ASSIGNMENT 2

Due 3 p.m. on November 13, 2025

Instructions. Hand-in is in electronic form via email to `sztachera@econ.uni-frankfurt.de`. If you submit multiple files, please combine them in a single *.zip* file. List clearly the names and student numbers of the group members. There should be sufficient documentation for third parties to execute the code. In each exercise, make additional assumptions if necessary and explain them.

Exercise 1 (up to Lecture 2) Finite-state Markov chain approximation.

Suppose that θ is governed by the following first-order autoregressive process:

$$\log \theta_t = 0.979 \log \theta_{t-1} + \epsilon_t \qquad \text{where} \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Assume the following value for the standard deviation of the error term: $\sigma_\epsilon = 0.0072$.

a) Code a routine that can execute the Tauchen method. The function should take as inputs the persistence parameter ρ, the mean μ, the variance of the innovations σ_ϵ^2, the Tauchen parameter m, and the number of desired grid points N. It should return the vector of states and the transition matrix. Clearly name this function.

b) Code a routine that can execute the Rouwenhorst method. The function should take as inputs the persistence parameter ρ, the mean μ, the variance of innovations σ_ϵ^2, and the number of desired grid points N. It should return the vector of states and the transition matrix. Clearly name this function.

c) Apply the Tauchen and Rouwenhorst methods to approximate the AR1 process for θ using your self-written code with a 7-state Markov chain (assume that the constant m in the Tauchen method is equal to 3). Report the implied values of θ and the transition probability matrix.

d) Simulate the discrete-valued Markov chain for 25,000 periods using the procedure described in the Lecture Slides. Use the simulated data to estimate an AR1 process for $\log \theta$. How do the estimated parameters match those of the AR1 process assumed above for the Tauchen and Rouwenhorst methods?
Hint: Use OLS to estimate the coefficients.

Exercise 2 (up to Lecture 2) **A simple stochastic Ramsey model.**

Consider the following model economy:

$$\max_{\{c_s, k_{s+1}\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \log(c_s)$$

such that

$$c_s + i_s = \theta k_s^{\alpha}$$
$$k_{s+1} = (1-\delta)k_s + i_s$$

where $\log \theta_s$ is generated by the process:

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t$$

with $\rho = 0.979$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_{\varepsilon} = 0.0072)$.
Assume that $\beta = 0.984$, $\alpha = 0.323$, and $\delta = 0.025$ (calibration follows King and Rebelo (2000)).

a) Discretize the stochastic process for aggregate productivity using the Rouwenhorst method with seven states.

b) For each discretized value of θ_i, solve the deterministic version of the economy, where θ_i stays constant. You can use your code from the previous assignment.

c) Solve the complete model with aggregate risk. You can use any method except VFI with full discretization, e.g., the EGM method. Plot the capital policy function for different values of θ. How do these policy functions compare with the policy functions you computed in the deterministic economy? How would your results change if you increased σ_{ε} to 0.2? Interpret your findings. *Hint: Use the code for the non-stochastic case as a foundation. You need to adjust only a few lines.*

d) Simulate the economy over $T = 51,000$ periods. Discard the first 1000 periods. Plot the simulated path of capital, consumption, and output for the last 2000 simulation periods.

e) Compute the stationary distribution of capital. What is the average level of capital in the economy? How does it compare to the steady-state capital stock of a deterministic economy with productivity equal to the unconditional expectation in the stochastic economy $\theta = \exp\left(\frac{\sigma_{\epsilon}^2}{2(1-\rho^2)}\right)$? Provide an economic intuition for the result.

f) Repeat exercises d) and e) for the risky economy with $\sigma_{\varepsilon} = 0.2$. How do your results change?

g) How does the stationary distribution change with the number of grid points for the stochastic process? What number of points is sufficient to get an accurate approximation of the distribution?

h) Calculate the dynamic Euler equation errors for the baseline economy with $\sigma_{\varepsilon} = 0.0072$ and 7 TFP states solved with your method of choice. Is the mean dynamic Euler equation error larger than the mean static Euler equation error? Why?