11. Markov chains

Generative Music AI

THE SOUND OF AI

upf. Universitat Pompeu Fabra Barcelona

MTG Music Technology Group

Overview

1. Intuition
2. Formalisation
3. Modelling music with MCs
4. Strengths and limitations

What’s a Markov chain (MC)?

- Mathematical system that undergoes transitions from one state to another

What’s a Markov chain (MC)?

- Mathematical system that undergoes transitions from one state to another
- Model sequence of events probabilistically

Key assumption

The next state depends only on the current state and not on the sequence of events that preceded it (memoryless)

Flipping a fair coin

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- 2 states: Head (H) or Tail (T)

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- Probability of getting H = 50%
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- Probability of getting H = 50%
- Probability of getting T = 50%
- Transition: Each flip of the coin

Flipping a fair coin

Current / next	Head	Tail
Head	0.5	0.5
Tail	0.5	0.5

Flipping a fair coin

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New coin flip is independent of previous flips

Weather patterns

- 3 states:   

Weather patterns

- 3 states:   
- Transitions: Weather change from one day to the next

Weather patterns

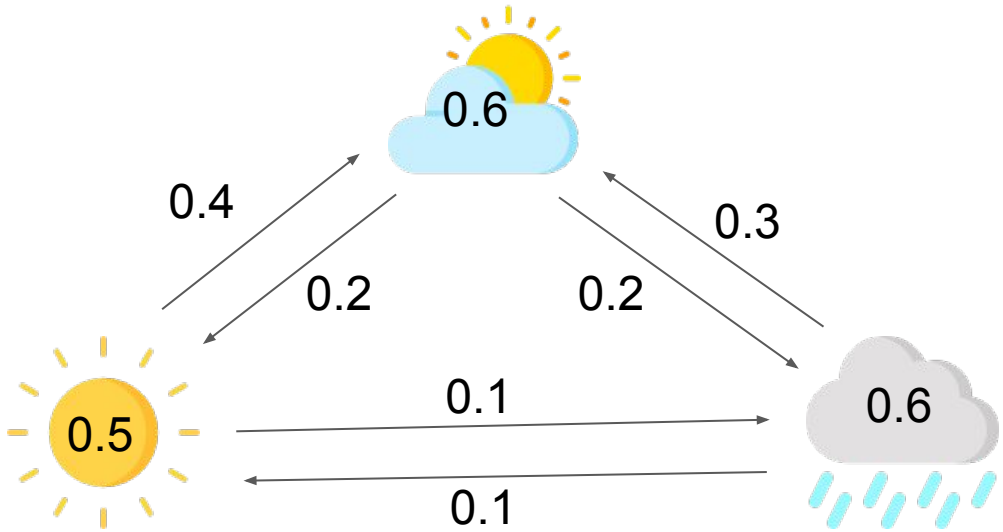
Current / next	☀️	⛅	🌧️
☀️	0.5	0.4	0.1
⛅	0.2	0.6	0.2
🌧️	0.1	0.3	0.6

Weather patterns

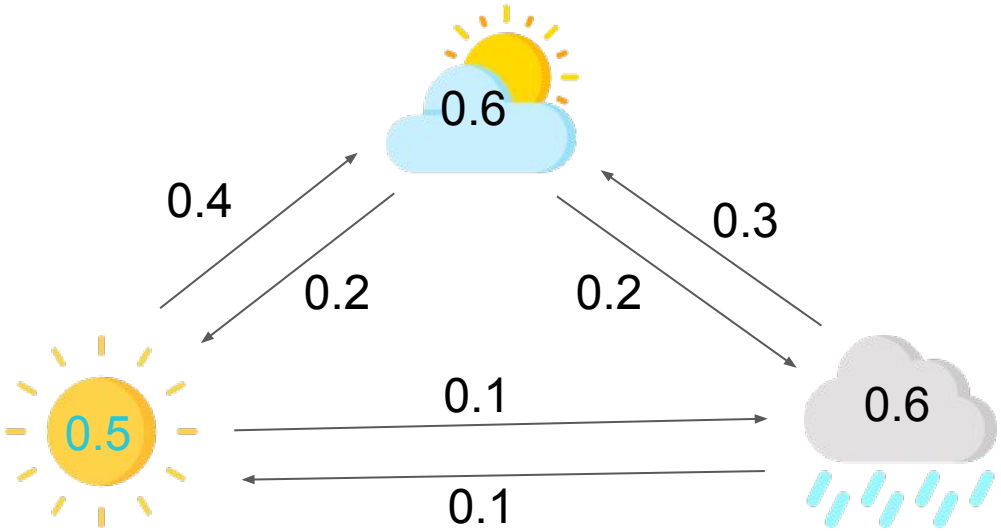
Current / next			
	0.5	0.4	0.1
	0.2	0.6	0.2
	0.1	0.3	0.6

Tot probability = 1.0

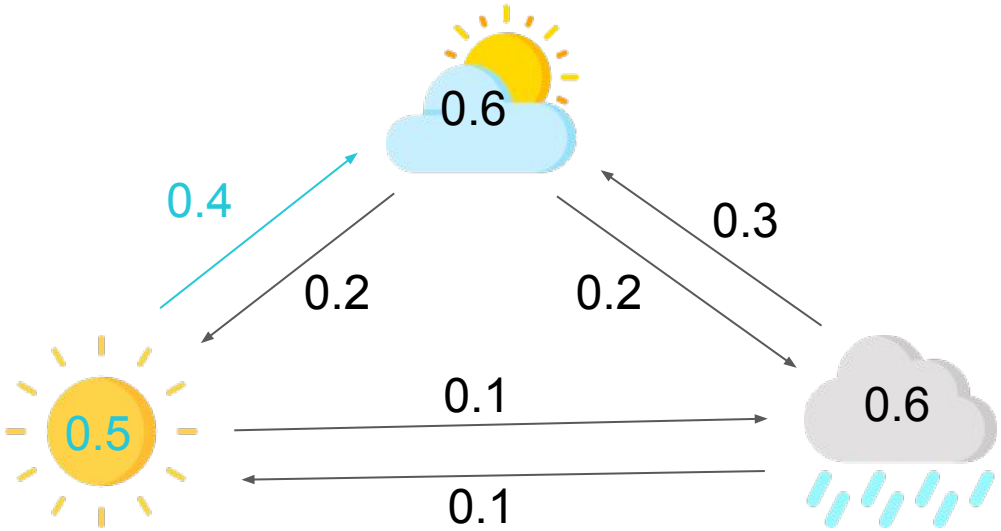
Weather patterns



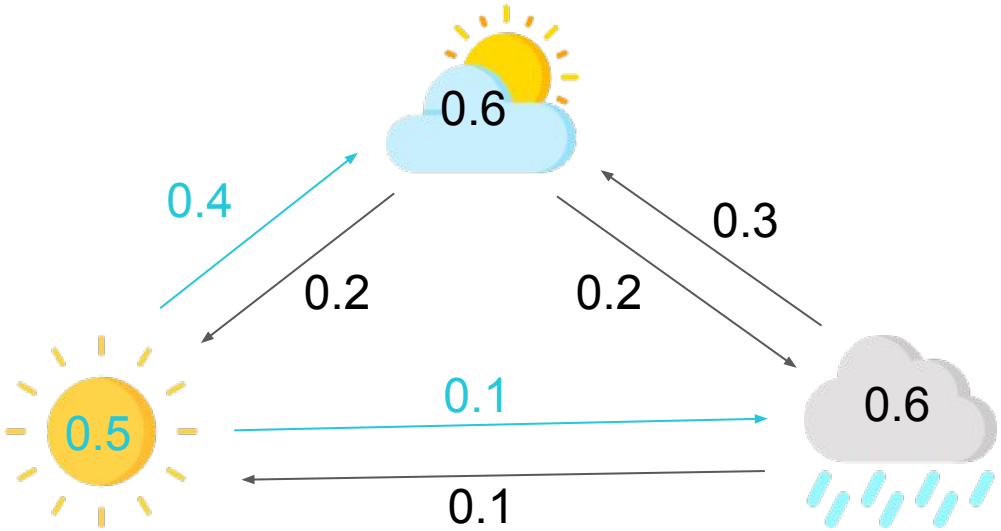
Weather patterns



Weather patterns



Weather patterns



Markov chains in the real world

MCs apply to various aspects of daily life, where the future state depends only on the current state

Markov chain formalisation

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- *States*: The possible conditions (e.g., weather condition, coin side, note)

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Markov chain formalisation

- *States*: The possible conditions (e.g., weather condition, coin side, note)
- *Initial probabilities*: Likelihood of starting the sequence in a state
- *Transition probabilities*: Likelihood of moving from one state to another

Formalising weather patterns

$$S = \{sunny, cloudy, rainy\}$$

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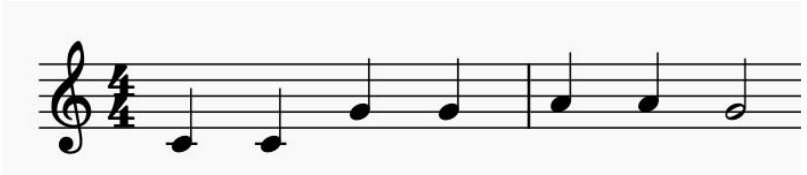
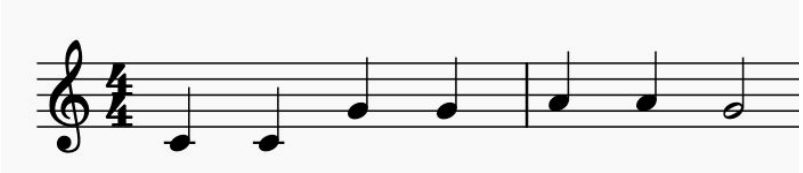
$$T_p = \begin{bmatrix} p_{ss} & p_{sc} & p_{sr} \\ p_{cs} & p_{cc} & p_{cr} \\ p_{rs} & p_{rc} & p_{rr} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

Modelling music with MCs



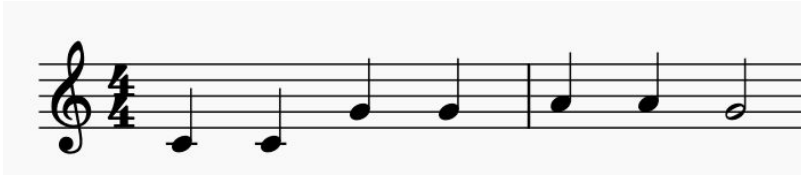
Modelling music with MCs

- Melody is a sequence of notes (duration + pitch)



Modelling music with MCs

- Melody is a sequence of notes (duration + pitch)
- Generate a melody based on the probability of one note following another



Modelling music with MCs

C | Am | F | G

Modelling music with MCs

- Chord progression is a sequence of chords

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- Chord progression is a sequence of chords
- Generate a chord progression based on the probability of one chord following another

C | Am | F | G

Melody generation with MCs

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- C major pentatonic scale

Melody generation with MCs

- C major pentatonic scale
- Simplifications:
  - Pitches in one octave
  - Focus on pitch, ignore durations

Melody generation with MCs

$$S = \{C, D, E, G, A\}$$

Melody generation with MCs

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$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

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$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

Melody generation with MCs

- First pitch

Melody generation with MCs

- First pitch
  a. Use Ip vector

$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice

$$I_p = \begin{pmatrix} p_C \\ p_D \\ p_E \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip

$$I_p = \begin{pmatrix} p_C \\ p_D \\ \boxed{p_E} \\ p_G \\ p_A \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.2 \\ 0.15 \\ 0.15 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use lp vector
  b. Roll dice
  c. Get pitch from lp
- Subsequent pitches

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice
  d. Get new pitch

$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E -> G

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
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  c. Roll dice
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$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E -> G -> A

Melody generation with MCs

- First pitch
  a. Use Ip vector
  b. Roll dice
  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
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$$T_p = \begin{pmatrix} p_{CC} & p_{CD} & p_{CE} & p_{CG} & p_{CA} \\ p_{DC} & p_{DD} & p_{DE} & p_{DG} & p_{DA} \\ p_{EC} & p_{ED} & p_{EE} & p_{EG} & p_{EA} \\ p_{GC} & p_{GD} & p_{GE} & p_{GG} & p_{GA} \\ p_{AC} & p_{AD} & p_{AE} & p_{AG} & p_{AA} \end{pmatrix} = \begin{pmatrix} 0 & 0.4 & 0.2 & 0.3 & 0.1 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.3 & 0 & 0.4 \\ 0.4 & 0.1 & 0.1 & 0.3 & 0 \end{pmatrix}$$

E -> G -> A -> G

Melody generation with MCs

- First pitch
  a. Use Ip vector
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  c. Get pitch from Ip
- Subsequent pitches
  a. Use Tp matrix
  b. Get to the row of the current pitch
  c. Roll dice
  d. Get new pitch

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E -> G -> A -> G -> C

What can you model with MCs?

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- Rhythms

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- Form

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- Octaves
- Dynamics (e.g., *piano*, *fortissimo*)
- Simple melodic patterns
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- Articulations (e.g, *staccato*, *legato*)
- Form
- …

Two modelling approaches

- 1 big MC with multiple parameters
  (e.g., duration + pitch together)

Two modelling approaches

- 1 big MC with multiple parameters (e.g., duration + pitches together)
- Multiple MCs in parallel, that handle different parameters (e.g., 1 MC for durations, 1 MC for pitches)

Where do we get probabilities?

- Manually from music domain experts
- Extract from musical corpus (learning)

Pros and cons of MCs

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- Simple
- Flexible
- Fun and creative
- OK for ambient

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- Simple
- Flexible
- Fun and creative
- OK for ambient

- Random walk
- Lack of musical context
- Bad for genres with strong musical direction

Key takeaways

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- MC can be used to generate many musical aspects
- Probabilities encoded manually, or learnt from corpus
- Simple and creative, but lack musical context

What’s up next?

Melody generation with Markov chains