

A SIMPLER ITERATIVE SOLUTION TO THE TOWERS OF HANOI PROBLEM

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An iterative solution to the Towers of Hanoi problem is given in [1]. There is another solution that was mentioned in old puzzle books, but that the author has not seen in print recently. It gives the same set of moves as the other iterative method and indeed the recursive method, but can be derived from a simpler set of principles. This iterative algorithm would provide an excellent validation problem for putative program simplification systems. To see this simple solution, first establish these standards:

1. Number the disks from 1 (the smallest) to N (the largest).
2. The three posts are ordered so that the concepts of moving a disk clockwise and counterclockwise are meaningful.

Now the whole solution derives from these three principles:

1. Move odd-numbered disks only clockwise and even-numbered disks only counterclockwise.
2. Do not move the same disk twice in succession.
3. Do not place a larger disk on top of a smaller one.

It can be shown by induction that there is only one legal move at any time, and that the whole pile moves in the optimal number of moves. Consider for example N=3. We will assume that clockwise means to the right.

1	
2	Initial position.
3	
A B C	
2	
3 1	1 moves clockwise.
A B C	
3 1 2	2 moves counterclockwise since 1 just moved.
A B C	
1	
3 2	This is the only legal move.
A B C	
1	
3 2	Move 3 clockwise since 1 just moved.
A B C	

1 3 2 The only legal move.
A B C

2
1 3 2 moves counterclockwise since 1 just moved.
A B C

1
2 Finished.
3
A B C

This is probably the simplest possible Towers of Hanoi solution. I look forward to an automatic program transformation system that can produce it from the standard recursive algorithm without human assistance.

References

- [1] H. Mayer and D. Perkins, "Towers of Hanoi Revisited, A Nonrecursive Surprise", ACM SIGPLAN Notices 19 (2), (February 1984) pp. 80-84.