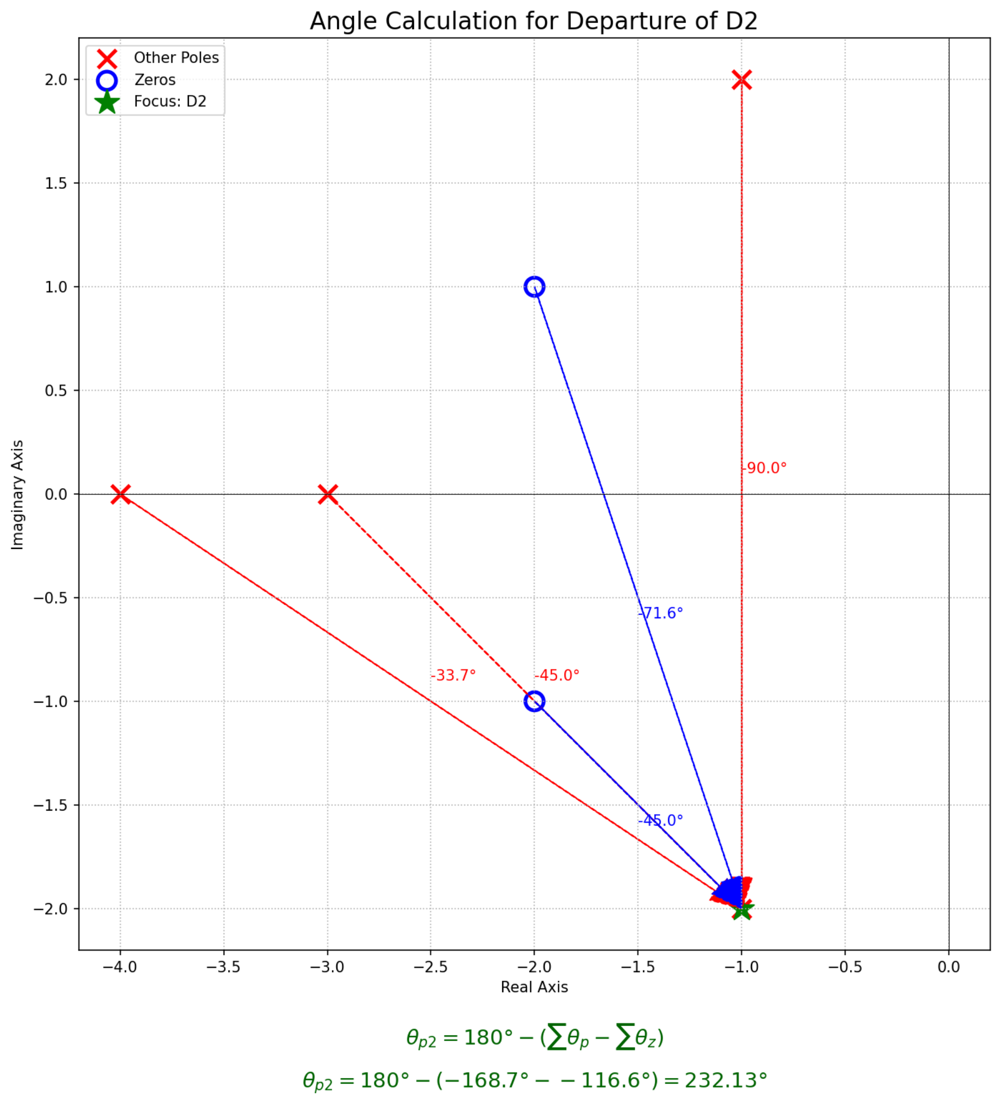
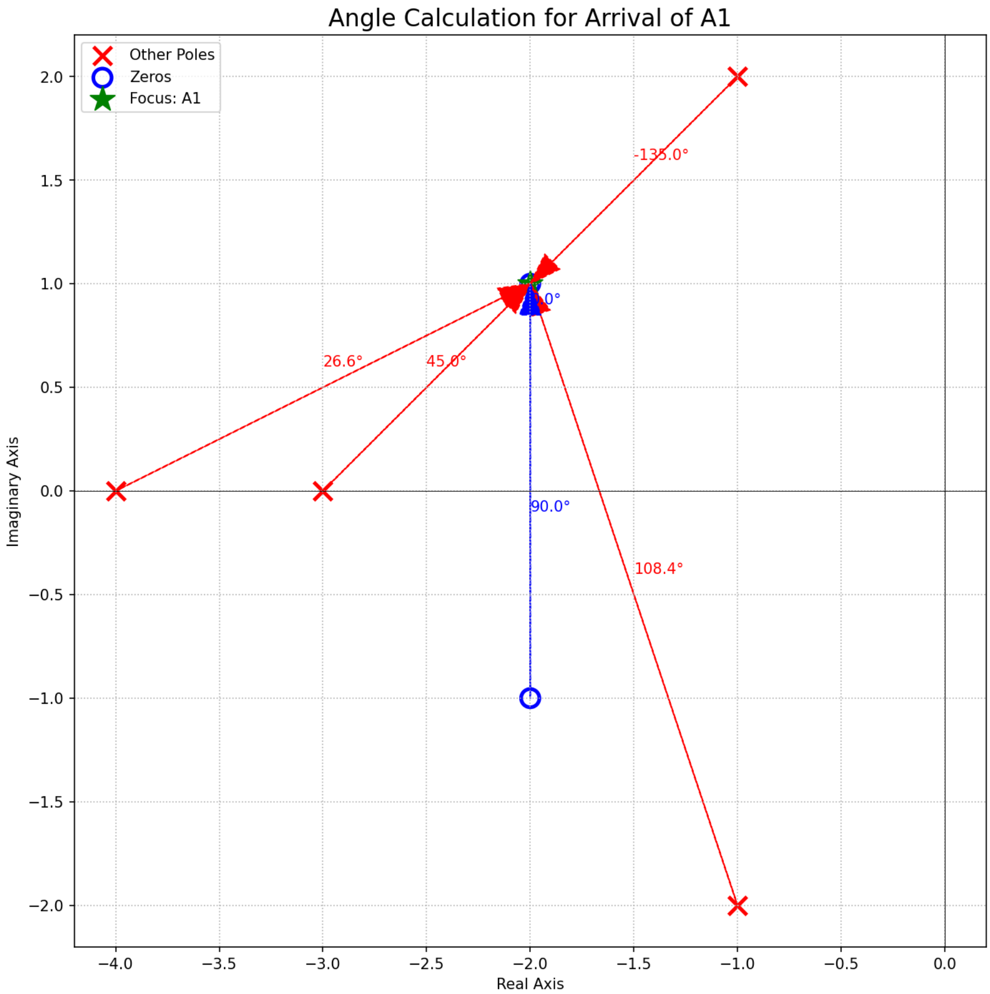


Figure 3: Visualization of the angle calculation for the departure of pole p2.



$$\theta_{z1} = 180° - (\sum \theta_z - \sum \theta_p)$$

$$\theta_{z1} = 180° - (90.0° - 45.0°) = 135.00°$$

Figure 4: Visualization of the angle calculation for the arrival of zero z1.

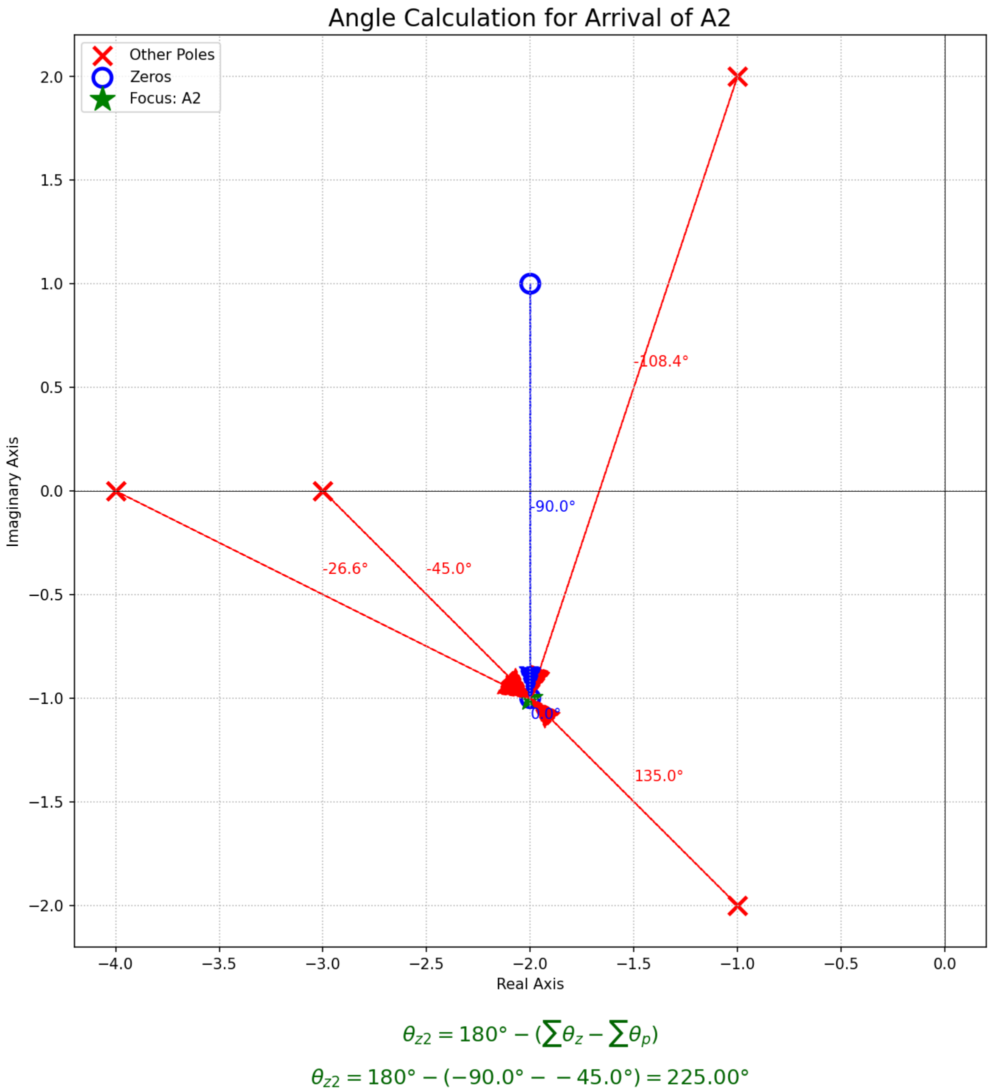


Figure 5: Visualization of the angle calculation for the arrival of zero z2.