Graph question bank for CSE101

These have been collected from books, other courses, and actual interview questions. Any starred question will not be asked in the exam.

Unless otherwise stated, assume a graph G is simple, undirected, and unweighted. Assume the graph is represented as an adjacency list. For any question, give a clear algorithm as pseudocode, and give a running time analysis.

1. A cycle is a sequence of vertices $v_1, v_2, \ldots, v_k$ such that for each i, (v_i, v_{i+1}) is an edge, and (v_k, v_0) is an edge. Determine if G has a cycle. Try using both BFS and DFS.

2. Given edge e, determine if G has a cycle involving e.

3. Determine if G contains *any* cycle. (Clearly, you can use the solution for the previous problem, but there's a more efficient method.)

4. A directed graph is called a DAG if it contains no directed cycles. Determine if a graph is a DAG.

5. * Prove that a graph G is bipartite iff it has no cycles of odd length. (Hint: follow the progress of BFS on G)

6. A vertex v is a cut or articulation vertex if its removal disconnects two other vertices. Equivalently, all paths (in G, before removal) from u to u' pass through v. Determine if v is a cut vertex. If so, find some pair (u, u') that got disconnected.

7. The proof of Dijkstra's algorithm fails when there are negative edges. Why?

8. We have three containers, whose sizes are 10 pints, 7 pints, and 4 pints. The 7-pint and 4-pint containers are full of water, while the 10-pint container is empty. We are only allowed one operation: pouring the contents on one container into another, stopping only when the source container is empty, or the destination container is empty. Determine if there is a sequence of pourings that leave exactly 2 pints in the 7- or 4- pint container.

   Model this as a graph problem. Trying solving the general version where there are three containers of integer capacities of a maximum of n. The starting configuration is provided, and we have some desired "end state". Determine if the "end state" can be achieved. Work out the running time.

9. Given a directed (unweighted) graph G, the *reverse* is obtained by simply reversing all edges. Assume G is represented as an adjacency list of out neighbors. Construct the reverse of G.

10. * An Euler tour is a cycle that can pass through each vertex multiple times, but must use each edge exactly once. Prove that an undirected graph has an Euler tour iff all vertices have even degree.

11. Using the previous theorem, give an algorithm that finds an Euler tour (if one exists).

12. A common recommendation feature used in social networking websites is the number of common friends. In an undirected graph G, find a pair (u, v) with the largest number of common neighbors.

13. * A directed graph G is singly connected if for any pair u, v of vertices that are connected (from u to v), there exists at most one simple path from u to v. Determine if a directed graph is singly connected. (Hint: DFS)

14. Consider a game of snakes and ladders. Determine the minimum number of throws required to win the game.

15. Biologists often construct a *food network* of species in an ecosystem. The vertices represent species, and a directed edge (u, v) means species u eats species v. An apex species is one that is not eaten by another species. Suppose you are give a list of all possible "eating" relationships (so a list of "u eats v"). Determine all apex species.

16. In a food network (described above), determine the effect of the extinction of a species. If a species v goes extinct, and there is some other species u that *only* eats v, then u will become extinct. This can cascade through the food network. Given a food network and a species v, determine all other species that will become extinct if v goes extinct.

17. A *triangle* is a set of edges $\{(u, v), (v, w), (w, u)\}$. (Equivalently, a triangle is a cycle of length 3.) Given a vertex s, we wish to count the number of distinct triangles in G that involve s. Get a solution with $O(n)$ space.

18. You are given a directed, unweighted graph G representing the power grid, as an adjacency list. Each vertex is a power station, and an edge (u, v) means that power goes from u to v. A source vertex denotes a power generator. A sink vertex denotes a power supplier, typically to a neighborhood. Your job is to understand the robustness of the power grid, under failure of a power station s. (Note that this may not be a source, it could be some internal vertex.) If s fails, it stops transmitting power to any of the outneighbors. This could potentially result in some sink node t

not receiving power at all, representing a power failure in a neighborhood. Thus, a sink node t will not receive power if every path from every source to t passes through the power station s. Design an algorithm that given G and a vertex s, determines all the sink vertices t (if any) that stop receiving power when s fails.

19. A directed graph G is semiconnected if for all pairs of vertices u, v, there is either a path from u to v, or from v to u (but not both). Give an algorithm to determine if G is semiconnected.

20. The diameter of a graph G is the largest shortest path distance in G (meaning, it is $\max_{u,v \in V} dist(u, v)$, where $dist(u, v)$ is the shortest path distance). Give an algorithm to compute the diameter of a undirected graph G. Give a more efficient algorithm to determine the diameter of an undirected tree T.