ASSIGNMENT 3

Due 3 p.m. on November 27, 2025

Instructions. Hand-in is in electronic form via email to `sztachera@econ.uni-frankfurt.de`. If you submit multiple files, please combine them in a single *.zip* file. List clearly the names and student numbers of the group members. There should be sufficient documentation for third parties to execute the code. In each exercise, make additional assumptions if necessary and explain them.

Exercise 1 (up to Lecture 3) Aiyagari (1994).

Consider the general equilibrium consumption-savings model, as in Aiyagari (1994), featuring uninsurable idiosyncratic income risk. Households solve the following utility maximization problem.

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} \; & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c) \\ \text{s.t. } & c_t + a_{t+1} = w x_t + (1+r) a_t \\ & a_{t+1} \geq 0 \end{aligned}$$

The utility function is of the CRRA form.

$$U(c) = \begin{cases} \log c & \text{if } \sigma = 1 \\ \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \end{cases}$$

Households exogenously supply working hours normalized to unity: $l_t = 1$. Efficiency units of labor x_t follows a Gaussian AR1 process:

$$\log x_t = (1-\rho)\mu_x + \rho \log x_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$$

There is a representative firm operating Cobb-Douglas technology: $Y_t = \theta K_t^{\alpha} L_t^{1-\alpha}$. Firms choose capital and labor. Capital depreciates at a rate δ. Markets clear:

$$\begin{aligned} A_t &= K_t \\ L_t^d &= L_t^s = 1 \\ Y_t &= C_t \end{aligned}$$

Assume the following parameter values: $\beta = 0.96$, $\sigma = 1$, $\rho = 0.9$, $\sigma_\varepsilon = 0.1743$, $\mu_x = -\frac{\sigma_\varepsilon^2}{2(1-\rho^2)}$, $\theta = 1$, $\alpha = 0.36$, $\delta = 0.08$.

a) Write down the household problem in a recursive form. Define the recursive competitive equilibrium of this economy. Explain why it is sufficient to focus on the asset market clearing.

b) Plot capital demand and capital supply on the (K, r) plane. Inspect the plot visually, and state the approximate equilibrium value of capital and the real interest rate. *Hint: To compute capital supply, calculate the wage rate implied by the interest rate r, and solve the household problem given r and w. Compute the capital distribution implied by the resulting policy functions and aggregate $K^s = \int_{\mathcal{A}\times\mathcal{X}} k_t(a, x) d\Theta(a, x)$.*

c) Implement a numerical algorithm finding the model's equilibrium capital and real interest rate. You can base your algorithm on the pseudo-code from Slide 16/38 of Lecture Slides 3.

d) Illustrate your algorithm's convergence path on the capital demand and supply plot, with capital on the x-axis and the real interest rate on the y-axis.

e) Change the relative risk aversion parameters σ to 2. How does the model equilibrium change? Why?

Exercise 2 (up to Lecture 3) **Fiscal Policy in Aiyagari (1994).**

Consider the general equilibrium consumption-savings model, as in Aiyagari (1994), featuring uninsurable idiosyncratic income risk.
The government runs a balanced budget and finances its expenditures through a linear tax rate on factor income.

$$G = Kr\tau + wL\tau$$

Households solve the following utility maximization problem.

$$\begin{aligned}
&\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c) \\
\text{s.t. } & c_t + a_{t+1} = (1-\tau)wx_t + (1 + (1-\tau)r)a_t \\
& a_{t+1} \geq 0 \\
& U(c) = \begin{cases} \log c & \text{if } \sigma = 1 \\ \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \end{cases}
\end{aligned}$$

Households exogenously supply working hours normalized to unity: $l_t = 1$. Efficiency units of labor x_t follows a Gaussian AR1 process:

$$\log x_t = (1-\rho)\mu_x + \rho \log x_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$$

There is a representative firm operating Cobb-Douglas technology: $Y_t = \theta K_t^{\alpha} L_t^{1-\alpha}$. Firms choose capital and labor. Capital depreciates at a rate δ. Markets clear:

$$\begin{aligned}
A_t &= K_t \\
L_t^d &= L_t^s = 1 \\
Y_t &= C_t + G_t
\end{aligned}$$

Assume the following parameter values: $\beta = 0.96$, $\sigma = 1$, $\rho = 0.9$, $\sigma_\varepsilon = 0.1743$, $\mu_x = -\frac{\sigma_\varepsilon^2}{2(1-\rho^2)}$, $\theta = 1$, $\alpha = 0.36$, $\delta = 0.08$, and $G = 0.25$.

a) Write down the household problem in a recursive form. Define the recursive competitive equilibrium of this economy.

b) Derive the first-order and envelope conditions of the recursive household problem.

c) Plot capital demand and capital supply on the (K, r) plane for different values of τ. How does taxation affect the capital market equilibrium? Separately plot the tax rate τ as a function of r (you can back out K from the capital demand schedule).

d) Implement a numerical algorithm to solve the model. You can base your algorithm on the pseudocode from Slide 36/37 of Lecture Slides 3. Plot your algorithm's convergence path on the capital demand and supply plot.