LUCID SURREALISM

EPISTEMIC INSURGENCY

Detecting Structured Perturbations in Bounded Random Walks via Topological and Spectral Diagnostics

Lucid Surrealism — May 2025 Draft for Research Collaboration

Abstract

We propose a domain-flexible method for detecting weak structure in bounded random walk data, inspired by concepts from Winiger's *PhaseScope* framework and grounded in established stochastic walk theory. The system integrates three analytical layers: (1) a delay-coordinate embedding over scalar converters derived from trial outcomes; (2) dynamic spectral diagnostics to identify transition regimes; and (3) controlled perturbation experiments to test the topological reachability of anomalous clusters. Persistent homology is used to detect topological signatures, and only embeddings passing observability and stability criteria are accepted. The architecture is agnostic to walk source or application domain, with modular support for different null models.

I. Overview and Analytical Setting

We consider bounded 1D random walks terminated upon hitting absorbing boundaries. Trials are composed of multiple walk segments; each complete trial is processed into a set of scalar summaries. These include:

- **Trial-level p-value** from walk completion statistics
- **Surprisal transform (SV)** from the p-value, centered on the null
- **Statistical converters** such as normalized run-length and autocorrelation metrics

The present implementation uses a cumulative distribution function (CDF) derived from first-passage time statistics of bounded 1D random walks under uniform IID bitstream conditions, as characterized in Wilber (2021). However, the null model is treated modularly and can be substituted for different domains with different entropy processes or walk geometries.

II. Delay Embedding and Observability

Trial-level scalar outputs are stacked into delay-coordinate vectors to approximate underlying state evolution. This embedding simulates a sensor array over trials:

Observability Validation:

- Embeddings are tested for geometric immersion using false-nearest-neighbor analysis and Jacobian conditioning
- Only embeddings with FNN ratio < 2% and well-behaved local condition numbers are retained

This ensures a stable topological manifold suitable for persistent homology.

III. Spectral Stability Layer

To detect transitions from noise to structure, we apply dynamic mode decomposition (DMD) to the sequence of trial-level scalars. This approximates the Koopman operator for trial dynamics.

Let be a sliding window of trials: Compute , and extract the leading eigenvalue .

A spectral softening alert is triggered when:

- Its gradient exceeds a preset threshold

These alerts gate downstream topological processing.

IV. Persistent Homology and Topological Alerts

Using the valid embedding vectors, we construct Vietoris–Rips filtrations (via Ripser++) and extract Betti-0 through Betti-2 features.

Distance Metric:

- Effective resistance on a trial-wise k-NN graph (k=10), following Damrich et al. (2024)

Normalization:

- Persistence values are z-scored using baseline distributions derived from 10^5 trials sampled under the domain’s null entropy model, as per Bobrowski & Skraba (2023)

Alert Condition:

- A topological anomaly is reported only if a Betti-1 intensity surge coincides (±5 trials) with a spectral flag

---

V. Controlled Perturbation Sweep

To evaluate the stability and reachability of emergent topological clusters, we conduct controlled perturbation experiments:

- Apply a deterministic bias to the entropy source
- Generate 1,000 trials per ε
- Project into the validated embedding
- Cluster using DBSCAN (ε=0.5)

Cluster Validation: Let be a persistent homology-identified cluster. Define

- : artefact
- : reachable structured mode

VI. Research Grounding

- **Null model structure:** Walk CDF drawn from Wilber (2021); modular for other entropy regimes
- **Embedding theory:** Sauer et al. (1991), extended to multi-scalar trial-level data
- **Spectral diagnostics:** Dynamic mode decomposition (Schmid, 2010)
- **TDA framework:** Damrich et al. (2024), Bobrowski & Skraba (2023), Carrière et al. (2023)

VII. Applications and Cross-Domain Relevance

- Sensor drift detection in industrial processes
- Regime-change detection in financial data
- Emergent behavior in synthetic sequence generation systems

VIII. Notes for Review

This document abstracts from proprietary software or specific experimental protocols. All methods are grounded in public research and structured to support domain substitution via modular null models. Collaborators are invited to suggest refinements or propose alternate trial formats.