Stochastic Processes and Application
Assignment - 1

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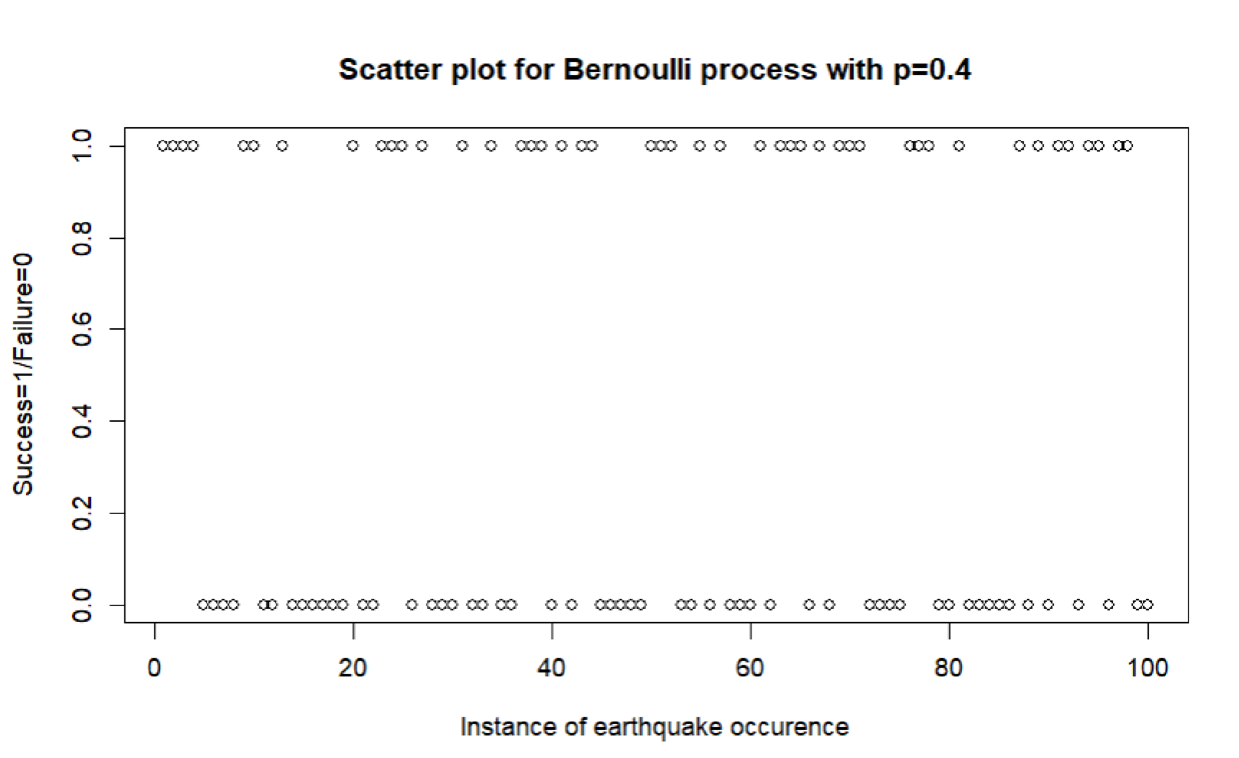
Q1) a) The study conducted on a particular site to record the magnitude of various earthquakes from 2000-2022 can be modeled as a Bernoulli process.
We can see that the conditions for the Bernoulli process are being satisfied -

1. There are only two possible outcomes for each trial, where each trial means an instance of an earthquake with a magnitude$\geqslant$6, which is a success, and otherwise, it is a failure.
2. It's given that each earthquake is independent of the others.
3. The probability of an earthquake with a magnitude$\geqslant$6 is equal to p for every trial.
4. The process is over discrete time t.
5. The process starts at time t=0, but the occurrence of an earthquake can only occur when t$\geqslant$1

The scatter plot is shown for the Bernoulli process with success probability p=0.4.
X-axis: Instance of occurrence of an earthquake.
Y-axis: Occurrence of success(Z_k=1) or failure(Z_k=0)

Plot:



b) Cumulative distribution of first inter-arrival time for time t

If success occurs at **t=1**

$\{X_1 = 1\} = \{Z_1 = 1\}$

$= \mathbf{p}$

If success occurs at **t=2**

$\{X_1 = 1\} = \{Z_2 = 1\}$

$= \{Z_1 = 0\}.\{Z_2 = 1\}$

$= \mathbf{(1\text{-}p).p}$

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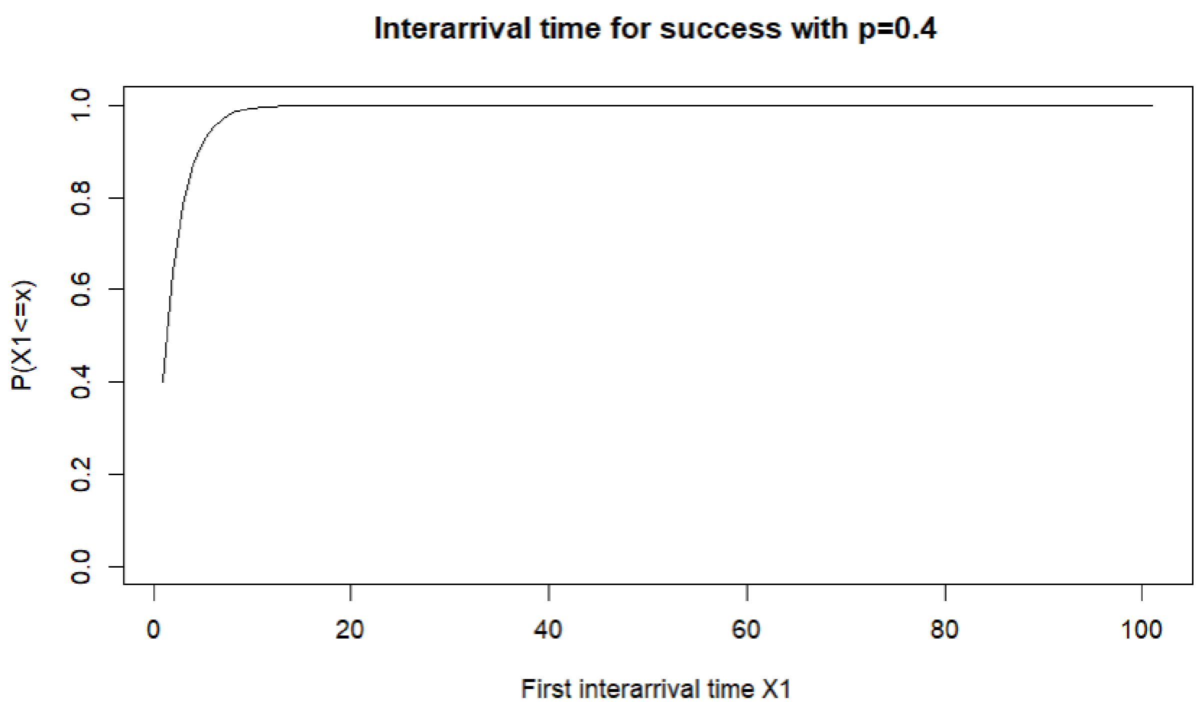
If success occurs at **t=k**

$\{X_1 = k\} = \{Z_k = 1\}$

$= \mathbf{\{1\text{-}p\}^{k\text{-}1},p}$

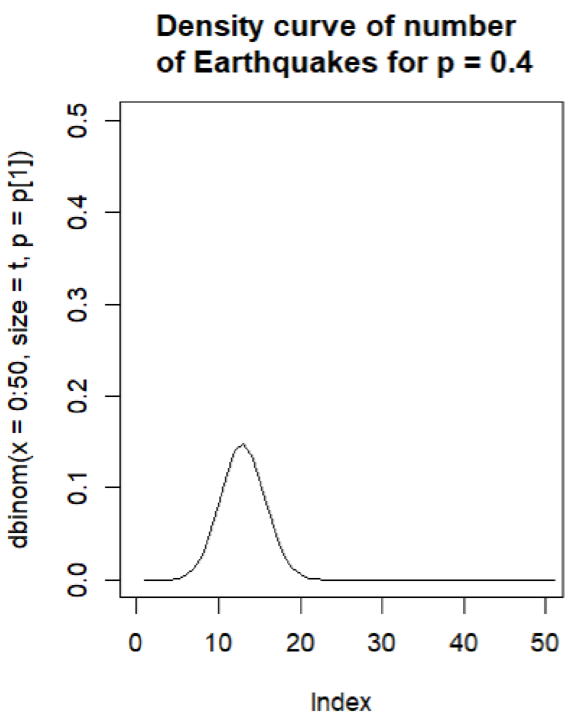
We can see that the first interval time here follows a geometric distribution.

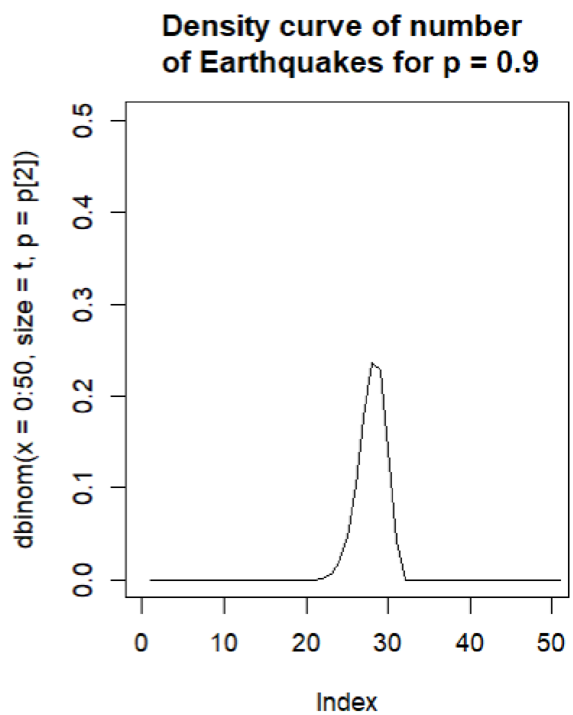
Plot:



c) Since we are studying the Bernoulli process for the number of occurrences of earthquakes till time **k,** it will now follow the binomial distribution.
We can see the variations in the graph with different values of p.

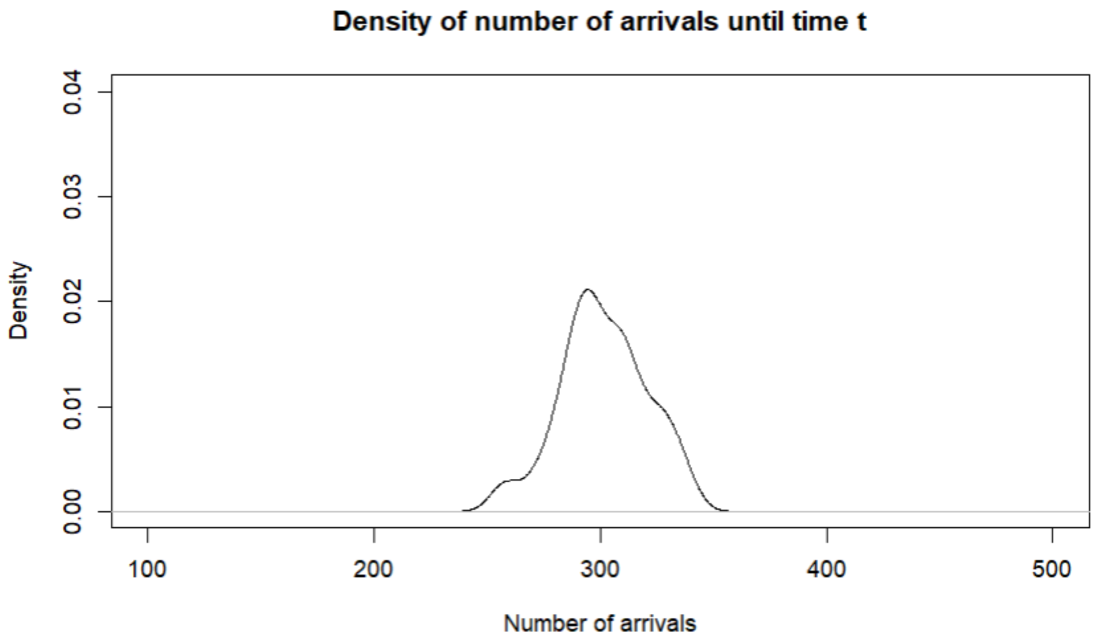
Plot:



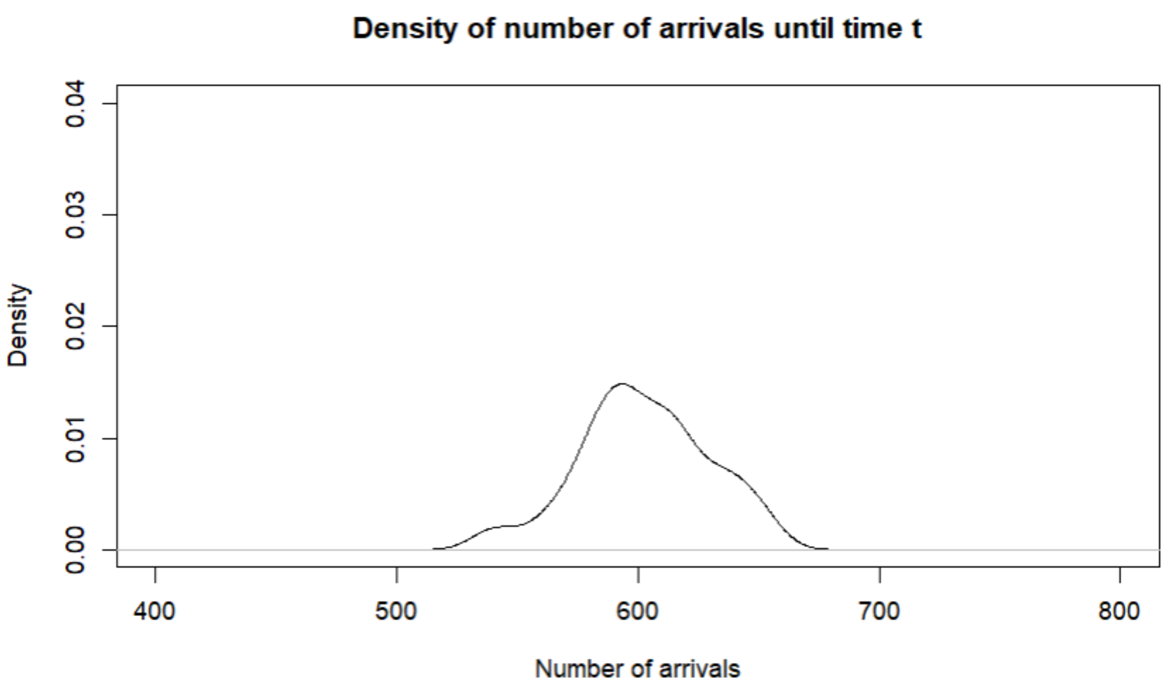


From the above two density curves for two different values of p, we can observe that as the probability increases, the curve shifts to its right, which means the number of successes increases.
Mathematically, the left graph, which is for p=0.4, is at its peak around 0.4*30 = 12, and the right graph is similarly at its peak around 0.9*30 = 27. Also, we can observe that the peak is higher for higher probability, hence more arrivals.
Hence, for p=0.9, the model will produce more arrivals than p=0.4 since the probability is higher.

Q2) a) It is given that the model follows the Poisson process.
Lambda, the average number of visitors per hour is 10.
Assuming **t** to be 30, the following graph is obtained,



b) When lambda = 20, and **t** is the same as in the previous part, we obtain the following graph.



We can see that as the value of lambda increased, the graph shifted to its right, which means the number of arrivals increased.

c) On the first website, the average number of visitors per hour is 10 (λ_1 = 10), and on the second website, which is **independent** of the first one, the average number of visitors per hour is 5 (λ_2 = 10).
Using MGF, we can simplify the model,
Let the total number of visitors be represented by λ.

Now MGF for a single website can be represented by the following

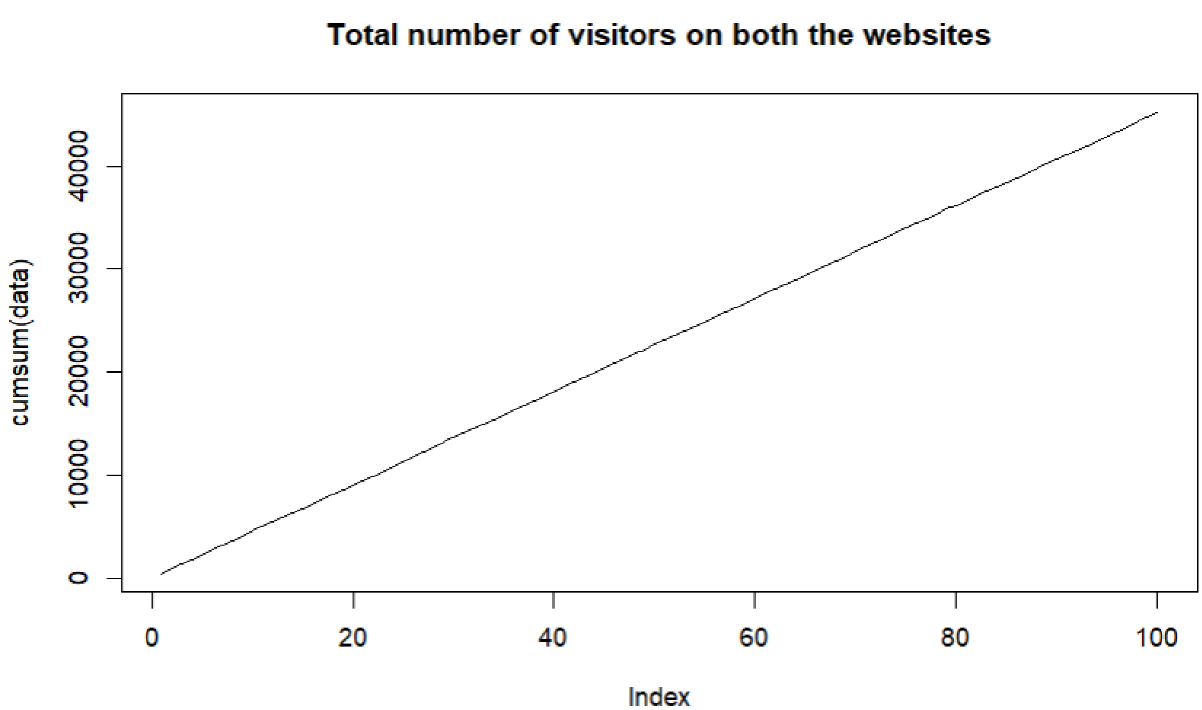
$$M_{N(t)}(s) = e^{\lambda t (e^\wedge s -1)} \qquad -(1)$$

Currently evaluating the MGFs for both websites.

$$\begin{aligned} M_{N(t)}(s) &= E[\, e^{N(t)s}] \\ &= E[\, e^{\{N1(t) + N2(t)\}s}] \\ &= M_{N1(t)}(s) + M_{N2(t)}(s) \\ &= e^{\lambda 1t(e^\wedge s -1)} * e^{\lambda 2t(e^\wedge s -1)} \\ &= e^{(\lambda 1+ \lambda 2)\, t\, (e^\wedge s -1)} \qquad -(2) \end{aligned}$$

Comparing 1 and 2 we get the MGF as $\mathbf{e^{\lambda t (e^\wedge s -1)}}$ where $\boldsymbol{\lambda} =$ $\lambda_1 + \lambda_2$
λ = 10+5
λ = 15

Plot:



Since the model follows the Poisson process, we know that **inter-arrival time** MGF is

$$= (\lambda/(\lambda-s))^n$$

which is also the MGF of the **exponential distribution**.
So the plot with λ = 10+5 will be,

Plot: