CS-174A Discussion 1A, Week 5

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@ https://github.com/luckiday/cs174a-1a-2021w (https://github.com/luckiday/cs174a-1a-2021w)

Logistics

- Online midterm instructions
- Midterm scope: slides02 to slides06.

Last time

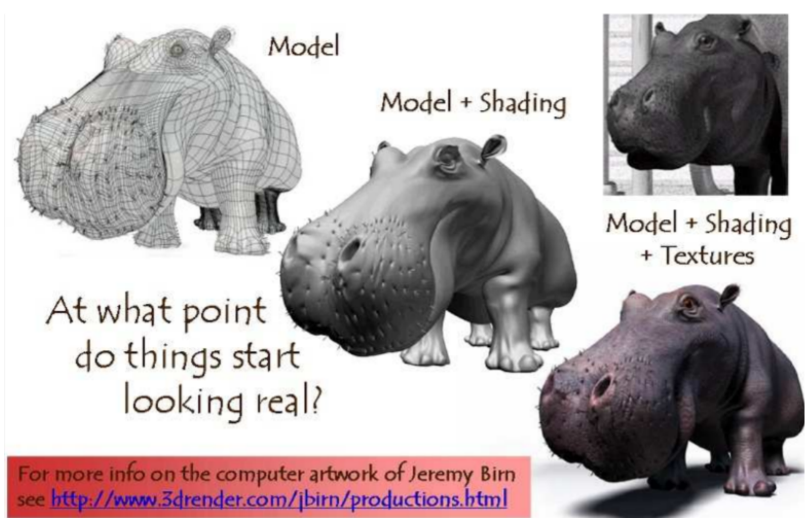
- Transformations: Pumpkin problem
- Change of basis
- Tiny graphics demos

Next up

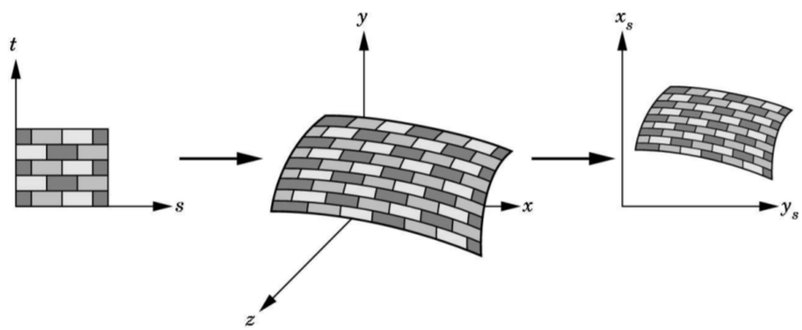
- Assignment 2
- Midterm review

Texture Mapping

- There are small surface details on real objects that need to be taken into account for better realism.

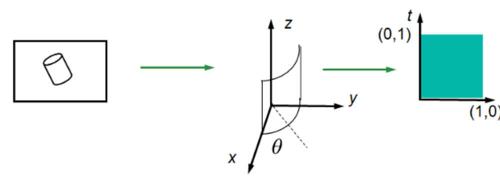


- To take these into account apply texture to our images to make the surfaces realistic
  - We can scan textures from real world or paint them
  - Store the texture as a 2D image
  - We map the texture to **object space** and then **screen space**



- From screen to texture:
  - Inverse the transform (s_x, s_y) to get the world (x, y, z)
  - With (x, y, z):

$$u = \tan^{-1}(y/x), \quad v = z$$
$$s = 2u/\pi, \quad t = v$$



Reminder: $\quad u = s\pi/2, \quad v = t$

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z$$

Surface parameters: $\quad u = \theta, \quad v = z$

Assignment 2

- Demo

Midterm Review

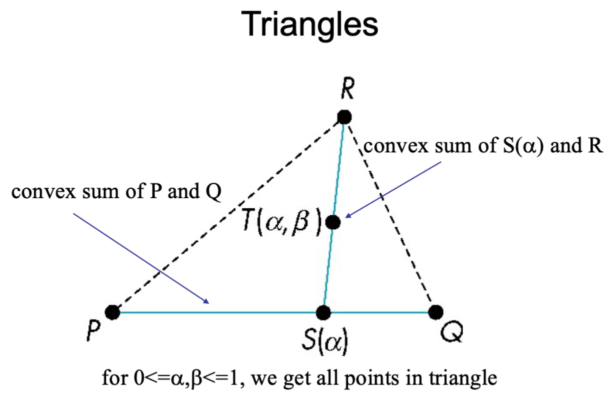
Q: What is the diff between affine combinations and convex combination of points?

Points and Vectors

Q: Parametric equations of line and triangle (barycentric coordinamtes)

A:

- *Line*: $P = \alpha_1 P_1 + \alpha_2 P_2$, and $\alpha_1 + \alpha_2 = 1$.
- *Triangle*:



A: *Barycentric coordinates*

Triangle is convex so any point inside can be represented as a convex sum

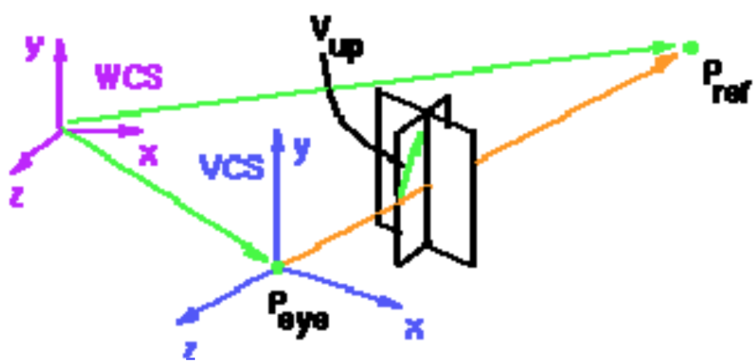
$$P(a1, a2, a3) = a1P + a2Q + a3R$$

where

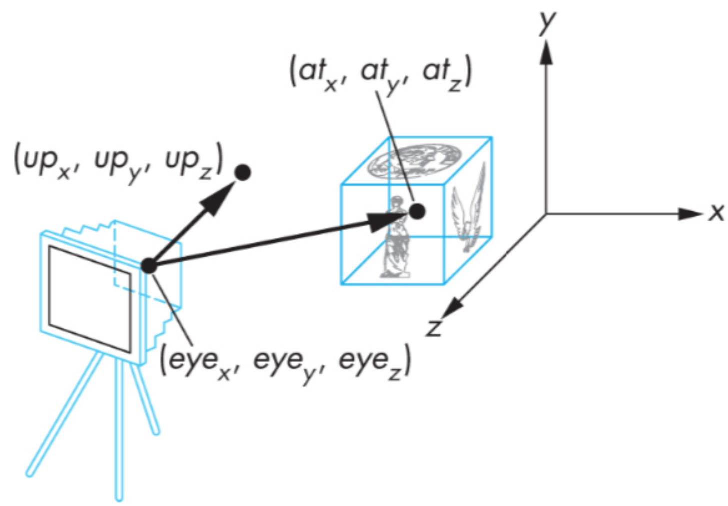
$$a1 + a2 + a3 = 1, a_i \geq 0$$

The representation is called the barycentric coordinate representation of P.

Change of Basis and Model-view transformation



Viewing



Projection

- Orthographic projection matrix:

$$N = \mathrm{ST} = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{l+r}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{b+t}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Perspective projection matrix:

$$P = NSH = \begin{pmatrix} \frac{n}{r} & 0 & 0 & 0 \\ 0 & \frac{n}{t} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

- Derivation

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

After perspective division, the point $(x, y, z, 1)$ goes to

$$x'' = -x/z, \quad y'' = -y/z, \quad z'' = -(a + b/z)$$

When $z = f, z'' = -1$; when $z = n, z'' = 1$.

Sample Questions

1. Q: Briefly describe what changes you would expect to see in the image with respect to the following changes in viewing parameters, all other params remaining unchanged:
    A. Half-angle-of-view decreases
    B. Aspect ratio increases
    C. COI moves closer to eye point
    D. Eye point moves away from COI
    E. Top vector becomes upside down
    F. Distance between hither and yon increases

1. Answers:
    A. Half-angle-of-view decreases: objects will project larger on window and viewport, because camera is now capturing lesser volume of the scene while image size remains same
    B. Aspect ratio increases: AR increases implies viewport became wider or the height decreased; if angle did not change then some objects may be clipped off; the final image will definitely look wider
    C. COI moves closer to eye point: no change in image
    D. Eye point moves away from COI: a different image will be grabbed because the location of eye changed
    E. Top vector becomes upside down: image will rotate 180
    F. Distance between hither and yon increases: depending on which direction H and Y moves, more or less objects will be included in the view volume

2. Prove that the perspective projection of a 3D point (x, y, z) onto the plane $z = d$ is given by $(\frac{x}{z}d, \frac{y}{z}d, d)$

3.

Given this pumpkin at (1,1), do the following:

model *= trans(x+2,y+2);
model *= rot_z(90);
model *= $scale_x$(-1);
model *= trans(x-1,y-1);