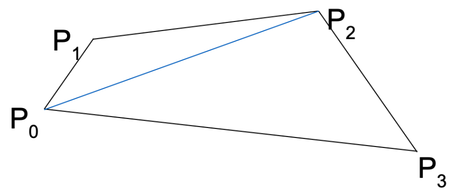
Newell's method

Goal: Need to find the normal for a possibly non-planar polygon.
Let's look at a quadrilateral as an example. Divide the quadrilateral into two triangles.



Find the normal for each triangle and add them together (this gives a scaled version of the average normal).
The normal for the triangle made up of P_0, P_1, and P_2 is,

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix}.$$

For the x component of the normal, the value is $(y_1-y_0)(z_2-z_0) - (y_2-y_0)(z_1-z_0)$.

Similarly, for the triangle made up of P_0, P_2, and P_3, the *x* component of the normal is $(y_2-y_0)(z_3-z_0) - (y_3-y_0)(z_2-z_0)$.

The total normal is the sum of the previous two expressions:

$$(y_1-y_0)(z_2-z_0) - (y_2-y_0)(z_1-z_0) +$$
$$(y_2-y_0)(z_3-z_0) - (y_3-y_0)(z_2-z_0) =$$
$$y_1z_2-y_1z_0-y_2z_1+y_0z_1+y_2z_3-y_0z_3-y_3z_2+y_3z_0$$

Add and subtract (giving a zero-sum change) terms of y_0z_0, y_1z_1, y_2z_2, y_3z_3:
$y_1z_2-y_1z_0-y_2z_1+y_0z_1+y_2z_3-y_0z_3-y_3z_2+ y_3z_0+y_0z_0-y_0z_0+y_1z_1-y_1z_1+y_2z_2-y_2z_2+ y_3z_3 -y_3z_3$
Rearrange terms for grouping:

$$y_0z_0+y_0z_1-y_1z_0-y_1z_1 +$$
$$y_1z_1+y_1z_2-y_2z_1-y_2z_2 +$$
$$y_2z_2+y_2z_3-y_3z_2-y_3z_3 +$$
$$y_3z_3+y_3z_0-y_0z_3-y_0z_0$$

Grouping the previous expression in the "right" way gives the normal as

$$(y_0-y_1)(z_0+z_1) + (y_1-y_2)(z_1+z_2) + (y_2-y_3)(z_2+z_3) + (y_3-y_0)(z_3+z_0)$$

This expression can be generalized to

$$\Sigma(y_i-y_j)(z_i+z_j) \text{ where } j=(i+1)\%n.$$

There are similar expressions for the *y* component ($\Sigma(z_i-z_j)(x_i+x_j)$) and the *z* component ($\Sigma(x_i-x_j)(y_i+y_j)$) of the normal.