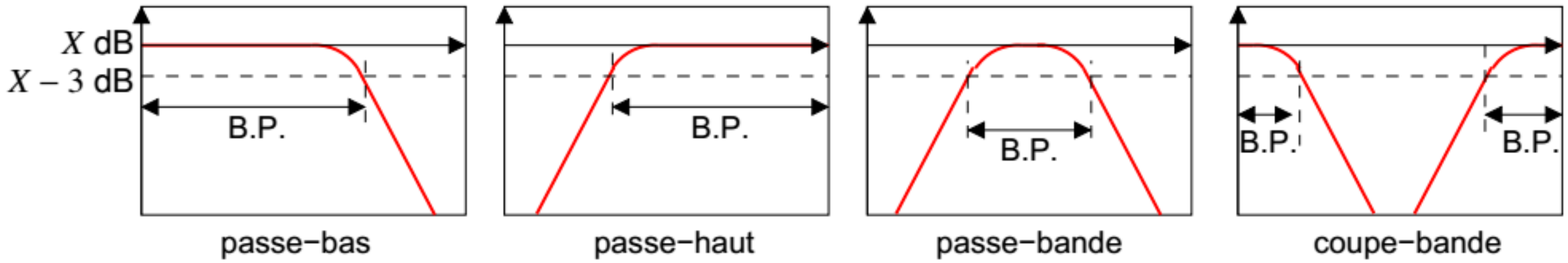
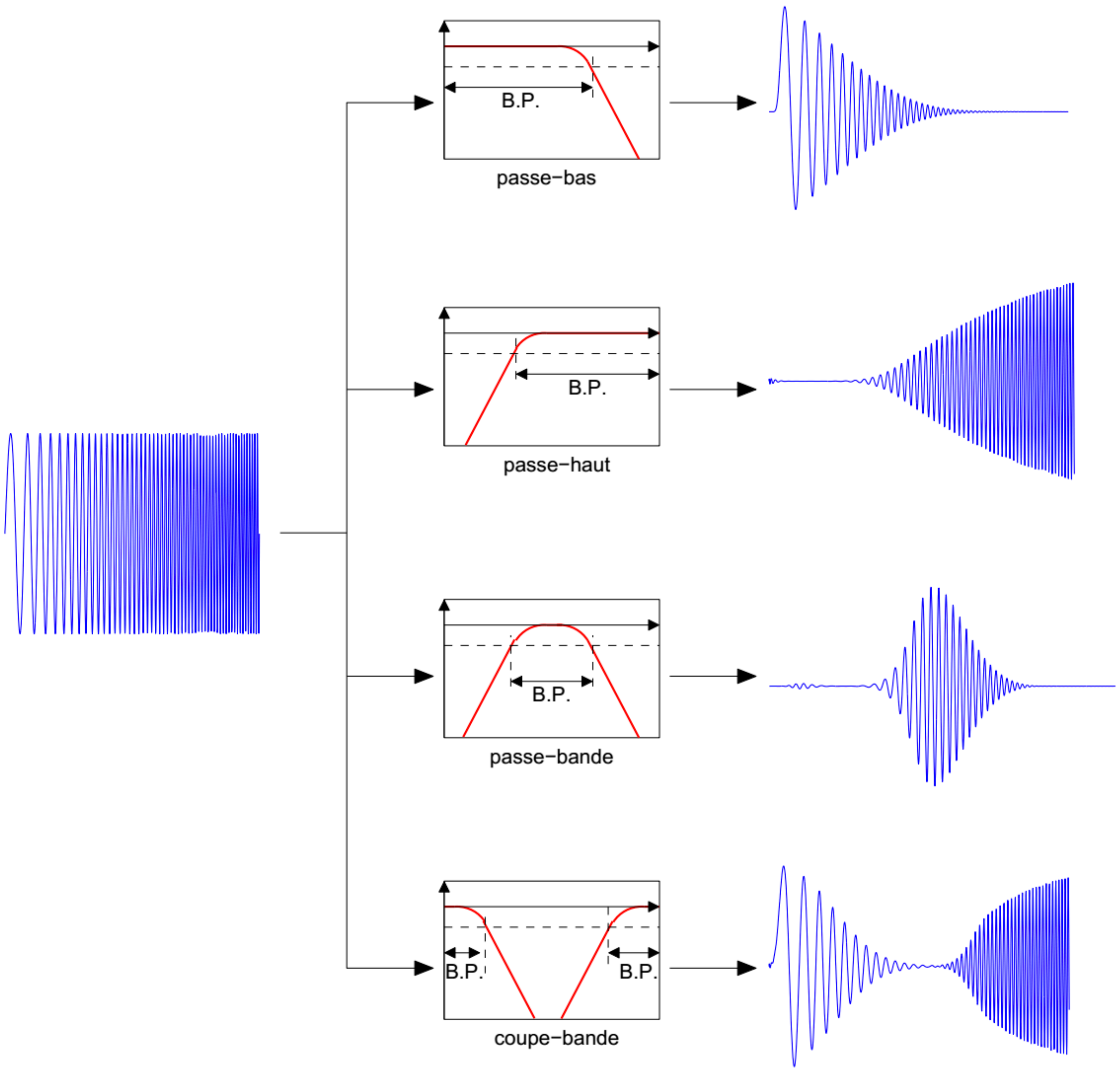
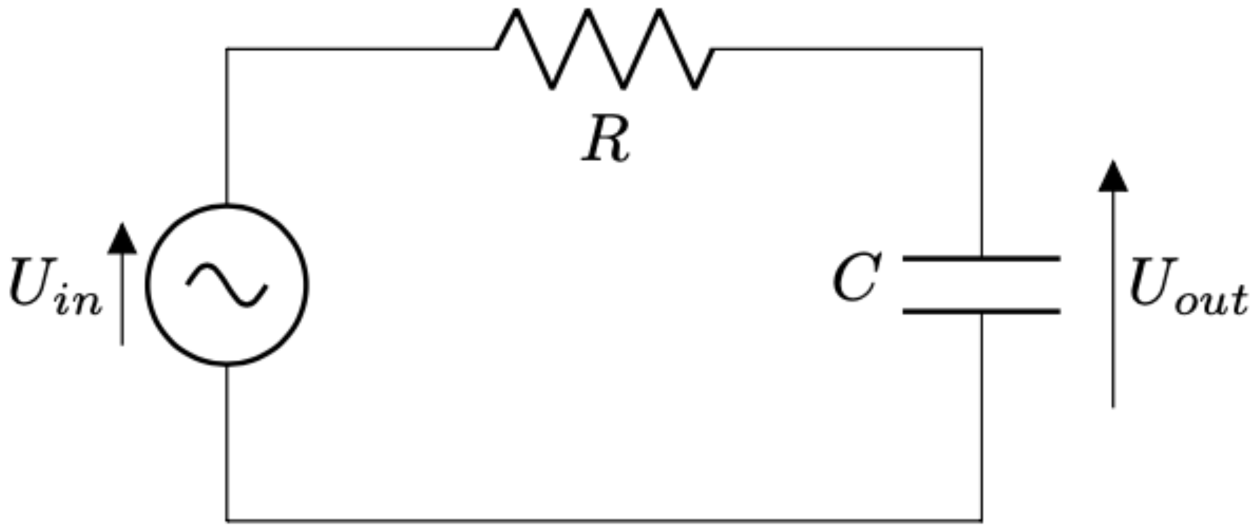
Principaux types de filtres



Principaux types de filtres



Filtres RC passe-bas

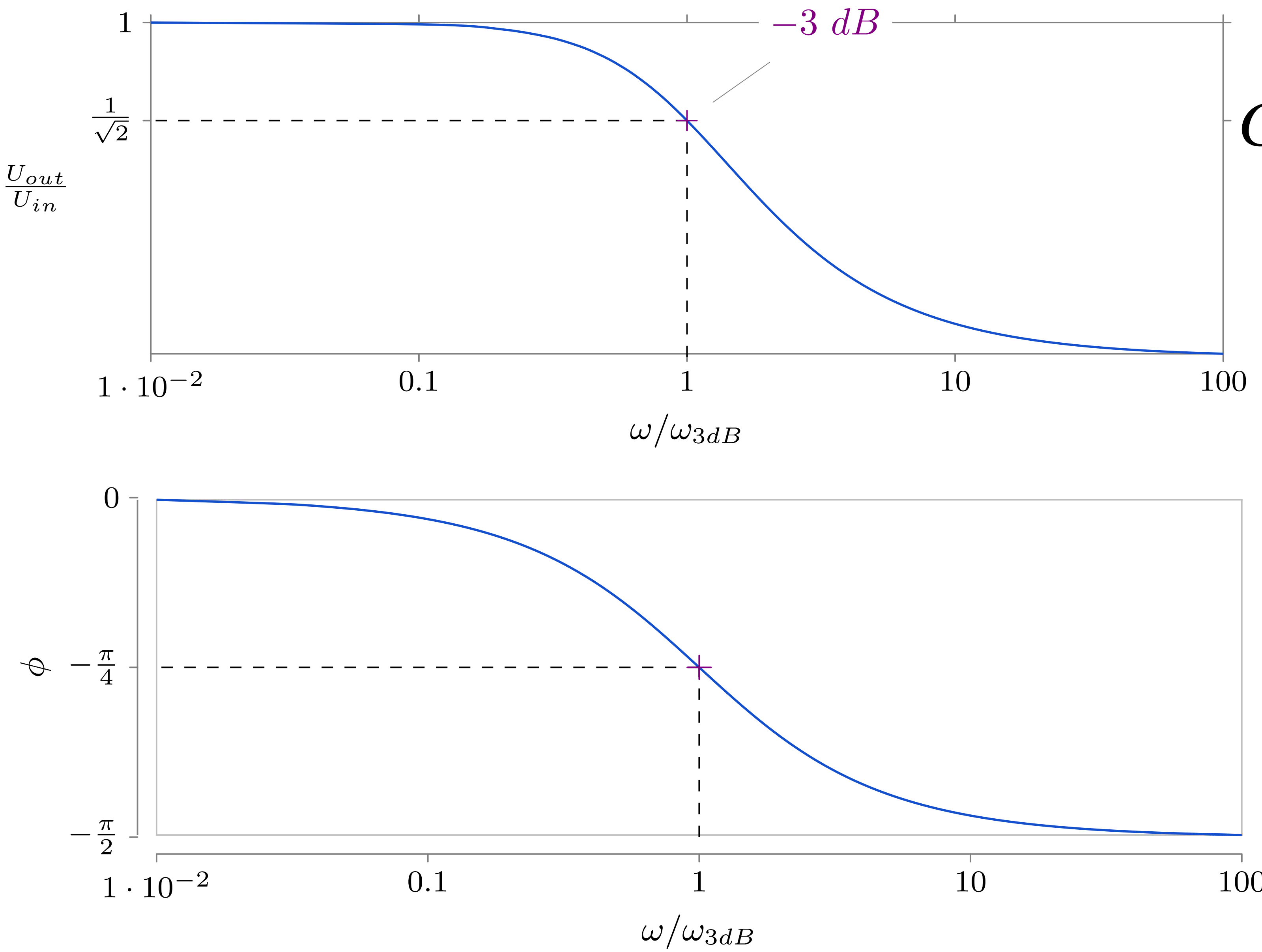


$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

$$G(\omega) = |\mathbf{H}(\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}},\ \phi(\omega) = -\arctan(\omega RC)$$

- $\lim\limits_{\omega \to 0} G(\omega) = 1$
- $\lim\limits_{\omega \to +\infty} G(\omega) = 0$
- $\lim\limits_{\omega \to 0} \phi(\omega) = 0$
- $\lim\limits_{\omega \to +\infty} \phi(\omega) = -\frac{\pi}{2}$

Filtres RC passe-bas



$$G_{dB} = 20\log_{10}\left(\frac{1}{\sqrt{1+\omega^2R^2C^2}}\right)$$

- $f_{-3dB} = \dfrac{1}{2\pi RC}$
- $\omega_{-3dB} = \dfrac{1}{RC}$

Bande-passante : $[0\ ;\ f_{-3dB}]$

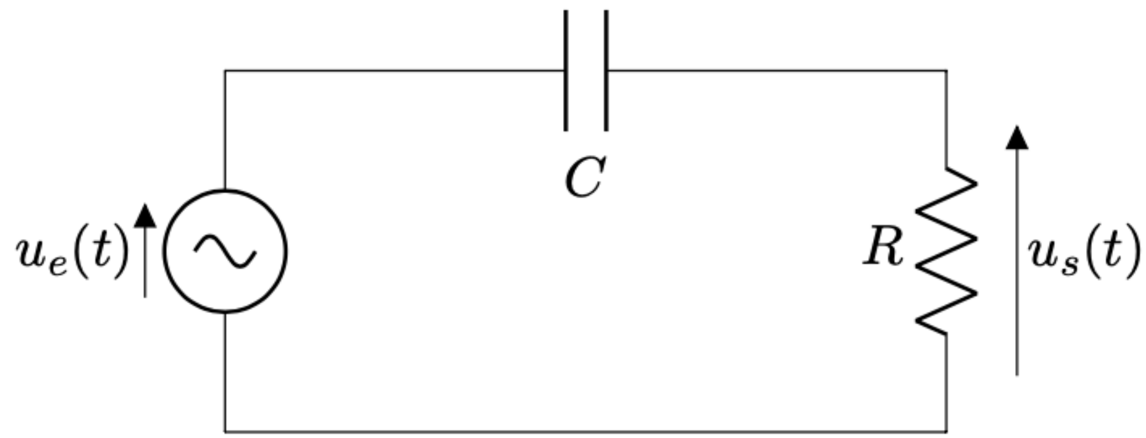
- $\lim\limits_{\omega\to 0} G(\omega) = 1$
- $\lim\limits_{\omega\to +\infty} G(\omega) = 0$
- $\lim\limits_{\omega\to \omega_{-3dB}} G(\omega) = \dfrac{1}{\sqrt{2}}$

- $\lim\limits_{\omega\to 0} \phi(\omega) = 0$
- $\lim\limits_{\omega\to +\infty} \phi(\omega) = -\dfrac{\pi}{2}$
- $\lim\limits_{\omega\to \omega_{-3dB}} \phi(\omega) = -\dfrac{\pi}{4}$

Question 3 Examen de mai 2019

3 **Filtre RC** (10 points)

On considère le filtre ci-dessous soumis à une tension d'entrée sinusoïdale $u_e(t)$ de pulsation ω :



Rappel :

Diviseur de tension $\frac{U_s}{U_e} = \frac{R_2}{R_1 + R_2}$

1. Donnez l'expression du module du gain $\frac{|u_s|}{|u_e|}$ en fonction de la pulsation ω de la tension d'entrée $u_e(t)$. **/2**

$$\frac{|u_s|}{|u_e|} = \frac{|Z_R|}{|Z_R + Z_C|} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{R}{\sqrt{\frac{\omega^2 R^2 C^2 + 1}{\omega^2 C^2}}}$$

$$\frac{|u_s|}{|u_e|} = \frac{R}{\sqrt{\frac{1 + \omega^2 R^2 C^2}{\omega^2 C^2}}} = \frac{R}{\frac{\sqrt{1 + \omega^2 R^2 C^2}}{\omega C}} = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}} = \frac{x}{\sqrt{1 + x^2}}$$

Question 3 Examen de mai 2019

2. Calculez le comportement asymptotique du filtre aux basses fréquences ($\omega \to 0$). **/2**

$$\lim_{\omega \to 0} \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}} = 0$$

Question 3 Examen de mai 2019

3. Calculez le comportement asymptotique du filtre aux hautes fréquences ($\omega \to \infty$). **/2**

$$\lim_{\omega \to +\infty} \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\lim_{x \to +\infty} \frac{x}{\sqrt{1 + x^2}} = \lim_{x \to +\infty} \frac{x}{\sqrt{x^2}} = \lim_{x \to +\infty} \frac{x}{x} = 1$$

Question 3 Examen de mai 2019

4. Déterminez la fréquence de coupure f_{-3dB} du filtre si $R = 10\ k\Omega$ et $C = 100\ nF$. **/2**

À la fréquence de coupure $\dfrac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} = \dfrac{1}{\sqrt{2}} \Leftrightarrow \omega RC = 1$

$$\omega_{-3db} = \frac{1}{RC},\ f_{-3dB} = \frac{1}{2\pi RC}$$

$$f_{-3dB} = \frac{1}{2\pi \cdot 10^4 \cdot 10^{-7}} \approx 159\ \text{Hz}$$

Question 3 Examen de mai 2019

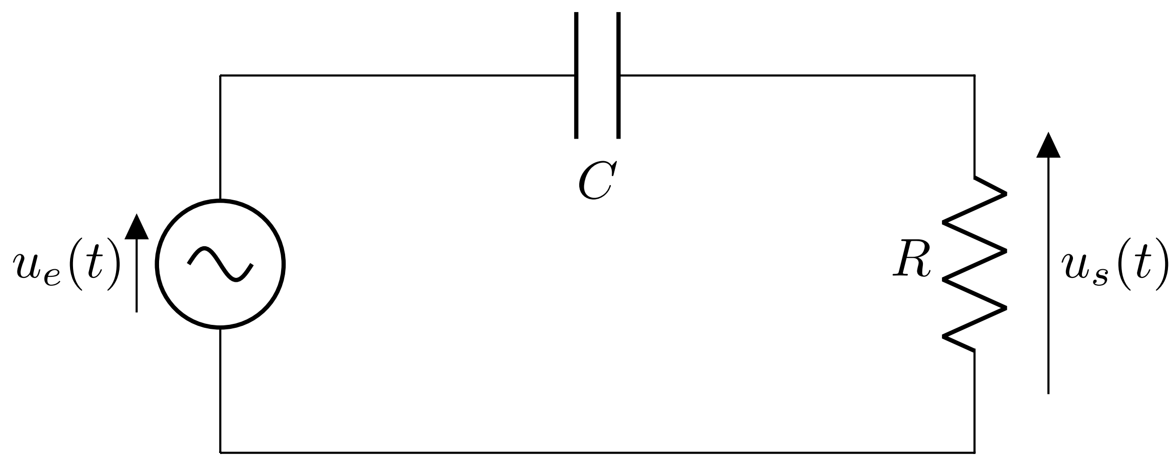
5. De quel type de filtre s'agit-il (passe-bas, passe-haut, coupe-bande, passe-bande) ?

/2

Filtre passe-haut car :

- $\lim_{\omega \to 0} G = 0$
- $\lim_{\omega \to +\infty} G = 1$

Filtres RC passe-haut



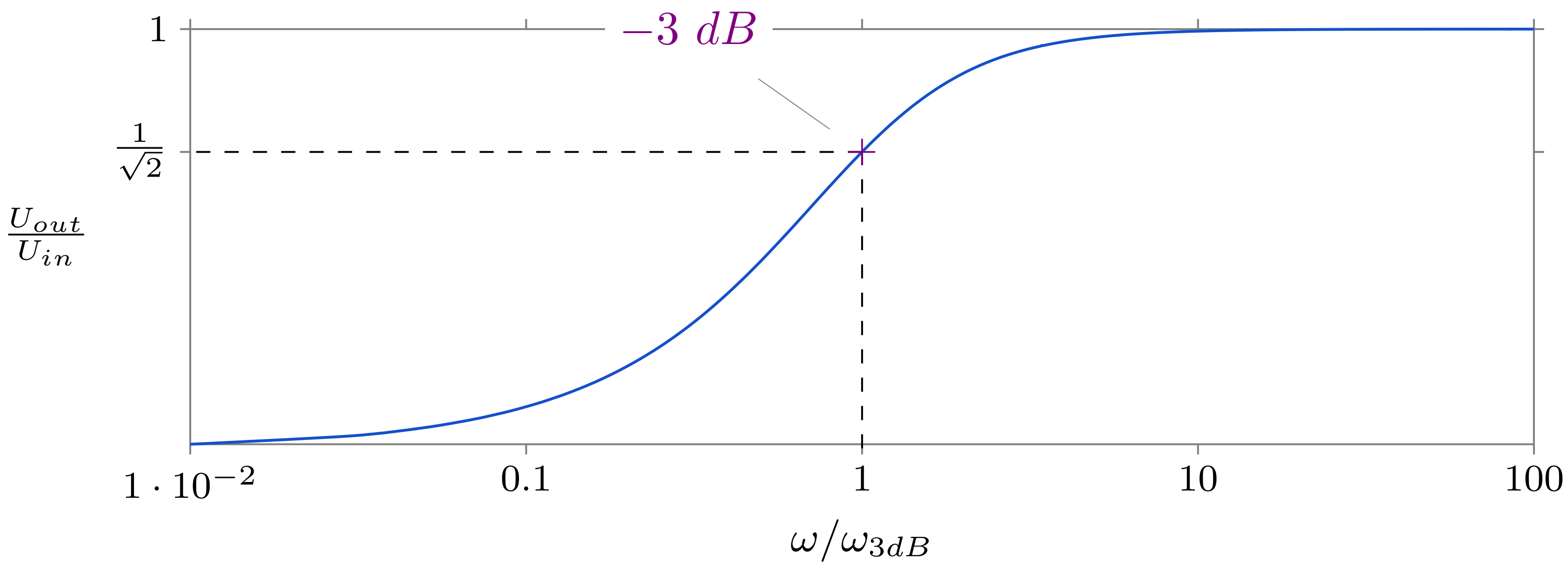
$$\mathbf{H}(\omega) = \frac{R}{R - \frac{j}{\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

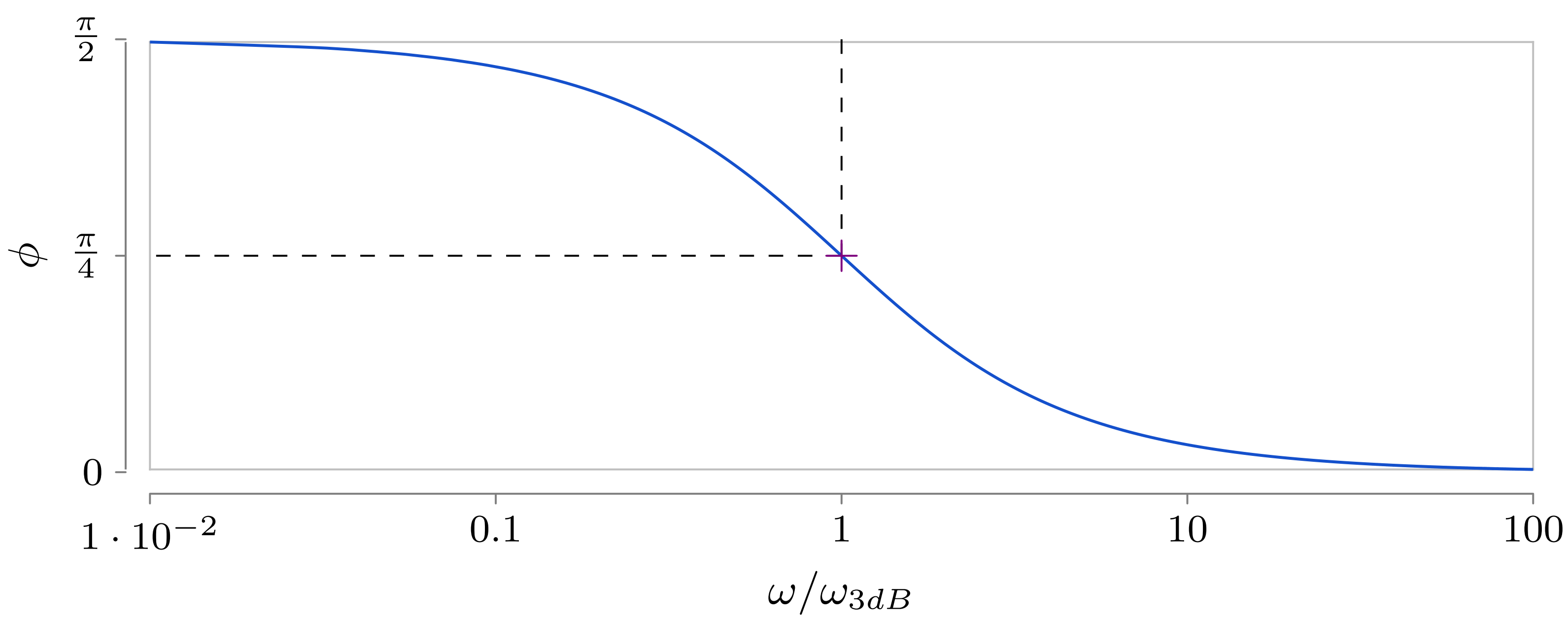
$$G(\omega) = \frac{|\mathbf{U_s}|}{|\mathbf{U_e}|}$$

$$G(\omega) = |\mathbf{H}(\omega)| = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}, \ \phi(\omega) = \arctan(\frac{1}{\omega RC})$$

- $\lim\limits_{\omega \to 0} G(\omega) = 0$
- $\lim\limits_{\omega \to +\infty} G(\omega) = 1$
- $\lim\limits_{\omega \to 0} \phi(\omega) = \frac{\pi}{2}$
- $\lim\limits_{\omega \to +\infty} \phi(\omega) = 0$

Filtres RC passe-haut





$$G_{dB} = 20\log_{10}(G)$$

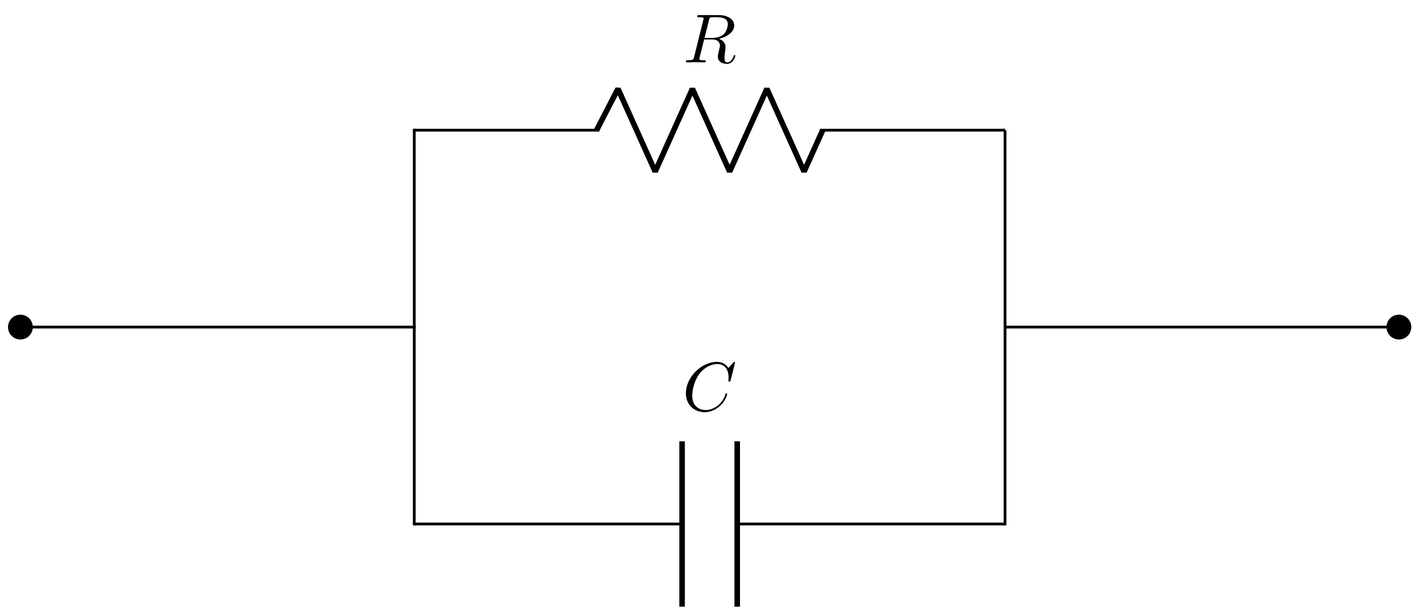
$$G_{dB} = 20\log_{10}\left(\frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}\right)$$

- $f_{-3dB} = \dfrac{1}{2\pi RC}$
- $\omega_{-3dB} = \dfrac{1}{RC}$

Bande-passante : $[f_{-3dB}\ ; +\infty[$

- $\lim_{\omega \to 0} G(\omega) = 0$
- $\lim_{\omega \to +\infty} G(\omega) = 1$
- $\lim_{\omega \to \omega_{-3dB}} G(\omega) = \dfrac{1}{\sqrt{2}}$
- $\lim_{\omega \to 0} \phi(\omega) = \dfrac{\pi}{2}$
- $\lim_{\omega \to +\infty} \phi(\omega) = 0$
- $\lim_{\omega \to \omega_{-3dB}} \phi(\omega) = \dfrac{\pi}{4}$

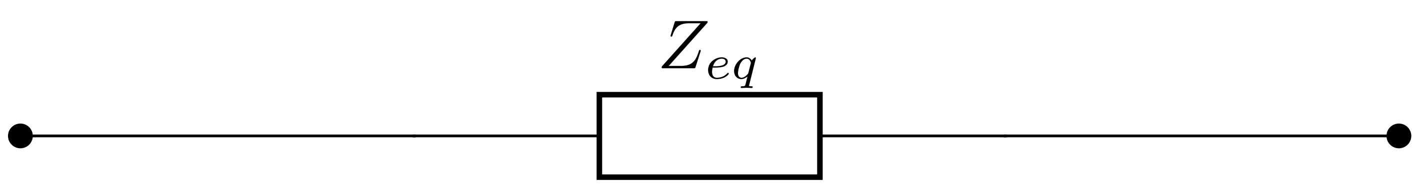
RC parallèle



$$Z_1 = R$$
$$Z_2 = -\frac{j}{\omega C}$$

$$j^2 = -1, \ \frac{1}{j} = -j$$

$$C = 2{,}20\ \mu\text{F},\ R = 330\ \Omega$$



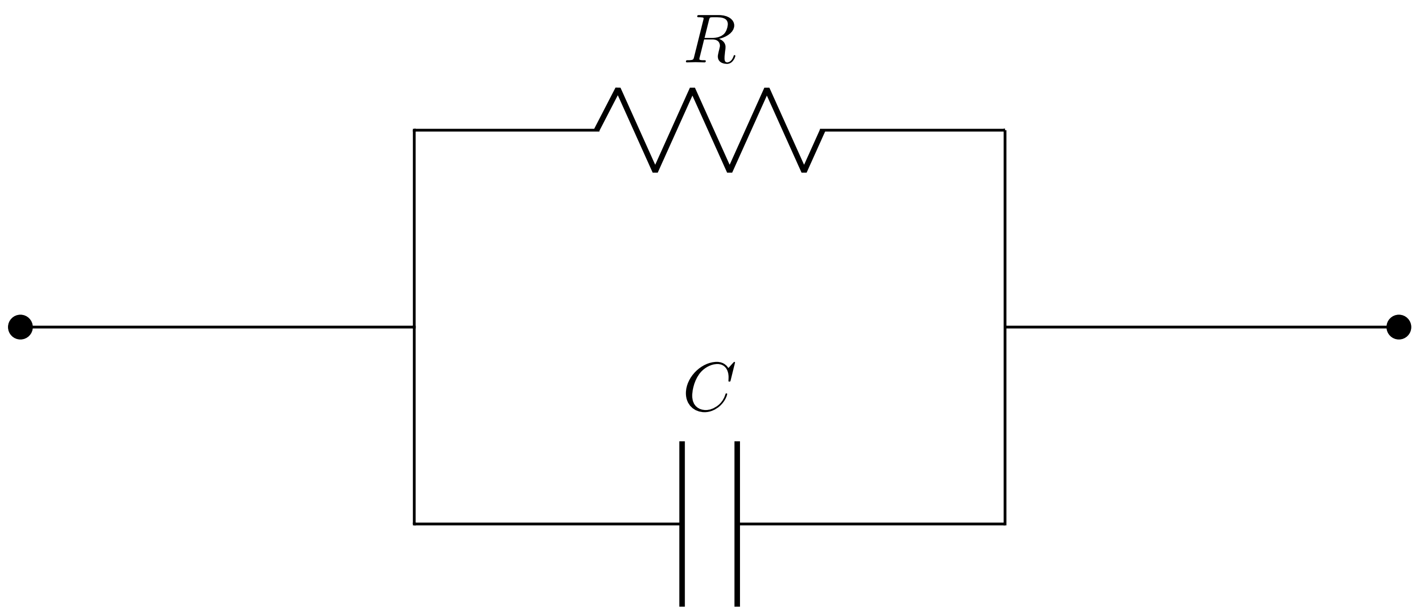
- $\dfrac{\mathbf{1}}{\mathbf{Z}} = \dfrac{\mathbf{1}}{\mathbf{Z_1}} + \dfrac{\mathbf{1}}{\mathbf{Z_2}} \Leftrightarrow \mathbf{Z} = \dfrac{\mathbf{Z_1} \cdot \mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}}$

$$\mathbf{Z_{eq}} = \frac{R \cdot (-j/\omega C)}{R - j/\omega C} \cdot \frac{-\omega C/j}{-\omega C/j} = \frac{R}{1 + j\omega RC}$$

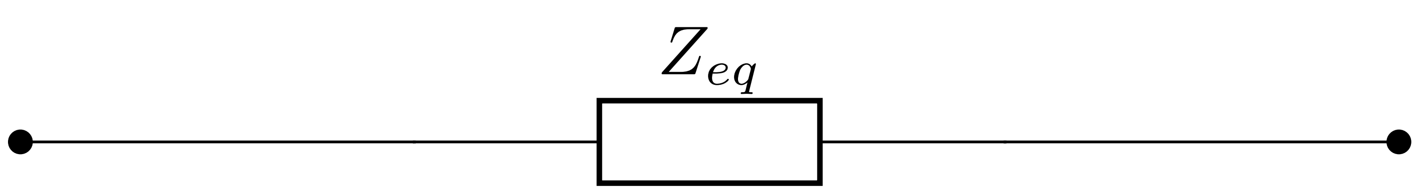
$$|\mathbf{Z_{eq}}| = \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\tan(\phi) = -\omega RC$$

RC parallèle



$C = 2{,}20\ \mu\mathrm{F}, R = 330\ \Omega, f = 100\ \mathrm{Hz}$



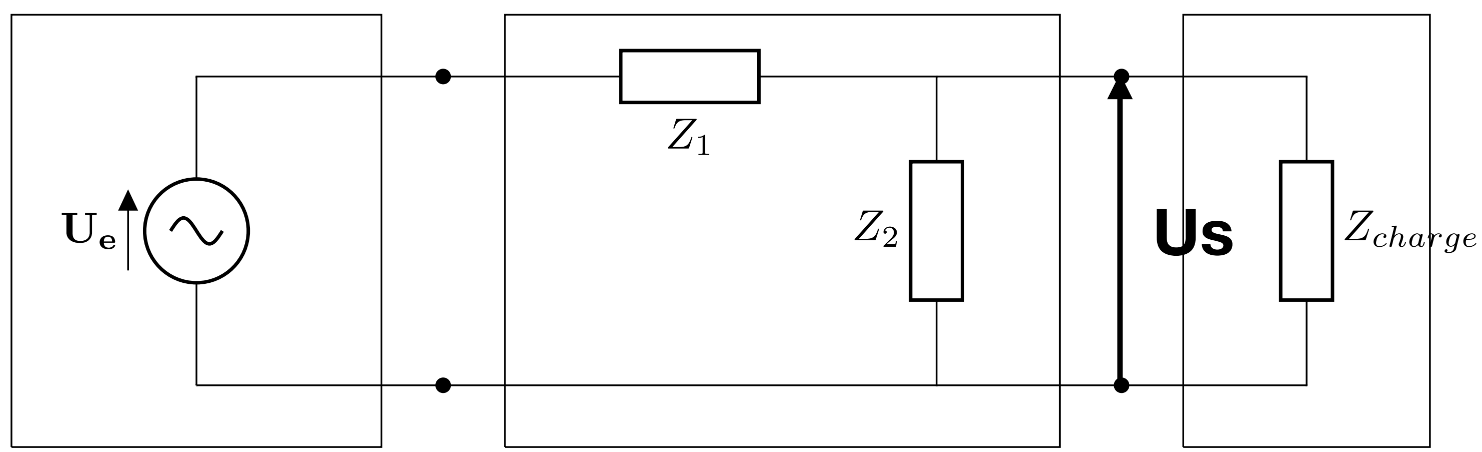
$$|\mathbf{Z_{eq}}| = \frac{R}{\sqrt{1+\omega^2R^2C^2}} = \frac{330}{\sqrt{1+(2\pi\cdot 100)^2\cdot(2{,}2\cdot 10^{-6})^2\cdot 330^2}} \approx 300\ \Omega$$

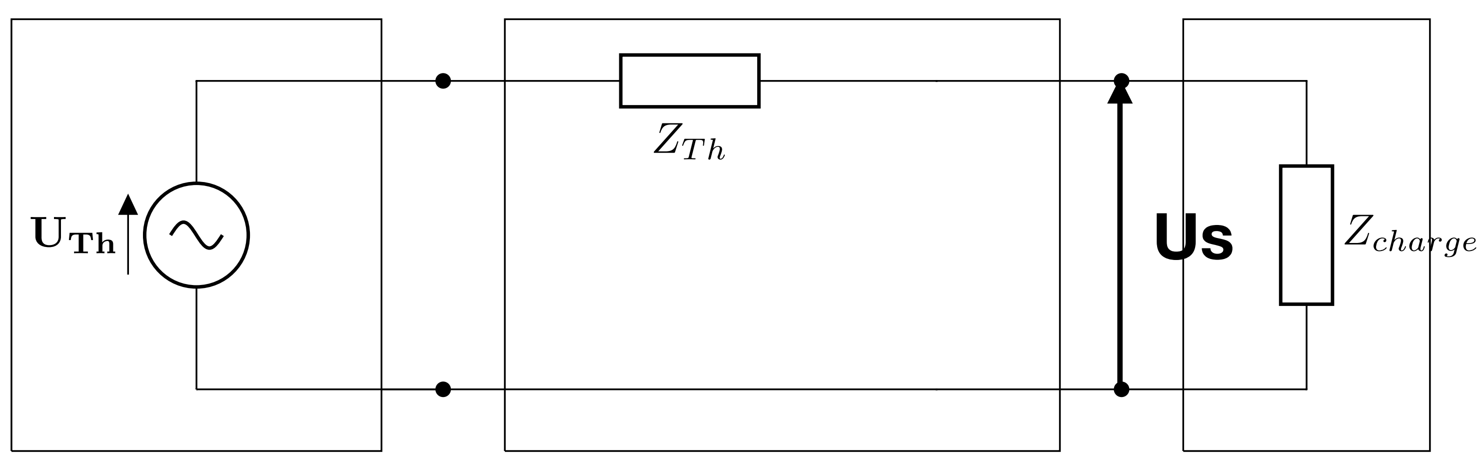
Théorème de Thévenin

- Un dipole électrique composé d'une ou plusieurs sources de tension et d'impédances est équivalent à un dipôle composé d'une source de tension $\mathbf{U_{Th}}$ en série avec une impédance $\mathbf{Z_{Th}}$

$$\mathbf{U_{Th}} = \mathbf{U_{circuit\ ouvert}},$$ (on remplace Z_{charge} par un interrupteur ouvert, pas de charge)

$$\mathbf{Z_{Th}} = \frac{\mathbf{U_{Th}}}{\mathbf{I_{court-circuit}}},$$ (on remplace Z_{charge} par un fil ou un interrupteur fermé)





Théorème de Thévenin

$\mathbf{U_{Th} = U_{circuit\ ouvert}}$, (on remplace Z_{charge} par un interrupteur ouvert, pas de charge)

$$\mathbf{Z_{Th} = \frac{U_{Th}}{I_{court-circuit}}}$$, (on remplace Z_{charge} par un fil ou un interrupteur fermé)

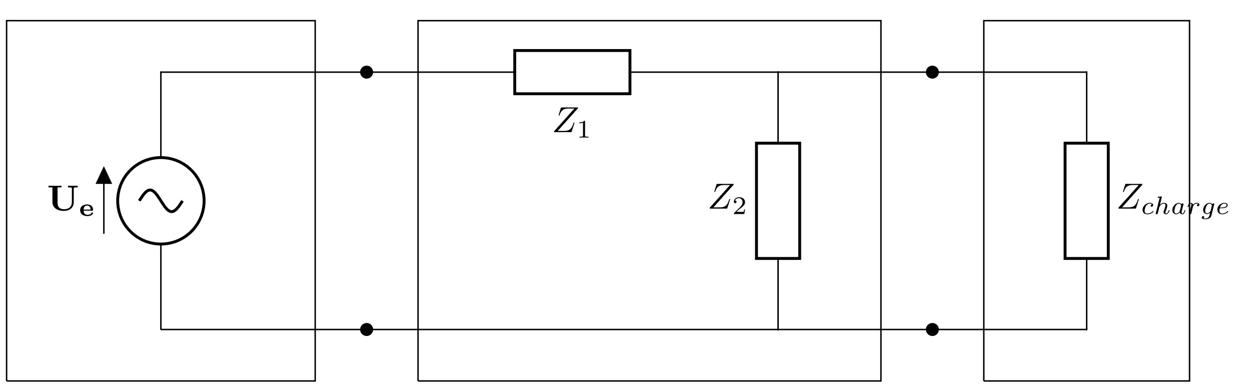
$$\mathbf{U_{Th}} = \frac{Z_2}{Z_1 + Z_2}\mathbf{U_e}$$

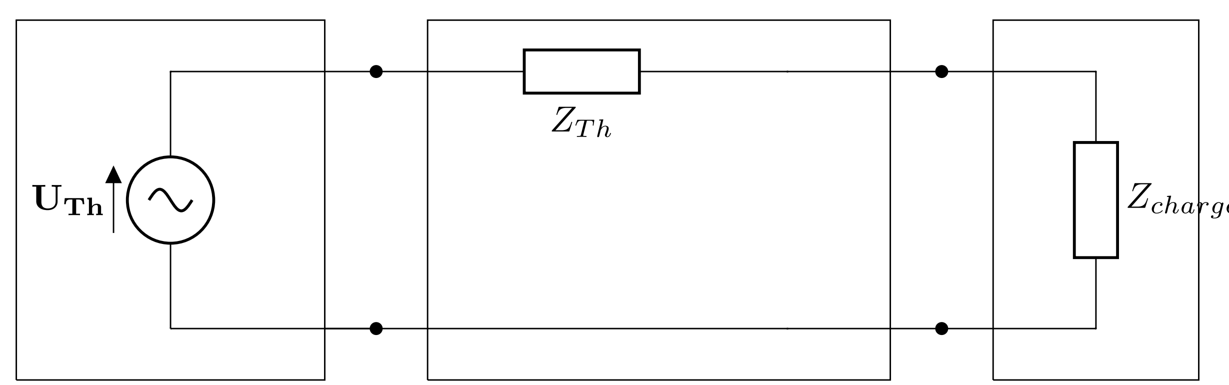
Dans notre cas, on peut montrer que $\mathbf{Z_{Th} = Z_1 // Z_2}$

$$\mathbf{I_{court-circuit} = \frac{U_e}{Z_1}}, \mathbf{Z_{Th} = \frac{U_{Th}}{I_{court-circuit}}} = \frac{Z_2}{Z_1 + Z_2}\mathbf{U_e} \cdot \frac{1}{\mathbf{U_e/Z_1}}$$

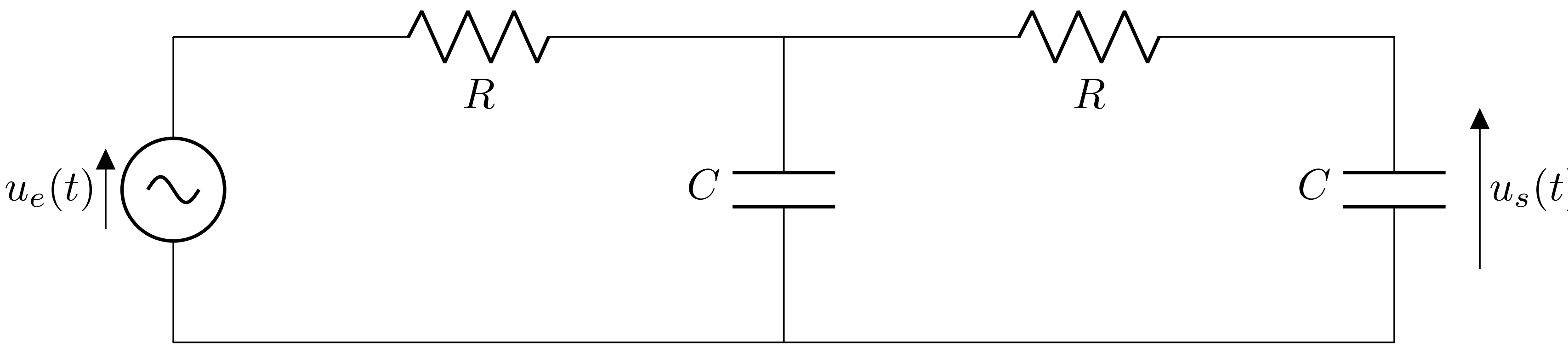
$$\mathbf{Z_{Th} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}}$$

$$\mathbf{U_s = \frac{Z_{charge}}{Z_{Th} + Z_{charge}} U_{Th}}$$



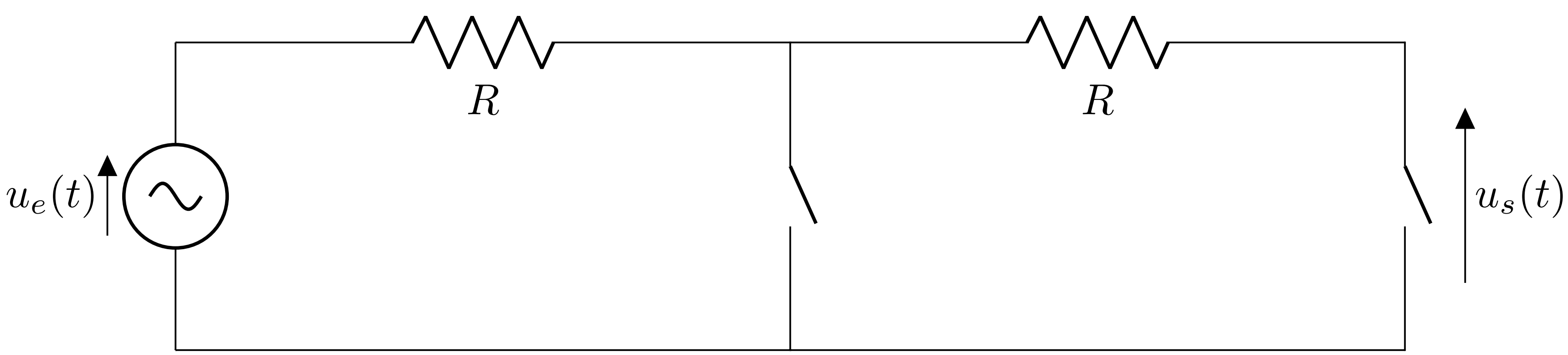


RC en cascade



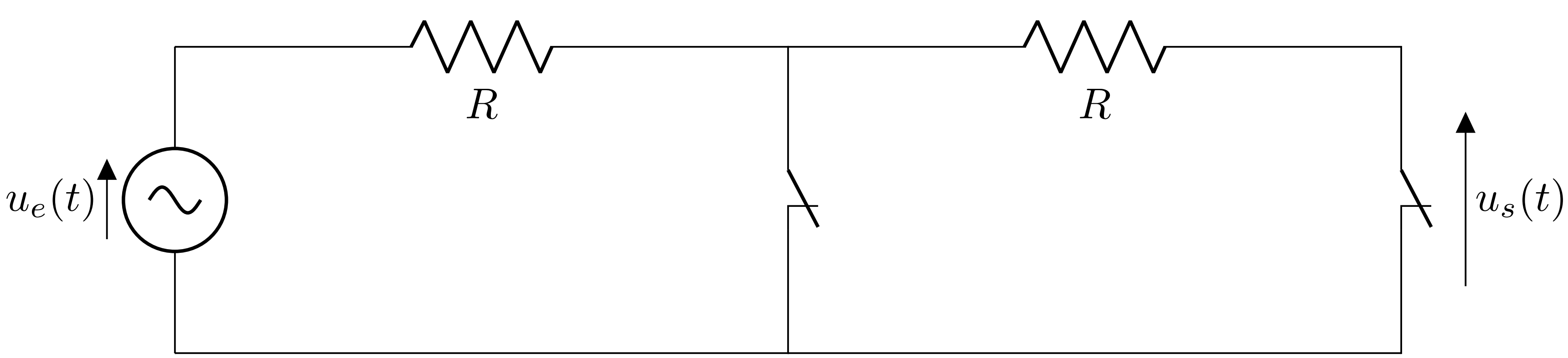
$$\mathbf{Z_C} = -\frac{j}{\omega C}$$
$$|\mathbf{Z_C}| = \frac{1}{\omega C}$$

À **basse fréquence**, C se comporte comme un interrupteur ouvert.



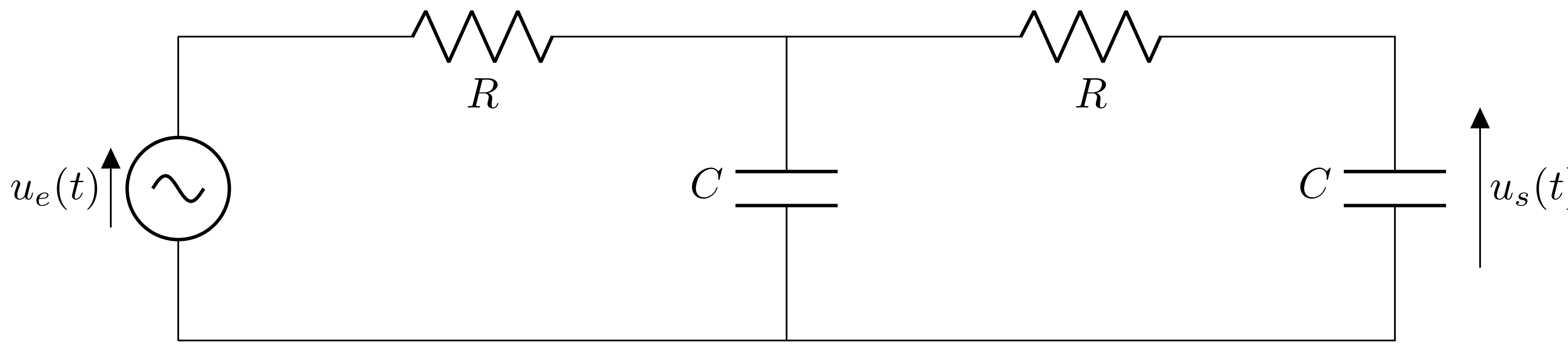
$$\lim_{\omega \to 0} U_s = U_e$$

À **haute fréquence**, C se comporte comme un interrupteur fermé.



$$\lim_{\omega \to +\infty} U_s = 0$$

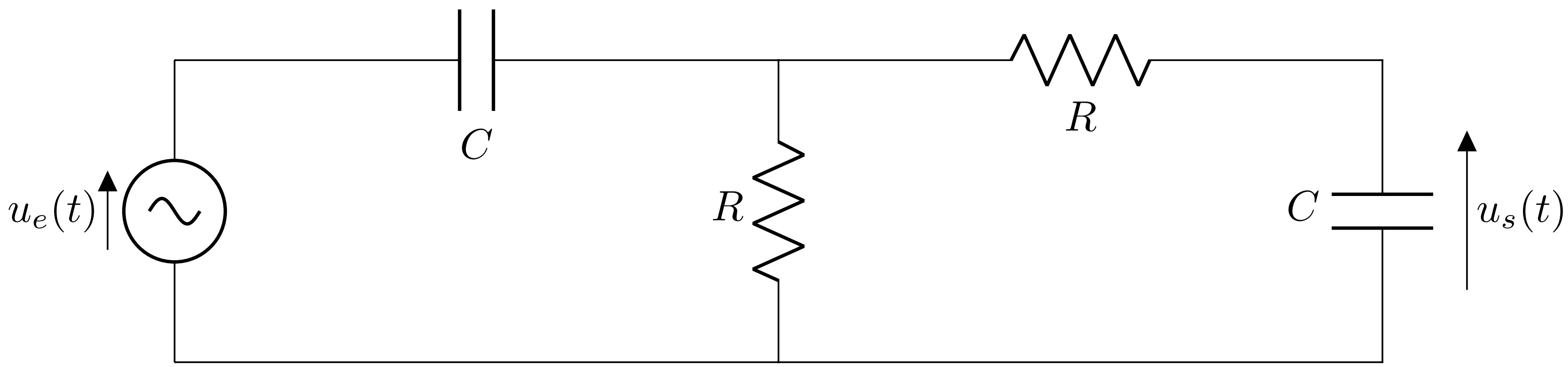
RC en cascade

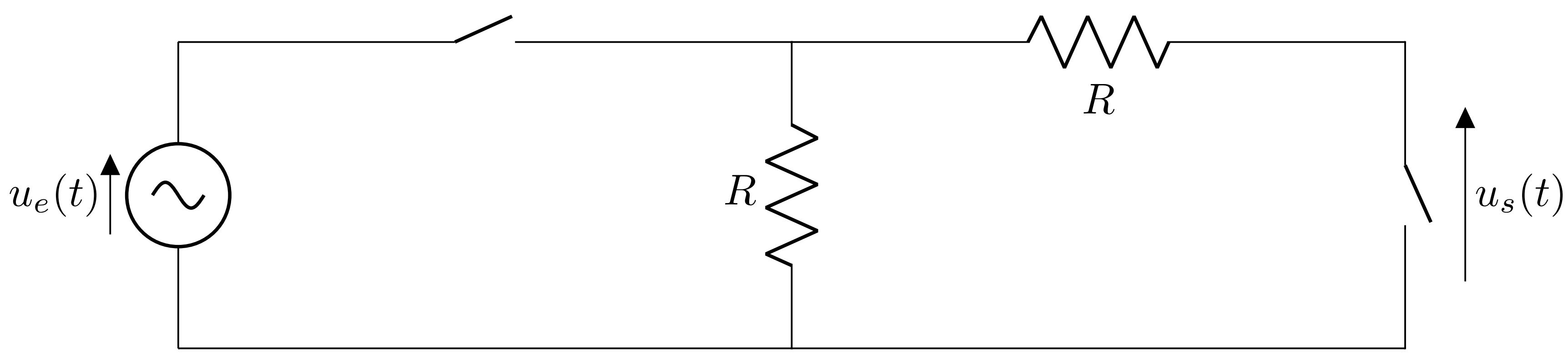


$$\mathbf{H}(\omega) = \frac{1}{1 + 3j\omega RC + (j\omega RC)^2}$$

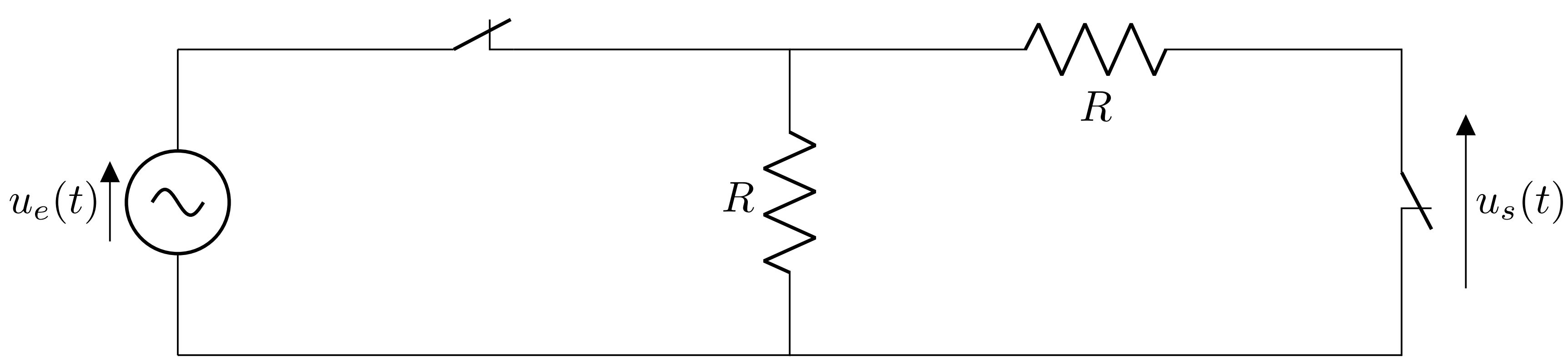
Essayez de démontrer ce résultat en utilisant le théorème de Thévenin.

RC en cascade





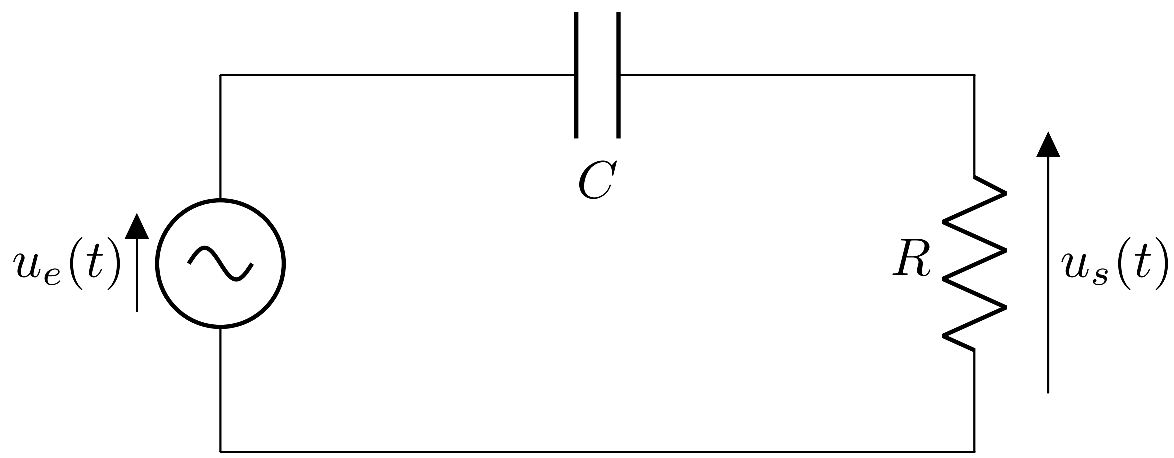
$$\lim_{\omega \to 0} U_s = 0$$



$$\lim_{\omega \to +\infty} U_s = U_e$$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Filtres RC passe-haut : simulation Qucs



$C = 100$ nF, $R = 10$ kΩ