單元 3: 反三角函數

3.1 反三角函數定義

三角函數: 角度求得數值　　$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$　　$\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$

反三角函數:數值求得角度　　$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$　　$\tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

三角函數與反三角函數互為反函數

許多教科書為避免與-1次方混淆，採用arcsin, arccos, arctan,... 表示反三角

三角函數: 角度得到數值

反三角函數: 數值得到角度

三角函數並非一對一，

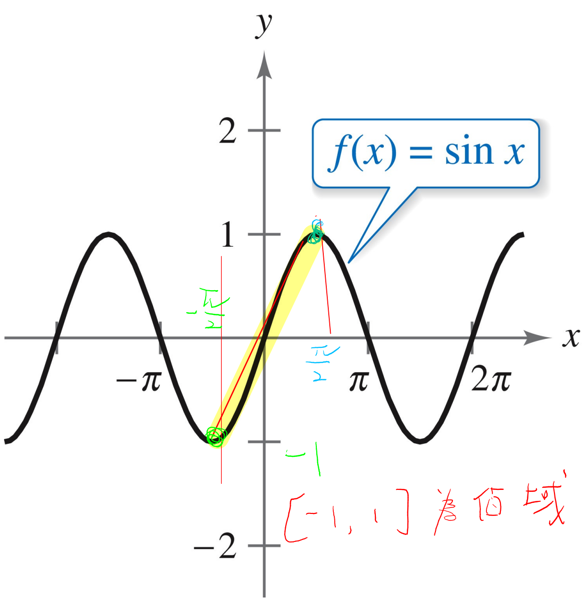
反三角之定義必須限制三角函數的定義域

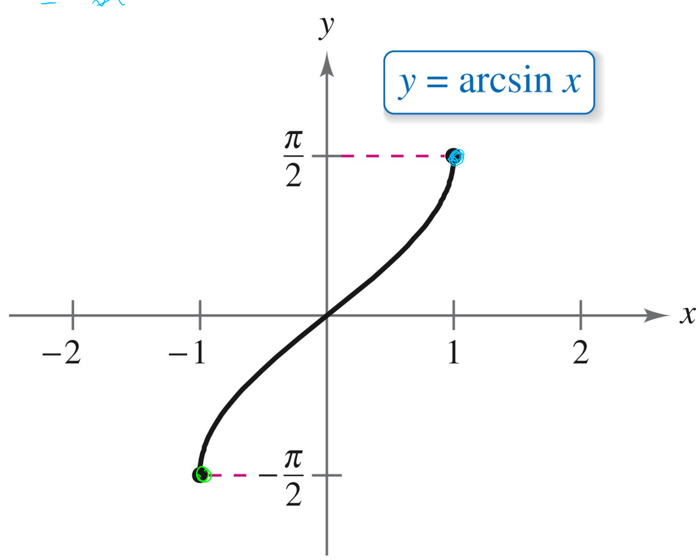
$\arcsin(x)$

$y = x^2$

$y = \sqrt{x}$

$= -\sqrt{x}$

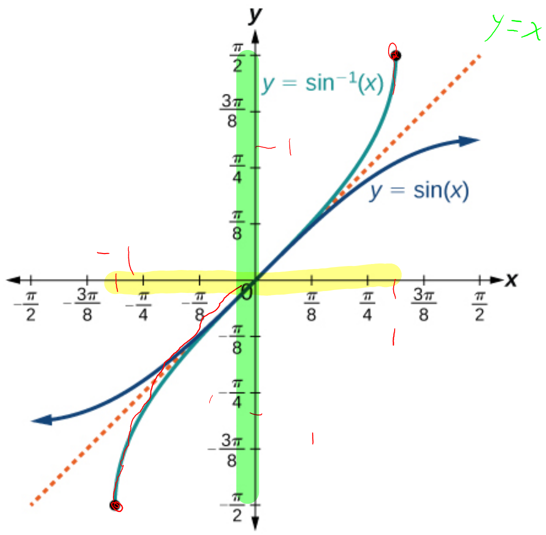




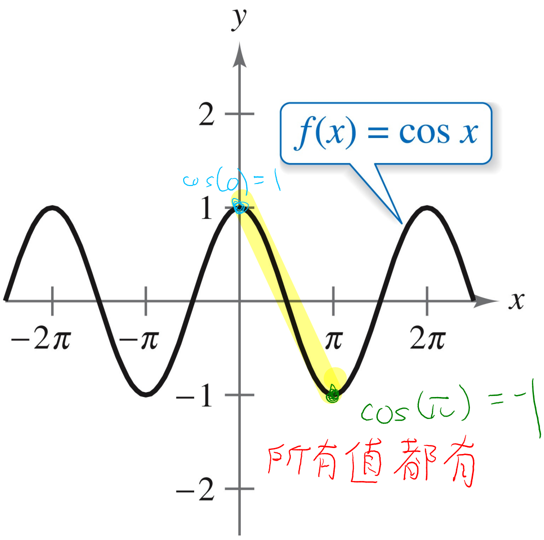
$\sin^{-1}(x)$ 值 ⇒ 角度

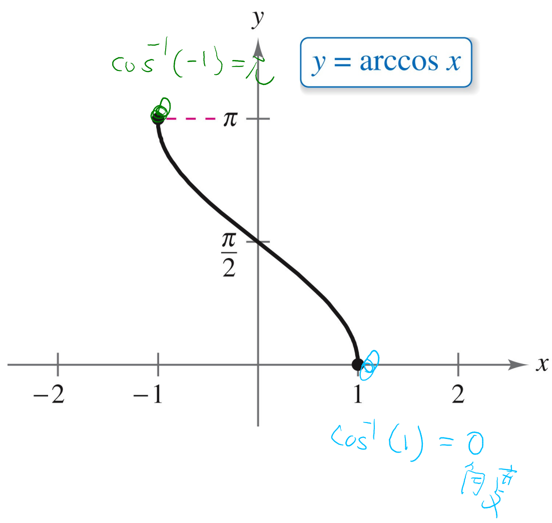
定義域 $[-1, 1]$

值域 $[-\frac{\pi}{2}, \frac{\pi}{2}]$



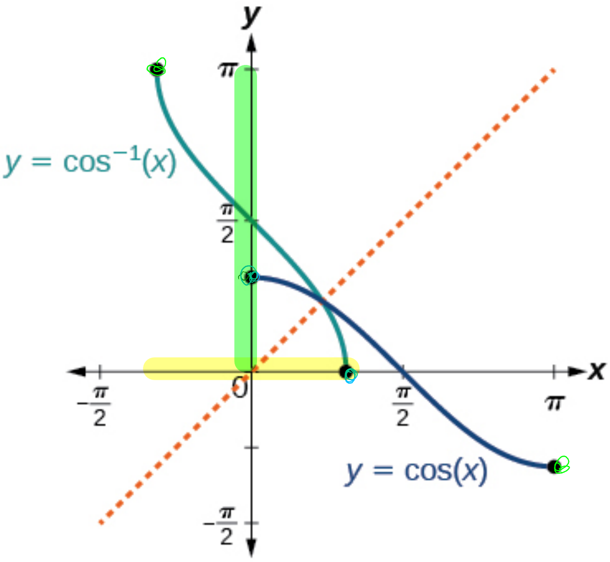
$\arccos(x)$



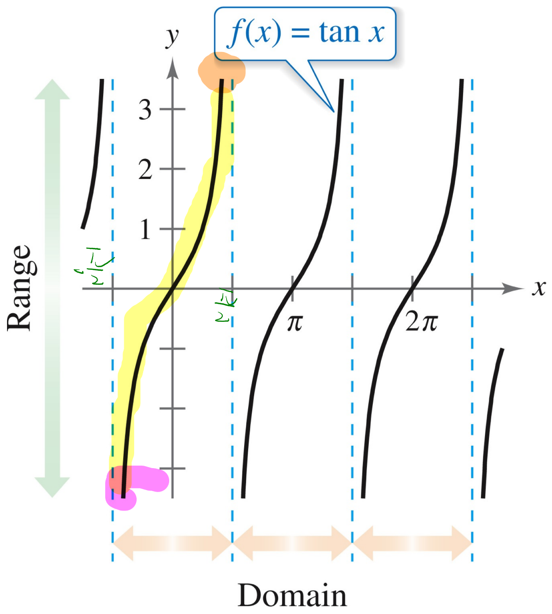


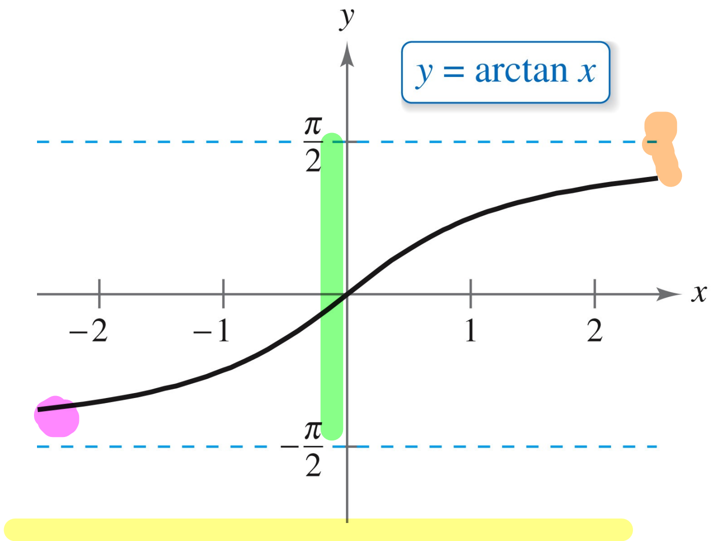
定義域 $[-1, 1]$

值域 $[0, \pi]$



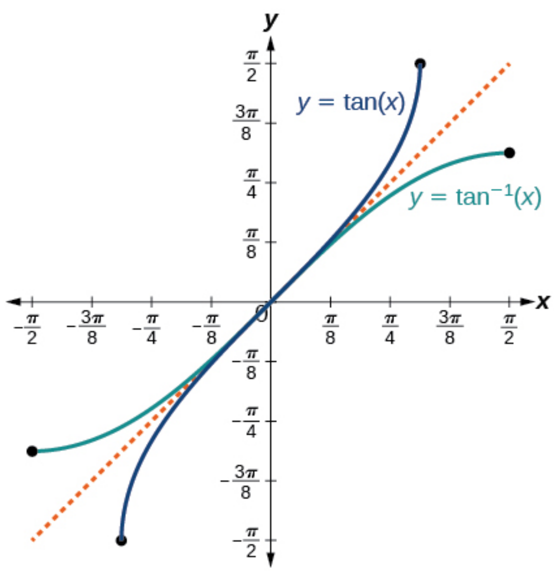
$\arctan(x)$





定義域 $(-\infty, \infty)$

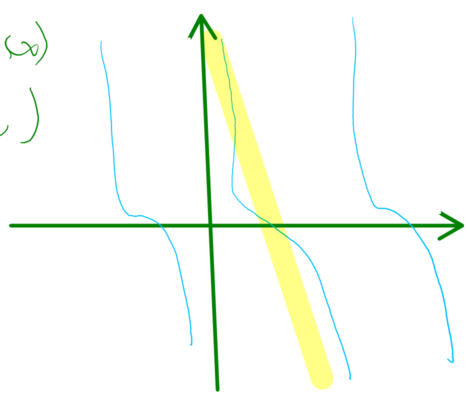
值域 $(-\frac{\pi}{2}, \frac{\pi}{2})$

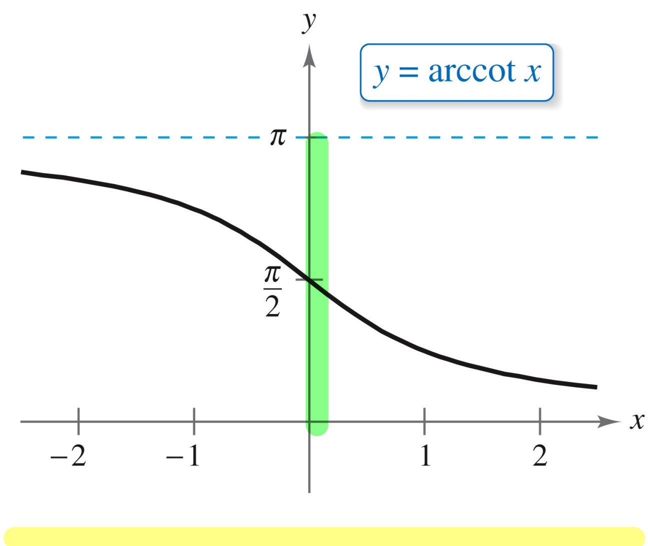


$\text{arccot}(x)$

定義域 $(-\infty, \infty)$

值域 $(0, \pi)$





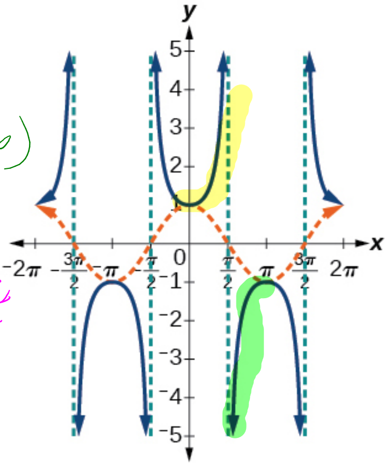
$\operatorname{arcsec}(x)$

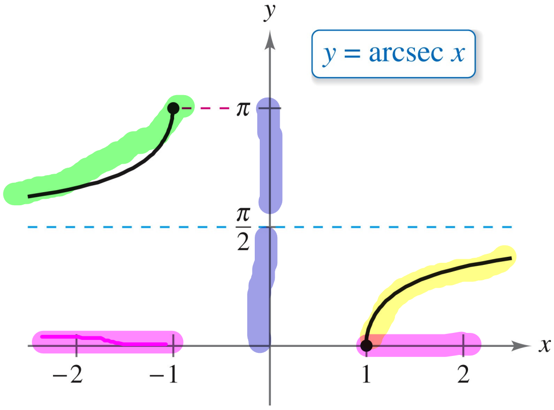
$|x| \geq 1$

定義域 $(-\infty,-1)\cup(1,\infty)$

值域 $[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$

$x \in [0,\pi],\ x \neq \frac{\pi}{2}$





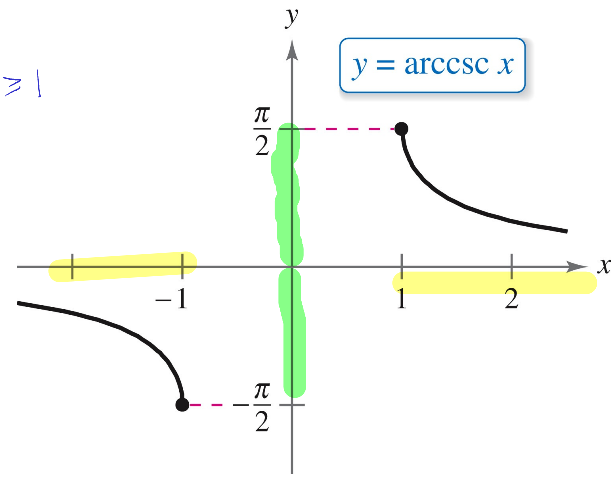
$\sec^{-1}$ 為不連續函數。微分與積分公式要加上絕對值

$\operatorname{arccsc}(x)$

定義域 $(-\infty,-1]\cup[1,\infty)$ $|x| \geq 1$

值域 $[-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$

$x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ $x \neq 0$



$\csc^{-1}$ 為不連續函數

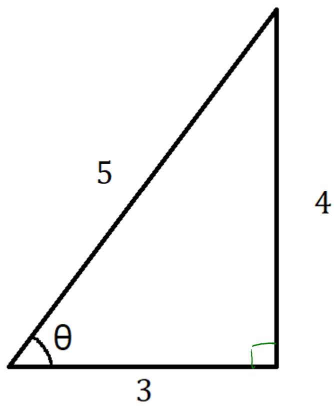
同角度不同的反三角表示

$$\sin\theta = \frac{4}{5} \quad \Rightarrow \quad \theta = \sin^{-1}\frac{4}{5}$$

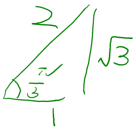
$$\cos\theta = \frac{3}{5} \quad \Rightarrow \quad \theta = \cos^{-1}\frac{3}{5}$$

$$\tan\theta = \frac{4}{3} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{4}{3}$$

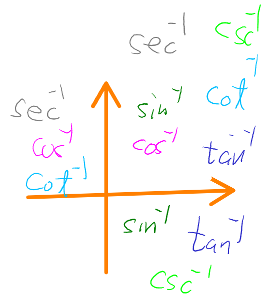
三者為同一角度



同一角度可使用不同的反三角函數表示



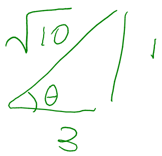
$\sin^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$ $\quad\cos^{-1}\dfrac{1}{2} = \dfrac{\pi}{3}$ $\quad\tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

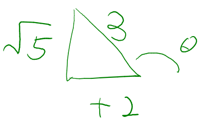


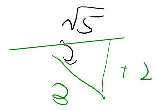
$\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = -\dfrac{\pi}{3}$ $\quad\cos^{-1}\left(-\dfrac{1}{2}\right) = \dfrac{2}{3}\pi$ $\quad\tan^{-1}\left(-\sqrt{3}\right) = -\dfrac{\pi}{3}$

$\sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$ $\quad\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$ $\quad\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$

$\sin\left(\tan^{-1}\left(\dfrac{1}{3}\right)\right) = \dfrac{1}{\sqrt{10}}$ $\quad\sin\left(\cos^{-1}\left(-\dfrac{2}{3}\right)\right) = \dfrac{\sqrt{5}}{3}$ $\quad\cos\left(\sin^{-1}\left(-\dfrac{2}{3}\right)\right) = \dfrac{\sqrt{5}}{3}$







3.2 反三角函數關係式

餘角

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$$

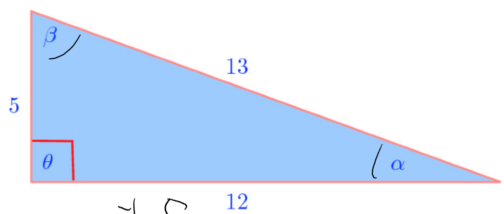
$\sin^{-1}\dfrac{5}{13} + \cos^{-1}\dfrac{5}{13} = \alpha + \beta = \dfrac{\pi}{2}$

$\sin\alpha = \dfrac{5}{13}$ $\quad\sin\beta = \dfrac{12}{13}$

$\cos\alpha = \dfrac{12}{13}$ $\quad\cos\beta = \dfrac{5}{13}$

$\tan\alpha = \dfrac{5}{12}$ $\quad\tan\beta = \dfrac{12}{5}$

$\cot\alpha = \dfrac{12}{5}$ $\quad\cot\beta = \dfrac{5}{12}$



$\tan^{-1}\dfrac{5}{12} + \cot^{-1}\dfrac{5}{12}$

$= \alpha + \beta = \dfrac{\pi}{2}$

負角度

$$\sin^{-1}(-x) = -\sin^{-1}x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$\tan^{-1}(-x) = -\tan^{-1}x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x$$

$$\csc^{-1}(-x) = -\csc^{-1}x$$