Discussion 1B.
04/10/2020.

1. Geometry review
2. Shapes in Computer Graphics.
3. * Quiz.
4. Team formation.

Geometry - Spaces

Defn. space → $\begin{cases} \text{Objects.} \\ \text{Operations.} \end{cases}$

- Vector Space

objects $\begin{cases} \text{Scalars} & : \text{numbers and "}\pm\text{" "}\div\text{"} \\ \text{vectors.} \end{cases}$

operations $\begin{cases} \text{vector + vector} \\ \text{scalar} \cdot \text{vector} \end{cases}$

$(\alpha, \beta, \nu, \cdots) \in S$ (Scalars)

$(u, v, w, \cdots) \in V$ (Vectors)

ex. $w = \alpha \cdot u + \beta \cdot v$

Defn. linear combination :

$$\alpha_1 \cdot u_1 + \alpha_2 \cdot u_2 + \cdots \alpha_n \cdot u_n$$

- Affine Space

Vector space +

Object : Point. (P, Q)

Operation : Point - Point subtraction.

$$P - Q = v.$$

Prop. $P + v \Rightarrow$ Point.

$P + \alpha \cdot v \Rightarrow$ Point.

$\underline{P + Q}$ ⊗

Prop. (Affine combination)

$\alpha_1 P_1 + \alpha_2 P_2 + \cdots + \alpha_n P_n \Rightarrow$ point

when $\alpha_1 + \alpha_2 + \cdots \alpha_n = 1$

Proof: $P_1 = \underline{P_0} + v_1$

$P_2 = P_0 + v_2$

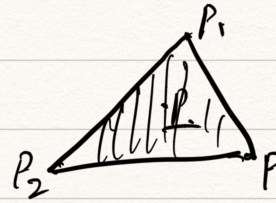
$\vdots$

$P_n = P_0 + v_n$

$= \alpha_1 (P_0 + v_1) + \alpha_2 (P_0 + v_2) \cdots + \alpha_n (P_0 + v_n)$

$= P_0 + \alpha_1 \cdot v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$

Dfn. Convex combination:



$P = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \cdots + \alpha_n P_n$

when $\alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_n = 1$

and $\alpha_1, \alpha_2, \alpha_3, \cdots \alpha_n \geq 0$

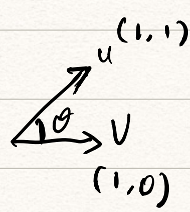
- Euclidean spaces:

vector space +

operation: inner (dot) product

Prop. $u \cdot v \Rightarrow$ scalar.

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$



ex. $u \cdot v = 1 \cdot 0 + 1 \cdot 1 = 1$

Prop. $u \cdot v > 0 \iff \cos\theta > 0, \ \theta < 90^\circ$

$u \cdot v = 0 \iff \cos\theta = 0, \ \theta = 90^\circ$

$u \cdot v < 0 \iff \cos\theta < 0, \ \theta > 90^\circ$

- Coordinates and Coordinate system

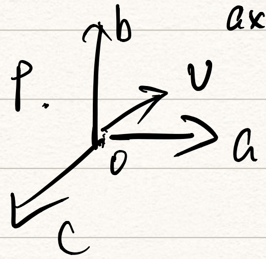
Homogeneous coordinates?

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{pmatrix} \qquad p = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}$$

Prop. $\begin{cases} p + v \Rightarrow \text{point.} \\ v_1 + v_2 \Rightarrow \text{vector} \\ \alpha_1 p_1 + \alpha_2 p_2 \Rightarrow \text{point when } \alpha_1 + \alpha_2 = 1 \end{cases}$

Coordinate system :

axis :



basis : a, b, c (vector) O (Point)

$$v = v_1 \cdot a + v_2 b + v_3 \cdot c$$

$$= \begin{pmatrix} a & b & c & O \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{pmatrix}$$

(4×4)

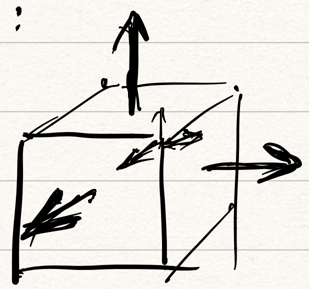
$$p = p_1 \cdot a + p_2 \cdot b + p_3 \cdot c + O$$

$$= (a \ b \ c \ O) \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}$$

usually $p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix}$

Question : How many points are needed to draw a cube?

Answer :



8 points × 3 normals / point

= 24 normals .