Query Expansion with Random Embeddings

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Query Expansion: Problem statement

- In the context of Information Retrieval, Query Expansion is a method to increase recall and/or precision by modifying a user query
- A specific case is the expansion of each term to related terms, increasing recall (and precision as a side effect)
- Example: User query: soccer, expanded query: soccer football maradona
- The Problem: Learn to expand terms

A General Approach

The distributional hypothesis

Words with similar distributional properties have similar meanings.

The geometric metaphor of meaning

Meanings are locations in a semantic space, and semantic similarity is proximity between the locations.

The Vector Space Model Approach

A solution

- An algebraic model for information retrieval and NLP
- Defines a vector space to represent terms
- Defines a dimension for each unique term
- Each term is represented by a semantic vector
- Each component of a semantic vector is the weight of the represented term in the direction of the dimension term
- Weights are a design decision, TF-IDF is widely used
- Compute the related terms of a given term as its k-nearest neighbors
- The vector distance/similarity measure is a design decision, cosine similarity is widely adopted

The Vector Space Model Approach

Some drawbacks

- Curse of dimensionality
- Sparse vectors
- Dynamic vector space

Alternatives

- Use documents as dimensions: Less dimensions, but still sparse vectors and still a dynamic space
- Use topics as dimensions: Less dimensions, dense vectors and fixed space, but how can we define topics and assign weights? Latent Semantic Analysis

The Random Projections Approach

Overcoming drawbacks

- Use a random low dimensional space
- Less dimensions, dense vectors, fixed space
- No semantic assigned to dimensions
- But, how are weights assigned?

The Random Projections Approach

A simple algorithm

- Assign a random k-dimensional vector to each term (index vector)
- Allocate a null k-dimensional vector to each term (semantic vector)
- For each sentence in the corpus compute bigrams (*left right*)
- For each bigram
  - $semantic(left) += index(right)$
  - $semantic(right) += index(left)$

The Random Projections Approach

A simple algorithm: a formal description

Given a text documents corpus D we define

$\mathbb{T} = \{\text{All terms in } D\}$, $t = |\mathbb{T}|$

$\mathbb{C} = \{\text{All contexts in } D\}$, where a context is a set of terms

t random k-dimensional vectors $r_i, 1 \leq i \leq t$, $\mathbb{R} = \{r_i\}$

t semantic k-dimensional vectors $s_i, 1 \leq i \leq t$, $\mathbb{S} = \{s_i\}$

A mapping $S : w \in \mathbb{T} \rightarrow \mathbb{S}$, which maps a term to its semantic vector

A mapping $R : w \in \mathbb{T} \rightarrow \mathbb{R}$, which maps a term to its random vector

A mapping $C : w \in \mathbb{T} \rightarrow \{\mathbb{C}\}$, which maps a term to a set containing all the contexts it appears

Then we can express the semantic vector assigned to the term w as:

$$s_w = \sum_{c \in C(w)} \sum_{p \in c} R(p) \tag{1}$$

The Random Projections Approach

Algebraic Interpretation

Defining a co-occurrence matrix $M \in \mathbb{Z}^{t \times t}$, $M_{ij} = \sum_{c \in C(t_i)} \mathbb{1}_c(t_j)$

Each cell i, j in M counts the amount of contexts in D containing the ith and jth terms

Introducing a random matrix $R \in \mathbb{Z}^{t \times k}$, $R_i = R(t_i)$, that is, all random vectors as rows, we can express a semantic vector as:

$$s_i = \sum_{j=1}^{t} R(t_j) M_{ij} = \sum_{j=1}^{t} R_j M_{ij} \tag{2}$$

If we also define the semantic matrix $S \in \mathbb{Z}^{t \times k}$ with $S_i = s_i$, that is, all semantic vectors as rows, we can finally write:

$$S = MR \tag{3}$$

The Random Projections Approach

Algebraic Interpretation

$$S = MR \quad (4)$$

- This equation is a mathematical expression of our initial algorithm
- Semantic vectors are just the matrix product of the VSM vectors and a random matrix
- The algorithm is simply reducing the dimensionality of the VSM vectors
- For certain distributions of R, R is a nearly orthogonal
- Then we can interpret this product as the projection of the VSM vectors onto a random lower dimensional space

The Random Projections Approach

Theoretical support

The Johnson and Lindenstrauss Lemma

Given $\epsilon > 0$ and an integer n, let k be a positive integer such that $k \geq k_0 = O\left(\epsilon^{-2} \log n\right)$. For every set $\mathbb{P}$ of n points in $\mathbb{R}^d$ there exists $f : \mathbb{R}^d \to \mathbb{R}^k$ such that for all $u, v \in \mathbb{P}$

$$(1-\epsilon)\|u-v\|^2 \leq \|f(u)-f(v)\|^2 \leq (1+\epsilon)\|u-v\|^2$$

The Random Projections Approach

Theoretical support

Achlioptas Theorem

Let $\mathbb{P}$ be an arbitrary set of n points in R^d represented as an $n \times d$ matrix A. Given $\epsilon, \beta > 0$ let $k_0 = \frac{4+2\beta}{\frac{\epsilon^2}{2} - \frac{\epsilon^3}{3}} \log n$. For integer $k \geq_0$, let $\mathbb{R}$ be a $d \times k$ random matrix with $R(i,j) = r_{ij}$, where $\{r_{ij}\}$ are independent random variables from either one of the following two probability distributions:

$$r_{i,j} = \begin{cases} +1 & \text{with probability } \frac{1}{2} \\ -1 & \text{with probability } \frac{1}{2} \end{cases} \quad r_{i,j} = \sqrt{3} \times \begin{cases} +1 & \text{with probability } \frac{1}{6} \\ 0 & \text{with probability } \frac{2}{3} \\ -1 & \text{with probability } \frac{1}{6} \end{cases}$$

Let $E = \frac{1}{\sqrt{k}} AR$. Let $f : \mathbb{R}^d \to \mathbb{R}^k$ map the i^{th} row of A to the i^{th} row of E. With probability at least $1 - n^{-\beta}, forallu, v \in \mathbb{P}$

$$(1-\epsilon)\|u-v\|^2 \leq \|f(u) - f(v)\|^2 \leq (1+\epsilon)\|u-v\|^2$$

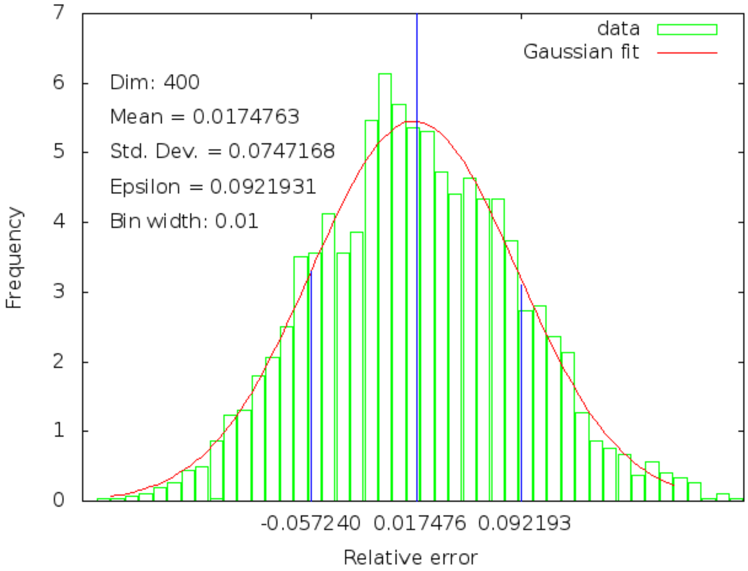
Empirical Study

Set up

- Natural language corpus with 140000 unique terms
- Filtered out low frequency terms, $n = 42905$
- Plain VSM vectors (original space is represented by the co-occurrence matrix)
- Evaluated random space dimension (k_0) ranging from 100 to 2000
- Random space generated with the dense Achlioptas distribution
- Pick two random vectors u, v, compute squared euclidean distance $\|u - v\|^2$ and $\|f(u) - f(v)\|^2$
- Compute relative error as $\frac{\|f(u)-f(v)\|^2 - \|u-v\|^2}{\|u-v\|^2}$
- Sample size: 3000

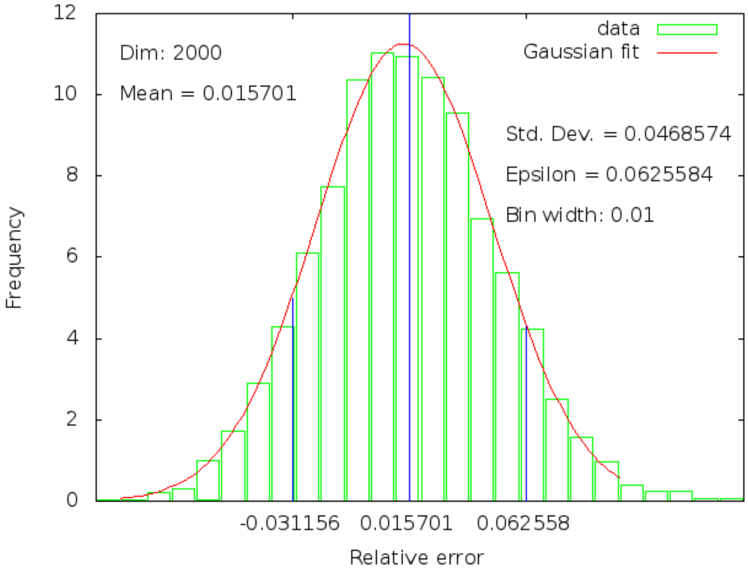
Empirical Study

Results



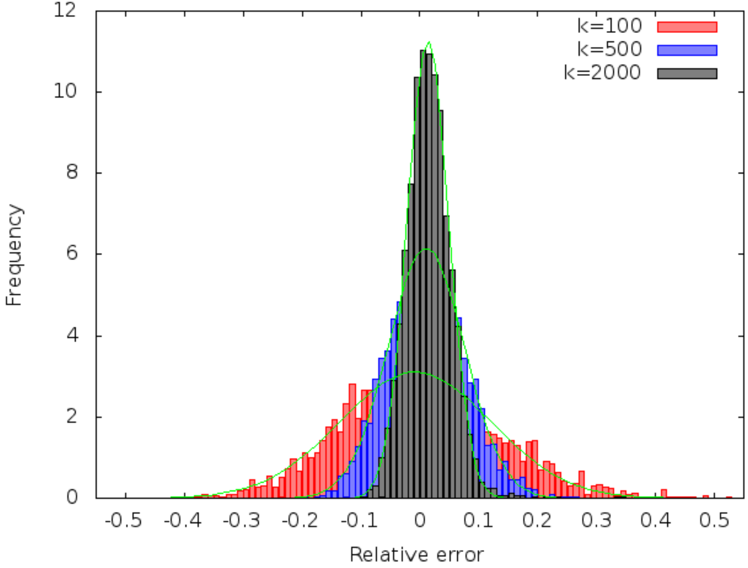
Empirical Study

Results



Empirical Study

Results



Empirical Study

Measuring ϵ

How can we compute ϵ from these histograms?

$$(1-\epsilon)\|u-v\|^2 \leq \|f(u)-f(v)\|^2 \leq (1+\epsilon)\|u-v\|^2$$

$$1-\epsilon \leq \frac{\|f(u)-f(v)\|^2}{\|u-v\|^2} \leq 1+\epsilon$$

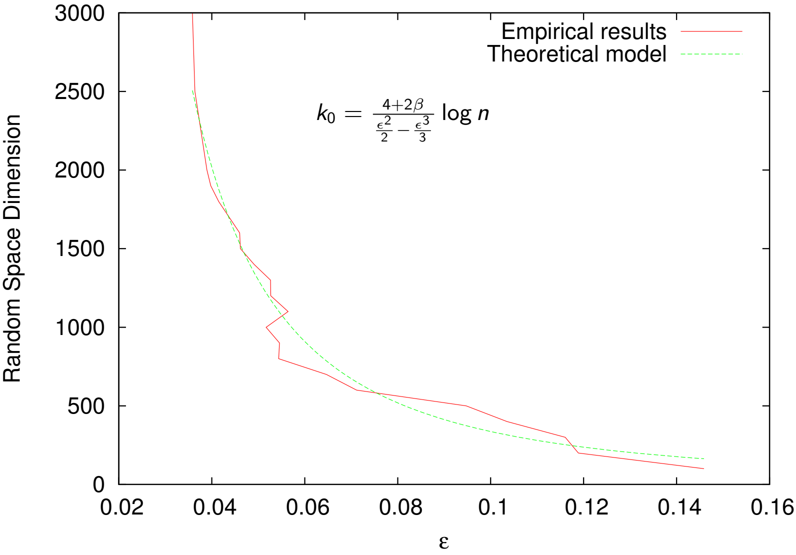
$$-\epsilon \leq \frac{\|f(u)-f(v)\|^2}{\|u-v\|^2} - 1 \leq \epsilon$$

$$-\epsilon \leq \frac{\|f(u)-f(v)\|^2 - \|u-v\|^2}{\|u-v\|^2} \leq \epsilon$$

$$\epsilon = \text{máx}(|\mu-\sigma|, |\mu+\sigma|)$$

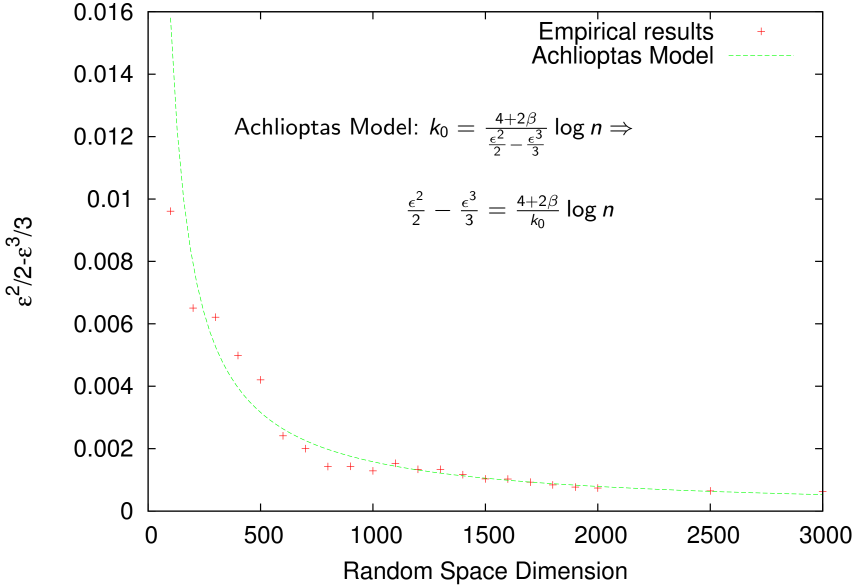
Empirical Study

Results



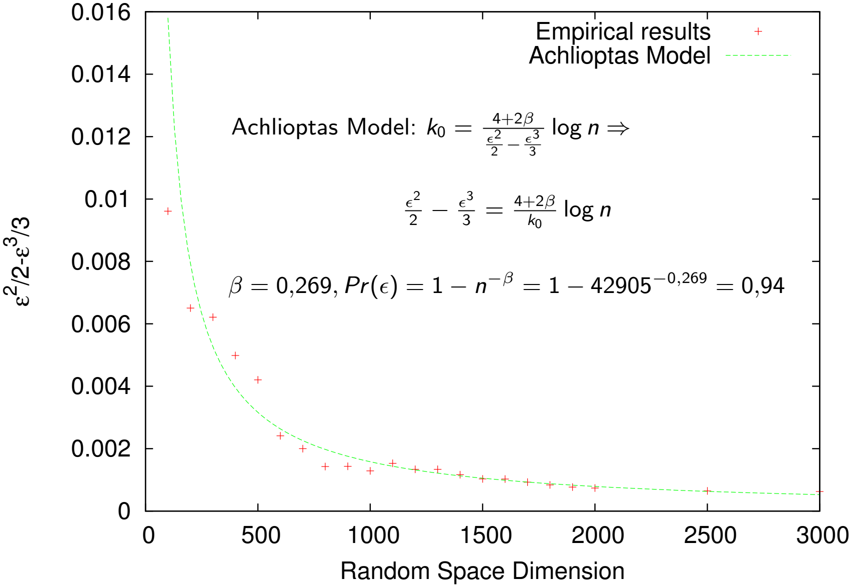
Empirical Study

Computing β



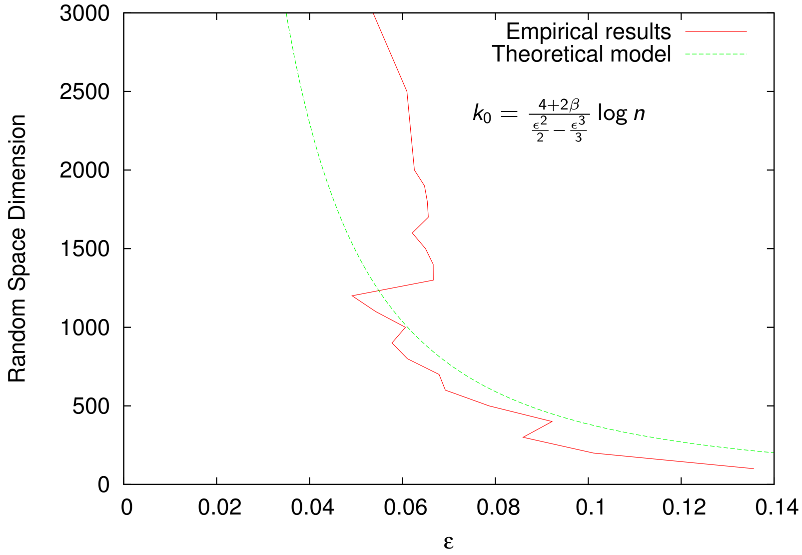
Empirical Study

Computing β



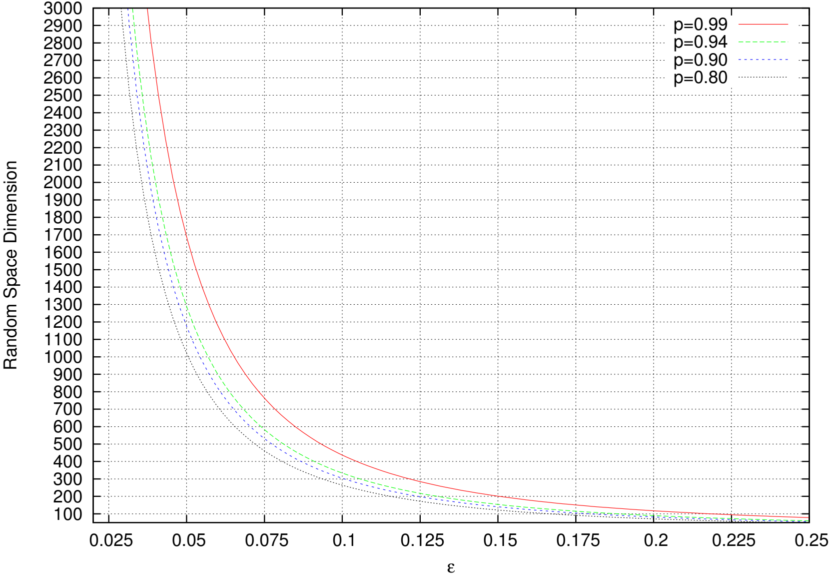
Empirical Study

Sparse projections



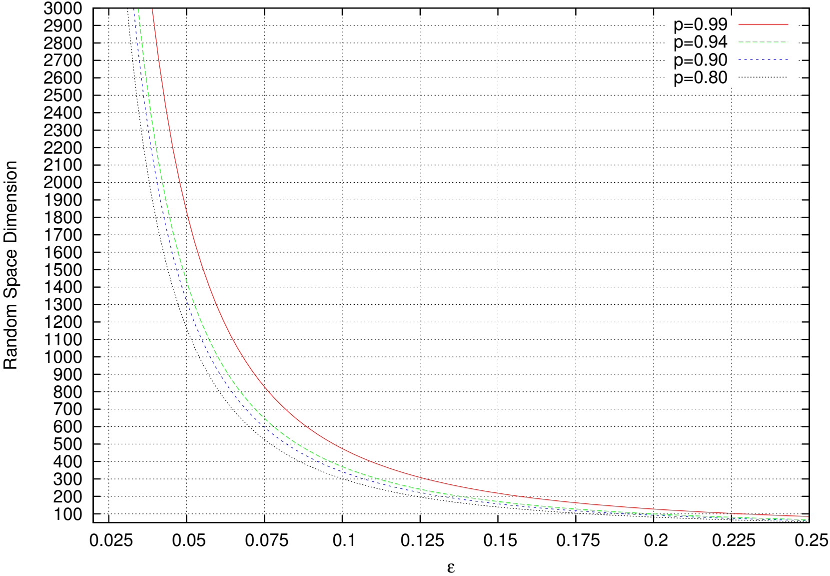
Trade offs

Dimensionality vs error vs certainty



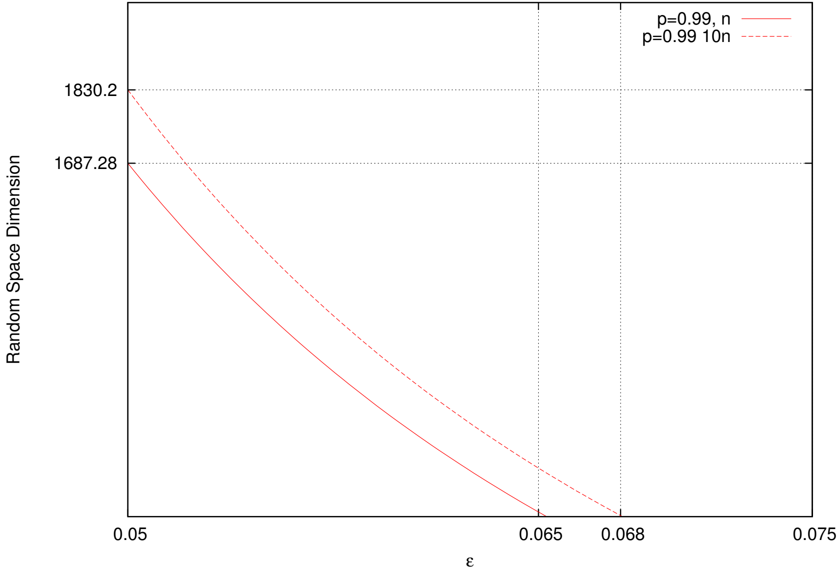
Trade offs

10 times more data



Trade offs

The effect of n



Random Projections Design

Selecting k_0

- Determine how much data is available: n
- Compute β for several certainty levels (99, 94, 90 typically)
- Compute and plot $k_0(\epsilon)$ for each p level
- Choose k_0 according to resources constraints and accepted ϵ
- More data? Start again. increase k_0, accept more error or more uncertainty

Random Projections Design

Selecting a random distribution

- Select k_0 using a dense random distribution
- No k_0 meets performance criteria?: Analyze sparsity.
- If data is indeed really sparse, consider matrix densification algorithms
- If k_0 meets performance criteria, evaluate a sparse random projection
- Achlioptas'distributions are not the only meeting JL property. Others could give better results

Random Projections Design

Weighting scheme

- Directional random vectors
- Distance permutations
- Reflective random projections

References

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