Lab 2 Report

Control of a Magnetically Levitated Ball
ECE311

Lab Group: 17

Name of Students: Alexander Mark, Kyu Bum Kim, Jack Khoueiri

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Output 1

To find the linearization of the model, we first find the equilibrium point of the magnetic levitating ball system.

equilibrium $f(\bar{x},\bar{u}) = 0$ $\quad \bar{x}_1 = \bar{y}$, $\bar{x}_3 > 0$

$$f(\bar{x},\bar{u}) = \begin{bmatrix} \bar{x}_2 \\ -\frac{k_m}{M}\frac{\bar{x}_3^2}{\bar{x}_1^2} + g \\ -\frac{R_a}{L_a}\bar{x}_3 + \frac{\bar{u}}{L_a} \end{bmatrix} = 0$$

$$\boxed{\therefore \bar{x}_2 = 0}$$

$$-\frac{k_m}{M}\frac{\bar{x}_3}{y} + g = 0 \qquad -\frac{R_a}{L_a}\bar{x}_3 + \frac{\bar{u}}{L_a} = 0$$

$$-\frac{k_m}{M}\frac{\bar{x}_3^2}{y^2} = -g \qquad \frac{\bar{u}}{L_a} = \frac{R_a}{L_a}\bar{x}_3$$

$$\bar{x}_3^2 = \frac{y^2 g M}{k_m} \qquad \bar{u} = R_a\bar{x}_3$$

$$\boxed{\bar{x}_3 = y\sqrt{\frac{gM}{k_m}}} \qquad \boxed{\bar{u} = R_a y\sqrt{\frac{gM}{k_m}}}$$

Figure 1: x and u Values at Equilibrium in Terms of Constant Values

In Figure 1, using the conditions given, we are able to find the equilibrium points of x and u in terms of the parameters of the model. Therefore, we are given:

$$\bar{x} = \left[\bar{y} \quad 0 \quad \bar{y}\sqrt{\frac{gM}{k_m}}\right]^T$$

$$\bar{u} = R_a \bar{y} \sqrt{\frac{gM}{k_m}}$$

We can now find the linearization of the system around $(\bar{x}, \bar{u})$:

$$\tilde{y} = y - \bar{y}$$

$$\tilde{x} = x - \bar{x} = \begin{bmatrix} x_1 - \bar{y} & x_2 & x_3 - \bar{y}\sqrt{\frac{gM}{k_m}} \end{bmatrix}^T$$

$$\tilde{u} = u - \bar{u} = u - R_a \bar{y} \sqrt{\frac{gM}{k_m}}$$

$$\textbf{(1)} \quad \tilde{x}_{dot} = A\tilde{x} + B\tilde{u}$$

$$\textbf{(2)} \quad y_{dot} = C\tilde{x} + D\tilde{u}$$

Where A,B,C,D are defined below

$$\frac{df}{dx} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2k_m}{M}\frac{x_3^2}{x_1^3} & 0 & -\frac{2k_m}{M}\frac{x_3}{x_1^2} \\ 0 & 0 & -\frac{R_a}{L_a} \end{bmatrix}$$

$$A = \frac{df}{dx}(\bar{x},\bar{u}) = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2k_m \bar{x}_3^2}{M \bar{x}_1^3} & 0 & -\frac{2k_m}{M}\frac{\bar{x}_3}{\bar{x}_1^2} \\ 0 & 0 & -\frac{R_a}{L_a} \end{bmatrix}$$

$$B = \frac{\partial f}{\partial u}(\bar{x},\bar{u}) = \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix}$$

$$C = \frac{\partial h}{\partial x}(\bar{x},\bar{u}) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D = \frac{\partial h}{\partial u}(\bar{x},\bar{u}) = 0$$

Figure 2: Computation of matrix A,B,C,D in (1) and (2)

The semicolons denote a new row in the matrix. $\overline{x}$ was defined earlier in this section.

$$A = \left[0\ 1\ 0;\ \frac{2k_m \overline{x}_3^2}{M\overline{x}_1^3}\ 0\ \frac{-2k_m \overline{x}_3}{M\overline{x}_1^2};\ 0\ 0\ \frac{-R_a}{L_a}\right]$$

$$B = \left[0\ 0\ \frac{1}{L_a}\right]^T$$

$$C = [1\ 0\ 0]$$

$$D = 0$$

Output 2

Figure 3 shown below shows the A and B matrices represented in equation (1) done through Matlab approximations and theoretical calculations respectively.

```
A =

         0    1.0000         0
  196.2000         0  -62.6418
         0         0  -60.0000


B =

         0
         0
   20.0000
```

```
A1 =

         0    1.0000         0
  196.2000         0  -62.6418
         0         0  -60.0000


B1 =

     0
     0
    20
```

Figure 3: Matlab (A,B) and Theoretical (A1,B1) Matrices of the System

As we can see from Figure 3, the values are equivalent in value to at least 4 significant figures. In order to see Matlab’s approximation error, we took their difference:

```
errorA =

   4.7481e-06


errorB =

   3.4795e-11
```

Figure 4: Error of Matlab vs Theoretical A & B Matrices

As we can see in figure 4, the difference between the theoretical and Matlab's approximation of the model is negligible. Theoretical calculations use infinite precision while Matlab uses a finite number of precision within their calculations resulting in a small margin of error.

```
poles =

  -60.0000
   14.0071
  -14.0071


eigA =

   14.0071
  -14.0071
  -60.0000
```

Figure 5: Eigenvalues of Theoretical Matrix A and Poles of G

As shown in figure 5, we observe that one of the eigenvalues exists on the real positive axis. Therefore we can determine that the signal will grow exponentially given a bounded input. Therefore the system is internally unstable. Because one of the poles of the transfer function is also on the real positive axis (figure 5), we can conclude that the system is also not BIBO stable.

In terms of physical representation of the system, we understand that a system is stable if we start near the equilibrium, we stay near the equilibrium. The equilibrium which we found in output 1 is when the ball is not accelerating due to a constant current within the electromagnet resulting in a net zero force at a specific distance (ball remains still).

However, since this system is unstable, we would observe that a change in displacement of the ball would cause the ball to either drop or move towards the magnet, deviating from the equilibrium and remaining that way.

Output 3

We determined that a value of K = 100, would make the closed-loop system BIBO stable as this value makes all zeros in 1-C*G in the OLHP (the zeros of this function are the poles of the closed-loop system transfer function). Figure 6 shows the controller's poles when K was set to 100.

```
K =

   100


controller_poles =

   1.0e+02 *

  -1.1730 + 0.0000i
  -0.2093 + 0.1793i
  -0.2093 - 0.1793i
  -0.0085 + 0.0000i
```

Figure 6: Controller Poles when K = 100

The plot in figure 7 shows the time response, the distance the metal ball is from the electromagnet over time, when the ball starts 0.15m away from the electromagnet and the controller has K = 100. We observe that this value of K does make the system converge to 0.1m.

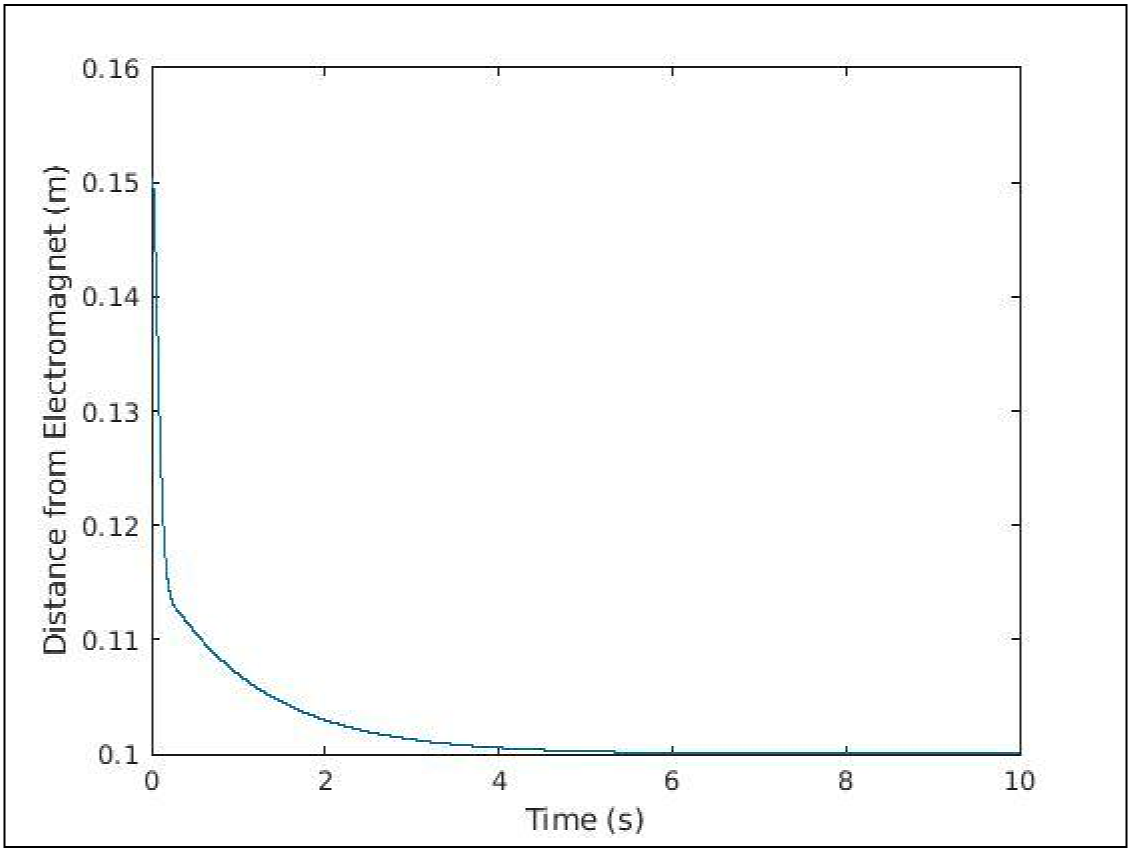

Figure 7: y(t) when y(0) = 0.15m and K = 100

The procedure we used to find a valid K was steadily increasing the value in increments of 10 and testing to see if the controller's poles were in the OLHP. The first K we saw that had this effect was at K = 100. When K was between 70 and 100, we saw that the controller caused the system to be unstable and made the ball diverge from the equilibrium.

As shown in figure 8 below, the range of initial conditions we found where the controller was successful was between 18m and 0.05m. Any initial condition outside of this range, caused the controller to not work. The controller does not work for initial conditions that are far from the equilibrium because it will overshoot too much when it nears the equilibrium value. In a physical sense, this means that the ball is hitting the magnet directly and the ODE solver would fail at this moment due to a division by 0 (y = 0), thus the controller is unable to control the ball to 0.15m.

```
upper_bound_init =

    18

lower_bound_init =

    0.0500
```

Figure 8: Upper bound and Lower Bound on Initial Conditions