CS-174A Discussion 1B, Week 3

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@ DODD 161 / Friday / 12:00pm-1:50pm

@ https://github.com/luckiday/cs174a-1b-2019f (https://github.com/luckiday/cs174a-1b-2019f)
(Short link: https://bit.ly/32Zt3sg (https://bit.ly/32Zt3sg))

Outline

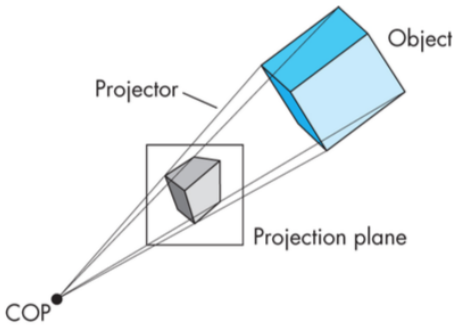
Viewing

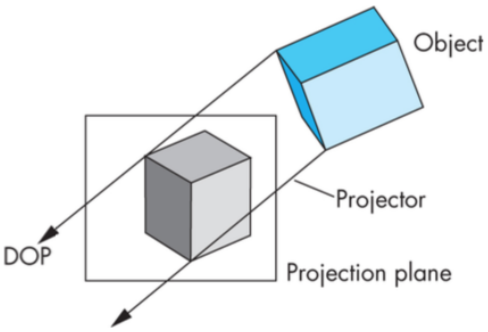
- Spaces
  - Model space
  - Objective/world space
  - Eye/camera space
  - Screen space
- Projections
  - Parallel
  - Perspective

Objective

Our goal is to

- Use transformations to project the vertices of objects onto the projection plane.
- Specifically we will create transformations to go from object to camera to clip coordinates.



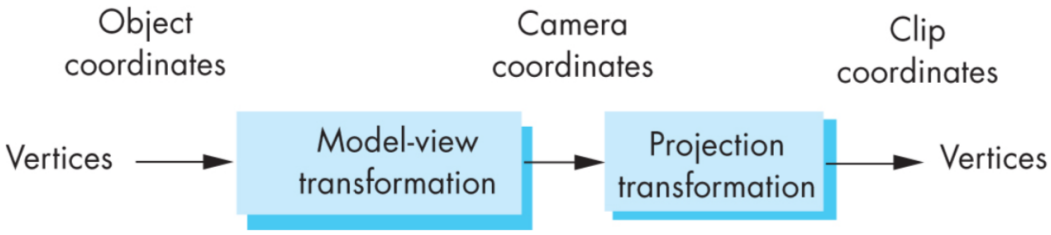


From World Space to Camera Space

Model-view transformation

- Does not take us all the way to clip coordinates.

- Later, we need a projection transformation for that.
- Model-view gets objects in front of the camera, potentially.
- A Projection defines which and how those objects will appear on the screen.



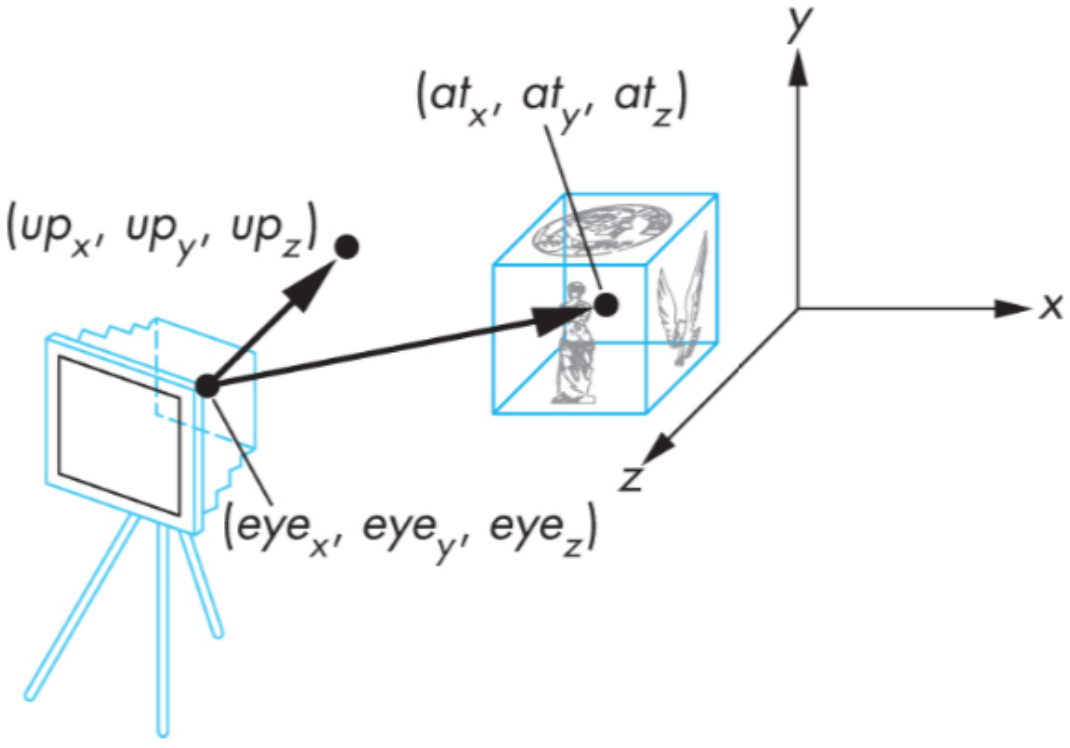
Positioning the (getting things in from of the) "camera"

- Recall that the default is "looking" down the $-z$ axis at the origin $(0, 0, 0)$.
  - This is equivalent to model-view set to the identity matrix.
- Remember, transformations are specified in reverse.
  - That means we specify the position of the camera first.

Look-at

We define three terms

- A point describing the location of the eye.
- A point the eye is looking at.
- An up direction for the camera.



Look-at

- The at and eye points give us
  - the view-plane-normal or vpn
- The up vector is usually $(0, 1, 0)$
  - Or, $(0, 1, 0, 0)$ in homogeneous coordinates!
- We then calculate the following

$$n = \frac{eye - at}{|eye - at|}, u = \frac{up \times n}{|up \times n|}, v = \frac{n \times u}{|n \times u|}$$

Look-at

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  - the view-plane-normal or vpn
- The up vector is usually $(0, 1, 0)$
  - Or, $(0, 1, 0, 0)$ in homogeneous coordinates!
- We then calculate the following

$$V = RT = \begin{pmatrix} u_x & u_y & u_z & -eye_x u_x - eye_y u_y - eye_z u_z \\ v_x & v_y & v_z & -eye_x v_x - eye_y v_y - eye_z v_z \\ n_x & n_y & n_z & -eye_x n_x - eye_y n_y - eye_z n_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Projection

- Perspective
- Parallel (orthographic)

Projections – Parallel (orthographic)

- Once in camera coordinates we need a projection transformation to get us to clip coordinates.
- The transformation matrix that gives us an orthographic projection is:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

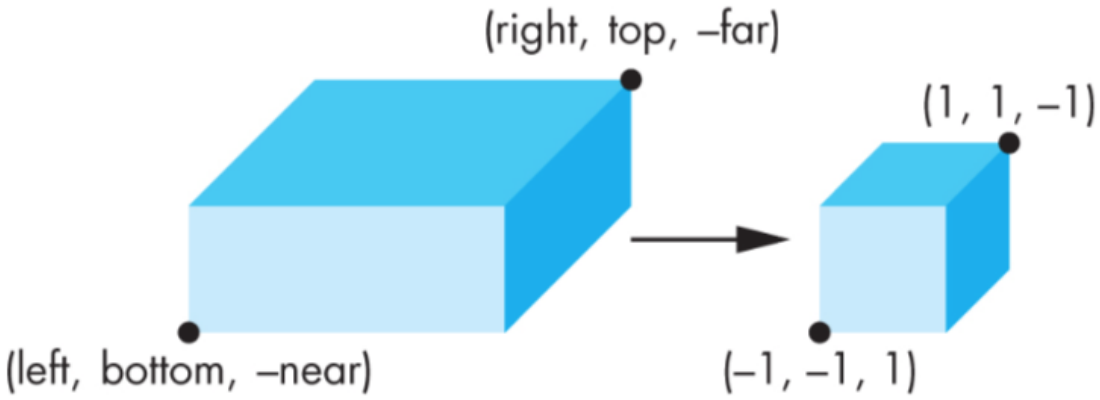
Projections – Parallel (orthographic)

- However, M is applied in the hardware after the vertex shader.
  - Which is in clip coordinates
- How do we “include” or “see” more of our scene?

Projections – Parallel (orthographic)

- We scale what we want to "include" to fit within the canonical view volume. i.e. (-1,1),(-1,1),(-1,1)
- Function in `tiny-graphics.js` :

```
orthographic(left, right, bottom, top, near, far)
```



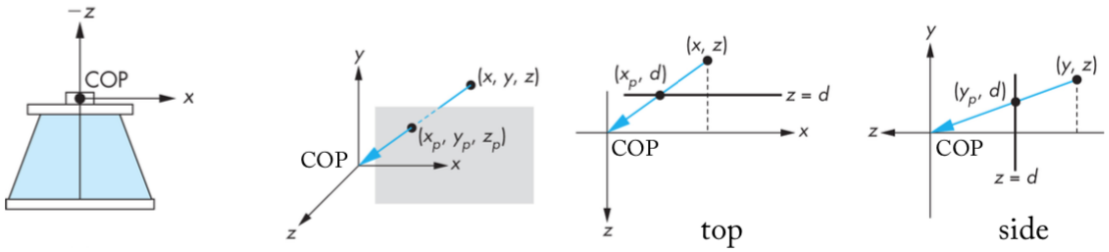
Projections – Parallel (orthographic)

$$N = \mathrm{ST} = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{l+r}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{b+t}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Projections – Perspective

- Basic symmetrical perspective projection
- The point (x, y, z) is projected through the projection plane to the eye point (or center of projection COP)
- We can compute the point of intersection with

$$x_p = \frac{x}{z/d}, y_p = \frac{y}{z/d}$$



Projections – Perspective

- The simple perspective projection matrix is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{pmatrix}$$

Projections – Perspective

- Uh oh, the homogeneous coordinate is no longer 1?

$$q = \begin{pmatrix} x \\ y \\ z \\ \frac{z}{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- We have to divide by the homogeneous coordinate to get back to 3D space.

$$q = \begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{x}{\frac{z}{d}} \\ \frac{y}{\frac{z}{d}} \\ d \\ 1 \end{pmatrix}$$

Projections – Perspective

Generalization!

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

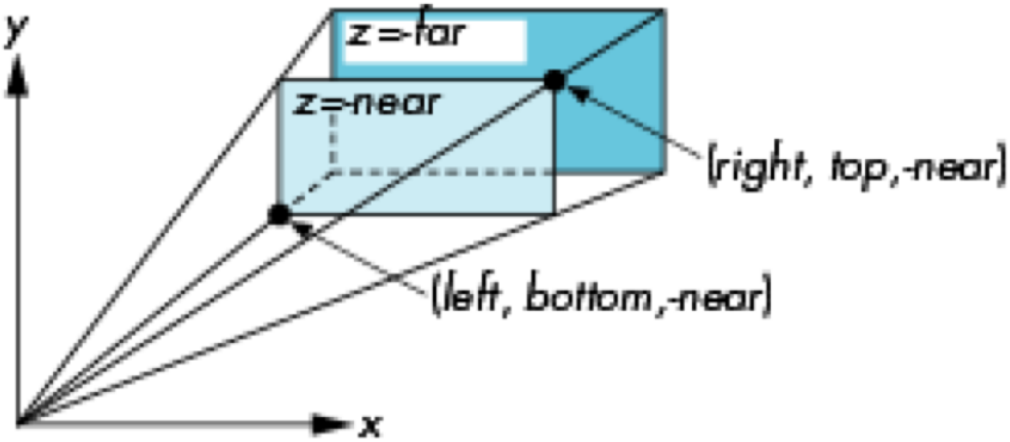
after perspective division, the point $(x, y, z, 1)$ goes to

$$x'' = -x/z, \quad y'' = -y/z, \quad z'' = -(a + b/z)$$

Projections – Perspective

- perspective projection matrix:

$$P = NSH = \begin{pmatrix} \frac{n}{r} & 0 & 0 & 0 \\ 0 & \frac{n}{t} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$



Projections – Perspective

- WebGL Perspective

```
frustum(left,right,bottom,top,near,far)
```

Projections – Perspective

- Function in `tiny-graphics.js` :

```
perspective(fov_y, aspect, near, far)
```