Lab 3 Report

Speed Control of a Simplified Car Model

ECE311

Lab Group: 17

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Output 1

In order to achieve SPEC 1, asymptotic tracking and disturbance rejection, we must fulfill the internal model principle (IMP). This means, our controller must satisfy two constraints:

1. C(s) must contain the same poles as D(s)
2. C(s)G(s) must contain the poles of R(s)

In this case, R(s) and D(s) can be derived using the Laplace transform:

$$R(s) = \frac{v_{des}}{s}$$

$$D(s) = \frac{\bar{d}}{s}$$

Both R(s) and D(s) have a pole at zero. Our controller, C(s) must contain a pole at zero to satisfy the first constraint above. Moreover, when C(s) has a pole at zero, C(s)G(s) has no zero-pole cancellations. Therefore, if C(s)G(s) contain the poles of R(s) it will satisfy the second constraint.

$$C(s) = \frac{C'(s)}{s}$$

SPEC 2 requires that the closed-loop system (input R(s), output Y(s), with no disturbance) be BIBO stable. In order for this to be satisfied, we require that Y(s) / R(s), when D(s) = 0, have all poles in the OLHP. This condition will also be met if the IMP is fulfilled because C(s)G(s) will contain the poles of R(s) and cancel any poles from R(s) that would cause Y(s) to be unstable (those that are in the closed right-hand plane). As a result, all poles of Y(s) / R(s) are now in the OLHP, meeting SPEC 2.

To use a PI controller, two specifications must be met as outlined in the lab. The requirements are:

a). The poles of $\frac{1}{1+C(s)G(s)}$ must contain a negative real part

b). C(s)G(s) must have no cancellations in the closed right-hand plane

To get these requirements, we calculate the following:

For b),

$$C(s)G(s) = \frac{K(T_I s+1)}{T_I s(s+a)} = \frac{K(T_I s+1)}{T_I s^2+aT_I s}$$

For a),

$$\frac{1}{1+C(s)G(s)} = \frac{1}{1+\frac{K(T_I s+1)}{T_I s^2+aT_I s}} = \frac{T_I s^2+aT_I s}{T_I s^2+aT_I s+K(T_I s+1)} = \frac{T_I s^2+aT_I s}{T_I s^2+aT_I s+KT_I s+K}$$

$$= \frac{T_I s^2+aT_I s}{T_I s^2+T_I(a+K)s+K} = \frac{s^2+as}{s^2+(a+K)s+\frac{K}{T_I}}$$

As we can see, C(s)G(s) has no illegal cancellations which satisfies (b). In order to satisfy (a), the poles must be in the OLHP. For second order polynomials, if all coefficients are positive or negative, we know that the zeros are all in the OLHP. Therefore:

$$a + K > 0 \text{ and } \frac{K}{T_I} > 0$$

Since the coefficient of s^2 is positive, K / T_I must also be positive. Therefore, K, T_I > 0 in order to satisfy (b). Thus, if K and T_I are positive, the PI controller satisfies constraint (a) and (b).

SPEC 3 specifies that all poles of the closed-loop system need to lie on the real axis so y(t) has no oscillatory behaviour. To meet this, we first define Y(s) / R(s):

Assuming D(s) = 0,

$$T(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)}$$

$$= \frac{K(T_I s+1)}{T_I s^2+(aT_I+KT_I)s + K}$$

$$= \frac{K(s+\frac{1}{T_I})}{s^2+(a+K)s+\frac{K}{T_I}}$$

In order to get SPEC 3, the poles of T(s) need to be in the OLHP and on the real axis. To determine K and T_I and to guarantee our poles have this property, we equate the coefficients of the denominator of T(s) with $s^2 + (p_1 + p_2)s + p_1p_2$, where $-p_1$ and $-p_2$ are negative real poles, to get:

$$p_1 + p_2 = a + K$$

$$p_1p_2 = \frac{K}{T_I}$$

From this, we observe that:

$$p_1 = a = \frac{1}{T_I} \rightarrow p_1 = T_I = \frac{1}{a}$$

$$p_2 = K$$

SPEC 5 says that when the desired reference speed $v_{des} = 14\ m/s$ and D(s) = 0, the control input signal u(t) should not exceed the bound $|u(t)| \leq 30\ m/s^2$. To do this, we first determine the transfer function U(s) / R(s):

Assuming D(s) = 0,

$$\frac{U(s)}{R(s)} = \frac{C(s)}{1+C(s)G(s)}$$

$$= \frac{s^2+as}{s^2+(a+K)s+\frac{K}{T_I}}\left(\frac{K(T_Is+1)}{T_Is}\right)$$

$$= \frac{\frac{K}{T_I}(s+a)(T_Is+1)}{s^2+(a+K)s+\frac{K}{T_I}}$$

We now calculate U(s) given $R(s) = \frac{v_{des}}{s} = \frac{14}{s}$:

$$U(s) = \frac{\frac{K}{T_I}(s+a)(T_Is+1)}{s^2+(a+K)s+\frac{K}{T_I}}\left(\frac{14}{s}\right) = \frac{\frac{14K}{T_I}(T_Is^2+(aT_I+1)s+a)}{s(s^2+(a+K)s+\frac{K}{T_I})}$$

Applying the initial value theorem, since U(s) is rational and strictly proper, we get the following:

$$u(0) = lim_{s\rightarrow\infty} sU(s)$$

$$= lim_{s\rightarrow\infty} \frac{\frac{14K}{T_I}(T_Is^2+(aT_I+1)s+a)}{(s^2+(a+K)s+\frac{K}{T_I})}$$

$$= 14K$$

Since we're assuming the largest acceleration will occur at t = 0, we need the following to hold true and we can determine the inequality K needs to satisfy to meet the initial condition:

$$|u(0)| < 30$$

$$14K < 30$$

$$K < \frac{15}{7}$$

With this, we can now find an upper bound on $p_1 + p_2$:

$$\text{Since } p_2 = K$$

$$p_2 < \frac{15}{7}$$

$$\text{And now since } p_1 = T_I = \frac{1}{a}$$

$$p_1 + p_2 < \frac{1}{a} + \frac{15}{7}$$

This condition on p_1 and p_2 will be used to tune the controller in output 2 to minimize settling time.

As stated in the lab handout, SPEC 4, the settling time when D(s) = 0 should be less than 6 seconds, will be resolved during tuning of p_1 and p_2 in the simulation.

Output 2

To satisfy SPEC 4, the settling time of the simple car model, we decided to choose our poles based off of our theoretical calculations made for SPEC 5 in Output 1 of this document.

$$p_1 = T_I = \frac{1}{a} = 1000$$

$$p_2 = K = \frac{15}{7}$$

This resulted in figures 1, 2, and 3.

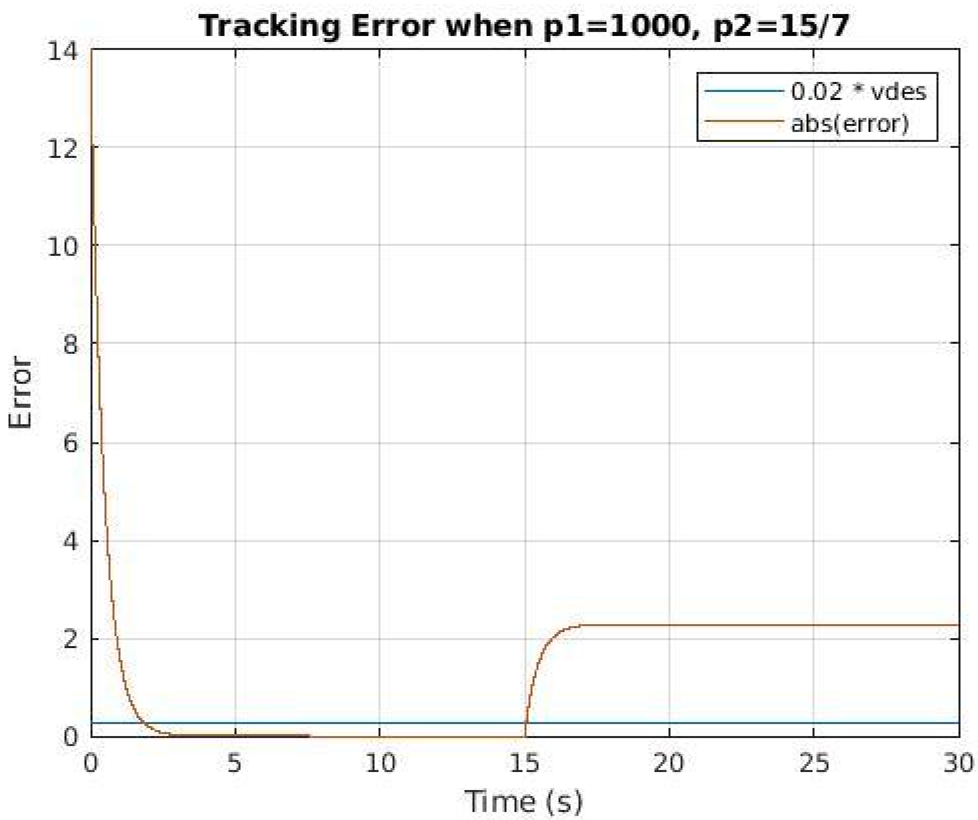

Figure 1: Tracking Error when p_1 = T_I = 1000 and p_2 = K = 15/7

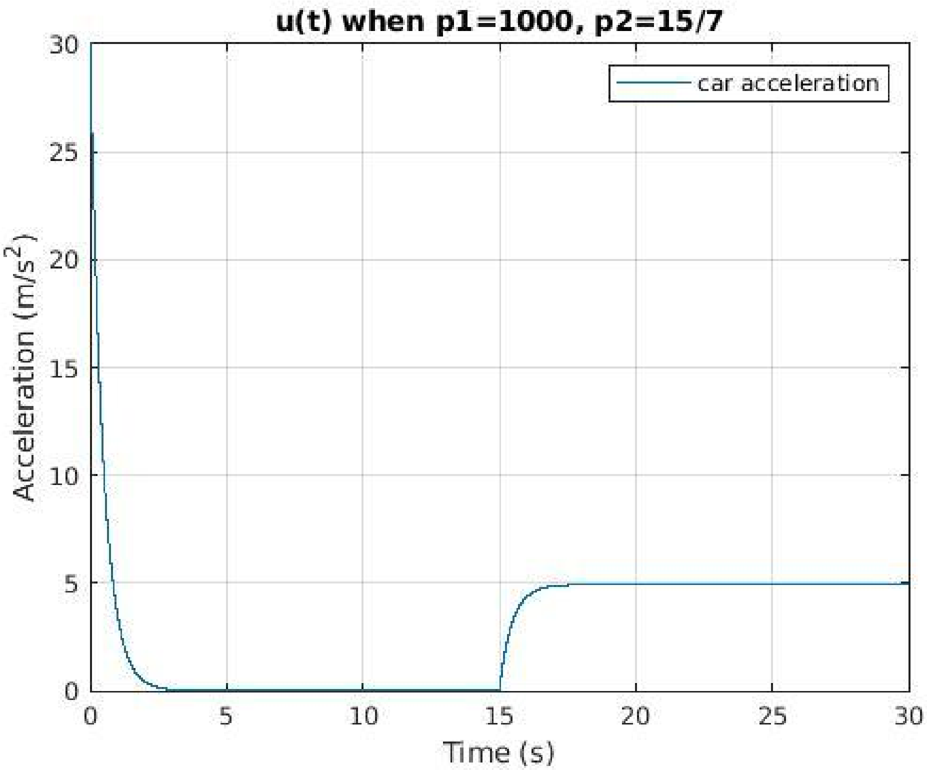

Figure 2: u(t) (ie. car acceleration) when p_1 = T_I = 1000 and p_2 = K = 15/7

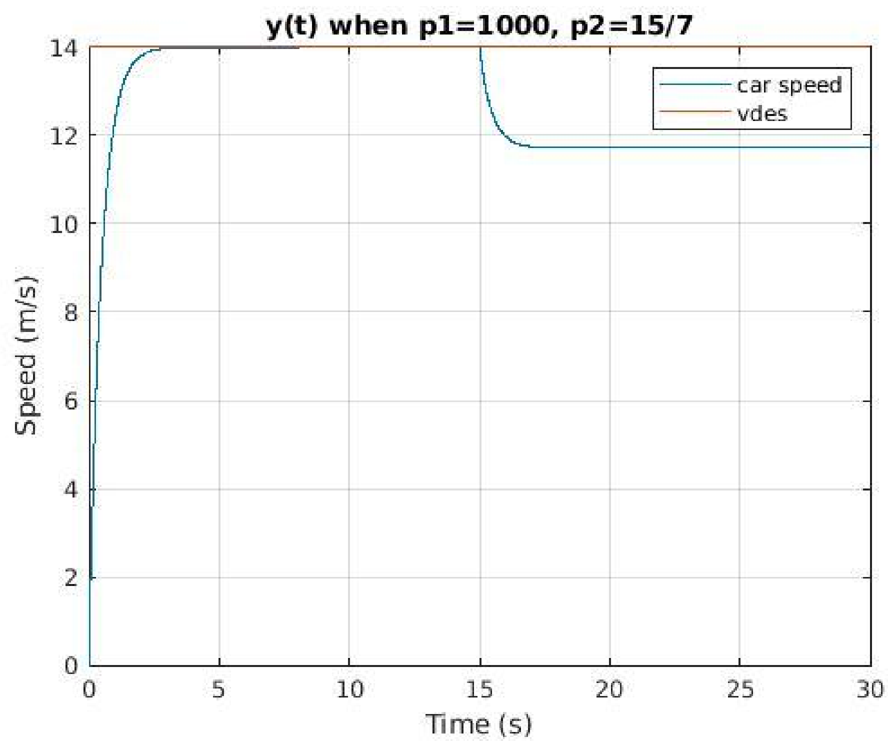

Figure 3: y(t) (ie. car speed) when p_1 = T_I = 1000 and p_2 = K = 15/7

Using Matlab's cursor tool, we were able to estimate the settling time when the disturbance is zero (horizontal road) and when the disturbance is activated at t=15s (incline road) as shown in figure 4.

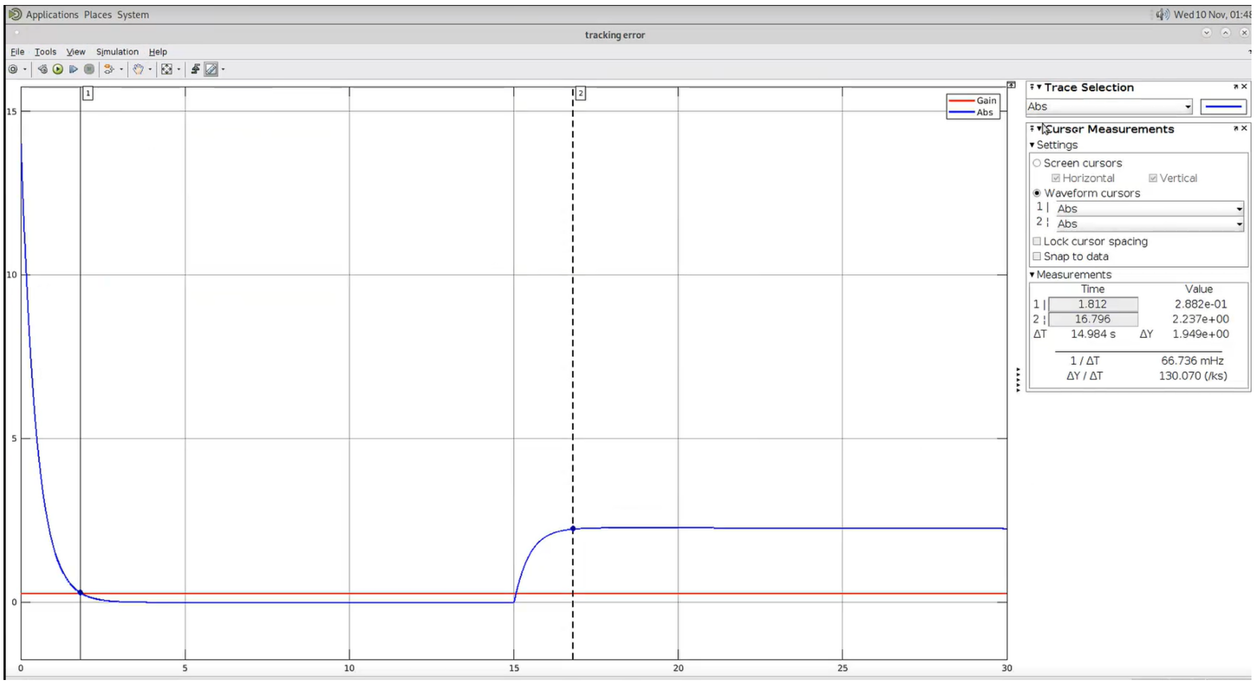

Figure 4: Settling Time for First 15 seconds and After Disturbance is Added
(p1 = 1000, p2 = 15/7)
(values are shown in the right window)

The settling time without disturbance is determined by the intersection between the lines *Gain* and *Abs*; *Gain* being 2% of v_{des}. Reading from cursor 1 from the Matlab cursor window, we conclude the settling to be around:

$$T_s^{flat} = 1.8s$$

To get the settling time when disturbance is present, we extended the simulation time until the tracking error graph crossed the *Gain* line. Then, using the cursor measurement tool again, we get a settling time of:

$$T_s^{incline} = 2122 - 15 = 2107s$$

With these values, we can conclude that all SPECs are met. SPEC 1 is satisfied as the error steadily approaches zero in the horizontal plane, despite the disturbance, as shown in figure 1. SPECs 2 and 3 are satisfied as shown in figure 3 as our acceleration values do not oscillate and are bounded by v_{des} = 14. The settling time is under 6s which satisfies SPEC 4 while our initial acceleration is 30m/s^2 (assuming p_2 is infinitesimally smaller than 15/7) which then satisfies SPEC 5.

To tune our settling time, we decided to change each pole at a time to their extremities within the bounds found in SPEC 5 of output 1. We decided to go with this strategy because real poles with multiplicity 1 in the Laplace domain represent an exponential within the time domain which gives us a general idea of what occurs in between the extremities.

As p_2 (K) approaches closer to zero, the settling time increases significantly. This is because, in the denominator of E(s), decreasing the second coefficient of the polynomial pushes the zero towards the origin, thus making it less negative. Therefore, the error will approach the steady state more gradually because in the time domain, the exponential is raised to a smaller number. As p_1 (1/a) approaches closer to zero, the settling time decreases. This is because the second coefficient is dependent on “*a*” and not its inverse. Therefore, decreasing p_1 increases “*a*” which results in a more negative real pole and a faster settling time.

With this knowledge in mind, while respecting the bounds we found for SPEC 5 ($p_1 + p_2 < \frac{1}{a} + \frac{15}{7}$), we obtain these pole values:

$$p_1 = T_I = 20$$

$$p_2 = K = \frac{15}{7}$$

Lower values of p_1 give us better settling times as expressed above, but as we go below 20, overshooting occurs, increasing our settling time. Therefore, we found the optimal pole to be at 20. The optimal settling time we obtained is below (figure 5) and was estimated using the cursor measurement tool:

$$T_s^{flat} = 1.54s$$

$$T_s^{incline} = 40.54s$$

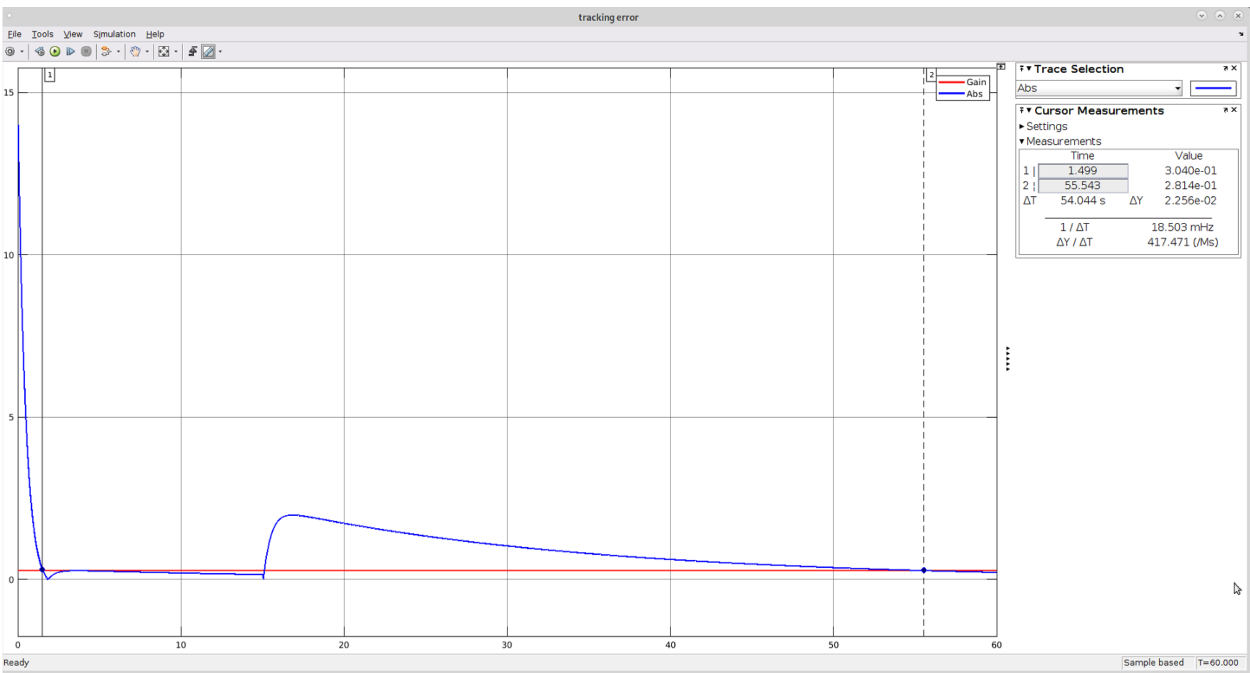

Figure 5: Settling Time for First 15 seconds and After Disturbance is Added

(p1 = 20, p2 = 15/7)

(values are shown in the right window)

We can deduce that using these poles, all SPECs are met. SPEC 1 is satisfied as the error steadily approaches zero in the horizontal plane, despite the disturbance, as shown in figure 6. SPECs 2 and 3 are satisfied as shown in figure 8 as our acceleration values do not oscillate and are bounded by v_{des} = 14. The settling time is under 6s when

D(s) = 0 which satisfies SPEC 4, while our initial acceleration is $30 m/s^2$ (assuming p_2 is infinitesimally smaller than 15/7) which then satisfies SPEC 5.

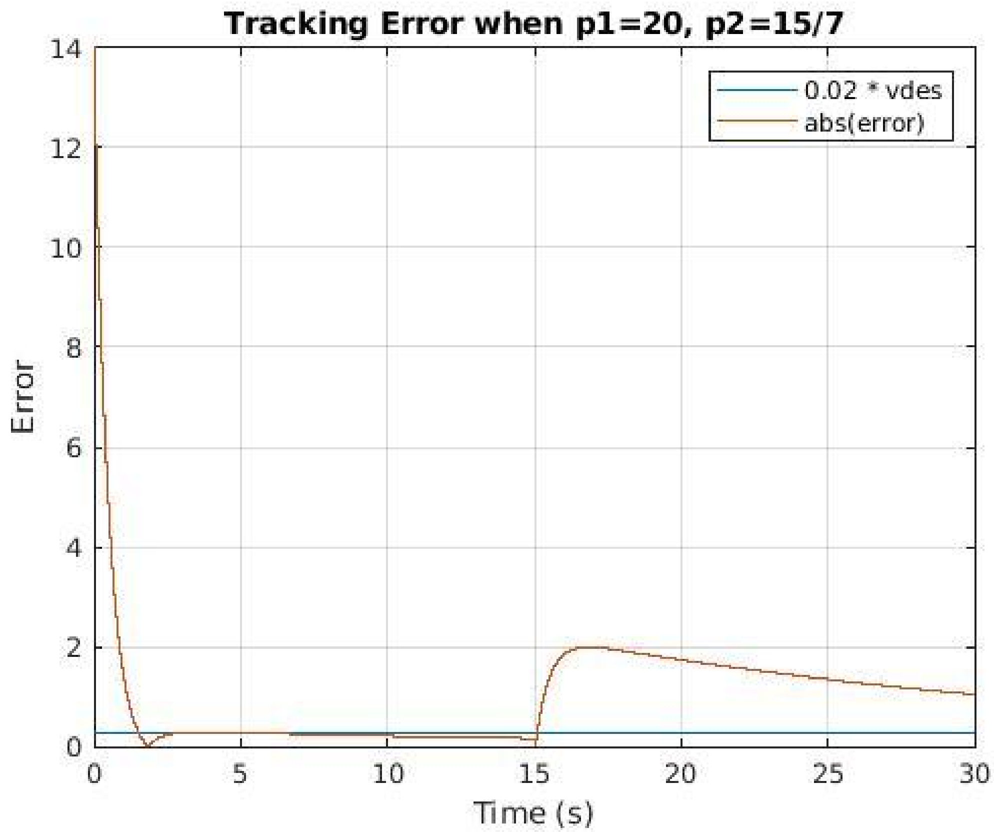

Figure 6: Tracking Error when $p_1 = T_I = 20$ and $p_2 = K = 15/7$

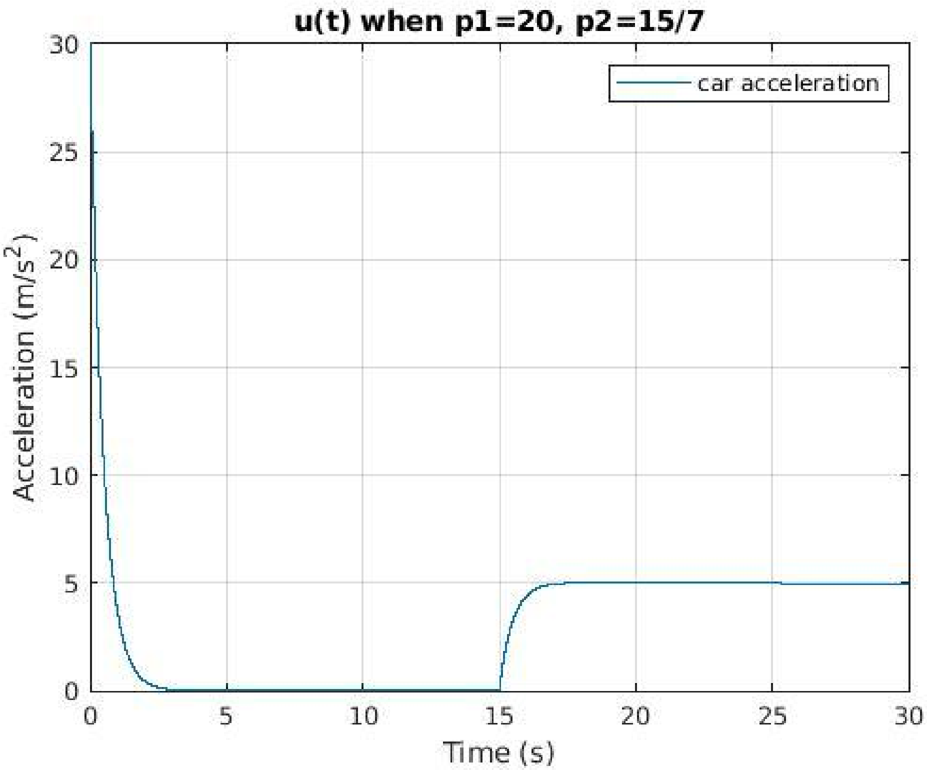

Figure 7: u(t) (ie. car acceleration) when $p_1 = T_I = 20$ and $p_2 = K = 15/7$

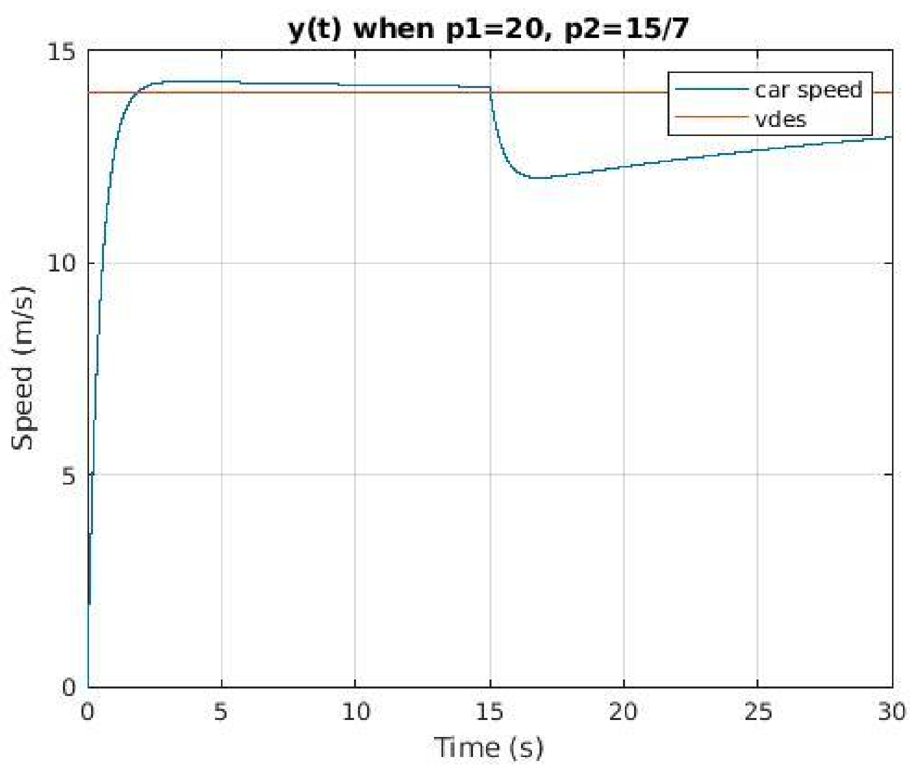

Figure 8: y(t) (ie. car speed) when $p_1 = T_I = 20$ and $p_2 = K = 15/7$

Using the *stepinfo* function, we verify that our empirically measured settling time (T_s^{flat}) is the same as the one found with *stepinfo*.

```
TStepResponseCharacteristics =

  struct with fields:

        RiseTime: 0.9625
    SettlingTime: 1.5180
     SettlingMin: 0.9002
     SettlingMax: 1.0199
       Overshoot: 1.9905
      Undershoot: 0
            Peak: 1.0199
        PeakTime: 3.6530
```

Figure 9: Settling Time Found from Calling stepinfo()

$$T_s^{stepinfo} = 1.5180s$$

As shown in figure 7 of our tuned controller, we can see that once the disturbance is present (t > 15s), the acceleration will again eventually reach a steady state value. We used the cursor measurement tool to find the value of the steady state value as shown in figure 10.

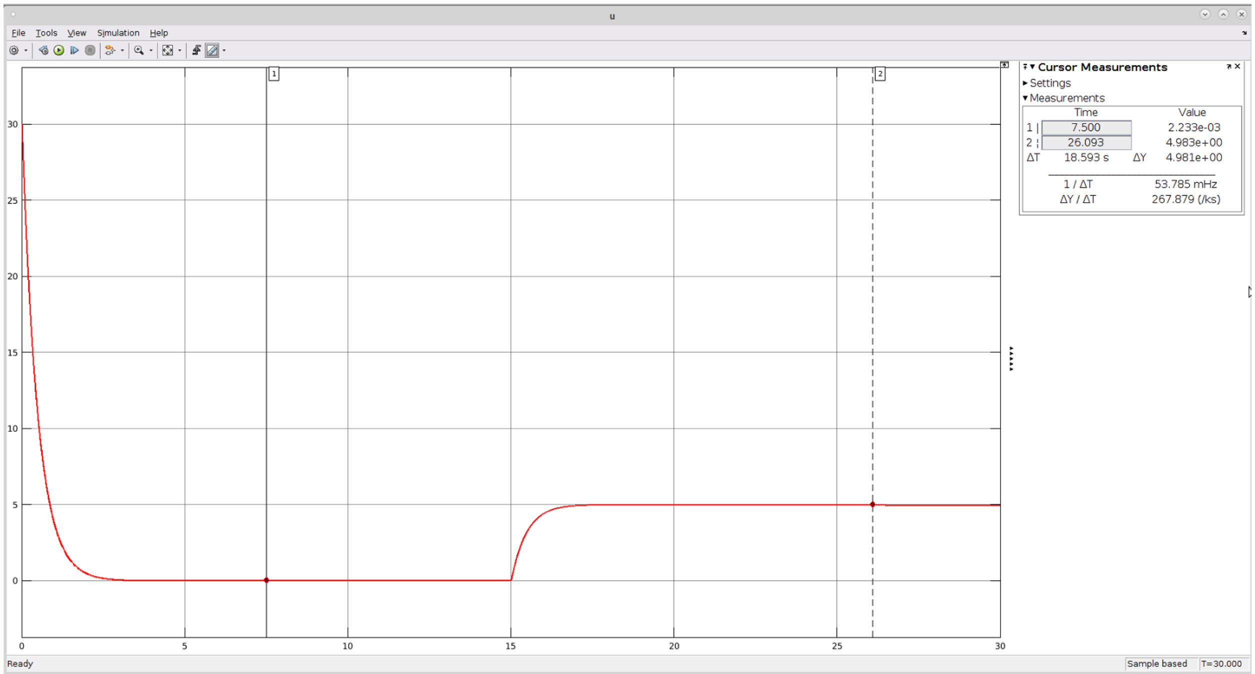

Figure 10: u(t) (ie. car acceleration) Settling at a Nonzero Value

(p_1 = 20, p_2 = 15/7)

(value is shown by cursor 2 in the right window)

We obtain the steady state value to be 4.983 m/s^2. Looking at the physical representation of this model, this is a result of the inclined hill. An acceleration of -4.9 m/s^2 is applied to the car due to gravity from the hill:

$$g \times sin(\Theta) = -\ 4.9\, m/s^2, \text{ where } \Theta = -\frac{\pi}{6}$$

Therefore, the car must apply an acceleration greater than 4.9 m/s^2 in magnitude against gravity to achieve v_{des} once again. This is why an acceleration of 4.983 is applied. This extra acceleration wasn't required before t = 15s because the disturbance was 0 and the car was on flat land (gravity was not working against the car).