Assignment 2, CIE4365 Coupled Processes
Programming the Richards' Equation
(Deadline Friday May 21, 2021)

15th April 2021

1 Programming Richard's Equation

- Several papers for background reading are provided on Blackboard under Readings, one of these an efficient and accurate approximation to derivatives, browse through these papers to obtain a better impression of the finesses of the Richards' equation.
- Use the Pressure Head form of the Richards' equation given by equation:

$$[C(h_w) + S_w S_s^w] \frac{\partial h_w}{\partial t} - \nabla \cdot [\mathbf{K}_{sat}^w k_{rw} (\nabla h_w + \nabla z)] = 0$$

- Use the van Genuchten equations in order to couple water content to capillary pressure head:

$$S_{eff}(h_c) = \begin{cases} [1 + (\alpha h_c)^n]^{-m} & h_c > 0 \\ 1 & h_c \leq 0 \end{cases}$$

  in which

$$S_{eff} = \frac{\theta_w - \theta_{res}}{\theta_{sat} - \theta_{res}}$$

  where θ_w is the volumetric water content, h_w is the water pressure head, θ_{res} is the residual volumetric water content, θ_{sat} is the saturated water content or total porosity, h_c is the capillary pressure head which we take equal to $h_c = h_a - h_w$ and α, n and m are empirical parameters. Often we take $m = 1 - \frac{1}{n}$ or $n = 1/(1 - m)$. Please note that we assume the air pressure head to be equal to zero everywhere: $h_a = 0$, see chapter 11 of Pinder and Celia in the additional readings.
- Use a simplified relative permeability term:

$$k_{rw} = (S_{eff})^3$$

- Derive the equation for the differential water capacity: $C(h_w) = \frac{\partial \theta_w}{\partial h_w}$, however you also may use the imaginary approximation from the paper;
- Implement a numerical solution using the solvers from Matlab or Python. Use the approach with and without the defining the Jacobian matrix and compare the difference.
- The initial condition for your problem is a soil column of a length of 1 m in equilibrium with a phreatic water level of 25 cm. Test your model for a number of other initial conditions. Soil properties can be found in Mayer and Hassanizadeh, chapter 2. But also using a logical reasoning from understanding the origin of capillary pressure and its relationship with capillary rise.
- Compare two lower boundary conditions: 1) a zero pressure head gradient (or gravity flow) and 2) a Robbins Boundary (mixed) condition with an external head of -1 m and a resistance term of 0.005 1/s . The top boundary condition is a situation a zero flux condition for a period of 25 days and after that a constant flux of -0.001 m/day for 200 days.
- Use the provided heat flow model as a template for this model.

2 Report

- Document your model with a small report (max 15 pages). In this report you should describe the model structure, the way you organized your input of parameters and initial conditions and what the structure of your model is (Pseudo Code).
  - Introduction
  - Basic equation
  - Method for discretisation
  - Pseudo Code
  - Description of specific functions (you may use the documentation in your code).
  - Test scenarios
    - Show that your model behaves as it should by running some standard test scenario's. Describe the test scenario's you chose. Explain why this would be a good test.
    - Check the mass balance of your approach. Explain why you may have a mass balance error.
  - Conclusions
- You should hand in a zip file with your code, and a pdf with your report.
- Upload your report to Brightspace via the group assignment.