單元 1: 函數與函數圖形

1.1 數學符號

N	自然數	(natural number)
Z	整數	integer
Q	有理數	
R	實數	

開區間(open interval) $x \in (a,b)$ $a < x < b$ 沒有邊界

閉區間(closed interval) $x \in [a,b]$ $a \le x \le b$

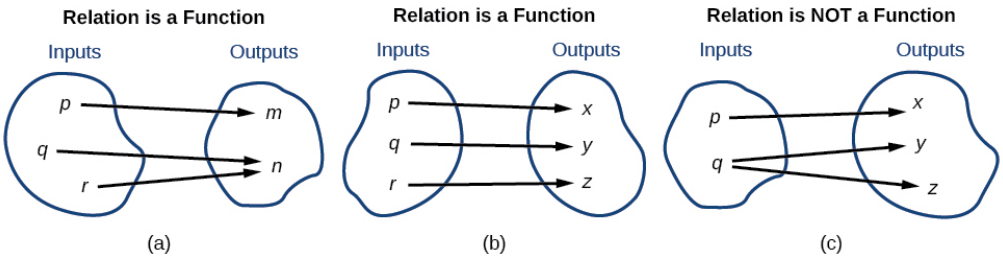
半開區間 $x \in [a,b)$ $a \le x < b$

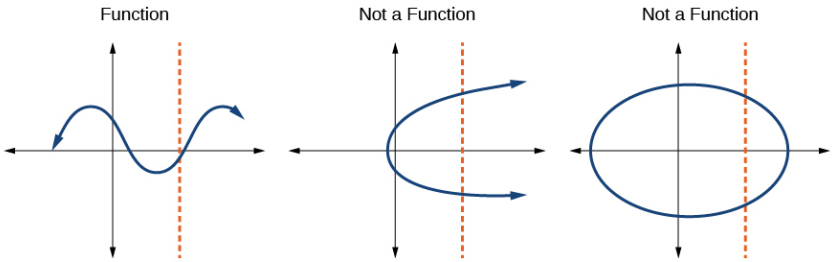
$x \in (a,b]$ $a < x \le b$

無限區間(infinite interval)

	Inequality Notation	Set-builder Notation	Interval Notation
	$5 < h \le 10$	$\{h \mid 5 < h \le 10\}$	$(5, 10]$
	$5 \le h < 10$	$\{h \mid 5 \le h < 10\}$	$[5, 10)$
	$5 < h < 10$	$\{h \mid 5 < h < 10\}$	$(5, 10)$
	$h < 10$	$\{h \mid h < 10\}$	$(-\infty, 10)$
	$h \ge 10$	$\{h \mid h \ge 10\}$	$[10, \infty)$
	All real numbers	$\mathbb{R}$	$(-\infty, \infty)$

1.2 函數的定義

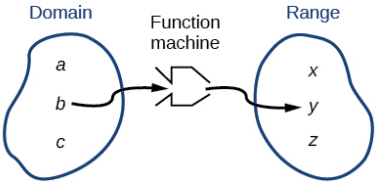


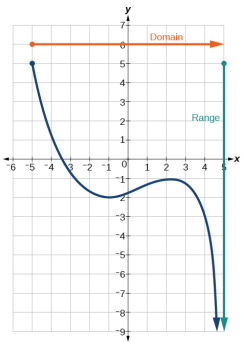


定義域與值域

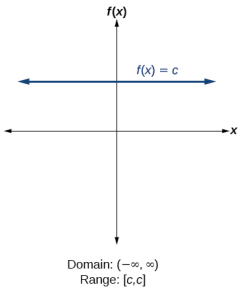
定義域(domain)

值域 (range)



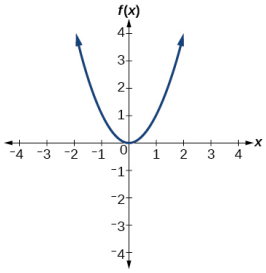


$f(x) = c$



Domain: $(-\infty, \infty)$
Range: $[c,c]$

$f(x) = x^2$

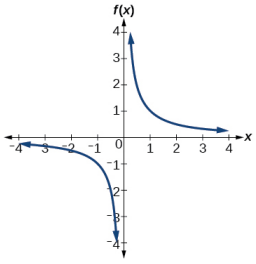


Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

$f(x) = \frac{1}{x}$

Domain=?

Range=?



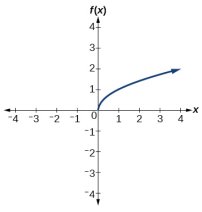
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

$f(x) = 2\sqrt{x+4}$

$f(x) = \sqrt{x}$

Domain=?

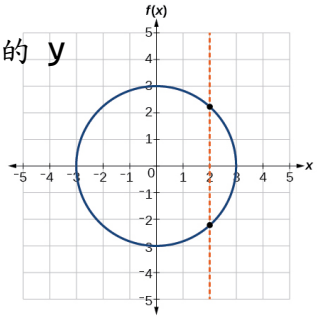
Range=?



Domain: $[0, \infty)$
Range: $[0, \infty)$

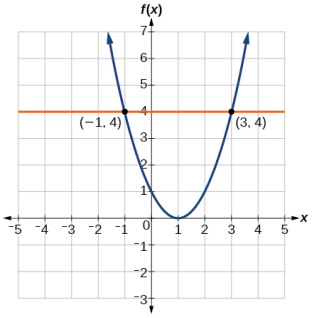
one-to-one 一對一

不同的x 得到不同的 y



函數: 垂直檢測

有兩點不是一對一



$y = (x-1)^2$

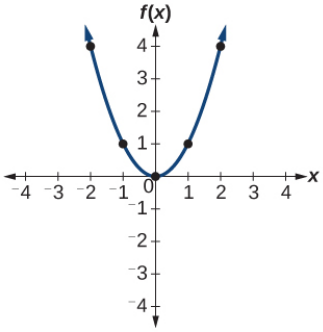
一對一函數: 垂直檢測+ 水平檢測

1.3 函數圖形

常見的函數

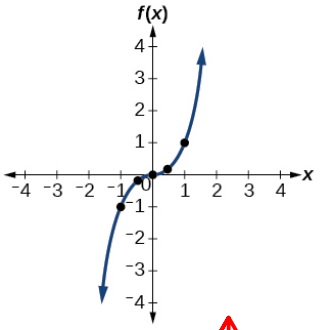
多項式

$y = x^2$

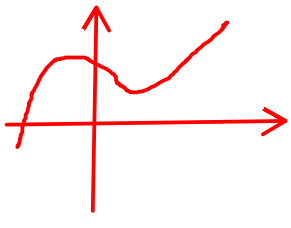


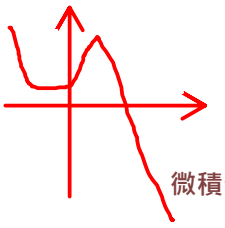
x	f(x)
−2	4
−1	1
0	0
1	1
2	4

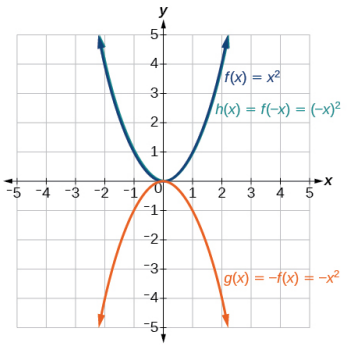
$y = x^3$

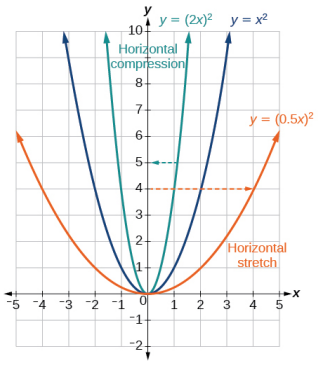


x	f(x)
−1	−1
−0.5	−0.125
0	0
0.5	0.125
1	1

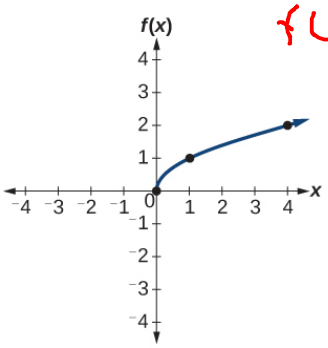






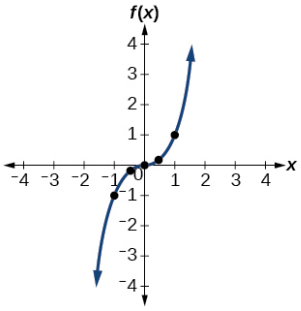


根式



$f(x) = \sqrt{x}$

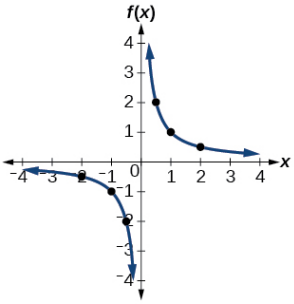
x	f(x)
0	0
1	1
4	2



x	f(x)
−1	−1
−0.5	−0.125
0	0
0.5	0.125
1	1

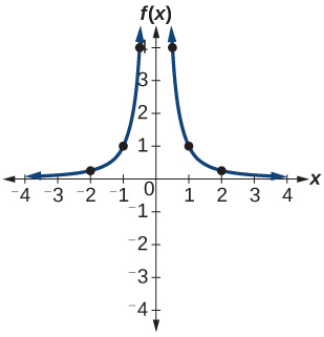
倒數

$f(x) = \frac{1}{x} = x^{-1}$



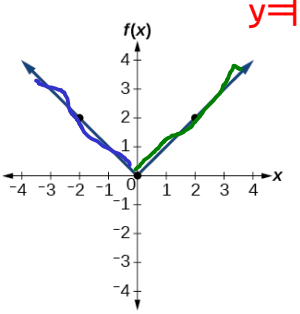
x	f(x)
−2	−0.5
−1	−1
−0.5	−2
0.5	2
1	1
2	0.5

$f(x) = \frac{1}{x^2} = x^{-2}$



x	f(x)
−2	0.25
−1	1
−0.5	4
0.5	4
1	1
2	0.25

絕對值



$y = |x|$

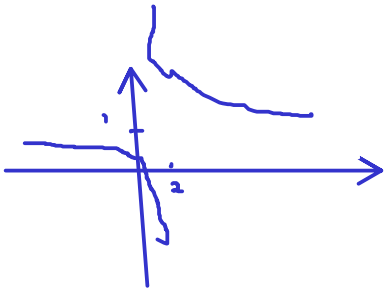
$x \geq 0 \quad y = x$

$x < 0 \quad y = -x$

x	f(x)
−2	2
0	0
2	2

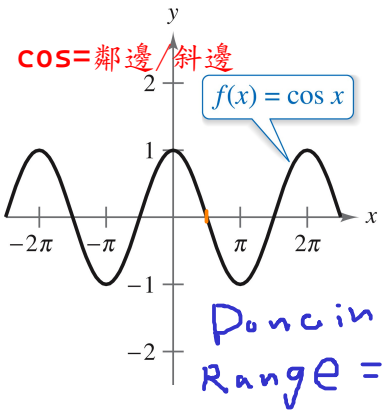
分式

$y = \frac{x-1}{x-2} = 1 + \frac{1}{x-2}$



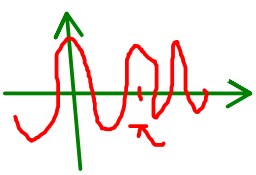
三角函數

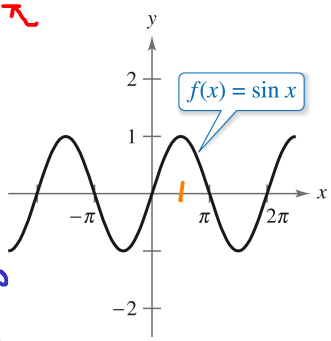
cos=鄰邊/斜邊



Domain $-\infty, \infty$

Range $= [-1, 1]$





sin=對邊/斜邊

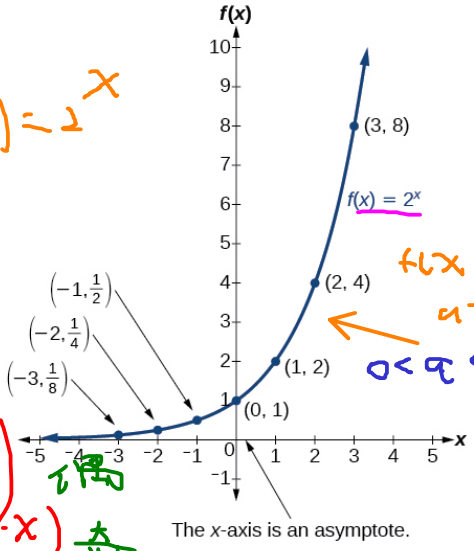
指數函數

$f(x) = 2^x$

$(e^x)' = e^x$

$\frac{1}{2}(2^x + 2^{-x})$ 偶

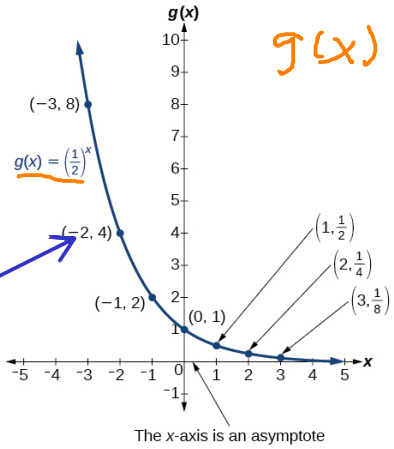
$\frac{1}{2}(2^x - 2^{-x})$ 奇



$f(x) = a^x$

$a > 1$

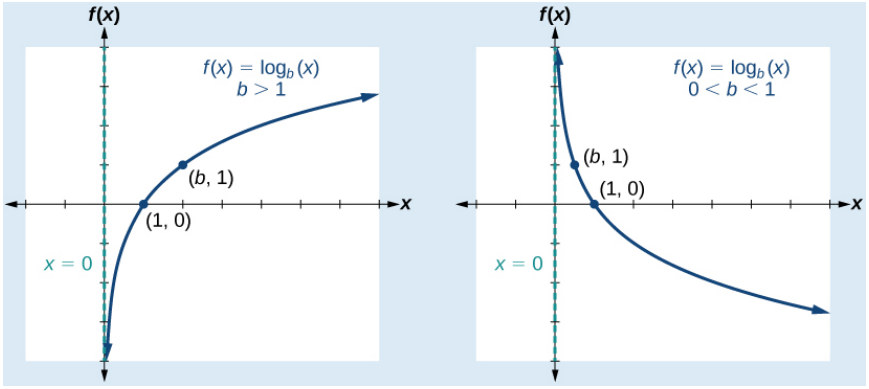
$0 < a < 1$

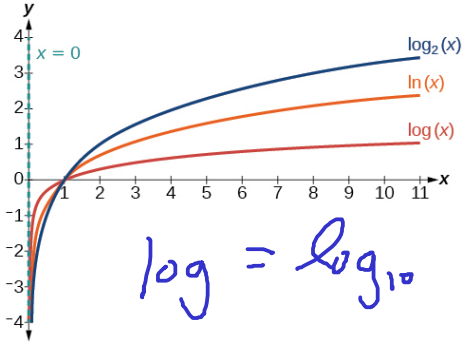


$g(x)$

$(-2)^x$

對數函數





$\log = \log_{10}$

$\ln(x) = \log_e(x)$

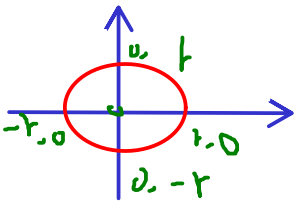
$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

$= \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

≈ 2.718281828

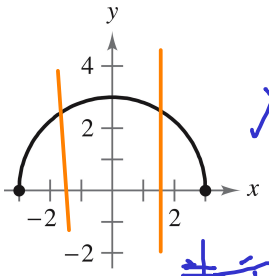
半圓

圓 $x^2+y^2=r^2$ $(0,0)$



$y^2=r^2-x^2$

$y=\sqrt{r^2-x^2}$



$y=\sqrt{9-x^2}$

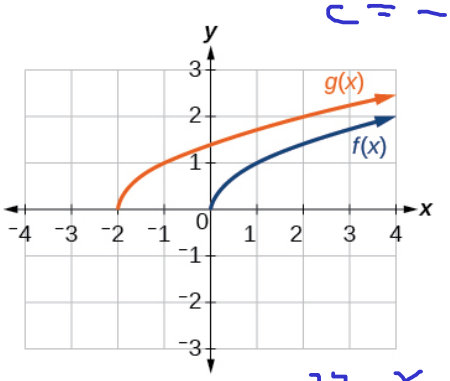
半徑=3

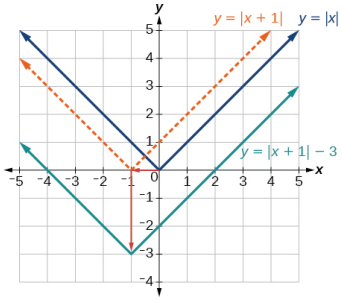
1.4 函數變數變換

位移

$g(x)=f(x+2)$

$c=-2$



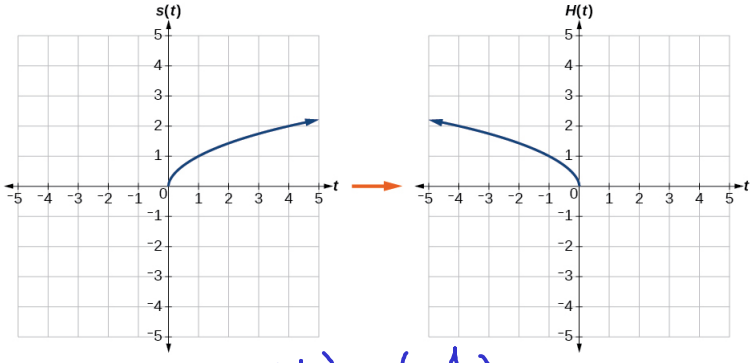


$x \to x-c$

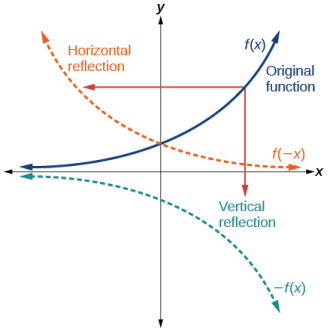
$c>0$ 右移

$c<0$ 左移

倒轉



$H(t)=s(-t)$



奇函數 與 偶函數

$x\ x^3\ x^5$ 奇函數

$f(x)=-f(-x)$

$f(0)=-f(0)$

偶函數

$1\ x^2\ x^4\ \cdots$

$f(x)=f(-x)$

$f(-x)$ 左右翻

$-f(-x)$ 再上下翻

左右翻 $\Rightarrow f(0)=0$

偶 $\frac{f(x)+f(-x)}{2}$

奇 $\frac{f(x)-f(-x)}{2}$

$x^3+x^2-2x \Rightarrow x^3-2x$ 奇

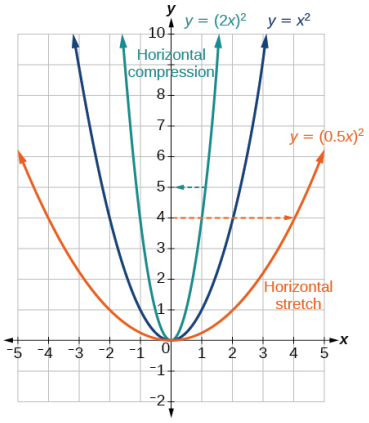
x^2 偶

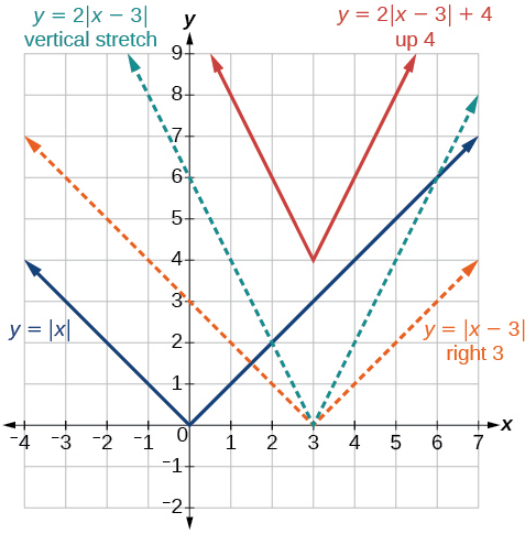
水平縮放

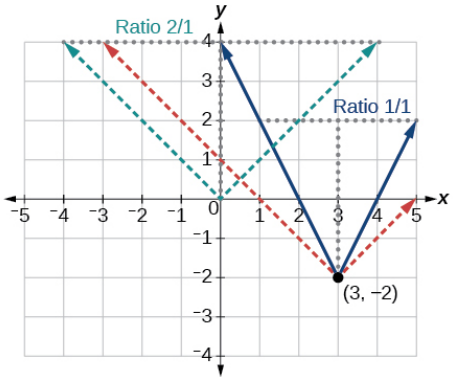
$g(x) = f(bx)$

$b > 1$　　壓縮

$0 < b < 1$　　伸長

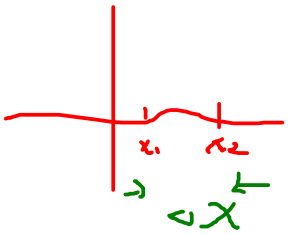




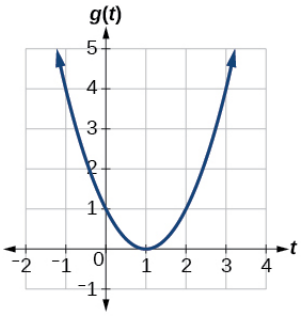


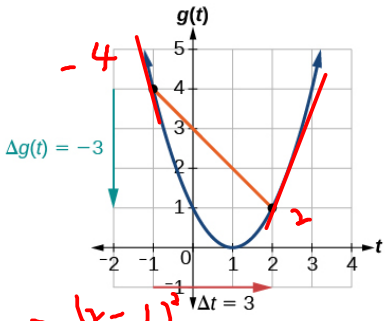
1.5 函數的變化率

平均變化率　$\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$



瞬間變化率　$\dfrac{dy}{dx} = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$





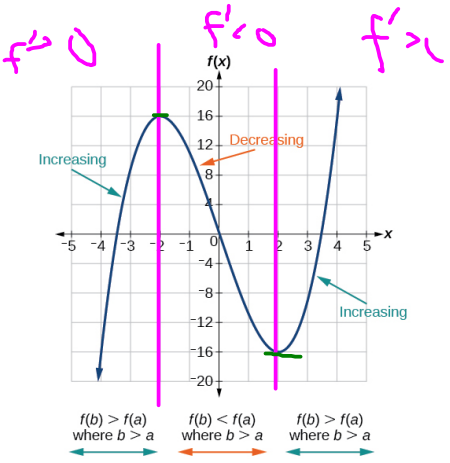
-4

$\dfrac{-3}{3} = -1$

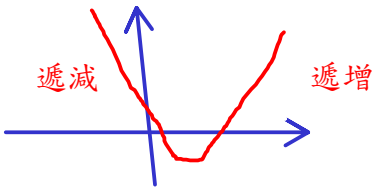
$f(x) = (x-1)^3$

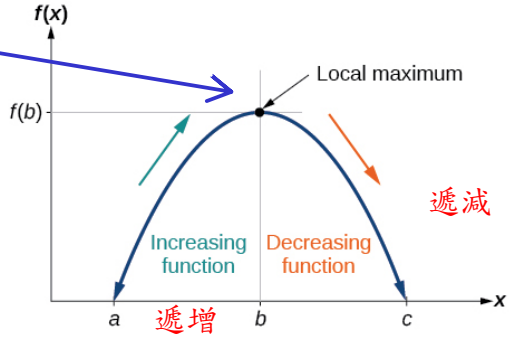
$f'(x) = 3(x-1)^2$

遞增與遞減



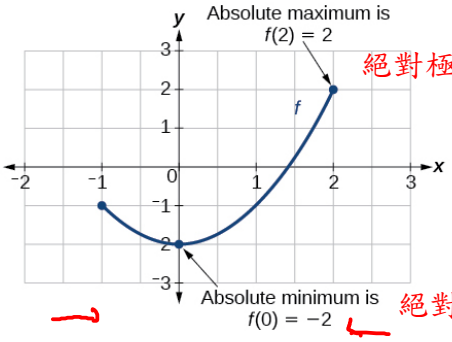
區域極值 (相對極大值、相對極小值)





絕對極值

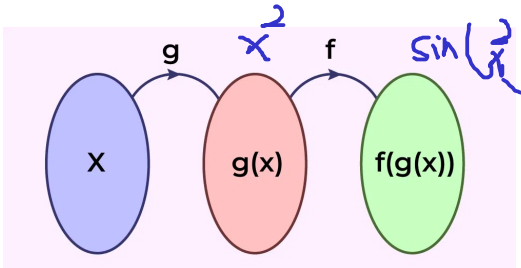
微分=0
邊界點



1.6 合成函數

$f \circ g(x)$

$f(g(x))$



$f(x) = \dfrac{5}{x-1}$ $\quad g(x) = \dfrac{4}{3x-2}$

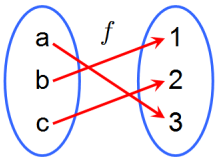
$f \circ g(x)$ $\quad$ domain=?

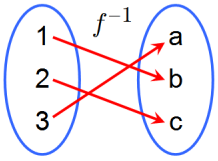
$x \neq 2$ $\quad x \neq \frac{2}{3}$

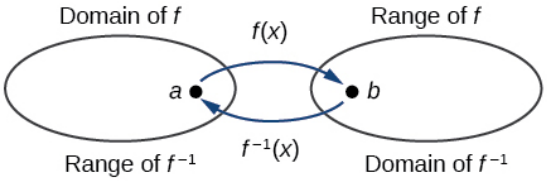
1.7 反函數

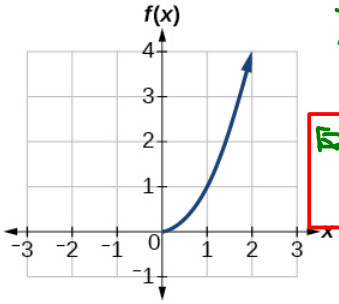
$y = f(x), \quad x = f^{-1}(y)$

一對一函數才存在反函數









$y = x^2$

$3 \to 9$

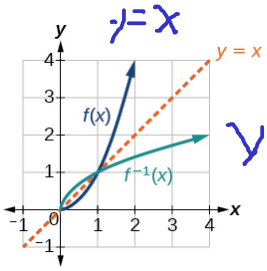
反函數

$9 \to 3$

$(-3)^2 = 9$

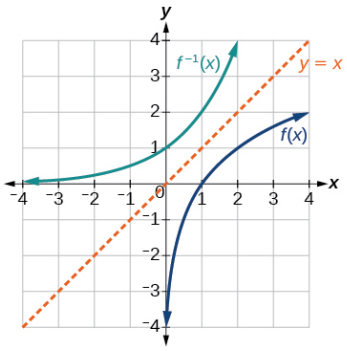
反函數

$y = x^2$



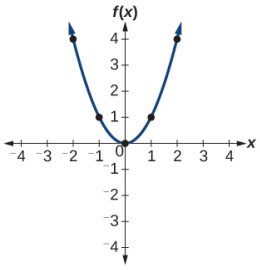
$y = \sqrt{x}$

$y = x^2 \quad x \geq 0$



指數與對數互為反函數

反函數求法



x	f(x)
−2	4
−1	1
0	0
1	1
2	4

不存在反函數

$f(x) = 2 + \sqrt{x-4}$

$x = 2 + \sqrt{y-4}$

$x - 2 = \sqrt{y-4}$

$(x-2)^2 = y - 4$

$y = (x-2)^2 + 4$