# 單元 3: 反三角函數

## 3.1 反三角函數定義

三角函數: 角度求得數值 $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

反三角函數:數值求得角度 $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

**三角函數與反三角函數互為反函數**

許多教科書為避免與-1次方混淆，採用arcsin, arccos, arctan,... 表示反三角

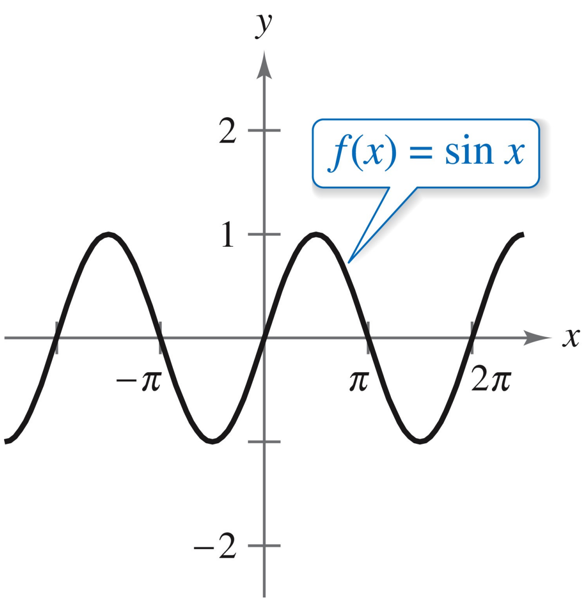
**三角函數: 角度得到數值**

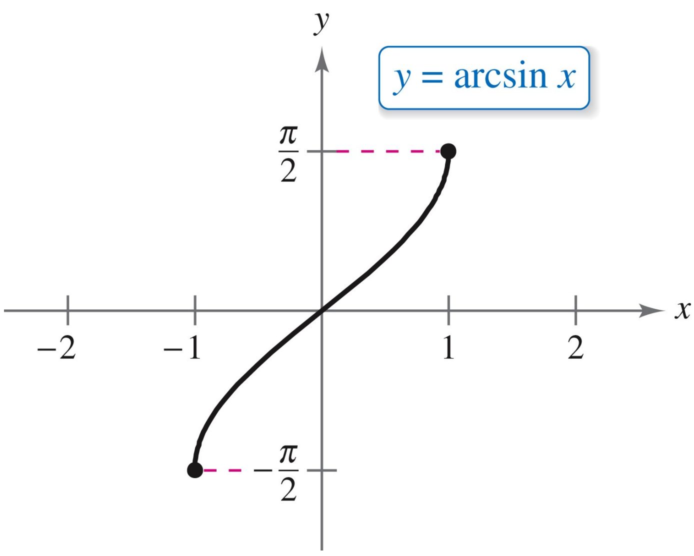
**反三角函數: 數值得到角度**

**三角函數並非一對一，**

反三角之定義必須限制三角函數的定義域

$\arcsin(x)$

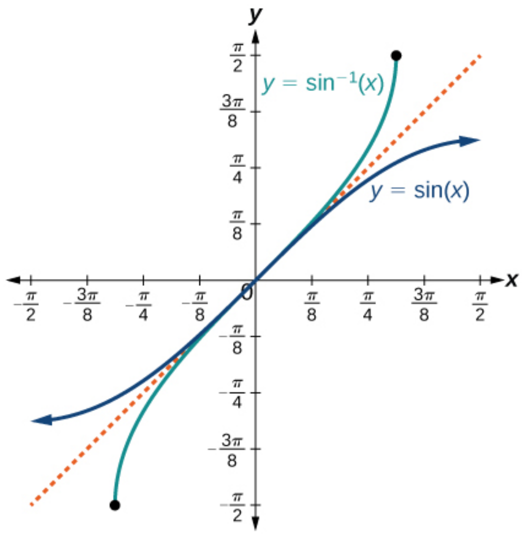


$\sin^{-1}$

**定義域**

**值域**

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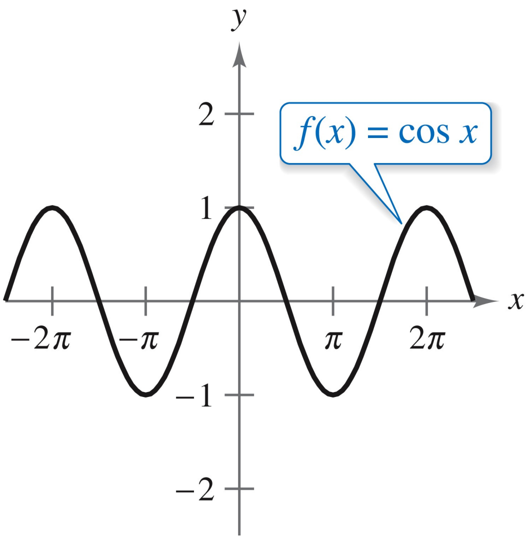
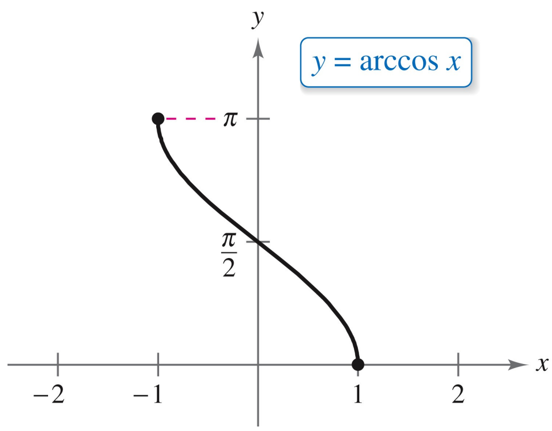
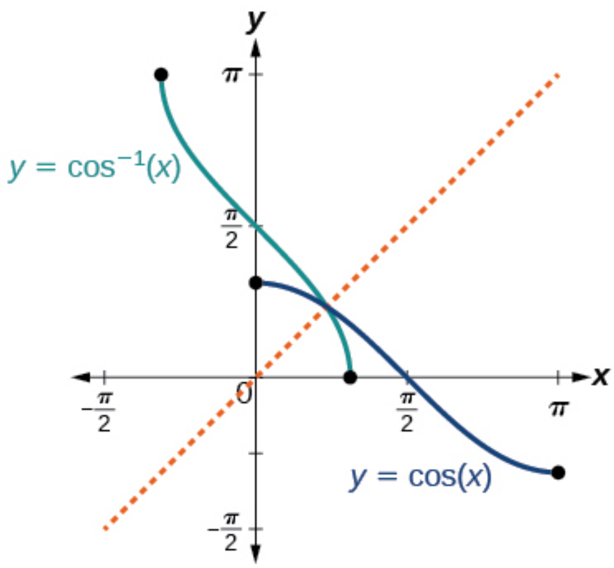
## $\arccos(x)$




**定義域**

**值域**

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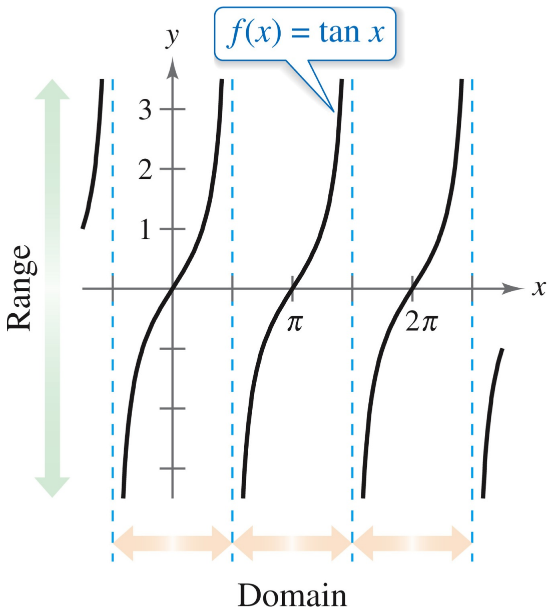
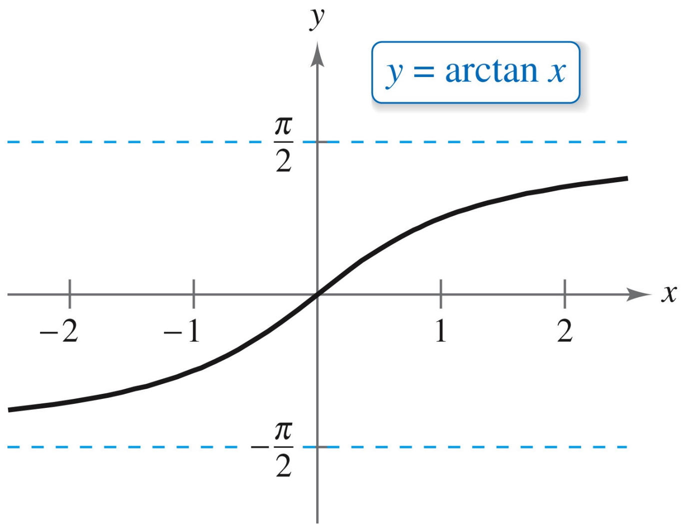
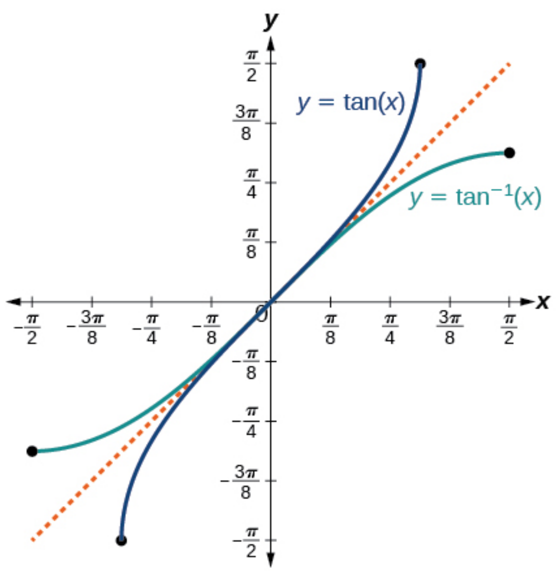
$\arctan(x)$




**定義域**

**值域**

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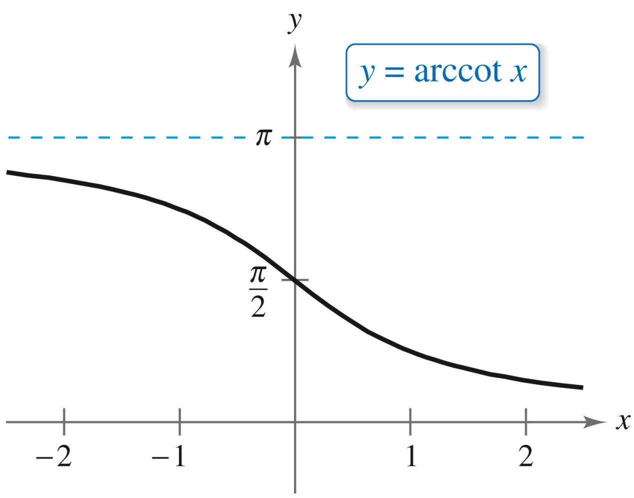
$\operatorname{arccot}(x)$

**定義域**

**值域**

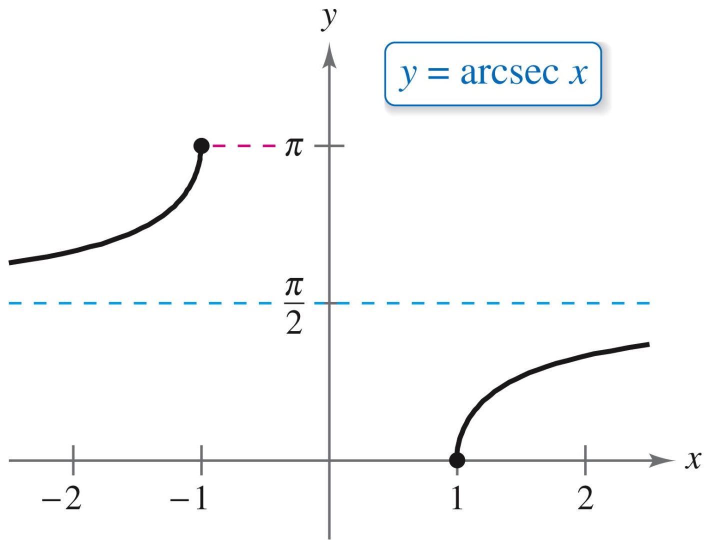


$\operatorname{arcsec}(x)$

定義域

值域

$\sec^{-1}$ 為不連續函數

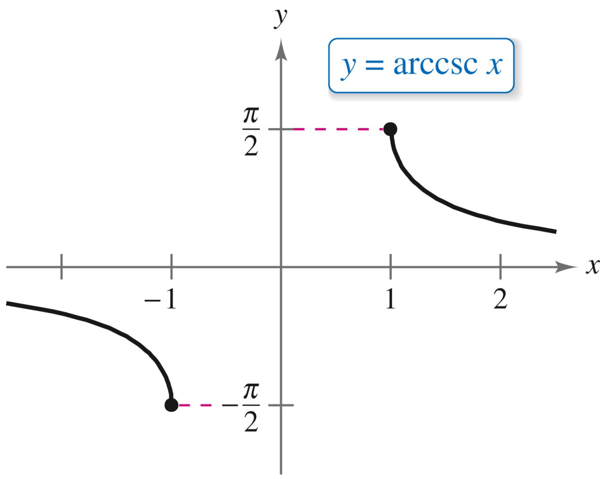



$\operatorname{arccsc}(x)$

定義域

值域

$\csc^{-1}$ 為不連續函數

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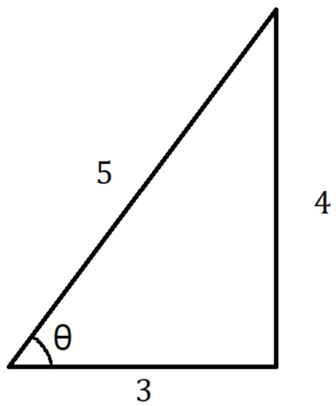
## 同角度不同的反三角表示

$$\sin\theta = \frac{4}{5} \quad \Rightarrow \quad \theta = \sin^{-1}\frac{4}{5}$$

$$\cos\theta = \frac{3}{5} \quad \Rightarrow \quad \theta = \cos^{-1}\frac{3}{5}$$

$$\tan\theta = \frac{4}{3} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{4}{3}$$




同一角度可使用不同的反三角函數表示

$\sin^{-1}\frac{\sqrt{3}}{2}$ $\cos^{-1}\frac{1}{2}$ $\tan^{-1}\sqrt{3}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\cos^{-1}\left(-\frac{1}{2}\right)$ $\tan^{-1}\left(-\sqrt{3}\right)$

$\sin^{-1}\left(-\frac{1}{2}\right)$ $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\sin\left(\tan^{-1}\left(\frac{1}{3}\right)\right)$ $\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$ $\cos\left(\sin^{-1}\left(-\frac{2}{3}\right)\right)$

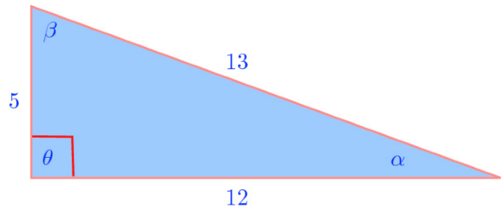
## 3.2 反三角函數關係式

### 餘角

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

$$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$$

$$\sec^{-1}x+\csc^{-1}x=\frac{\pi}{2}$$



## 負角度

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\csc^{-1}(-x) = -\csc^{-1} x$$